II.

$$H = \underbrace{\frac{\hat{p}_x^2}{2m} + V(\hat{x})}_{H_0} \underbrace{- B_y \hat{\mu}_y}_{H_B} \quad , \quad \begin{array}{ll} V(x) = 0 & \text{für } x \in [0,L] \\ V(x) = \infty & \text{sonst} \end{array}$$

1) $H_0 |\varphi_n\rangle = E_n |\varphi_n\rangle$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \quad , n \in \mathbb{N}$$

Eigenzustände von H_B mit $\hat{\mu}_y |\pm\rangle_y = \pm \mu_0 |\pm\rangle_y$

↳ Eigenzustände von $\hat{H}$: $(|\phi_n\rangle \otimes |\pm\rangle_y)$

$$H(|\phi_n\rangle \otimes |\pm\rangle_y) = H_0(|\phi_n\rangle \otimes |\pm\rangle_y) + H_B(|\phi_n\rangle \otimes |\pm\rangle_y)$$

$$= H_0|\phi_n\rangle \otimes \mathbb{1}|\pm\rangle_y + \mathbb{1}|\phi_n\rangle \otimes H_B|\pm\rangle_y$$

$$= E_n(|\phi_n\rangle \otimes |\pm\rangle_y) - B_y(\pm\mu_0 |\phi_n\rangle \otimes |\pm\rangle_y)$$

$$= (E_n \mp B_y\mu_0)|\phi_n\rangle \otimes |\pm\rangle_y$$

$\Rightarrow$ Grundzustand $|\phi_1\rangle \otimes |+\rangle_y$ mit EW: $E_0 - \mu_0 B_y$

2) zu $t=0$ mit $|\psi(t=0)\rangle$

$$|\psi(t=0)\rangle = N \underbrace{(|\phi_1\rangle + |\phi_2\rangle)}_{\text{Eigenzustände von } H_0} \otimes \underbrace{(|+\rangle_z + |-\rangle_z)}_{\text{EZ von } \hat{\mu}_z}$$

Normierung

$$\langle \Psi(0) | \Psi(0) \rangle = |N|^2 \left(\langle \phi_1 | \phi_1 \rangle + \langle \phi_2 | \phi_2 \rangle \right) \cdot \left(\langle + | + \rangle + \langle - | - \rangle \right)$$

$$= |N|^2 \cdot 4 \quad \Rightarrow \quad N = 1/2$$

Zeitentwicklung

$$|\phi_1\rangle \rightarrow e^{-i \frac{E_1 t}{\hbar}} |\phi_1\rangle \qquad |+\rangle_z = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \right) = \frac{1}{\sqrt{2}} \left(|+\rangle_y + |-\rangle_y \right)$$

$$|\phi_2\rangle \rightarrow e^{-i \frac{E_2 t}{\hbar}} |\phi_2\rangle \qquad |-\rangle_z = \frac{1}{\sqrt{2}} \left(-i |+\rangle_y + i |-\rangle_y \right)$$

$$|+\rangle_z \rightarrow \frac{1}{\sqrt{2}} \left(\exp\left(-i \left(-\frac{B_y \mu_0}{\hbar}\right) t\right) |+\rangle_y + \exp\left(-i \left(\frac{B_y \mu_0}{\hbar}\right) t\right) |-\rangle_y \right)$$

$$|-\rangle_z \rightarrow \frac{1}{\sqrt{2}} \left(-i \exp\left(-i \left(-\frac{B_y \mu_0}{\hbar}\right) t\right) |+\rangle_y + i \exp\left(-i \left(\frac{B_y \mu_0}{\hbar}\right) t\right) |-\rangle_y \right)$$

$$|+\rangle_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ +i \end{pmatrix} = \frac{1}{2} \left((1+i) \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + (1-i) \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right)$$

$$= \frac{1+i}{2} |+\rangle_x + \frac{1-i}{2} |-\rangle_x$$

$$|-\rangle_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} = \frac{1-i}{2} |+\rangle_x + \frac{1+i}{2} |-\rangle_x$$

$$\Rightarrow \quad |+\rangle_z + |-\rangle_z \rightarrow |+\rangle_x \frac{1}{\sqrt{2}} \left(e^{i \frac{B_y \mu_0}{\hbar} t} + e^{-i \frac{B_y \mu_0}{\hbar} t} \right) + |-\rangle_x \frac{1}{\sqrt{2}} \left(-i e^{i \frac{B_y \mu_0}{\hbar} t} + i e^{i \frac{B_y \mu_0}{\hbar} t} \right)$$

$\Rightarrow$

i) $$\langle\psi(t)|\hat{\mu}_x|\psi(t)\rangle = \frac{1}{2}\left(e^{-i\alpha t}+e^{i\alpha t}\right)\left(e^{i\alpha t}+e^{-i\alpha t}\right){}_x\langle+|\hat{\mu}_x|+\rangle_x$$
$$+\frac{1}{2}\left(ie^{-i\alpha t}-ie^{i\alpha t}\right)\left(-ie^{i\alpha t}+ie^{-i\alpha t}\right){}_x\langle-|\hat{\mu}_x|-\rangle_x$$

mit $\alpha = \frac{B_y\mu_0}{\hbar}$

$$=\left(e^{2i\alpha t}+e^{-2i\alpha t}\right)\mu_0$$
$$=2\mu_0\cos\left(\frac{2B_y\mu_0}{\hbar}t\right)$$

