

# nuSIprop: solving for the astrophysical propagation of self-interacting neutrinos

Ivan Esteban, Sujata Pandey, Vedran Brdar, John F. Beacom

The code **nuSIprop** numerically evolves a self-interacting astrophysical neutrino spectrum. As detailed in the companion paper, the evolution equations for the comoving differential density of neutrinos plus antineutrinos<sup>1</sup> of mass eigenstate  $i$ ,  $\tilde{n}_i(t, E_\nu) \equiv \frac{dn_i(t, E_\nu)}{dE_\nu}$ , read

$$\begin{aligned} \frac{\partial \tilde{n}_i(t, E_\nu)}{\partial t} = & \frac{\partial}{\partial E_\nu} [H(t) E_\nu \tilde{n}_i(t, E_\nu)] + \mathcal{L}_i(t, E_\nu) - \tilde{n}_i(t, E_\nu) \sum_j n_j^t \sigma_{ij}(E_\nu) \\ & + \sum_{jkl} n_j^t \int_{E_\nu}^\infty dE'_\nu \tilde{n}_k(t, E'_\nu) \frac{d\sigma_{jk \rightarrow il}}{dE_\nu}(E'_\nu, E_\nu). \end{aligned} \quad (1)$$

Here  $H(t)$  is the Hubble parameter as a function of time  $t$ ,  $\mathcal{L}_i(t, E_\nu)$  is the production rate of neutrinos with mass eigenstate  $i$  and energy  $E_\nu$ ,  $\sigma_{ij}(E_\nu)$  is the absorption cross section of an incident neutrino with mass eigenstate  $i$  and energy  $E_\nu$  on a target neutrino with mass eigenstate  $j$ , and  $\sigma_{jk \rightarrow il}(E'_\nu, E_\nu)$  is the cross section for an incident neutrino with mass eigenstate  $j$  and energy  $E'_\nu$  on a target neutrino with mass eigenstate  $k$  to generate a *detectable* neutrino with mass eigenstate  $i$  and energy  $E_\nu$  and a neutrino with mass eigenstate  $l$ . Here *detectable* means that, for Dirac neutrinos, neutrinos must be left-handed and antineutrinos right-handed. In both terms,  $n_i^t$  is the CνB density of the mass eigenstate  $i$ .

As we are dealing with propagation over cosmological scales, it is simpler to express all quantities as a function of redshift  $z$  ( $\frac{\partial}{\partial t} = -H(z) \cdot (1+z) \frac{\partial}{\partial z}$ ). Furthermore, we can absorb the cosmological redshift factor  $\frac{\partial}{\partial E_\nu} [H(t) E_\nu \tilde{n}_i(t, E_\nu)]$  by defining

$$Z_i(z, E_\nu) \equiv (1+z) \tilde{n}_i(z, E_\nu [1+z]). \quad (2)$$

Notice that at  $z = 0$ ,  $Z_i(0, E_\nu) = \tilde{n}_i(0, E_\nu)$ . The evolution equations for  $Z_i$  then read

$$\begin{aligned} -H(z) \frac{\partial Z_i(z, E_\nu)}{\partial z} = & \mathcal{L}_i(z, E_\nu (1+z)) - \frac{Z_i(z, E_\nu) \sum_j n_j^t \sigma_{ij}(E_\nu)}{1+z} \\ & + \sum_{jkl} n_j^t \int_{E_\nu}^\infty d\tilde{E}_\nu Z_k(t, \tilde{E}_\nu) \frac{d\sigma_{jk \rightarrow il}}{dE_\nu (1+z)}(\tilde{E}_\nu [1+z], E_\nu [1+z]). \end{aligned} \quad (3)$$

To numerically solve these equations, we divide the neutrino energy range in bins inside which  $Z_i(z, E_\nu)$  is assumed to be constant. We will denote

$$Z_i^k(z) \equiv \int_{E_{k-1/2}}^{E_{k+1/2}} dE_\nu Z_i(z, E_\nu) = Z_i(z, E_k) \Delta E_k, \quad (4)$$

where  $E_{k-1/2}$  and  $E_{k+1/2} \equiv E_{k-1/2} + \Delta E_k$  are the energy bin limits. If we discretize redshift in

