$$|\psi(t)\rangle = U(t, t_0)\,|\psi(t_0)\rangle$$

$$U(t, t_0) = e^{-i/\hbar \; H(t-t_0)}$$ H nicht explizit zeitabhängig

$$U(t, t_0) = e^{-i/\hbar \int_{t_0}^{t} H(\tau)\,d\tau}$$ H explizit zeitabhängig

1. $U(t_0, t_0) = \mathbb{1}$
2. $U^\dagger(t, t_0)\,U(t, t_0) = \mathbb{1}$ $\qquad U^\dagger = U^{-1}$
3. $U(t, t_0) = U(t, t')\,U(t', t_0)$ $\qquad \forall\ t'$

$$|\psi_1(t)\rangle = e^{-i/\hbar\; \hat{H}(t-t_1)}\,|\psi_1(t_0)\rangle$$ Eigenzustand zu $\hat{H}$

$$= e^{-i/\hbar\; E_1(t-t_1)}\,|\psi_1(t_0)\rangle$$

Blatt 3,

1)

iii) $\Delta p_x \geq \frac{\hbar}{2\delta x} = 10\,\mathrm{eV}/c$ mit $\delta x = 10\,\mathrm{nm}$

iv) Möglicherweise lässt sich Gauß dranfitten.

4|

$$[L_i, L_c] = [L_i, \varepsilon^{abc} r_a p_c]$$

$$= \varepsilon^{abc} [L_i, r_a p_c]$$

$$= \varepsilon^{abc} \Big(\underbrace{[L_i, r_a]}_{i\hbar \varepsilon^{iak} r_?} p_c + r_a [L_i, p_c] \Big)$$

$$= i\hbar \left[(-\delta\delta + \delta\delta) r p + r (\delta\delta - \delta\delta) p \right]$$

$$= i\hbar \quad \ldots$$

$$[V(|r|), L_j] = [V(|r|), \varepsilon_{jlk} r_i p_k]$$

$$= V(|r|)\, \varepsilon_{jik} r_i \frac{\hbar}{i} \partial_k - \varepsilon_{jik} r_i \frac{\hbar}{i} \partial_k V(|r|)$$

$$= \varepsilon_{jik} r_i \frac{\hbar}{i} \left(V(|r|)\, \partial_k - V(|r|)\, \partial_k - \frac{\partial V(|r|)}{\partial r_k} \right)$$

$$= \varepsilon_{jik}\, i\hbar\, r_i \frac{\partial V(|r|)}{\partial |r|} \frac{\partial |r|}{\partial r_k}$$

$$= \varepsilon_{jik}\, i\hbar\, r_i \frac{r_k}{|r|} \frac{\partial V(|r|)}{\partial |r|}$$

$$= i\hbar (\vec{r} \times \vec{r})_j \frac{1}{|r|} \frac{\partial V(|r|)}{\partial |r|} = 0$$

Blatt 4,

1)

$$1 \overset{!}{=} \int \varphi^*_{a/s}(x)\, \varphi_{a/s}(x)\, dx$$

$$= N^2 \int (\varphi_0 \pm \varphi_1)^* (\varphi_0 \pm \varphi_1)\, dx$$

$$= N^2 \int \underbrace{|\varphi_0|^2}_{1} + \underbrace{|\varphi_1|^2}_{1} \pm \cancel{\varphi_0^* \varphi_1}^{\,0} \pm \cancel{\varphi_1^* \varphi_0}^{\,0}\, dx$$

$$\Rightarrow 2N^2 = 1 \qquad \Rightarrow N = \frac{1}{\sqrt{2}}$$

3(a)

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x}) + V(\hat{y}) + \frac{1}{2} m\omega^2 \hat{z}^2$$

$$V(x_i) = \begin{cases} 0 & \text{für } -L < x_i < L \\ \infty & \text{sonst} \end{cases}$$

$$\psi(\vec{r}) = \psi_1(x)\, \psi_2(y)\, \psi_3(z)$$

$$\hat{H}\, \psi(\vec{r}) = E\, \psi(\vec{r})$$

$$\Leftrightarrow -\frac{\hbar^2}{2m}\left[(\partial_x^2 \psi_x)\psi_y \psi_z + \psi_x (\partial_y^2 \psi_y)\psi_z + \psi_x \psi_y\, \partial_z^2 \psi_z\right]$$

$$+ V_x \psi_x \psi_y \psi_z + \psi_x V_y \psi_y \psi_z + \psi_x \psi_y V_z \psi_z = E\, \psi_x \psi_y \psi_z$$

