$$\xi \sim \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\xi_2 \sim \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix}$$

$$\xi + \xi_2 = \eta \sim \begin{pmatrix} 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ \frac{1}{36} & \frac{2}{36} & \frac{3}{36} & \frac{4}{36} & \frac{5}{36} & \frac{6}{36} & \frac{5}{36} & \frac{4}{36} & \frac{3}{36} & \frac{2}{36} & \frac{1}{36} \end{pmatrix}$$

$\forall\ k = 2, \ldots, 7$

$$p\left(\eta = k\right) =$$

$$p\left(\xi + \xi_2 = k\right) = p\left(\bigcup_{t=1}^{k-1} \{(\xi, \xi_2) : \xi = t,\ \xi_2 = k - t - 1\}\right) = \sum_{t=1}^{k-1} p\left(\xi = t,\ \xi_2 = k - t - 1\right) =$$

$$\sum_{t=1}^{k-1} p\left(\xi = t\right) p\left(\xi_2 = k - t - 1\right) = \sum_{t=1}^{k-1} p\left(\xi = t\right) p\left(\xi_2 = k - t - 1\right) = \left(k - 1\right) \frac{1}{6} * \frac{1}{6} = \frac{\left(k - 1\right)}{36},$$

$\forall\ k = 8, \ldots, 12$

$$p\left(\eta = k\right) = p\left(\xi + \xi_2 = k\right) = p\left(\bigcup_{t=k-6}^{6} \{(\xi, \xi_2) : \xi = t,\ \xi_2 = k - t\}\right) =$$

$$\sum_{t=k-6}^{6} p\left(\xi = t,\ \xi_2 = k - t - 1\right) = \sum_{t=k-6}^{6} p\left(\xi = t\right) p\left(\xi_2 = k - t - 1\right) =$$

$$\sum_{t=k-6}^{6} p\left(\xi = t\right) p\left(\xi_2 = k - t - 1\right) = \left(12 - k + 1\right) \frac{1}{6} * \frac{1}{6} = \frac{\left(13 - k\right)}{36},$$

$\forall\ n = 1, \ldots, 6;\ k = n + 1, \ldots, 7$

$$p\left(\eta = k \mid \xi = n\right) =$$

$$\frac{p\left(\xi + \xi_2 = k,\ \xi = n\right)}{p\ (\xi = n)} = \frac{p\left(\xi_2 = k - n,\ \xi = n\right)}{p\ (\xi = n)} = \frac{p\left(\xi_2 = k - n\right) * p\ (\xi = n)}{p\ (\xi = n)} = p\left(\xi_2 = k - n\right) = \frac{1}{6}$$

$\forall\ n = 1, \ldots, 6;\ k = 1, \ldots, n$

$$p\left(\eta = k \mid \xi = n\right) = \frac{p\left(\xi + \xi_2 = k,\ \xi = n\right)}{p\ (\xi = n)} =$$

$$\frac{p\left(\xi_2 = k - n,\ \xi = n\right)}{p\ (\xi = n)} = \frac{p\left(\xi_2 = k - n\right) * p\ (\xi = n)}{p\ (\xi = n)} = p\left(\xi_2 = k - n\right) = 0, \text{ т.к. } k - n < 1$$

$\forall\ k = 8, \ldots, 12;\ n = k - 6, \ldots, 6$

$$p\left(\eta = k \mid \xi = n\right) =$$

$$\frac{p\left(\xi + \xi_2 = k,\ \xi = n\right)}{p\ (\xi = n)} = \frac{p\left(\xi_2 = k - n,\ \xi = n\right)}{p\ (\xi = n)} = \frac{p\left(\xi_2 = k - n\right) * p\ (\xi = n)}{p\ (\xi = n)} = p\left(\xi_2 = k - n\right) = \frac{1}{6}$$

$\forall\ k = 8, \ldots, 12;\ n = 1, \ldots, k - 6 - 1$

$$p\left(\eta = k \mid \xi = n\right) = \frac{p\left(\xi + \xi_2 = k,\ \xi = n\right)}{p\ (\xi = n)} =$$
$$\frac{p\left(\xi_2 = k - n,\ \xi = n\right)}{p\ (\xi = n)} = \frac{p\left(\xi_2 = k - n\right) * p\ (\xi = n)}{p\ (\xi = n)} = p\left(\xi_2 = k - n\right) = 0,\ \text{т.к.}\ k - n > 6$$

Способ 1.

Уберём из рассмотрения невосместные события

$$I\left(n,\ k\right) = I\left(\xi = n;\ \eta = k\right) =$$
$$\log \frac{p\left(\eta = k \mid \xi = n\right)}{p\left(\eta = k\right)} = \begin{cases} \log \frac{1/6}{\frac{(k-1)}{36}}, & n = 1,\ \ldots,\ 6,\ k = n + 1,\ \ldots,\ 7 \\ \log \frac{1/6}{\frac{(13-k)}{36}}, & k = 8,\ \ldots,\ 12,\ n = k - 6,\ \ldots,\ 6 \end{cases} =$$
$$\begin{cases} \log 6 - \log\left(k - 1\right), & n = 1,\ \ldots,\ 6,\ k = n + 1,\ \ldots,\ 7 \\ \log 6 - \log\left(13 - k\right), & k = 8,\ \ldots,\ 12,\ n = k - 6,\ \ldots,\ 6 \end{cases}$$

