

$$\mathbf{H} || \mathbf{z}: \\ \mathbf{H} = 2 J S_1 S_2 - \mu_B g H S_z$$

$$\mathbf{S} = 0 \left( 2 J S_1 S_2 = -\frac{15}{2} J \right):$$

$$\mathbf{e}_1 = 0 \\ \Delta = 2 J$$

$$\mathbf{S} = 1 \left( 2 J S_1 S_2 = -\frac{11}{2} J \right):$$

```
In[97]:= mat2 = Δ * IdentityMatrix[3] - g μ H * DiagonalMatrix[{1, 0, -1}] - e * IdentityMatrix[3] ;
```

```
mat2 // MatrixForm
```

Out[98]//MatrixForm=

$$\begin{pmatrix} -e + \Delta - g H \mu & 0 & 0 \\ 0 & -e + \Delta & 0 \\ 0 & 0 & -e + \Delta + g H \mu \end{pmatrix}$$

```
In[99]:= det2 = Det[mat2]
```

```
Out[99]= -e^3 + 3 e^2 Δ - 3 e Δ^2 + Δ^3 + e g^2 H^2 μ^2 - g^2 H^2 Δ μ^2
```

```
In[100]:= Reduce[det2 == 0, e]
```

```
Out[100]= e == Δ || e == Δ - g H μ || e == Δ + g H μ
```

$$\mathbf{S} = 2 \left( 2 J S_1 S_2 = -\frac{3}{2} J \right):$$

```
In[104]:= mat3 = 3 Δ * IdentityMatrix[5] -
g μ H * DiagonalMatrix[{2, 1, 0, -1, -2}] - e * IdentityMatrix[5] ;
mat3 // MatrixForm
```

Out[105]//MatrixForm=

$$\begin{pmatrix} -e + 3 \Delta - 2 g H \mu & 0 & 0 & 0 & 0 \\ 0 & -e + 3 \Delta - g H \mu & 0 & 0 & 0 \\ 0 & 0 & -e + 3 \Delta & 0 & 0 \\ 0 & 0 & 0 & -e + 3 \Delta + g H \mu & 0 \\ 0 & 0 & 0 & 0 & -e + 3 \Delta + 2 g H \mu \end{pmatrix}$$

```
In[106]:= det3 = Det[mat3]
```

```
Out[106]= (-e + 3 Δ) (-e + 3 Δ + g H μ) (-e + 3 Δ + 2 g H μ) (e^2 - 6 e Δ + 9 Δ^2 + 3 e g H μ - 9 g H Δ μ + 2 g^2 H^2 μ^2)
```

In[107]:= **Reduce**[**det**<sub>3</sub> == 0, e]  
 [привести]

Out[107]=  $e == 3\Delta \mid \mid e == 3\Delta - 2gH\mu \mid \mid e == 3\Delta - gH\mu \mid \mid e == 3\Delta + gH\mu \mid \mid e == 3\Delta + 2gH\mu$

$$S = 3 \left( 2JS_1, S_2 = -\frac{9}{2}J \right):$$

In[108]:= **mat**<sub>4</sub> = 6Δ \* **IdentityMatrix**[7] -  
 [единичная матрица]  
 gμH \* **DiagonalMatrix**[{3, 2, 1, 0, -1, -2, -3}] - e \* **IdentityMatrix**[7];  
 [диагональная матрица] [единичная матрица]  
**mat**<sub>4</sub> // **MatrixForm**  
 [матричная форма]

Out[109]//MatrixForm=

$$\begin{pmatrix} -e + 6\Delta - 3gH\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -e + 6\Delta - 2gH\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -e + 6\Delta - gH\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -e + 6\Delta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -e + 6\Delta + gH\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -e + 6\Delta + 2gH\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -e + 6\Delta - 3gH\mu \end{pmatrix}$$

In[110]:= **det**<sub>4</sub> = **Det**[**mat**<sub>4</sub>]  
 [детерминант]

Out[110]=  $(-e + 6\Delta)(-e + 6\Delta - gH\mu)(-e + 6\Delta + gH\mu)(-e + 6\Delta + 2gH\mu)(-e + 6\Delta - 3gH\mu)(e^2 - 12e\Delta + 36\Delta^2 + 5e gH\mu - 30gH\Delta\mu + 6g^2H^2\mu^2)$