ii) $$\langle\psi(t)|\hat{\mu}_y|\psi(t)\rangle = \frac{(1+i)(1-i)}{2}{}_y\langle+|\hat{\mu}_y|+\rangle_y+\frac{(1-i)(1+i)}{2}{}_y\langle-|\hat{\mu}_y|-\rangle_y$$
$$=0$$

iii) $$|+\rangle_z+|-\rangle_z \longrightarrow |+\rangle_z\left(\frac{1-i}{2}e^{i\alpha t}+\frac{1+i}{2}e^{-i\alpha t}\right)+|-\rangle_z\left(\frac{1+i}{2}e^{i\alpha t}+\frac{1-i}{2}|-\rangle_z\right)$$

$$\langle\psi(t)|\hat{\mu}_z|\psi t)\rangle = \mu_0\left(\frac{1-i}{2}e^{i\alpha t}+\frac{1+i}{2}\right)^2-\mu_0\left(\frac{1+i}{2}e^{i\alpha t}+\frac{1-i}{2}e^{-i\alpha t}\right)^2$$

$$=2\mu_0\frac{1}{2i}\left(e^{2i\alpha t}-e^{-2i\alpha t}\right)$$
$$=2\mu_0\sin\left(\frac{2B_y\mu_0}{\hbar}t\right)$$

III. $\hat{H} = \frac{1}{2m}\left(\hat{p} - \frac{e}{c}\hat{A}(\hat{r})\right)^2 , \quad \hat{A} = -B_0 \hat{y}\, \vec{e}_x$

3. Enegieniveau?

$\{\hat{H}, \hat{p}_x, \hat{p}_y\}$ VSkO , $\psi(\vec{r}) = \psi(x,y,z)$

$$= e^{i(p_x \cdot x + p_y \cdot y)/\hbar}\, \phi(y)$$

$$\hat{H}\psi(\hat{r}) = E\psi(\hat{r})$$

$$\frac{1}{2m}\left(p_x + \frac{eB_0 y}{c}\right)^2 \overset{1}{\cancel{e^{ixp_x/\hbar}}}\, \overset{1}{\cancel{e^{izp_z/\hbar}}}\, \phi(y)$$

$$-\frac{\hbar^2}{2m}\partial_y^2\, \overset{1}{\cancel{e^{ixp_x/\hbar}}}\, \overset{1}{\cancel{e^{izp_z/\hbar}}}\, \phi(y)$$

$$+\frac{p_z^2}{2m}\, \overset{1}{\cancel{e^{ixp_x/\hbar}}}\, \overset{1}{\cancel{e^{izp_z/\hbar}}}\, \phi(y)$$

$$= E \cdot \overset{1}{\cancel{e^{ixp_x/\hbar}}}\, \overset{1}{\cancel{e^{ip_z z/\hbar}}}\, \phi(y)$$

$$\Rightarrow -\frac{\hbar^2}{2m}\partial_y^2\phi + \frac{m}{1}\underbrace{\left(\frac{eB_0}{mc}\right)^2}_{\omega^2}\underbrace{\left(y + \frac{c}{eB_0}p_x\right)^2}_{:=\tilde{y}}\phi = \underbrace{\left(E - \frac{p_x^2}{2m}\right)}_{:=\tilde{E}}\phi$$

$-\frac{\hbar^2}{2m}\frac{\partial^2\phi}{\partial y^2} + \frac{m}{2}\omega^2\tilde{y}^2\phi = \tilde{E}\phi$ Eigengl. eines harm. Oszillators in y-Richtung

$$\Rightarrow \tilde{E} = (n + \tfrac{1}{2})\hbar\omega = E - \frac{p_z^2}{2m}$$

$$\Rightarrow E = \frac{p_z^2}{2m} + (n + \tfrac{1}{2})\hbar\omega$$

Landau Niveaus

IV. c) Eigenfunktion von $\hat{H}_S$

$$\phi_S \sim e^{i\vec{P}\cdot\vec{R}/\hbar}$$

Die stationären Zustände von H faktorisieren in EZ von $\hat{H}_S$ und $\hat{H}_{rel}$

$\Rightarrow$ stationäre Zustände von $\hat{H}$:

$$\Psi(r,R) \sim e^{i\vec{P}\cdot\vec{R}/\hbar}\,\phi(r)$$

II.c) $\Delta J_x^2 = \langle J_x^2 \rangle - \langle J_x \rangle^2$

Symmetrie $\Rightarrow \Delta J_x = \Delta J_y \qquad \Delta J_x^2 = \frac{1}{2}(\Delta J_x^2 + \Delta J_y^2)$

$$\Delta J_x^2 = \frac{1}{2}\left(\langle J_x^2 \rangle + \langle J_y^2 \rangle - \langle J_x \rangle^2 - \langle J_y \rangle^2\right)$$

$$= \frac{1}{2}\Big(\underbrace{\langle J_x^2 + J_y^2 \rangle}_{= J^2 - J_z^2} - \frac{1}{4}\underbrace{\langle J_+ + J_- \rangle^2}_{= \langle J_+ \rangle^2 + \langle J_- \rangle^2 + \langle J_+ \rangle\langle J_- \rangle + \langle J_- \rangle \langle J_+ \rangle} + \frac{1}{4}\langle J_+ - J_- \rangle^2\Big)$$

$\langle m | J_+ | m \rangle^2 = 0$

$$= \frac{1}{2}\left(\langle J^2 - J_z^2 \rangle\right)$$

$$= \frac{1}{2}\left(\hbar^2 j(j+1) - \hbar^2 m^2\right)$$

$$\Rightarrow \Delta J_x = \Delta J_y = \hbar \sqrt{\frac{j(j+1) - m^2}{2}}$$