

$$\xi \sim \left( \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{array} \right)$$

$$\xi_2 \sim \left( \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{array} \right)$$

$$\xi + \xi_2 = \eta \sim \left( \begin{array}{cccccccccccc} 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ \frac{1}{36} & \frac{2}{36} & \frac{3}{36} & \frac{4}{36} & \frac{5}{36} & \frac{6}{36} & \frac{5}{36} & \frac{4}{36} & \frac{3}{36} & \frac{2}{36} & \frac{1}{36} \end{array} \right)$$

$$(1, 12)$$

$$\forall k = 2, \dots, 7$$

$$p(\eta = k) =$$

$$p(\xi + \xi_2 = k) = p\left(\bigcup_{t=1}^{k-1} \{(\xi, \xi_2) : \xi = t, \xi_2 = k - t - 1\}\right) = \sum_{t=1}^{k-1} p(\xi = t, \xi_2 = k - t - 1) =$$

$$\sum_{t=1}^{k-1} p(\xi = t) p(\xi_2 = k - t - 1) = \sum_{t=1}^{k-1} p(\xi = t) p(\xi_2 = k - t - 1) = (k-1) \frac{1}{6} * \frac{1}{6} = \frac{(k-1)}{36},$$

$$\forall k = 8, \dots, 12$$

$$p(\eta = k) = p(\xi + \xi_2 = k) = p\left(\bigcup_{t=k-6}^6 \{(\xi, \xi_2) : \xi = t, \xi_2 = k - t\}\right) =$$

$$\sum_{t=k-6}^6 p(\xi = t, \xi_2 = k - t - 1) = \sum_{t=k-6}^6 p(\xi = t) p(\xi_2 = k - t - 1) =$$

$$\sum_{t=k-6}^6 p(\xi = t) p(\xi_2 = k - t - 1) = (12 - k + 1) \frac{1}{6} * \frac{1}{6} = \frac{(13 - k)}{36},$$

$$\forall k = 2, \dots, 7; n = 1, \dots, 6$$

$$p(\eta = k | \xi = n) =$$

$$\frac{p(\xi + \xi_2 = k, \xi = n)}{p(\xi = n)} = \frac{p(\xi_2 = k - n, \xi = n)}{p(\xi = n)} = \frac{p(\xi_2 = k - n) * p(\xi = n)}{p(\xi = n)} = p(\xi_2 = k - n) = \frac{1}{6}$$

$$\forall k = 8, \dots, 12; n = k - 6, \dots, 6$$

$$p(\eta = k | \xi = n) =$$

$$\frac{p(\xi + \xi_2 = k, \xi = n)}{p(\xi = n)} = \frac{p(\xi_2 = k - n, \xi = n)}{p(\xi = n)} = \frac{p(\xi_2 = k - n) * p(\xi = n)}{p(\xi = n)} = p(\xi_2 = k - n) = \frac{1}{6}$$

$$\forall k = 8, \dots, 12; n = 1, \dots, k - 6 - 1$$

$$p(\eta = k | \xi = n) = \frac{p(\xi + \xi_2 = k, \xi = n)}{p(\xi = n)} =$$

$$\frac{p(\xi_2 = k - n, \xi = n)}{p(\xi = n)} = \frac{p(\xi_2 = k - n) * p(\xi = n)}{p(\xi = n)} = p(\xi_2 = k - n) = 0, \text{ т.к. } k - n > 6$$

Уберём из рассмотрения несовместные события

$$I(n, k) = I(\xi = n; \eta = k) = \log \frac{p(\eta = k | \xi = n)}{p(\eta = k)} = \begin{cases} \log \frac{1/6}{\frac{(k-1)}{36}}, & k = 2, \dots, 7, n = 1, \dots, 6 \\ \log \frac{1/6}{\frac{(13-k)}{36}}, & k = 8, \dots, 12, n = k-6, \dots, 6 \end{cases} = \begin{cases} \log 6 - \log(k-1), & k = 2, \dots, 7, n = 1, \dots, 6 \\ \log 6 - \log(13-k), & k = 8, \dots, 12, n = k-6, \dots, 6 \end{cases}$$

$$I_{\xi; \eta} \sim \left( \begin{array}{ccccccc} I(1; 2) & I(1; 3) & \dots & I(6; 12) \\ p(\xi = 1) * p(\eta = 2 | \xi = 1) & p(\xi = 1) * p(\eta = 3 | \xi = 1) & \dots & p(\xi = 6) * p(\eta = 12 | \xi = 6) \end{array} \right)$$

$$I(\xi; \eta) = \mathbb{E}[I_{\xi; \eta}] = \sum_{j=1}^7 \sum_{i=1}^6 I(i, j) * p(\xi = i) * p(\eta = j | \xi = i) + \sum_{j=8}^{12} \sum_{i=j-6}^6 I(i, j) * p(\xi = i) * p(\eta = j | \xi = i) =$$

$$\sum_{j=1}^7 \sum_{i=1}^6 (\log 6 - \log(j-1)) * \frac{1}{6} * \frac{1}{6} + \sum_{j=8}^{12} \sum_{i=j-6}^6 (\log 6 - \log(13-j)) * \frac{1}{6} * \frac{1}{6} =$$

$$\log 6 - \frac{1}{36} \sum_{j=2}^7 6 * \log(j-1) + \frac{1}{36} \sum_{j=8}^{12} ((13-j) * (\log 6 - \log(13-j))) =$$

$$\log 6 - \frac{1}{6} \log(6!) + \frac{5}{36} * \log 6 \sum_{j=8}^{12} (13-j) - \frac{1}{36} \sum_{j=8}^{12} (13-j) \log(13-j) =$$

$$\log 6 - \frac{1}{6} \log(720) + \frac{25}{12} * \log 6 - \frac{1}{36} \sum_{j=8}^{12} (13-j) \log(13-j) =$$

$$\frac{37}{12} * \log 6 - \frac{1}{6} \log(720) - \frac{1}{36} (\log(5^5 * 4^4 * 3^3 * 2^2)) =$$

$$\frac{37}{12} * \log 6 - \frac{1}{6} \log(720) - \frac{1}{36} \left( \log\left(\frac{5!}{0!}\right) + \log\left(\frac{5!}{1!}\right) + \log\left(\frac{5!}{2!}\right) + \log\left(\frac{5!}{3!}\right) + \log\left(\frac{5!}{4!}\right) \right) =$$

$$\frac{37}{12} * \log 6 - \frac{1}{6} \log(720) - \frac{1}{36} (\log(5 * 5!) - \log(2! 3! 4!)) =$$

$$\frac{37}{12} * \log 6 + \frac{1}{36} \log(228) - \left( \frac{1}{6} \log(720) + \frac{1}{36} \log(600) \right)$$