In[111]:= **Reduce**[**det**<sub>4</sub> == 0, e]  
 [привести]

Out[111]=  $e == 6\Delta \mid \mid e == 3(2\Delta - gH\mu) \mid \mid e == 2(3\Delta - gH\mu) \mid \mid e == 6\Delta - gH\mu \mid \mid e == 3(2\Delta + gH\mu) \mid \mid e == 2(3\Delta + gH\mu) \mid \mid e == 6\Delta + gH\mu$

$$H \parallel x: \\ H = 2JS_1 S_2 - \mu_B gHS_x$$

$$S = 0 \left( 2JS_1, S_2 = -\frac{15}{2}J \right):$$

$$e_1 = 0 \\ \Delta = 2J$$

$$S = 1 \left( 2JS_1, S_2 = -\frac{11}{2}J \right):$$

```
In[112]:= matx2 = Δ * IdentityMatrix[3] -
           |единичная матрица
           g μ H * (DiagonalMatrix[{ $\frac{\sqrt{2}}{2}$ ,  $\frac{\sqrt{2}}{2}$ }, 1] + DiagonalMatrix[{ $\frac{\sqrt{2}}{2}$ ,  $\frac{\sqrt{2}}{2}$ }, -1]) -
           |диагональная матрица |диагональная матрица
           e * IdentityMatrix[3] ;
           |единичная матрица
matx2 // MatrixForm
           |матричная форма
```

Out[113]//MatrixForm=

$$\begin{pmatrix} -e + \Delta & -\frac{g H \mu}{\sqrt{2}} & 0 \\ -\frac{g H \mu}{\sqrt{2}} & -e + \Delta & -\frac{g H \mu}{\sqrt{2}} \\ 0 & -\frac{g H \mu}{\sqrt{2}} & -e + \Delta \end{pmatrix}$$

```
In[114]:= detx2 = Det[matx2] // Expand
           |детерминант |раскрыт
```

Out[114]=  $-e^3 + 3 e^2 \Delta - 3 e \Delta^2 + \Delta^3 + e g^2 H^2 \mu^2 - g^2 H^2 \Delta \mu^2$

```
In[115]:= Reduce[detx2 == 0, e]
           |привести
```

Out[115]=  $e == \Delta \mid \mid e == \Delta - g H \mu \mid \mid e == \Delta + g H \mu$

$$\mathbf{S} = 2 \left( 2 \mathbf{J} \mathbf{S}_1, \mathbf{S}_2 = -\frac{3}{2} \mathbf{J} \right):$$

```
In[116]:= matx3 = 3 Δ * IdentityMatrix[5] - g μ H * (DiagonalMatrix[{1,  $\frac{\sqrt{6}}{2}$ ,  $\frac{\sqrt{6}}{2}$ , 1}, 1] +
           |единичная матрица |диагональная матрица
           DiagonalMatrix[{1,  $\frac{\sqrt{6}}{2}$ ,  $\frac{\sqrt{6}}{2}$ , 1}, -1]) - e * IdentityMatrix[5] ;
           |диагональная матрица |единичная матрица
```

```
matx3 //
MatrixForm
|матричная форма
```

Out[117]//MatrixForm=

$$\begin{pmatrix} -e + 3 \Delta & -g H \mu & 0 & 0 & 0 \\ -g H \mu & -e + 3 \Delta & -\sqrt{\frac{3}{2}} g H \mu & 0 & 0 \\ 0 & -\sqrt{\frac{3}{2}} g H \mu & -e + 3 \Delta & -\sqrt{\frac{3}{2}} g H \mu & 0 \\ 0 & 0 & -\sqrt{\frac{3}{2}} g H \mu & -e + 3 \Delta & -g H \mu \\ 0 & 0 & 0 & -g H \mu & -e + 3 \Delta \end{pmatrix}$$

```
In[118]:= detx3 = Det[matx3] // Expand
           |детерминант |раскрыт
```

Out[118]=  $-e^5 + 15 e^4 \Delta - 90 e^3 \Delta^2 + 270 e^2 \Delta^3 - 405 e \Delta^4 + 243 \Delta^5 + 5 e^3 g^2 H^2 \mu^2 -$   
 $45 e^2 g^2 H^2 \Delta \mu^2 + 135 e g^2 H^2 \Delta^2 \mu^2 - 135 g^2 H^2 \Delta^3 \mu^2 - 4 e g^4 H^4 \mu^4 + 12 g^4 H^4 \Delta \mu^4$

