Fast algorithms and numerical methods for the solution of Boundary Element Methods

Session 4: Boundary Integral Equations

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Principle of derivation of Boundary Integral Representation

Boundary-value problem over Ω : $\begin{cases} \mathcal{L}u + f = 0 & \text{in } \Omega, \\ u = g_1 & \text{on } \partial\Omega_D, \\ T^n(u) = g_2 & \text{on } \partial\Omega_N. \end{cases}$

where u: unknown; g_1, g_2 and source f given. T^n: first-order partial differential operator, linear with respect to n. $\mathcal{L}$ linear second-order partial differential operator

- $\mathcal{L}$ and T^n assumed to satisfy the reciprocity identity

$$\int_\Omega (\mathcal{L}u.v - \mathcal{L}v.u)dV = \int_{\partial\Omega} (T^n(u).v - T^n(v).u)dS$$

- G: fundamental solution (point source f applied at x $\notin \partial\Omega$)

$$\mathcal{L}G(\boldsymbol{x},\boldsymbol{y}) + \delta(\boldsymbol{y}-\boldsymbol{x}) = 0 \quad \text{in } \Omega$$

- Property of the Dirac distribution

$$\int_\Omega \delta(\boldsymbol{y}-\boldsymbol{x})u(\boldsymbol{y})dV_y = \kappa u(\boldsymbol{x}) \quad (\kappa = 1 \; if \; \boldsymbol{x} \in \Omega, \quad \kappa = 0 \; if \; \boldsymbol{x} \notin \Omega)$$

- Integral Representation formula: $x \notin \partial\Omega$

$$\kappa u(\boldsymbol{x}) = \int_\Omega f(\boldsymbol{y})G(\boldsymbol{x},\boldsymbol{y})dV_y + \int_{\partial\Omega} (u(\boldsymbol{y})T^n G(\boldsymbol{x},\boldsymbol{y}) - T^n u(\boldsymbol{y})G(\boldsymbol{x},\boldsymbol{y}))dS_y$$

Integral Representation for Helmholtz equation

Scattering problen in Ω_ℓ ($\ell = e$ or $\ell = i$): $u \in H^1(\Omega_1)$

$$\Delta u + \omega^2 u = 0 \text{ dans } \Omega_\ell$$

Rigorously we could only assume $u \in H^1(\Delta; \Omega_1)$ and use duality parings $< .,. >_{H^{-1/2},H^{1/2}}$ instead of integrals on Γ

Notations: $\mathbf{n}$ normal from Ω_i to Ω_e. f^i/f^e interior/exterior traces

$$[f]_\Gamma = f^i - f^e$$

We know that

$$\Delta G(\mathbf{y},\mathbf{x}) + \omega^2 G(\mathbf{y},\mathbf{x}) = -\delta_x(\mathbf{y}) \quad G(\mathbf{y},\mathbf{x}) = G(r) = \frac{1}{4\pi r} e^{i\omega r} \text{ with } r = |\mathbf{x}-\mathbf{y}|$$

Representation Theorem

(a) If u is solution in Ω_i then

$$\int_\Gamma \left(G(\mathbf{x},\mathbf{y}) \left(\frac{\partial u}{\partial n} \right)^i (\mathbf{y}) - \frac{\partial G}{\partial n_y}(\mathbf{x},\mathbf{y}) u^i(\mathbf{y}) \right) d\gamma_y = \begin{cases} u(\mathbf{x}) \text{ if } \mathbf{x} \in \Omega_i \\ 0 \text{ if } \mathbf{x} \in \Omega_e \end{cases}$$

(b) If u is solution in Ω_e then (with the normal still from Ω_i to Ω_e)

$$\int_\Gamma \left(-G(\mathbf{x},\mathbf{y}) \left(\frac{\partial u}{\partial n} \right)^e (\mathbf{y}) + \frac{\partial G}{\partial n_y}(\mathbf{x},\mathbf{y}) u^e(\mathbf{y}) \right) d\gamma_y = \begin{cases} 0 \text{ if } \mathbf{x} \in \Omega_i \\ u(\mathbf{x}) \text{ if } \mathbf{x} \in \Omega_e \end{cases}$$

(c) If u is solution in $\Omega_i \cup \Omega_e$ then

$$\forall \mathbf{x} \in \Omega_i \cup \Omega_e,\ u(\mathbf{x}) = \int_\Gamma \left(G(\mathbf{x},\mathbf{y}) \left[\frac{\partial u}{\partial n} \right]_\Gamma (\mathbf{y}) - \frac{\partial G}{\partial n_y}(\mathbf{x},\mathbf{y}) [u]_\Gamma(\mathbf{y}) \right) d\gamma_y$$

What is next?