$$I_{\xi;\eta} \sim \begin{pmatrix} I\left(1;\ 2\right) & I\left(1;\ 3\right) & \ldots & I\left(6;\ 12\right) \\ p\ \left(\xi = 1\right) * p\left(\eta = 2 \mid \xi = 1\right) & p\left(\xi = 1\right) * p\left(\eta = 3 \mid \xi = 1\right) & \ldots & p\ (\xi = 6) * p\left(\eta = 12 \mid \xi = 6\right) \end{pmatrix}$$

$$I\ (\xi;\ \eta) =$$
$$\mathbb{E}\left[I_{\xi;\eta}\right] =$$
$$\sum_{i=1}^{6} \sum_{j=i+1}^{7} I\left(i,\ j\right) * p\left(\xi = i\right) * p\left(\eta = j \mid \xi = i\right) + \sum_{j=8}^{12} \sum_{i=j-6}^{6} I\left(i,\ j\right) * p\left(\xi = i\right) * p\left(\eta = j \mid \xi = i\right) =$$

$$\sum_{i=1}^{6} \sum_{j=i+1}^{7} \left(\log 6 - \log\left(j - 1\right)\right) * \frac{1}{6} * \frac{1}{6} + \sum_{j=8}^{12} \sum_{i=j-6}^{6} \left(\log 6 - \log\left(13 - j\right)\right) * \frac{1}{6} * \frac{1}{6} =$$

$$\frac{21}{36} \log 6 - \frac{1}{36} \sum_{i=1}^{6} \sum_{j=i+1}^{7} \log\left(j - 1\right) + \frac{15}{36} \log 6 - \frac{1}{36} \sum_{j=8}^{12} \sum_{i=j-6}^{6} \log\left(13 - j\right) =$$

$$\log 6 - \frac{1}{36} \left(\sum_{i=1}^{6} \left(\log i + \log\left(i + 1\right) + \ldots + \log 6\right) + \sum_{j=8}^{12} \left(13 - j\right) \log\left(13 - j\right)\right) =$$

$$\log 6 - \frac{1}{36}$$
$$\left(\left(2 \log 2 + 3 \log 3 + 4 \log 4 + 5 \log 5 + 6 \log 6\right) + \left(5 \log 5 + 4 \log 4 + 3 \log 3 + 2 \log 2\right)\right) =$$

$$\frac{5 * \log 6}{6} - \frac{5 * \log 5}{18} - \frac{4 * \log 4}{9} - \frac{\log 3}{6} - \frac{\log 2}{9}$$

Способ 2.

$$H\ (A_\xi) = \log 6$$

$$H\ (B_\eta) = -\sum_{k=2}^{7} p\left(\eta = k\right) * \log p\left(\eta = k\right) - \sum_{k=8}^{12} p\left(\eta = k\right) * \log p\left(\eta = k\right) =$$

$$-\sum_{k=2}^{7} \frac{\left(k - 1\right)}{36} * \log \frac{\left(k - 1\right)}{36} - \sum_{k=8}^{12} \frac{\left(13 - k\right)}{36} * \log \frac{\left(13 - k\right)}{36} =$$

$$\frac{-1}{36} * \left(\sum_{k=2}^{7} (k-1) * \log (k-1) - \sum_{k=2}^{7} (k-1) * \log 36 + \sum_{k=8}^{12} (13-k) * \log (13-k) - \sum_{k=8}^{12} (13-k) * \log 36\right) =$$

$$\frac{-1}{36} * \left(\left(2 \log 2 + 3 \log 3 + 4 \log 4 + 5 \log 5 + 6 \log 6\right) - 21 * \log 36 + \left(2 \log 2 + 3 \log 3 + 4 \log 4 + 5 \log 5\right) - 15 * \log 36\right) =$$

$$\frac{-1}{36} * \left(4 \log 2 + 6 \log 3 + 8 \log 4 + 10 \log 5 + 6 \log 6 - 36 * \log 36\right) =$$

$$\log 36 - \frac{\log 2}{9} - \frac{\log 3}{6} - \frac{4 * \log 4}{9} - \frac{5 * \log 5}{18}$$

$$H(AB) = -\sum_{n=1}^{6} \sum_{k=n+1}^{7} p(\eta = k, \xi = n) * \log p(\eta = k, \xi = n) - \sum_{k=8}^{12} \sum_{n=k-6}^{6} p(\eta = k, \xi = n) * \log p(\eta = k, \xi = n) =$$

$$-\sum_{n=1}^{6} \sum_{k=1+n}^{7} p(\xi_2 = k-n) * p(\xi = n) * \log\left(p(\xi_2 = k-n) * p(\xi = n)\right) - \sum_{k=8}^{12} \sum_{n=k-6}^{6} p(\xi_2 = k-n) * p(\xi = n) * \log\left(p(\xi_2 = k-n) * p(\xi = n)\right) =$$

$$-\sum_{n=1}^{6} \sum_{k=n+1}^{7} \frac{1}{6} * \frac{1}{6} * \log\left(\frac{1}{6} * \frac{1}{6}\right) - \sum_{k=8}^{12} \sum_{n=k-6}^{6} \frac{1}{6} * \frac{1}{6} * \log\left(\frac{1}{6} * \frac{1}{6}\right) = \log 36$$

$$I(A_\xi, B_\eta) = H(A_\xi) + H(B_\eta) - H(AB) = \log 6 - \frac{\log 2}{9} - \frac{\log 3}{6} - \frac{4 * \log 4}{9} - \frac{5 * \log 5}{18} - \frac{\log 6}{6} = \frac{5 * \log 6}{6} - \frac{5 * \log 5}{18} - \frac{4 * \log 4}{9} - \frac{\log 3}{6} - \frac{\log 2}{9}$$