nuSIprop: solving for the astrophysical propagation of self-interacting neutrinos

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The code `nuSIprop` numerically evolves a self-interacting astrophysical neutrino spectrum. As detailed in the companion paper, the evolution equations for the comoving differential density of neutrinos plus antineutrinos[1] of mass eigenstate i, $\tilde{n}_i(t, E_\nu) \equiv \frac{\mathrm{d}n_i(t,E_\nu)}{\mathrm{d}E_\nu}$, read

$$
\begin{aligned}
\frac{\partial \tilde{n}_i(t, E_\nu)}{\partial t} =& \frac{\partial}{\partial E_\nu} \left[H(t)\, E_\nu\, \tilde{n}_i(t, E_\nu) \right] + \mathcal{L}_i(t, E_\nu) - \tilde{n}_i(t, E_\nu) \sum_j n_j^t \sigma_{ij}(E_\nu) \\
&+ \sum_{jkl} n_j^t \int_{E_\nu}^{\infty} dE_\nu' \tilde{n}_k(t, E_\nu') \frac{d\sigma_{jk \to il}}{dE_\nu}(E_\nu', E_\nu)\,.
\end{aligned}
\tag{1}
$$

Here $H(t)$ is the Hubble parameter as a function of time t, $\mathcal{L}_i(t, E_\nu)$ is the production rate of neutrinos with mass eigenstate i and energy E_ν, $\sigma_{ij}(E_\nu)$ is the absorption cross section of an incident neutrino with mass eigenstate i and energy E_ν on a target neutrino with mass eigenstate j, and $\sigma_{jk \to il}(E_\nu', E_\nu)$ is the cross section for an incident neutrino with mass eigenstate j and energy E_ν' on a target neutrino with mass eigenstate k to generate a *detectable* neutrino with mass eigenstate i and energy E_ν and a neutrino with mass eigenstate l. Here *detectable* means that, for Dirac neutrinos, neutrinos must be left-handed and antineutrinos right-handed. In both terms, n_i^t is the CνB density of the mass eigenstate i.

As we are dealing with propagation over cosmological scales, it is simpler to express all quantities as a function of redshift z ($\frac{\partial}{\partial t} = -H(z) \cdot (1+z)\frac{\partial}{\partial z}$). Furthermore, we can absorb the cosmological redshift factor $\frac{\partial}{\partial E_\nu} \left[H(t)\, E_\nu\, \tilde{n}_i(t, E_\nu) \right]$ by defining

$$
Z_i(z, E_\nu) \equiv (1+z)\tilde{n}_i(z, E_\nu[1+z])\,.
\tag{2}
$$

Notice that at $z = 0$, $Z_i(0, E_\nu) = \tilde{n}_i(0, E_\nu)$. The evolution equations for Z_i then read

$$
\begin{aligned}
-H(z)\frac{\partial Z_i(z, E_\nu)}{\partial z} =& \mathcal{L}_i(z, E_\nu(1+z)) - \frac{Z_i(z, E_\nu) \sum_j n_j^t \sigma_{ij}(E_\nu)}{1+z} \\
&+ \sum_{jkl} n_j^t \int_{E_\nu}^{\infty} d\tilde{E}_\nu Z_k(t, \tilde{E}_\nu) \frac{d\sigma_{jk \to il}}{dE_\nu(1+z)}(\tilde{E}_\nu[1+z], E_\nu[1+z])\,.
\end{aligned}
\tag{3}
$$

To numerically solve these equations, we divide the neutrino energy range in bins inside which $Z_i(z, E_\nu)$ is assumed to be constant. We will denote

$$
Z_i^k(z) \equiv \int_{E_{k-1/2}}^{E_{k+1/2}} \mathrm{d}E_\nu\, Z_i(z, E_\nu) = Z_i(z, E_k)\Delta E_k\,,
\tag{4}
$$

where $E_{k-1/2}$ and $E_{k+1/2} \equiv E_{k-1/2} + \Delta E_k$ are the energy bin limits. If we discretize redshift in