For exterior problems, we have

$$u(\mathbf{x}) = \int_\Gamma \left(-G(\mathbf{x},\mathbf{y}) \left(\frac{\partial u}{\partial n}\right)^e (\mathbf{y}) + \frac{\partial G}{\partial n_y}(\mathbf{x},\mathbf{y}) u^e(\mathbf{y}) \right) d\gamma_y, \quad \forall \mathbf{x} \in \Omega_e, \mathbf{x} \notin \Gamma$$

☺ u in the domain is known only from values on boundary

☺ Reduction of computational costs and memory requirements

⚠ It is a Boundary Integral Representation (not an equation)

⚠ You need to obtain the traces of u on Γ (with an equation)

The Boundary Element Method is a two steps methods

1. Resolution of a boundary integral equation to obtain traces (TP2)
2. Application of the boundary integral representation to obtain solution in the domain (TP1)

Single and double layer potentials

Single layer potential: q with enough regularity, e.g., $q \in C^0(\Gamma)$

$$\mathcal{S}q(\mathbf{x}) = \int_\Gamma G(\mathbf{x}-\mathbf{y})q(\mathbf{y})d\gamma_y, \quad \forall \mathbf{x} \in \Omega_i \cup \Omega_e$$

Double layer potential: φ with enough regularity, e.g., $\varphi \in C^0(\Gamma)$

$$\mathcal{D}\varphi(\mathbf{x}) = \int_\Gamma \frac{\partial G}{\partial n_y}(\mathbf{x}-\mathbf{y})\varphi(\mathbf{y})d\gamma_y, \quad \forall \mathbf{x} \in \Omega_i \cup \Omega_e$$

Boundary Integral Representations: if u sol. of Helmholtz eq. in $\Omega_i \cup \Omega_e$

$$(a) \quad \mathcal{S}\gamma_1^i u(\mathbf{x}) - \mathcal{D}\gamma_0^i u(\mathbf{x}) = \begin{cases} u(\mathbf{x}) \text{ if } \mathbf{x} \in \Omega_i \\ 0 \text{ if } \mathbf{x} \in \Omega_e \end{cases}$$

$$(b) \quad -\mathcal{S}\gamma_1^e u(\mathbf{x}) + \mathcal{D}\gamma_0^e u(\mathbf{x}) = \begin{cases} 0 \text{ if } \mathbf{x} \in \Omega_i \\ u(\mathbf{x}) \text{ if } \mathbf{x} \in \Omega_e \end{cases}$$

$$(c) \quad \forall \mathbf{x} \in \Omega_i \cup \Omega_e,\ u(\mathbf{x}) = \mathcal{S}\left[\frac{\partial u}{\partial n}\right]_\Gamma (\mathbf{x}) - \mathcal{D}[u]_\Gamma(\mathbf{x})$$

with $\mathbf{n}$ normal from Ω_i to Ω_e

Some other boundary integral representations

- If u is solution of the Helmholtz equation in $\Omega_i \cup \Omega_e$ then it can be represented with single and double layer potentials.
- On the other hand, if we pick a function q or φ, then, we can verify that the single $\mathcal{S}q$ and double layer $\mathcal{D}\varphi$ potentials are solutions of the Helmholtz equation in $\Omega_i \cup \Omega_e$.

It is how we can derive various boundary integral representations.

Use of boundary conditions

We need to derive boundary integral equations and use the boundary conditions to solve the correct PDE.

We need the traces of the single and double layer potentials.