In[119]:= **Reduce**[**detx<sub>3</sub>** == 0, e]  
 [привести]

Out[119]=  $e == 3 \Delta \mid \mid e == 3 \Delta - 2 g H \mu \mid \mid e == 3 \Delta - g H \mu \mid \mid e == 3 \Delta + g H \mu \mid \mid e == 3 \Delta + 2 g H \mu$

$$S = 3 \left( 2 J S_1, S_2 = \frac{9}{2} J \right):$$

In[120]:= **matx<sub>4</sub>** = 6  $\Delta$  \* **IdentityMatrix**[7] -  
 [единичная матрица]

$g \mu H * \left( \text{DiagonalMatrix} \left[ \left\{ \frac{\sqrt{6}}{2}, \frac{\sqrt{10}}{2}, \frac{\sqrt{12}}{2}, \frac{\sqrt{12}}{2}, \frac{\sqrt{10}}{2}, \frac{\sqrt{6}}{2} \right\}, 1 \right] + \text{DiagonalMatrix} \left[ \right. \right.$   
 [диагональная матрица] [диагональная матрица]  
 $\left. \left. \left\{ \frac{\sqrt{6}}{2}, \frac{\sqrt{10}}{2}, \frac{\sqrt{12}}{2}, \frac{\sqrt{12}}{2}, \frac{\sqrt{10}}{2}, \frac{\sqrt{6}}{2} \right\}, -1 \right] \right) - e * \text{IdentityMatrix}[7];$   
 [единичная матрица]

**matx<sub>4</sub>** //  
**MatrixForm**  
 [матричная форма]

Out[121]/MatrixForm=

$$\begin{pmatrix} -e + 6\Delta & -\sqrt{\frac{3}{2}} g H \mu & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{\frac{3}{2}} g H \mu & -e + 6\Delta & -\sqrt{\frac{5}{2}} g H \mu & 0 & 0 & 0 & 0 \\ 0 & -\sqrt{\frac{5}{2}} g H \mu & -e + 6\Delta & -\sqrt{3} g H \mu & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{3} g H \mu & -e + 6\Delta & -\sqrt{3} g H \mu & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{3} g H \mu & -e + 6\Delta & -\sqrt{\frac{5}{2}} g H \mu & 0 \\ 0 & 0 & 0 & 0 & -\sqrt{\frac{5}{2}} g H \mu & -e + 6\Delta & -\sqrt{\frac{3}{2}} g H \mu \\ 0 & 0 & 0 & 0 & 0 & -\sqrt{\frac{3}{2}} g H \mu & -e + 6\Delta \end{pmatrix}$$

In[122]:= **detx<sub>4</sub>** = **Det**[**matx<sub>4</sub>**] // **Expand**  
 [детерминант] [раскрыть]

Out[122]=  $-e^7 + 42 e^6 \Delta - 756 e^5 \Delta^2 + 7560 e^4 \Delta^3 - 45360 e^3 \Delta^4 + 163296 e^2 \Delta^5 -$   
 $326592 e \Delta^6 + 279936 \Delta^7 + 14 e^5 g^2 H^2 \mu^2 - 420 e^4 g^2 H^2 \Delta \mu^2 + 5040 e^3 g^2 H^2 \Delta^2 \mu^2 -$   
 $30240 e^2 g^2 H^2 \Delta^3 \mu^2 + 90720 e g^2 H^2 \Delta^4 \mu^2 - 108864 g^2 H^2 \Delta^5 \mu^2 - 49 e^3 g^4 H^4 \mu^4 +$   
 $882 e^2 g^4 H^4 \Delta \mu^4 - 5292 e g^4 H^4 \Delta^2 \mu^4 + 10584 g^4 H^4 \Delta^3 \mu^4 + 36 e g^6 H^6 \mu^6 - 216 g^6 H^6 \Delta \mu^6$

In[123]:= **Reduce**[**detx<sub>4</sub>** == 0, e]  
 [привести]

