nuSIprop: solving for the astrophysical propagation of self-interacting neutrinos

Ivan Esteban, Sujata Pandey, Vedran Brdar, John F. Beacom

The code `nuSIprop` numerically evolves a self-interacting astrophysical neutrino spectrum. As detailed in the companion paper, the evolution equations for the comoving differential number density of neutrinos plus antineutrinos[1] of mass eigenstate i, $\tilde{n}_i(t, E_\nu) \equiv \frac{\mathrm{d}n_i(t,E_\nu)}{\mathrm{d}E_\nu}$, read

$$\begin{aligned}\frac{\partial \tilde{n}_i(t, E_\nu)}{\partial t} =& \frac{\partial}{\partial E_\nu}\left[H(t)\, E_\nu\, \tilde{n}_i(t, E_\nu)\right] + \mathcal{L}_i(t, E_\nu) - \tilde{n}_i(t, E_\nu) \sum_j n_j^t \sigma_{ij}(E_\nu) \\ &+ \sum_{jkl} n_j^t \int_{E_\nu}^{\infty} dE'_\nu \tilde{n}_k(t, E'_\nu) \frac{d\sigma_{jk\to il}}{dE_\nu}(E'_\nu, E_\nu)\,.\end{aligned} \tag{1}$$

Here $H(t)$ is the Hubble parameter as a function of time t, $\mathcal{L}_i(t, E_\nu)$ is the production rate of neutrinos with mass eigenstate i and energy E_ν, $\sigma_{ij}(E_\nu)$ is the absorption cross section of an incident ν_i with energy E_ν on a target ν_j, and $\sigma_{jk\to il}(E'_\nu, E_\nu)$ is the cross section for an incident ν_j with energy E'_ν on a target ν_k to generate a *detectable* ν_i with energy E_ν, along with a ν_j. *Detectable* means that, for Dirac neutrinos, neutrinos must be left-handed and antineutrinos right-handed. For Majorana neutrinos, all final states are detectable. In both terms, $n_i^t \simeq 2 \times 56(1+z)\,\mathrm{cm}^{-3}$ is the CνB density of the mass eigenstate i.

As we are dealing with propagation over cosmological scales, it is simpler to express all quantities as a function of redshift z ($\frac{\partial}{\partial t} = -H(z)\cdot(1+z)\frac{\partial}{\partial z}$). Furthermore, we can absorb the cosmological redshift factor $\frac{\partial}{\partial E_\nu}\left[H(t)\, E_\nu\, \tilde{n}_i(t, E_\nu)\right]$ by defining

$$Z_i(z, E_\nu) \equiv (1+z)\tilde{n}_i(z, E_\nu[1+z])\,. \tag{2}$$

Notice that at $z = 0$, $Z_i(0, E_\nu) = \tilde{n}_i(0, E_\nu)$. The evolution equations for Z_i then read

$$\begin{aligned}-H(z)\frac{\partial Z_i(z, E_\nu)}{\partial z} =& \mathcal{L}_i(z, E_\nu(1+z)) - \frac{Z_i(z, E_\nu)\sum_j n_j^t \sigma_{ij}(E_\nu)}{1+z} \\ &+ \sum_{jkl} n_j^t \int_{E_\nu}^{\infty} d\tilde{E}_\nu Z_k(t, \tilde{E}_\nu) \frac{d\sigma_{jk\to il}}{dE_\nu(1+z)}(\tilde{E}_\nu[1+z], E_\nu[1+z])\,.\end{aligned} \tag{3}$$

To numerically solve these equations, we divide the neutrino energy range in bins inside which $Z_i(z, E_\nu)$ is assumed to be constant. We will denote

$$Z_i^k(z) \equiv \int_{E_{k-1/2}}^{E_{k+1/2}} \mathrm{d}E_\nu\, Z_i(z, E_\nu) = Z_i(z, E_k)\Delta E_k\,, \tag{4}$$

where $E_{k-1/2}$ and $E_{k+1/2} \equiv E_{k-1/2} + \Delta E_k$ are the energy bin limits. If we discretize redshift in nodes z_α and integrate Eq. (3) in energy from $E_{k-1/2}$ to $E_{k+1/2}$,[2] we obtain