Traces of the single layer potential

(i) The single layer potential is continuous across Γ and has the following traces:

$$\boxed{\gamma_0^i(\mathcal{S}q)(\mathbf{x}) = \gamma_0^e(\mathcal{S}q)(\mathbf{x}) = \int_\Gamma G(\mathbf{x}-\mathbf{y})q(\mathbf{y})d\gamma_y := Sq(\mathbf{x}), \quad \forall \mathbf{x} \in \Gamma}$$

(ii) The normal derivative of the single layer potential is discontinuous across Γ:

$$\boxed{\begin{aligned} \gamma_1^i(\mathcal{S}q)(\mathbf{x}) &= \frac{1}{2}q(\mathbf{x}) + D'q(\mathbf{x}), \quad \forall \mathbf{x} \in \Gamma \\ \gamma_1^e(\mathcal{S}q)(\mathbf{x}) &= -\frac{1}{2}q(\mathbf{x}) + D'q(\mathbf{x}), \quad \forall \mathbf{x} \in \Gamma \end{aligned}}$$

with

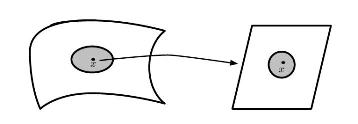
$$D'q(\mathbf{x}) = \int_\Gamma \frac{\partial G}{\partial n_x}(\mathbf{x},\mathbf{y})q(\mathbf{y})d\gamma_y, \quad \forall \mathbf{x} \in \Gamma$$

Proof of (i)

$$\mathbf{x}_\varepsilon = \mathbf{x} + \varepsilon \boldsymbol{n}_x \in \Omega_\varepsilon (\varepsilon > 0)$$

$$\mathcal{S}q(\mathbf{x}_\varepsilon) = \int_\Gamma G(\mathbf{x}_\varepsilon - \mathbf{y}) q(\mathbf{y}) d\gamma_y = \int_\Gamma \frac{1}{4\pi |\mathbf{x}_\varepsilon - \mathbf{y}|} e^{i\omega |\mathbf{x}_\varepsilon - \mathbf{y}|} q(\mathbf{y}) d\gamma_y$$

When $\varepsilon \to 0$: weak singularity. Assuming Γ with enough regularity, we can show that it integrable. When $\mathbf{x} \in \Gamma$ and $\mathbf{y} \in \Gamma_x$ (neighbourhood of $\mathbf{x}$), we can approximate locally the integral on Γ_x by an integral on a small disk around $\mathbf{x}$ on the tangent plane.



The singularity is similar to

$$\int_0^{r_0} \int_0^{2\pi} \frac{1}{4\pi r} e^{i\omega r} q(re^{i\theta}) r dr d\theta$$

The singularity cancels out and $G(\mathbf{x}, \mathbf{y}) q(\mathbf{y})$ is integrable.

Proof of (i)

To apply the dominated convergence theorem and writes

$$\lim_{\varepsilon \to 0} \mathcal{S}q(\mathbf{x}_\varepsilon) = \int_\Gamma G(\mathbf{x} - \mathbf{y})q(\mathbf{y})d\gamma_y$$

we look for $f \in L^1$ such that

$$|G(\mathbf{x}_\varepsilon, \mathbf{y})q(\mathbf{y})| \leq f$$

If Γ is plane: $\forall \mathbf{y} \in \Gamma$, $\mathbf{y} - \mathbf{x}$ is orthogonal to $\mathbf{x}_\varepsilon - \mathbf{x}$ such that

$$|\mathbf{x}_\varepsilon - \mathbf{y}| \geq |\mathbf{x} - \mathbf{y}| \text{ and } |G(\mathbf{x}_\varepsilon, \mathbf{y})| \leq |G(\mathbf{x}, \mathbf{y})| \in L^1$$

The proof in the general case use a parameterization of Γ and the tangent plane.

Proof of (ii)

We consider

$$\nabla_{\mathbf{x}_\varepsilon} \int_\Gamma G(\mathbf{x}_\varepsilon, \mathbf{y}) q(\mathbf{y}) d\gamma_y . \boldsymbol{n}_x$$

If we could apply the dominated convergence theorem we would end with

$$D' q(\mathbf{x}) = \int_\Gamma \frac{\partial G}{\partial n_x}(\mathbf{x}, \mathbf{y}) q(\mathbf{y}) d\gamma_y$$

Does this integral make sense? The singularity is now in $1/|\mathbf{x}-\mathbf{y}|^2$

$$\frac{\partial G}{\partial n_y}(\mathbf{x}, \mathbf{y}) = -\frac{1}{4\pi|\mathbf{x}-\mathbf{y}|}(\frac{1}{|\mathbf{x}-\mathbf{y}|} - i\omega) e^{i\omega|\mathbf{x}-\mathbf{y}|} \frac{(\mathbf{x}-\mathbf{y}, \boldsymbol{n}_x)}{|\mathbf{x}-\mathbf{y}|}$$

But if Γ has enough regularity

$$\frac{(\mathbf{x}-\mathbf{y}, \boldsymbol{n}_x)}{|\mathbf{x}-\mathbf{y}|} = O(|\mathbf{x}-\mathbf{y}|)$$

and the singularity is integrable. We cannot however find a L^1 function. The proof is too technical to be presented.

