Project Proposal: Isoperimetric Enclosures and Optimal Curves

Team Composition: Charels Hugo, Installé Arthur, Caeyman Hamza

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1 Introduction

The isoperimetric problem is a classical question in geometry: among all curves of a given fixed perimeter, which one encloses the maximum area? This fundamental problem has applications in various fields such as physics, optimization, and geometric design.

In this project, we aim to explore a specific variant of the isoperimetric problem: maximizing the area enclosed by a convex curve made up of circular arcs, given a fixed perimeter P, while enclosing a set of points S. This problem is discussed in the paper *Isoperimetric Enclosures* [1], which provides a complete characterization of the solution as well as an approximation algorithm for computing the optimal curve.

2 Project Objectives

The goal of this project is to:

1. Explain the properties of the optimal curve, including the use of Cauchy's Arm Lemma to show that the solution must be convex.

2. Implement the algorithm provided in the paper to compute the optimal curve using the farthest-point Voronoi diagram.

3. Create an interactive tool that allows users to adjust the perimeter P and observe how the optimal curve changes.

4. Discuss the computational hardness of exact solutions, based on the results in Sections 4 and 5 of the paper, and demonstrate these through interactive visualizations.

3 Proposed Methodology

3.1 Part 1: Explaining the Problem and the Properties of the Optimal Curve

The first part of the project will focus on understanding and explaining the properties of the optimal curve. In particular, we will explore how convex curves composed of circular arcs maximize the enclosed area for a fixed perimeter. Cauchy's Arm Lemma, which helps prove that the optimal curve is convex, will be explained through visual aids.

We will create an interactive visual tool using JavaScript (with p5.js) that demonstrates how non-convex curves can be "straightened" to form convex ones, increasing the enclosed area.

3.2 Part 2: Implementing the Approximation Algorithm

In the second part, we will implement the algorithm from the paper that approximates the optimal curve using the farthest-point Voronoi diagram of the points in S. This will involve:

- Computing the farthest-point Voronoi diagram for a set of points.
- Generating a convex curve made up of circular arcs based on the perimeter P.
- Allowing users to modify P using a slider and visualizing the changes to the curve in real-time.

3.3 Part 3: Hardness Results and Approximation

In the final part of the project, we will explore the hardness of exactly computing the optimal curve. We will provide visual examples to show when the problem becomes computationally difficult and explain why approximation algorithms are necessary in such cases.

4 Accountability

Hamza Caeyman will be in charge of the part 1, Arthur Installé of the part 2 and Hugo Charels for the part 3, but we will all collaborate in each parts together.

5 Conclusion

This project will provide an in-depth exploration of a classical problem in computational geometry, using modern algorithmic approaches to solve and visualize it. The interactive tool will allow users to explore the geometric properties of isoperimetric curves and understand their complexity.

References

[1] Greg Aloupis, Luis Barba, Jean-Lou De Carufel, Stefan Langerman, and Diane L. Souvaine. Isoperimetric enclosures. *Graphs and Combinatorics*, 31(2):361–392, Mar 2015.