$$\begin{aligned}-\frac{H(z_\alpha)}{z_{\alpha+1} - z_\alpha}\left[Z_i^k(z_{\alpha+1}) - Z_i^k(z_\alpha)\right] =& \mathcal{L}_i^k(z_\alpha) - \Gamma_i^k(z_{\alpha+1})\frac{Z_i^k(z_{\alpha+1})}{\Delta E_k} + \sum_j \tilde{\alpha}_{ij}^k(z_{\alpha+1})\frac{Z_j^k(z_{\alpha+1})}{\Delta E_k} \\ &+ \sum_j \sum_{k'>k} \alpha_{ij}^{k,k'}(z_{\alpha+1})\frac{Z_j^{k'}(z_{\alpha+1})}{\Delta E_{k'}}\,,\end{aligned} \tag{5}$$

[1] In what follows, under explicitly stated, when we mention neutrinos we also refer to the corresponding antineutrinos.

[2] This will guarantee that the algorithm will be well-behaved even if the cross section has sharp features as a function of energy.

with

$$\mathcal{L}_i^k(z_\alpha) \equiv \int_{E_{k-1/2}}^{E_{k+1/2}} \mathcal{L}_i(z_\alpha, E_\nu[1+z_\alpha])\,\mathrm{d}E_\nu \,, \tag{6}$$

$$\Gamma_i^k(z_{\alpha+1}) \equiv \sum_j \frac{n_j^t(z_{\alpha+1})}{1+z_{\alpha+1}} \int_{E_{k-1/2}}^{E_{k+1/2}} \sigma_{ij}(E_\nu[1+z_{\alpha+1}])\,\mathrm{d}E_\nu \,, \tag{7}$$

$$\tilde{\alpha}_{ij}^k(z_{\alpha+1}) \equiv \sum_{j_1,j_2} n_j^t(z_{\alpha+1}) \int_{E_{k-1/2}}^{E_{k+1/2}} \mathrm{d}E_\nu \int_{E}^{E_{k+1/2}} \mathrm{d}\tilde{E}_\nu \frac{\mathrm{d}\sigma_{j\,j_1\to i\,j_2}}{\mathrm{d}E_\nu(1+z_{\alpha+1})}(\tilde{E}_\nu[1+z_{\alpha+1}], E_\nu[1+z_{\alpha+1}])\,, \tag{8}$$

$$\alpha_{ij}^{k,k'}(z_{\alpha+1}) \equiv \sum_{j_1,j_2} n_j^t(z_{\alpha+1}) \int_{E_{k-1/2}}^{E_{k+1/2}} \mathrm{d}E_\nu \int_{E_{k'-1/2}}^{E_{k'+1/2}} \mathrm{d}\tilde{E}_\nu \frac{\mathrm{d}\sigma_{j\,j_1\to i\,j_2}}{\mathrm{d}E_\nu(1+z_{\alpha+1})}(\tilde{E}_\nu[1+z_{\alpha+1}], E_\nu[1+z_{\alpha+1}])\,. \tag{9}$$

We have followed `arXiv:astro-ph/9604098` and adopted a first order implicit scheme to improve numerical convergence. All the integrals (except for double scalar production, see `README.md`) can be done analytically, which dramatically reduces computation time.

Once $\mathcal{L}_i^k$, Γ_i^k, $\tilde{\alpha}_{ij}^k$ and $\alpha_{ij}^{k,k'}$ are known, Eq. (5) is a system of linear equations that, for known values of $Z_i^k(z_\alpha)$, can be numerically solved to obtain $Z_i^k(z_{\alpha+1})$. This procedure can be iterated until we obtain $Z_i^k(z=0)$, i.e., the present-day neutrino flux.

We have assumed $\mathcal{L}_i$ to follow a power-law in energy, with a redshift dependence proportional to the Star Formation Rate. We include sources up to $z=5$, as this already takes into account more than 95% of the total neutrino flux. Thus, we evolve the equations starting from $z=5$ with the initial condition $\tilde{n}_i(z=5, E_\nu)=0$, and the goal is to evaluate $\tilde{n}_i(0, E_\nu)$, the neutrino spectrum at Earth at energy E_ν.