Traces of the double layer potential

(i) The double layer potential is discontinuous across Γ :

$$\boxed{\begin{aligned} \gamma_0^i(\mathcal{D}\varphi)(\mathbf{x}) &= -\frac{1}{2}\varphi(\mathbf{x}) + D\varphi(\mathbf{x}), \quad \forall \mathbf{x} \in \Gamma \\ \gamma_0^e(\mathcal{D}\varphi)(\mathbf{x}) &= \frac{1}{2}\varphi(\mathbf{x}) + D\varphi(\mathbf{x}), \quad \forall \mathbf{x} \in \Gamma \end{aligned}}$$

with

$$D\varphi(\mathbf{x}) = \int_\Gamma \frac{\partial G}{\partial n_y}(\mathbf{x}, \mathbf{y})\varphi(\mathbf{y})d\gamma_y, \quad \forall \mathbf{x} \in \Gamma$$

(ii) The normal derivative of the double layer potential is continuous across Γ :

$$\boxed{\gamma_1^i(\mathcal{D}\varphi)(\mathbf{x}) = \gamma_1^e(\mathcal{D}\varphi)(\mathbf{x}) := N\varphi(\mathbf{x}), \quad \forall \mathbf{x} \in \Gamma}$$

Some remarks

- D and D' are adjoints and have the same singularity
- N is hypersingular with singularity in $1/r^3$

$$N\varphi(\mathbf{x}) = \lim_{\varepsilon \to 0} \nabla_{\mathbf{x}_\varepsilon} \int_\Gamma \frac{\partial G}{\partial \mathbf{n}_y}(\mathbf{x}_\varepsilon, \mathbf{y})\varphi(\mathbf{y}) d\gamma_y . \boldsymbol{n}_x \neq \int_\Gamma \frac{\partial^2 G}{\partial \mathbf{n}_x \partial \mathbf{n}_y}(\mathbf{x}, \mathbf{y})\varphi(\mathbf{y}) d\gamma_y$$

Dirichlet Interior problem with natural traces

Boundary Integral representation with single and double layer potentials for which the densities are the natural traces of the solution

$$u(\mathbf{x}) = \mathcal{S}\gamma_1^i u(\mathbf{x}) - \mathcal{D}\gamma_0^i u(\mathbf{x}) \quad \forall \mathbf{x} \in \Omega_i, \quad \mathbf{x} \notin \Gamma$$

By using the trace theorems

$$u^i(\mathbf{x}) = S\gamma_1^i u(\mathbf{x}) - (-\frac{I}{2} + D)\gamma_0^i u(\mathbf{x}) \quad \forall \mathbf{x} \in \Gamma$$

We use the boundary condition $u^i = u_d$ and set $q = \gamma_1^i u$ such that the Boundary Integral Equation to solve is

$$Sq = (\frac{I}{2} + D)u_d \text{ on } \Gamma$$

Dirichlet Interior problem with a continuation

u is extended in Ω_e such that it still satisfies Helmholtz equation in Ω_e

$$u(\mathbf{x}) = \mathcal{S}\left[\frac{\partial u}{\partial n}\right]_\Gamma(\mathbf{x}) - \mathcal{D}[u]_\Gamma(\mathbf{x}) \quad \forall \mathbf{x} \in \Omega_i \cup \Omega_e, \quad \mathbf{x} \notin \Gamma$$

Continuation with 0 such that $[u]_\Gamma = \gamma_0^i u$ and $\left[\frac{\partial u}{\partial n}\right]_\Gamma = \gamma_1^i u$ (ends with equation for natural traces)