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[1]In what follows, under explicitly stated, when we mention neutrinos we also refer to the corresponding antineutrinos.

nodes z_α and integrate Eq. (3) in energy from $E_{k-1/2}$ to $E_{k+1/2}$,[2] we obtain

$$-\frac{H(z_\alpha)}{z_{\alpha+1} - z_\alpha} \left[Z_i^k(z_{\alpha+1}) - Z_i^k(z_\alpha) \right] = \mathcal{L}_i^k(z_\alpha) - \Gamma_i^k(z_{\alpha+1}) \frac{Z_i^k(z_{\alpha+1})}{\Delta E_k} + \sum_j \tilde{\alpha}_{ij}^k(z_{\alpha+1}) \frac{Z_j^k(z_{\alpha+1})}{\Delta E_k}$$
$$+ \sum_j \sum_{k'>k} \alpha_{ij}^{k,k'}(z_{\alpha+1}) \frac{Z_j^{k'}(z_{\alpha+1})}{\Delta E_{k'}} \, , \tag{5}$$

with

$$\mathcal{L}_i^k(z_\alpha) \equiv \int_{E_{k-1/2}}^{E_{k+1/2}} \mathcal{L}_i(z_\alpha, E_\nu[1+z_\alpha]) \, \mathrm{d}E_\nu \, , \tag{6}$$

$$\Gamma_i^k(z_{\alpha+1}) \equiv \sum_j \frac{n_j^t(z_{\alpha+1})}{1+z_{\alpha+1}} \int_{E_{k-1/2}}^{E_{k+1/2}} \sigma_{ij}(E_\nu[1+z_{\alpha+1}]) \, \mathrm{d}E_\nu \, , \tag{7}$$

$$\tilde{\alpha}_{ij}^k(z_{\alpha+1}) \equiv \sum_{j_1,j_2} n_j^t(z_{\alpha+1}) \int_{E_{k-1/2}}^{E_{k+1/2}} \mathrm{d}E_\nu \int_E^{E_{k+1/2}} \mathrm{d}\tilde{E}_\nu \frac{\mathrm{d}\sigma_{j\,j_1 \to i\,j_2}}{\mathrm{d}E_\nu(1+z_{\alpha+1})}(\tilde{E}_\nu[1+z_{\alpha+1}], E_\nu[1+z_{\alpha+1}]) \, , \tag{8}$$

$$\alpha_{ij}^{k,k'}(z_{\alpha+1}) \equiv \sum_{j_1,j_2} n_j^t(z_{\alpha+1}) \int_{E_{k-1/2}}^{E_{k+1/2}} \mathrm{d}E_\nu \int_{E_{k'-1/2}}^{E_{k'+1/2}} \mathrm{d}\tilde{E}_\nu \frac{\mathrm{d}\sigma_{j\,j_1 \to i\,j_2}}{\mathrm{d}E_\nu(1+z_{\alpha+1})}(\tilde{E}_\nu[1+z_{\alpha+1}], E_\nu[1+z_{\alpha+1}]) \, . \tag{9}$$

We have followed `arXiv:astro-ph/9604098` and adopted a first order implicit scheme to improve numerical convergence.

Once $\mathcal{L}_i^k$, Γ_i^k, $\tilde{\alpha}_{ij}^k$ and $\alpha_{ij}^{k,k'}$ are known, Eq. (5) is a system of linear equations that, for known values of $Z_i^k(z_\alpha)$, can be numerically solved to obtain $Z_i^k(z_{\alpha+1})$. This procedure can be iterated until we obtain $Z_i^k(z=0)$, i.e., the present-day neutrino flux.

We assume $\mathcal{L}_i$ to follow a power-law in energy, with a redshift dependence proportional to the Star Formation Rate. We include sources up to $z \sim 5$, as this already takes into account more than 95% of the total neutrino flux.

With this, all the integrals (except for double scalar production, see `README.md`) can be done analytically, which dramatically reduces computation time.

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[2]This will guarantee that the algorithm will be well-behaved even if the cross section has sharp features as a function of energy.