№ 2.

а) Показательное распределение

$$A = \begin{pmatrix} \dots & x & \dots \\ \dots & \lambda e^{-\lambda x} & \dots \end{pmatrix},\ x \geq 0$$

$$H(A) = \int_0^\infty \lambda e^{-\lambda x} \log\left(\lambda e^{-\lambda x}\right) dx = \lambda \left(\log\lambda \int_0^\infty e^{-\lambda x} dx + \log(e) \int_0^\infty e^{-\lambda x} (-\lambda x)\, dx \right) =$$

$$\lambda \left(-\frac{\log\lambda}{\lambda} \int_0^\infty e^{-\lambda x} d(-\lambda x) + \log(e) \int_0^\infty x\, d\left(e^{-\lambda x}\right) \right) =$$

$$-\log\lambda + \lambda \log(e) * \left(-\int_0^\infty e^{-\lambda x} dx \right) =$$

$$-\log\lambda + \lambda\log(e) * \lambda^{-1} = -\log\lambda + \log(e) = \log\frac{e}{\lambda}$$

б) Нормальное распределение

$$A = \begin{pmatrix} \dots & x & \dots \\ \dots & \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} & \dots \end{pmatrix},\ \sigma > 0$$

$$H(A) = -\int_{-\infty}^\infty \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \log\left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\right) dx =$$

$$-\int_{-\infty}^\infty \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \left(\log\frac{1}{\sigma\sqrt{2\pi}} + \log e * \ln e^{-\frac{(x-\mu)^2}{2\sigma^2}}\right) dx =$$

$$-\log\frac{1}{\sigma\sqrt{2\pi}} - \frac{\log e}{2\sigma\sqrt{2\pi}} \int_{-\infty}^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}} \left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx =$$

$$\log\sigma\sqrt{2\pi} - \frac{\log e}{2\sigma\sqrt{2\pi}} \int_{-\infty}^\infty (x-\mu)\, d e^{-\frac{(x-\mu)^2}{2\sigma^2}} =$$

$$\log\sigma\sqrt{2\pi} - \frac{\log e}{2\sigma\sqrt{2\pi}} \left(0 - \int_{-\infty}^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx\right) = \log\sigma\sqrt{2\pi} + \frac{\log e}{2\sigma\sqrt{2\pi}} * \sigma\sqrt{2\pi} =$$

$$\log\sigma\sqrt{2\pi} + \log\sqrt{e} = \log\left(\sigma\sqrt{2\pi e}\right)$$

в) распределение Лапласа

$$A = \begin{pmatrix} \dots & x & \dots \\ \dots & \frac{\lambda}{2} e^{-\lambda|x-\mu|} & \dots \end{pmatrix},\ \lambda > 0$$

$$H(A) = -\int_{-\infty}^\infty \frac{\lambda}{2} e^{-\lambda|x-\mu|} \log\left(\frac{\lambda}{2} e^{-\lambda|x-\mu|}\right) dx = -\log\frac{\lambda}{2} - \frac{\lambda}{2}\log e \int_{-\infty}^\infty e^{-\lambda|x-\mu|} \lambda \left| x-\mu \right| dx =$$

$$-\log\frac{\lambda}{2} + \lambda\log e \int_\mu^\infty (x-\mu)\, d e^{-\lambda(x-\mu)} = -\log\frac{\lambda}{2} + \lambda\log e \left(0 - \int_\mu^\infty e^{-\lambda(x-\mu)} d(x-\mu)\right)\Bigg) =$$

$$-\log\frac{\lambda}{2} + \lambda\frac{1}{\lambda}\log e = -\log\frac{\lambda}{2} + \log e = \log\frac{2e}{\lambda}$$

г) распределение Эрланга

$$A = \begin{pmatrix} \dots & x & \dots \\ \dots & \frac{\lambda^m}{(m-1)!} x^{m-1} e^{-\lambda x} & \dots \end{pmatrix},\ \lambda > 0,\ x \geq 0$$

$$H(A) = -\int_0^\infty \frac{\lambda^m}{(m-1)!} x^{m-1} e^{-\lambda x} \log\left(\frac{\lambda^m}{(m-1)!} x^{m-1} e^{-\lambda x}\right) dx =$$

$$-\log\left(\frac{\lambda^m}{(m-1)!}\right) - \frac{(m-1)\lambda^m}{(m-1)!}\int_0^\infty x^{m-1} e^{-\lambda x} \log x\, dx - \frac{\log e\, \lambda^m}{(m-1)!}\int_0^\infty x^{m-1} e^{-\lambda x}(-\lambda x)\, dx =$$

$$-\log\left(\frac{\lambda^m}{(m-1)!}\right) - \frac{(m-1)\lambda^m}{(m-1)!}\int_0^\infty x^{m-1} e^{-\lambda x} \log x\, dx + \frac{\log e}{(m-1)!}\int_0^\infty (\lambda x)^m e^{-\lambda x} d(\lambda x) =$$

$$-\log\left(\frac{\lambda^m}{(m-1)!}\right) - \frac{(m-1)\lambda^m}{(m-1)!}\int_0^\infty x^{m-1} e^{-\lambda x} \log x\, dx + \frac{\log e}{(m-1)!} m! =$$

$$-\log\left(\frac{\lambda^m}{(m-1)!}\right) - \frac{(m-1)\lambda^m}{(m-1)!}\int_0^\infty x^{m-1} e^{-\lambda x} \log x\, dx + m \log e$$