Out[123]=  $e == 6 \Delta \mid \mid e == 3 (2 \Delta - g H \mu) \mid \mid e == 2 (3 \Delta - g H \mu) \mid \mid$   
 $e == 6 \Delta - g H \mu \mid \mid e == 3 (2 \Delta + g H \mu) \mid \mid e == 2 (3 \Delta + g H \mu) \mid \mid e == 6 \Delta + g H \mu$

**H**||y:

$$H = 2JS_1 S_2 - \mu_B g H S_y$$

$$S = 0 \left( 2JS_1 S_2 = -\frac{15}{2} J \right) :$$

$$e_1 = 0$$

$$\Delta = 2J$$

$$S = 1 \left( 2JS_1 S_2 = -\frac{11}{2} J \right) :$$

```
In[124]:= maty2 = Δ * IdentityMatrix[3] - g μ H * (DiagonalMatrix[{ $\frac{-i \sqrt{2}}{2}$ ,  $\frac{-i \sqrt{2}}{2}$ }, 1] +  

  DiagonalMatrix[{ $\frac{i \sqrt{2}}{2}$ ,  $\frac{i \sqrt{2}}{2}$ }, -1]) - e * IdentityMatrix[3];
```

```
maty2 //  
MatrixForm  
|матричная форма
```

```
Out[125]//MatrixForm=
```

$$\begin{pmatrix} -e + \Delta & \frac{i g H \mu}{\sqrt{2}} & 0 \\ -\frac{i g H \mu}{\sqrt{2}} & -e + \Delta & \frac{i g H \mu}{\sqrt{2}} \\ 0 & -\frac{i g H \mu}{\sqrt{2}} & -e + \Delta \end{pmatrix}$$

```
In[126]:= dety2 = Det[maty2] // Expand  
|детерминант |раскрыт
```

```
Out[126]= -e^3 + 3 e^2 Δ - 3 e Δ^2 + Δ^3 + e g^2 H^2 μ^2 - g^2 H^2 Δ μ^2
```

```
In[127]:= Reduce[dety2 == 0, e]  
|привести
```

```
Out[127]= e == Δ || e == Δ - g H μ || e == Δ + g H μ
```

$$S = 2 \left( 2JS_1 S_2 = -\frac{3}{2} J \right) :$$

In[128]:= **maty<sub>3</sub>** =

$$3 \Delta * \text{IdentityMatrix}[5] - g \mu H * \left( \text{DiagonalMatrix}\left[\{-i * 1, \frac{-i \sqrt{6}}{2}, \frac{-i \sqrt{6}}{2}, -i * 1\}, 1\right] + \right. \\ \left. \text{DiagonalMatrix}\left[\{i * 1, \frac{i \sqrt{6}}{2}, \frac{i \sqrt{6}}{2}, i * 1\}, -1\right] \right) - e * \text{IdentityMatrix}[5];$$

**maty<sub>3</sub>** //  
**MatrixForm**  
матричная форма

Out[129]//MatrixForm=

$$\begin{pmatrix} -e + 3 \Delta & i g H \mu & 0 & 0 & 0 \\ -i g H \mu & -e + 3 \Delta & i \sqrt{\frac{3}{2}} g H \mu & 0 & 0 \\ 0 & -i \sqrt{\frac{3}{2}} g H \mu & -e + 3 \Delta & i \sqrt{\frac{3}{2}} g H \mu & 0 \\ 0 & 0 & -i \sqrt{\frac{3}{2}} g H \mu & -e + 3 \Delta & i g H \mu \\ 0 & 0 & 0 & -i g H \mu & -e + 3 \Delta \end{pmatrix}$$

In[130]:= **dety<sub>3</sub>** = **Det[maty<sub>3</sub>]** // **Expand**  
детерминант раскрыт

$$\text{Out[130]} = -e^5 + 15 e^4 \Delta - 90 e^3 \Delta^2 + 270 e^2 \Delta^3 - 405 e \Delta^4 + 243 \Delta^5 + 5 e^3 g^2 H^2 \mu^2 - \\ 45 e^2 g^2 H^2 \Delta \mu^2 + 135 e g^2 H^2 \Delta^2 \mu^2 - 135 g^2 H^2 \Delta^3 \mu^2 - 4 e g^4 H^4 \mu^4 + 12 g^4 H^4 \Delta \mu^4$$

