2.c)

$$\psi(x,t) = \underbrace{\left(\frac{1}{\pi a_0^2}\right)^{1/4} \frac{1}{\sqrt{1+i\omega t}}}_{①} \underbrace{\exp\left(-\frac{(x+d)^2}{2a_0^2}\frac{1}{1+i\omega t}\right)}_{②}$$

gesucht: $n(x,t) = |\psi(x,t)|^2$

① $\Rightarrow \left|\left(\frac{1}{\pi a_0^2}\right)^{1/4} \frac{1}{\sqrt{1+i\omega t}}\right|^2$

$\frac{1}{a_0\sqrt{\pi}}$ $\quad$ $\sqrt{\left|\frac{1}{1+i\omega t}\right|^2} = \left|\frac{1-i\omega t}{1+\omega^2 t^2}\right|$

$$= \frac{1}{1+\omega^2 t^2}\sqrt{(1-i\omega t)(1+i\omega t)}$$

$$= \frac{1}{\sqrt{1+\omega^2 t^2}}$$

②

$$\Rightarrow \tilde{\psi}(x,t) = \exp\left(-\frac{(x+d)^2}{2a_0^2}\cdot\frac{1}{1+i\omega t}\right)$$

$$\tilde{\psi}^*(x,t) = \exp\left(-\frac{(x+d)^2}{2a_0^2}\frac{1}{1-i\omega t}\right)$$

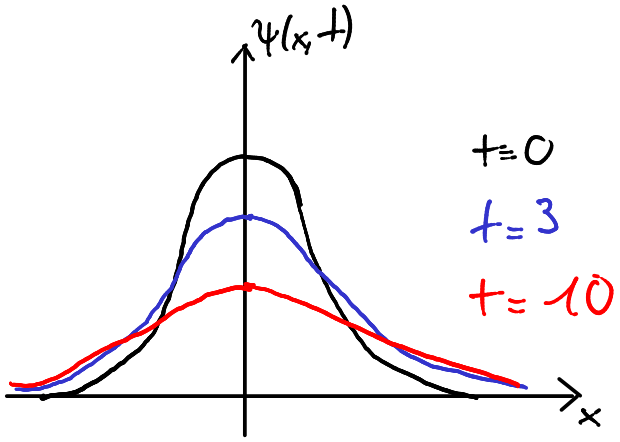
$$|\tilde{\psi}(x,t)\,\tilde{\psi}^*(x,t)| = \exp\left(-\frac{(x+d)^2}{2a_0^2}\underbrace{\left(\frac{1}{1+i\omega t}+\frac{1}{1-i\omega t}\right)}\right)$$

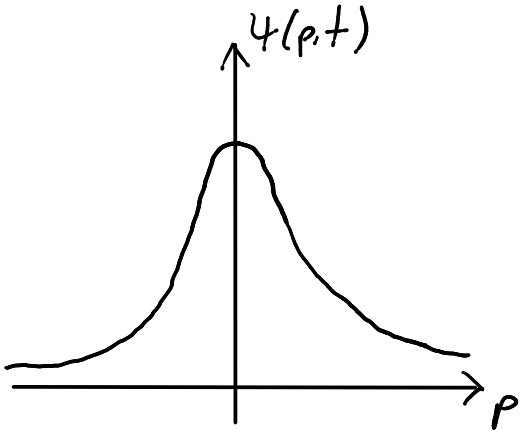
$$= \frac{1+i\omega t+1-i\omega t}{1+\omega^2 t^2}$$

$$= \frac{2}{1+\omega^2 t^2}$$

$$\Rightarrow \quad n(x,t) = \frac{1}{a_0\sqrt{\pi}} \frac{1}{\sqrt{1+\omega^2 t^2}} \exp\left(-\frac{(x+d)^2}{a_0} \frac{1}{1+\omega^2 t^2}\right)$$

2.d)





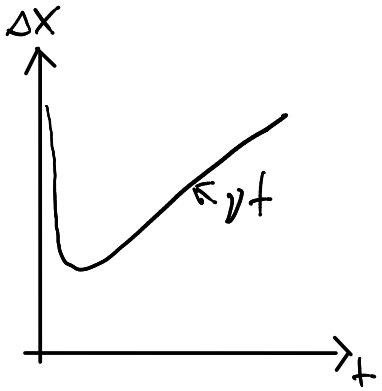
$$(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$$= \int_{-\infty}^{\infty} (x')^2 \frac{1}{a_0\sqrt{\pi}} \frac{1}{\sqrt{1+\omega^2 t^2}} \exp\left(-\frac{(x')^2}{a_0^2} \frac{1}{1+\omega^2 t^2}\right) dx'$$

$$= \frac{a_0^2}{2}(1+\omega^2 t^2)$$

$$\Delta x = \frac{a_0}{\sqrt{2}} \sqrt{1+\omega^2 t^2}$$

$$= \frac{a_0}{\sqrt{2}} \left[\omega t + \frac{1}{2\omega t} + \mathcal{O}\left(\frac{1}{t^3}\right)\right]$$



3.a|

$$n(x,t) = |\psi(x,t)|^2$$

$$\psi(x,t) = \psi_1(x,t) + \psi_2(x,t)$$

Sei $\psi(x,t) = \psi$

$$\alpha = \sqrt{\frac{1}{\pi a_0^2 (1+\omega^2 t^2)}} = \sqrt{\frac{1}{\pi \beta}}$$

$$\Rightarrow n(x,t) = |\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + \psi_1^* \psi_2 + \psi_2^* \psi_1$$

$$= \alpha \left[\exp\left(\frac{-(x+d)^2}{\beta}\right) + \exp\left(-\frac{(x-d)^2}{\beta}\right) \right.$$

$$+ \exp\left(-\frac{x^2+d^2+2ixd\omega t}{\beta} + i\phi\right)$$

$$\left. + \exp\left(-\frac{x^2+d^2+2ixd\omega t}{\beta} - i\phi\right) \right]$$

$$= 4\alpha e^{-\frac{x^2+d^2}{\beta}} \left[\cosh\left(\frac{2xd}{\beta}\right) + \cos\left(\frac{2xd\omega t}{\beta} - \phi\right) \right]$$

3.b|

$$\delta x = \frac{\pi a_0^2 (1+\omega^2 t^2)}{\omega t d} \approx \frac{\pi a_0^2 \omega t}{d} \quad , \omega t \gg 1$$