---

<sup>1</sup>In what follows, under explicitly stated, when we mention neutrinos we also refer to the corresponding antineutrinos.

nodes  $z_\alpha$  and integrate Eq. (3) in energy from  $E_{k-1/2}$  to  $E_{k+1/2}$ ,<sup>2</sup> we obtain

$$\begin{aligned}
-\frac{H(z_\alpha)}{z_{\alpha+1} - z_\alpha} [Z_i^k(z_{\alpha+1}) - Z_i^k(z_\alpha)] = & \mathcal{L}_i^k(z_\alpha) - \Gamma_i^k(z_{\alpha+1}) \frac{Z_i^k(z_{\alpha+1})}{\Delta E_k} + \sum_j \tilde{\alpha}_{ij}^k(z_{\alpha+1}) \frac{Z_j^k(z_{\alpha+1})}{\Delta E_k} \\
& + \sum_j \sum_{k' > k} \alpha_{ij}^{k,k'}(z_{\alpha+1}) \frac{Z_j^{k'}(z_{\alpha+1})}{\Delta E_{k'}},
\end{aligned} \tag{5}$$

with

$$\mathcal{L}_i^k(z_\alpha) \equiv \int_{E_{k-1/2}}^{E_{k+1/2}} \mathcal{L}_i(z_\alpha, E_\nu [1 + z_\alpha]) dE_\nu, \tag{6}$$

$$\Gamma_i^k(z_{\alpha+1}) \equiv \sum_j \frac{n_j^t(z_{\alpha+1})}{1 + z_{\alpha+1}} \int_{E_{k-1/2}}^{E_{k+1/2}} \sigma_{ij}(E_\nu [1 + z_{\alpha+1}]) dE_\nu, \tag{7}$$

$$\tilde{\alpha}_{ij}^k(z_{\alpha+1}) \equiv \sum_{j_1, j_2} n_j^t(z_{\alpha+1}) \int_{E_{k-1/2}}^{E_{k+1/2}} dE_\nu \int_E^{E_{k+1/2}} d\tilde{E}_\nu \frac{d\sigma_{j j_1 \rightarrow i j_2}}{dE_\nu (1 + z_{\alpha+1})} (\tilde{E}_\nu [1 + z_{\alpha+1}], E_\nu [1 + z_{\alpha+1}]), \tag{8}$$

$$\alpha_{ij}^{k,k'}(z_{\alpha+1}) \equiv \sum_{j_1, j_2} n_j^t(z_{\alpha+1}) \int_{E_{k-1/2}}^{E_{k+1/2}} dE_\nu \int_{E_{k'-1/2}}^{E_{k'+1/2}} d\tilde{E}_\nu \frac{d\sigma_{j j_1 \rightarrow i j_2}}{dE_\nu (1 + z_{\alpha+1})} (\tilde{E}_\nu [1 + z_{\alpha+1}], E_\nu [1 + z_{\alpha+1}]). \tag{9}$$

We have followed [arXiv:astro-ph/9604098](#) and adopted a first order implicit scheme to improve numerical convergence.

Once  $\mathcal{L}_i^k$ ,  $\Gamma_i^k$ ,  $\tilde{\alpha}_{ij}^k$  and  $\alpha_{ij}^{k,k'}$  are known, Eq. (5) is a system of linear equations that, for known values of  $Z_i^k(z_\alpha)$ , can be numerically solved to obtain  $Z_i^k(z_{\alpha+1})$ . This procedure can be iterated until we obtain  $Z_i^k(z = 0)$ , i.e., the present-day neutrino flux.

We assume  $\mathcal{L}_i$  to follow a power-law in energy, with a redshift dependence proportional to the Star Formation Rate. We include sources up to  $z = 5$ , as this already takes into account more than 95% of the total neutrino flux. Thus, we evolve the equations starting from  $z = 5$  with the initial condition  $\tilde{n}_i(z = 5, E_\nu) = 0$ , and the goal is to evaluate  $\tilde{n}_i(0, E_\nu)$ , the neutrino spectrum at Earth at energy  $E_\nu$ .

With this, all the integrals (except for double scalar production, see [README.md](#)) can be done analytically, which dramatically reduces computation time.

---

<sup>2</sup>This will guarantee that the algorithm will be well-behaved even if the cross section has sharp features as a function of energy.