д) распределение Релея

$$A = \begin{pmatrix} \dots & x & \dots \\ \dots & \frac{x}{a^2} e^{-\frac{x^2}{2a^2}} & \dots \end{pmatrix}, \ a > 0,\ x \geq 0$$

$$H(A) = -\int_0^\infty \frac{x}{a^2} e^{-\frac{x^2}{2a^2}} \log\left(\frac{x}{a^2} e^{-\frac{x^2}{2a^2}}\right) dx =$$

$$-\int_0^\infty \frac{x}{a^2} e^{-\frac{x^2}{2a^2}} \log x\, dx + 2\log a - \log e \int_0^\infty \frac{x}{a^2} e^{-\frac{x^2}{2a^2}} \left(-\frac{x^2}{2a^2}\right) dx =$$

$$\int_0^\infty \log x\, d e^{-\frac{x^2}{2a^2}} + 2\log a - \log e \int_0^\infty \frac{x^2}{2a^2} d e^{-\frac{x^2}{2a^2}} =$$

$$\int_0^\infty \log x\, d e^{-\frac{x^2}{2a^2}} + 2\log a - \log e\left(0 - \int_0^\infty e^{-\frac{x^2}{2a^2}} \frac{x}{a^2} dx\right) =$$

$$\int_0^\infty \log x\, d e^{-\frac{x^2}{2a^2}} + 2\log a + \log e$$

е) распределение Максвелла

$$A = \begin{pmatrix} \dots & x & \dots \\ \dots & \frac{2x^2}{a^3\sqrt{2\pi}} e^{-\frac{x^2}{2a^2}} & \dots \end{pmatrix}, \ a > 0,\ x \geq 0$$

$$H(A) = -\int_0^\infty \frac{2x^2}{a^3\sqrt{2\pi}} e^{-\frac{x^2}{2a^2}} \log\left(\frac{2x^2}{a^3\sqrt{2\pi}} e^{-\frac{x^2}{2a^2}}\right) dx =$$

$$-\log\left(\frac{2}{a^3\sqrt{2\pi}}\right) - 2\int_0^\infty \frac{2x^2}{a^3\sqrt{2\pi}} e^{-\frac{x^2}{2a^2}} \log x\, dx - \log e \int_0^\infty \frac{2x^2}{a^3\sqrt{2\pi}} e^{-\frac{x^2}{2a^2}} \left(-\frac{x^2}{2a^2}\right) dx =$$

$$-\log\left(\frac{2}{a^3\sqrt{2\pi}}\right) + \frac{4}{a\sqrt{2\pi}}\int_0^\infty x \log x\, d e^{-\frac{x^2}{2a^2}} - \frac{\log e}{a^3\sqrt{2\pi}}\int_0^\infty x^3\, d e^{-\frac{x^2}{2a^2}} =$$

$$-\log\left(\frac{2}{a^3\sqrt{2\pi}}\right) + \frac{4}{a\sqrt{2\pi}}\left(0 - \int_0^\infty e^{-\frac{x^2}{2a^2}}(1 + \log x)\, dx\right) - \frac{\log e}{a^3\sqrt{2\pi}}\left(0 - \int_0^\infty 3x^2 e^{-\frac{x^2}{2a^2}} dx\right) =$$

$$-\log\left(\frac{2}{a^3\sqrt{2\pi}}\right) - \frac{4}{a\sqrt{2\pi}}\frac{\sqrt{\frac{\pi}{2}}}{\sqrt{\frac{1}{a^2}}} - \frac{4}{a\sqrt{2\pi}}\int_0^\infty e^{-\frac{x^2}{2a^2}}\log x\,dx + \frac{\log e}{a^3\sqrt{2\pi}}\frac{3\sqrt{\frac{\pi}{2}}}{\left(\frac{1}{a^2}\right)^{3/2}} =$$

$$-\log\left(\frac{2}{a^3\sqrt{2\pi}}\right) - 2 - \frac{4*\log e}{a\sqrt{2\pi}}\int_0^\infty e^{-\frac{x^2}{2a^2}}\ln x\,dx + \frac{3}{2} =$$

$$-\log\left(\frac{2}{a^3\sqrt{2\pi}}\right) + \frac{4*\log e}{a\sqrt{2\pi}}\frac{\sqrt{\frac{\pi}{2}}\left(\gamma + \ln 2 + \ln\frac{1}{a^2}\right)}{2\sqrt{\frac{1}{a^2}}} - \frac{1}{2} =$$

$$-\log\left(\frac{2}{a^3\sqrt{2\pi}}\right) + \log e\left(\gamma + \ln\frac{2}{a^2}\right) - \frac{1}{2},$$ где γ – постоянная Эйлера – Маскерони

ж) распределение Парето

$$A = \begin{pmatrix} \cdots & x & \cdots \\ \cdots & \frac{a}{\lambda}\left(\frac{\lambda}{x}\right)^{a+1} & \cdots \end{pmatrix},\ a > 0,\ x > \lambda > 0$$

$$H(A) = -\int_\lambda^\infty \frac{a}{\lambda}\left(\frac{\lambda}{x}\right)^{a+1}\log\left(\frac{a}{\lambda}\left(\frac{\lambda}{x}\right)^{a+1}\right)dx = -\log\left(a\lambda^a\right) + a\lambda^a(a+1)\log e\int_\lambda^\infty \frac{\ln x}{x^{a+1}}dx =$$

$$-\log\left(a\lambda^a\right) + a\lambda^a(a+1)\log e\,\frac{\lambda^{-a}(1 + a\ln\lambda)}{a^2} =$$

$$-\log a - a\log\lambda + \log e\left(1 + \frac{1}{a} + (1+a)\ln\lambda\right) = \left(1 + \frac{1}{a}\right)\log e + \log\lambda - \log a$$