I in Dirac Schreibweise

$$\sigma_1: \quad |+\rangle_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \quad , \quad |-\rangle_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

$$\sigma_2: \quad |+\rangle_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \quad , \quad |-\rangle_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

$$\sigma_3: \quad |+\rangle_3 = \begin{pmatrix}1\\0\end{pmatrix} \quad , \quad |-\rangle_3 = \begin{pmatrix}0\\1\end{pmatrix}$$

$$|\psi\rangle_1 = \frac{1}{\sqrt{2}}\left(|+\rangle_1 + |-\rangle_1\right) \quad , \quad |\psi\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

$$|\psi\rangle_2 = \frac{1}{\sqrt{2}}\left(|+\rangle_2 + |-\rangle_2\right)$$

$$|\psi\rangle_3 = |+\rangle$$

$${}_1\langle\psi|\sigma_i|\psi\rangle_1$$

$$\sigma_1: \quad {}_1\langle\psi|\sigma_1|\psi\rangle_1 = \frac{1}{2}\Big[{}_1\langle+|\sigma_1|+\rangle_1 + \underbrace{\cancel{{}_1\langle+|\sigma_1|-\rangle_1}}_{0}$$

0, da orthonormal

$$+ \cancel{{}_1\langle-|\sigma_1|+\rangle} + {}_1\langle-|\sigma_1|-\rangle_1\Big]$$

$$= \frac{1}{2}\Big[(+1)\underbrace{{}_1\langle+|+\rangle_1}_{1} + (-1)\underbrace{{}_1\langle-|-\rangle_1}_{1}\Big]$$

$$= 0$$

III.

a) $(\hat{A})_{ij} = \langle i | \hat{A} | j \rangle \quad \rightarrow \quad \langle 1 | \hat{A} | 1 \rangle = 5$

d) $|\psi\rangle = a|1\rangle + b|2\rangle$

$$\langle \psi | A | \psi \rangle = \langle \psi | A \left(a|1\rangle + b|2\rangle \right)$$

$$= \langle \psi | \Big[a\big(5|1\rangle + \underbrace{a|2\rangle}_{\beta=0}\big) + b\big(\underbrace{\beta|1\rangle}_{\beta=0} + i|3\rangle\big) \Big]$$

$$= \left(\langle 1 | a^* + \cancel{\langle 2 | \beta^*} \right) \left(5a|1\rangle + \cancel{ib|3\rangle} \right)$$

$$= 5a^* a \langle 1 | 1 \rangle$$

$$= 5|a|^2$$

e) $|\lambda_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = |1\rangle$

$\lambda_1 = 1$

$\lambda_2 = \frac{\gamma}{2} \pm \sqrt{\frac{\gamma^2}{4} + 1}$

$\{|1\rangle, |2\rangle, |3\rangle\}$

$N_{2,3} = \sqrt{1 + \lambda_{2,3}^2}$

$$|\lambda_2\rangle = \frac{1}{N_2} \begin{pmatrix} 0 \\ 1 \\ i\lambda_2 \end{pmatrix}$$

$$= \frac{1}{N_2} |2\rangle + \frac{i\lambda_2}{N_2} |3\rangle$$

$$|\lambda_3\rangle = \frac{1}{N_3} \begin{pmatrix} 0 \\ 1 \\ i\lambda_3 \end{pmatrix} = \frac{1}{N_3} |2\rangle + \frac{i\lambda_3}{N_3} |3\rangle$$

$$|\psi\rangle = a|1\rangle + b|2\rangle$$

$$|\psi\rangle = \sum_n \langle\lambda_n|\psi\rangle|\lambda_n\rangle$$

$$\Rightarrow \quad |2\rangle = \cancelto{0}{\langle\lambda_1|2\rangle}|\lambda_1\rangle + \underbrace{\langle\lambda_2|2\rangle}_{1/N_2}|\lambda_2\rangle + \underbrace{\langle\lambda_3|3\rangle}_{1/N_3}|\lambda_3\rangle$$

$$\Rightarrow |\psi\rangle = a|1\rangle + b|2\rangle$$

$$= a|\lambda_1\rangle + \frac{b}{N_2}|\lambda_2\rangle + \frac{b}{N_3}|\lambda_3\rangle$$

$$\Rightarrow \langle\psi|\hat{A}|\psi\rangle = \langle\psi|\hat{A}\left(a|\lambda_1\rangle + \frac{b}{N_2}|\lambda_2\rangle + \frac{b}{N_3}|\lambda_3\rangle\right)$$

$$= \langle\psi|\underbrace{\left(a\lambda_1|\lambda_1\rangle + \frac{b}{N_2}\lambda_2|\lambda_2\rangle + \frac{b}{N_3}\lambda_3|\lambda_3\rangle\right)}_{=|\gamma\rangle}$$

$$= \left(a^*\langle\lambda_1| + \left(\frac{b}{N_2}\right)^*\langle\lambda_2| + \left(\frac{b}{N_3}\right)^*\langle\lambda_3|\right)\cdot|\gamma\rangle$$

$$= |a|^2\lambda_1\langle\lambda_1|\lambda_1\rangle + \frac{b^2\lambda_2}{N_2^2}\langle\lambda_2|\lambda_2\rangle + \frac{b^2\lambda_3}{N_3^2}\langle\lambda_3|\lambda_3\rangle$$

$$= \int \psi^*(x)\,\hat{A}\,\psi(x)\,d^3x$$

IV.

b) $\langle 1|\hat{P}|1\rangle = \frac{1}{2}$, $\langle 1|P^2|1\rangle = \frac{1}{4}$

$P|1\rangle = \frac{1}{2}|1\rangle + \alpha|2\rangle$

2-Niveau-System

$\hookrightarrow \{|1\rangle, |2\rangle\}$

$\langle 1|P|2\rangle = \frac{1}{2}\langle 1|2\rangle + \alpha^*\langle 2|2\rangle = \alpha^*$

P hermitesch:

$\langle 2|P|1\rangle = \alpha$

$P|2\rangle = \alpha^*|1\rangle + \beta|2\rangle$

$$
\begin{aligned}
P^2|1\rangle &= P(P|1\rangle) \\
&= \frac{1}{2}P|1\rangle + \alpha P|2\rangle \\
&= \frac{1}{4}|1\rangle + \frac{\alpha}{2}|2\rangle + |\alpha|^2|1\rangle + \alpha\beta|2\rangle \\
&= \left(\frac{1}{4} + |\alpha|^2\right)|1\rangle + \left(\frac{\alpha}{2} + \alpha\beta\right)|2\rangle
\end{aligned}
$$

Da $\langle 1|P^2|1\rangle = \frac{1}{4}$ $\Rightarrow$ $|\alpha|^2 = 0$