# mat-mek4270 oblig1

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## 1 1.2.1. The Dirichlet problem

See the first page of pictures for finding w

## 2 1.2.3. Exact solution

Check that
$u(x, y, t) = e^{i(k_x x + k_y y + \omega t)}$

Satisifies the wave equation
$\frac{\delta^2 u}{\delta t^2} = c^2 \nabla^2 u$

First we find the separate second derivatives of u
$\frac{\delta^2 u}{\delta t^2} = \omega^2 e^{i(k_x x + k_y y + \omega t)}$
$\frac{\delta^2 u}{\delta x^2} = k_x^2 e^{i(k_x x + k_y y + \omega t)}$
$\frac{\delta^2 u}{\delta y^2} = k_y^2 e^{i(k_x x + k_y y + \omega t)}$

And we get that
$\frac{\delta^2 u}{\delta t^2} = c^2 \nabla^2 u$

Becomes
$\omega^2 = c^2(k_x^2 + k_y^2)$
$\omega = c\sqrt{k_x^2 + k_y^2}$

Which is the dispersion coefficient we found in task 1.2.1. u therefore satisfies the wave equation.
□

## 3 1.2.4 Dispersion coefficient

See next page for scan. I was not writing this in latex. I also failed. I ended up with an answer of the type 1 = 1, but prioritized getting the rest done over this. But I figured I would include the attempt.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$ Matlum 4270 Øy 1

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\cancel{\frac{\partial^2 u}{\partial t}} : u(t,x,y) = \sin(k_x x)\sin(k_y y)\cos(\omega t)$$

$$\frac{\partial u}{\partial t} : \sin(k_x x)\sin(k_y y)\,\omega \sin(\omega t)(-1)$$

$$\frac{\partial^2 u}{\partial t^2} : \sin(k_x x)\sin(k_y y)\,\omega^2 \cos(\omega t)(-1)$$

$$\frac{\partial u}{\partial x} : k_x \cos(k_x x)\sin(k_y y)\cos(\omega t)$$

$$\frac{\partial^2 u}{\partial x^2} : -k_x^2 \sin(k_x x)\sin(k_y y)\cos(\omega t)$$

$$\frac{\partial u}{\partial y} : \sin(k_x x)\, k_y \cos(k_y y)\cos \omega t$$

$$\frac{\partial^2 u}{\partial y^2} : \sin(k_x x)\, k_y^2 (-1)\sin k_y y \cos \omega t$$

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$$\cancel{\sin(k_x x)\sin(k_y y)\cos(\omega t)}(-1)\,\omega^2$$

$$\cancel{\sin(k_x x)\sin(k_y y)\cos(\omega t)}(-1)\,k_x^2$$

$$\cancel{\sin(k_x x)\sin k_y y \cos \omega t}\;(-1)\,k_y^2$$

$$\omega^2 = c^2(k_x^2 + k_y^2)$$

$$\omega = c\sqrt{k_x^2 + k_y^2}$$

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$$u_{ij}^n = e^{i(kh(i+j)-\tilde{\omega} n\Delta t)}$$

$$\underbrace{\frac{u_{ij}^{n+1}-2u_{ij}^{n}+u_{ij}^{n-1}}{\Delta t^2}}_{1} = c^2\left(\underbrace{\frac{u_{i+1j}^{n}-2u_{ij}^{n}+u_{i-1j}^{n}}{h^2}}_{2} + \underbrace{\frac{u_{ij+1}^{n}-2u_{ij}^{n}+u_{ij-1}^{n}}{h^2}}_{3}\right)$$

1 $$\frac{e^{i(kh(i+j)-\tilde{\omega}(n+1)\Delta t)} - 2e^{i(kh(i+j))-\tilde{\omega}(n)\Delta t)} - e^{i(kh(i+j)\ldots}}{\Delta t^2}$$

$$e^{ikh(i+j)}\left(\cancel{e^{-i\Delta t\tilde{\omega}(n+1+n+n-1)}}\left(e^{n+1}-2e^{n}+e^{n-1}\right)^{-i\Delta t\tilde{\omega}}\right)$$

2 $$\frac{e^{ikh(i+1+j)-\tilde{\omega}n\Delta t)} - 2e^{ikh(i+j))-\tilde{\omega}n\Delta t} + e^{ikh(i-1+j))-\tilde{\omega}n\Delta t}}{h^2}$$

$$e^{-i\tilde{\omega}n\Delta t}\left(\cancel{e^{ikh}}\left(e^{i+j+1}-2e^{i+j}+e^{i+j-1}\right)^{ikh}\right)$$

3 $$e^{ikh(i+j+1)-\tilde{\omega}n\Delta t} - 2e^{ikh(i+j))-\tilde{\omega}n\Delta t} + e^{ikh(i+j-1)-\tilde{\omega}n\Delta t}$$

$$e^{-i\tilde{\omega}n\Delta t}\left(e^{i+j+1}-2e^{i+j}+e^{i+j-1}\right)^{ikh}$$

$C=\frac{c\Delta t}{h}$ $$e^{ikh(i+j)}\left(e^{n+1}-2e^{n}+e^{n-1}\right)^{-i\Delta t\tilde{\omega}} = C^2\, e^{-i\tilde{\omega}n\Delta t}\left(e^{i+j+1}-2e^{i+j}+e^{i+j-1}\right)^{ikh}$$

$C=\frac{1}{\sqrt{2}}$

$C^2 = \frac{c^2\Delta t^2}{h^2} = 2$

$$e^{ikh(i+j)}\left(e^{-i\Delta t\tilde{\omega}(n+1)} - 2e^{-i\Delta t\tilde{\omega}n} + e^{-i\Delta t\tilde{\omega}(n-1)}\right)$$

$$ikh(i+j)\left[-i\Delta t\tilde{\omega}(n+1)(n-2) - i\Delta t\tilde{\omega}n + i\Delta t\tilde{\omega}(n-1)\right]$$

$$-i\tilde{\omega}n\Delta t\left[ikh(i+j+1)(n-2) + (i+j)ikh + (i+j-1)\ \ldots\right]$$

1 ln(1)

$i(hh(i+j)-\tilde{\omega}(n+1)\Delta t + \ln -2 + i(hh(i+j)-\tilde{\omega}n\Delta t + i(hh(i+j)-\tilde{\omega}\Delta t...$

~~$i(hh(i+j)+1-\tilde{\omega}(n+1)...$~~

Bar

$\ln(2e^2)$

$= \ln -2\ln e$

$3ihh(i+j)+\ln -2 - \tilde{\omega}\Delta t(n+1+n+n-1)$

$3ihh(i+j+\ln -2 - \tilde{\omega}\Delta t(3n)$

2/3 bli det same

ln(2 eller 3)

$ihh(i+j+1) - \omega n\Delta t) + \ln -2 + ihh(i+j) - \tilde{\omega}n\Delta t + ihh(i+j-1)...$

$ihh(i+j+1+i+j+i+j-1) + 3\tilde{\omega}n\Delta t + \ln -2$

$ihh(3i+3j) + 3\tilde{\omega}n\Delta t + \ln -2$

~~$1 = c^2(2+3)$~~

$\frac{1}{\Delta t^2} = \frac{c^2\Delta t^2}{h^2}(2+3) = C^2 = 2$

~~$1 = c^2(2+2)$~~

$1 = 2/3$

$\ln 1 = \ln 2/3$

$3ihh(i+j) + \ln -2 - \tilde{\omega}\Delta t(3n) = ihh(3i+3j) + 3\tilde{\omega}n\Delta t + \ln -2$

. Stemmer.