$$-\frac{\hbar^2}{2m}\partial_{x_i}^2 \psi_{x_i} + V_{x_i}\psi_{x_i} = E_i\psi_{x_i} \qquad \text{mit } E = E_1 + E_2 + E_3$$

$$\Rightarrow \psi''_{x_i} = \frac{2m}{\hbar^2}(V-E)\psi_{x_i}$$

$i = 1,2$: Kastenpotential mit $V_0 = +\infty$

ψ_{x_i}: sinus / cosinus $\qquad \psi_{x_i} = A\sin(kx) + B\cos(kx)$

Randbed: $\psi_{x_i}(x_i = \pm L) = 0 \Rightarrow k = \frac{(2n+1)\pi}{2L} \quad n \in \mathbb{N},\ A = 0$

Normierung: $|B|^2 \int_{-L}^{L} \cos^2(kx)\,dx = |B|^2 L \overset{!}{=} 1$

$$\Rightarrow B = \frac{1}{\sqrt{L}}$$

$i = 3$: Harm. Oszillator: Eigenfunktion siehe H4(1)

$$\Rightarrow \psi_0(\vec{r}) = \psi_1(x, n_1 = 0)\,\psi_2(y, n_2 = 0)\,\psi_3(z, n_3 = 0)$$

$$= \frac{1}{\sqrt{2}}\cos\left(\frac{\pi x}{2L}\right)\frac{1}{\sqrt{2}}\cos\left(\frac{\pi y}{2L}\right)\frac{\pi^{-1/4}}{\sqrt{a}}\,e^{-t^2/2a^2}$$

b) $i = 1,2$: $\frac{\hbar^2}{2m}\left(\frac{n\pi}{2L}\right)^2\psi_i + V_i\psi_i = E_i\psi_i$

$$\Rightarrow E_i = \frac{\hbar^2}{2m}\left(\frac{n\pi}{2L}\right)^2$$

$i = 3$: $E_3 = \hbar\omega(n + 1/2) = \frac{\hbar^2}{a^2 m}\left(n + \frac{1}{2}\right)$

$\hookrightarrow \quad E = \frac{\hbar^2}{m}\left(\frac{\pi^2}{8L^2}(n_1^2+n_2^2) + \frac{1}{a^2}(n_3+1/2)\right)$

$a \ll L \quad , \quad a = \sqrt{\frac{\hbar}{m\omega}}$

$E_0 = \frac{\hbar^2}{m}\left(\frac{\pi^2}{4L^2} + \frac{1}{2a^2}\right) \qquad (n_1, n_2) = (1,1)$

$E_1 = \frac{\hbar^2}{m}\left(\frac{5}{8}\frac{\pi^2}{L^2} + \frac{1}{2a^2}\right) \qquad (n_1, n_2) = (1,1), (2,1)$

$E_2 = \frac{\hbar^2}{m}\left(\frac{\pi^2}{L^2} + \frac{1}{2a^2}\right) \qquad (n_1, n_2) = (2,2)$

c) $\phi_0(x_i) = \left(\frac{1}{\pi a_0^2}\right)^{1/4} \exp\left[-\frac{x_i^2}{2a_0^2}\right]$

Wahrscheinlichkeitsdichte $n(\vec{r},t) = |\psi(\vec{r},t)|^2$

Teilchen zu t am Ort $r_0 = (x,0,0)$

$L \to L' = +\infty$

$n(r_0,t) = |\psi_x(r_0,t)|^2 \, |\psi_y(r_0,t)|^2 \, |\psi_z(r_0,t)|^2$

$i = 1,2 \quad : \quad |\psi_i(x_i)|^2 = \frac{1}{a_0^2\sqrt{\pi(1+\omega^2t^2)}} \exp\left(-\frac{x_i^2}{a_0^2(1+\omega^2t^2)}\right)$

$i = 3 \quad : \quad \psi_3(z,t) = \phi_0(z) e^{-i/\hbar E_0 t}$

$$\hookrightarrow |\psi_3(z,t)|^2 = \left(\frac{1}{\pi a_0^2}\right)^{1/4} e^{-\frac{2z^2}{2a_0^2}}$$

$$|\psi(r_0,t)|^2 = \frac{1}{a_0^4 \pi (1+\omega^2 t^2)} \left(\frac{1}{\pi a_0^2}\right)^{1/2} \exp\left[-\frac{x^2}{a_0^2(1+\omega^2 t^2)}\right]$$