In[131]:= **Reduce[dety<sub>3</sub> == 0, e]**  
привести

$$\text{Out[131]} = e == 3 \Delta \mid \mid e == 3 \Delta - 2 g H \mu \mid \mid e == 3 \Delta - g H \mu \mid \mid e == 3 \Delta + g H \mu \mid \mid e == 3 \Delta + 2 g H \mu$$

$$S = 3 \left( 2 J S_1 S_2 = \frac{9}{2} J \right):$$

In[132]:= **maty<sub>4</sub>** = 6 Δ \* IdentityMatrix[7] - g μ H \*  
[единичная матрица]

$$\left( \text{DiagonalMatrix}\left[\left\{\frac{-i\sqrt{6}}{2}, \frac{-i\sqrt{10}}{2}, \frac{-i\sqrt{12}}{2}, \frac{-i\sqrt{12}}{2}, \frac{-i\sqrt{10}}{2}, \frac{-i\sqrt{6}}{2}\right\}, 1\right] + \right. \\ \left. \text{DiagonalMatrix}\left[\left\{\frac{i\sqrt{6}}{2}, \frac{i\sqrt{10}}{2}, \frac{i\sqrt{12}}{2}, \frac{i\sqrt{12}}{2}, \frac{i\sqrt{10}}{2}, \frac{i\sqrt{6}}{2}\right\}, \right. \right. \\ \left. \left. -1\right]\right) - e * \text{IdentityMatrix}[7];$$

[единичная матрица]

**maty<sub>4</sub>** //  
**MatrixForm**  
[матричная форма]

Out[133]//MatrixForm=

$$\begin{pmatrix} -e + 6\Delta & i\sqrt{\frac{3}{2}}gH\mu & 0 & 0 & 0 & 0 & 0 \\ -i\sqrt{\frac{3}{2}}gH\mu & -e + 6\Delta & i\sqrt{\frac{5}{2}}gH\mu & 0 & 0 & 0 & 0 \\ 0 & -i\sqrt{\frac{5}{2}}gH\mu & -e + 6\Delta & i\sqrt{3}gH\mu & 0 & 0 & 0 \\ 0 & 0 & -i\sqrt{3}gH\mu & -e + 6\Delta & i\sqrt{3}gH\mu & 0 & 0 \\ 0 & 0 & 0 & -i\sqrt{3}gH\mu & -e + 6\Delta & i\sqrt{\frac{5}{2}}gH\mu & 0 \\ 0 & 0 & 0 & 0 & -i\sqrt{\frac{5}{2}}gH\mu & -e + 6\Delta & i\sqrt{\frac{3}{2}}gH\mu \\ 0 & 0 & 0 & 0 & 0 & -i\sqrt{\frac{3}{2}}gH\mu & -e + 6\Delta \end{pmatrix}$$

In[134]:= **dety<sub>4</sub>** = Det[maty<sub>4</sub>] // Expand  
[детерминант] [раскрыт]

Out[134]=  $-e^7 + 42e^6\Delta - 756e^5\Delta^2 + 7560e^4\Delta^3 - 45360e^3\Delta^4 + 163296e^2\Delta^5 -$   
 $326592e\Delta^6 + 279936\Delta^7 + 14e^5g^2H^2\mu^2 - 420e^4g^2H^2\Delta\mu^2 + 5040e^3g^2H^2\Delta^2\mu^2 -$   
 $30240e^2g^2H^2\Delta^3\mu^2 + 90720e^2g^2H^2\Delta^4\mu^2 - 108864g^2H^2\Delta^5\mu^2 - 49e^3g^4H^4\mu^4 +$   
 $882e^2g^4H^4\Delta\mu^4 - 5292e^2g^4H^4\Delta^2\mu^4 + 10584g^4H^4\Delta^3\mu^4 + 36e^6g^6H^6\mu^6 - 216g^6H^6\Delta\mu^6$

In[135]:= **Reduce[dety<sub>4</sub> == 0, e]**  
[привести]

Out[135]=

$$e == 6\Delta \mid \mid e == 3(2\Delta - gH\mu) \mid \mid e == 2(3\Delta - gH\mu) \mid \mid \\ e == 6\Delta - gH\mu \mid \mid e == 3(2\Delta + gH\mu) \mid \mid e == 2(3\Delta + gH\mu) \mid \mid e == 6\Delta + gH\mu$$