Dirichlet Interior problem with a continuation

u is extended in Ω_e such that it still satisfies Helmholtz equation in Ω_e

$$u(\mathbf{x}) = \mathcal{S}\left[\frac{\partial u}{\partial n}\right]_\Gamma(\mathbf{x}) - \mathcal{D}[u]_\Gamma(\mathbf{x}) \quad \forall \mathbf{x} \in \Omega_i \cup \Omega_e, \quad \mathbf{x} \notin \Gamma$$

Continuation with 0 such that $[u]_\Gamma = \gamma_0^i u$ and $\left[\frac{\partial u}{\partial n}\right]_\Gamma = \gamma_1^i u$ (ends with equation for natural traces)

Continuation by continuity: $[u]_\Gamma = 0$, we note $q = \left[\frac{\partial u}{\partial n}\right]_\Gamma(\mathbf{x})$

Representation: $u(\mathbf{x}) = \mathcal{S}q \quad \forall \mathbf{x} \in \Omega_i \cup \Omega_e, \quad \mathbf{x} \notin \Gamma$

Equation: $u_d = Sq$

Dirichlet Interior problem with a continuation

u is extended in Ω_e such that it still satisfies Helmholtz equation in Ω_e

$$u(\mathbf{x}) = \mathcal{S}\left[\frac{\partial u}{\partial n}\right]_\Gamma(\mathbf{x}) - \mathcal{D}[u]_\Gamma(\mathbf{x}) \quad \forall \mathbf{x} \in \Omega_i \cup \Omega_e, \quad \mathbf{x} \notin \Gamma$$

Continuation with 0 such that $[u]_\Gamma = \gamma_0^i u$ and $\left[\frac{\partial u}{\partial n}\right]_\Gamma = \gamma_1^i u$ (ends with equation for natural traces)

Continuation by continuity: $[u]_\Gamma = 0$, we note $q = \left[\frac{\partial u}{\partial n}\right]_\Gamma(\mathbf{x})$

Representation: $u(\mathbf{x}) = \mathcal{S}q \quad \forall \mathbf{x} \in \Omega_i \cup \Omega_e, \quad \mathbf{x} \notin \Gamma$

Equation: $u_d = Sq$

Continuation by continuity of normal deriv. $\left[\frac{\partial u}{\partial n}\right]_\Gamma = 0, \varphi = [u]_\Gamma$

Repres.: $u(\mathbf{x}) = -\mathcal{D}\varphi(\mathbf{x}) \quad \forall \mathbf{x} \in \Omega_i \cup \Omega_e, \quad \mathbf{x} \notin \Gamma$

Equation: $u_d = (\frac{I}{2} - D)\varphi$

Neumann Interior problem with natural traces

Boundary Integral representation with single and double layer potentials for which the densities are the natural traces of the solution

$$u(\mathbf{x}) = \mathcal{S}\gamma_1^i u(\mathbf{x}) - \mathcal{D}\gamma_0^i u(\mathbf{x}) \quad \forall \mathbf{x} \in \Omega_i, \quad \mathbf{x} \notin \Gamma$$

By using the trace theorems

$$\gamma_1^i u(\mathbf{x}) = (\frac{I}{2} + D')\gamma_1^i u(\mathbf{x}) - N\gamma_0^i u(\mathbf{x}) \quad \forall \mathbf{x} \in \Gamma$$

We use the boundary condition $\gamma_1^i u = u_n$ and set $\varphi = \gamma_0^i u$ such that the Boundary Integral Equation to solve is

$$N\varphi = (-\frac{I}{2} + D')u_n \text{ on } \Gamma$$

Neumann Interior problem with a continuation

u is extended in Ω_e such that it still satisfies Helmholtz equation in Ω_e

$$u(\mathbf{x}) = \mathcal{S}\left[\frac{\partial u}{\partial n}\right]_\Gamma(\mathbf{x}) - \mathcal{D}[u]_\Gamma(\mathbf{x}) \quad \forall \mathbf{x} \in \Omega_i \cup \Omega_e, \quad \mathbf{x} \notin \Gamma$$

Continuation with 0 such that $[u]_\Gamma = \gamma_0^i u$ and $\left[\frac{\partial u}{\partial n}\right]_\Gamma = \gamma_1^i u$ (ends with equation for natural traces)

Neumann Interior problem with a continuation

u is extended in Ω_e such that it still satisfies Helmholtz equation in Ω_e

$$u(\mathbf{x}) = \mathcal{S}\left[\frac{\partial u}{\partial n}\right]_\Gamma(\mathbf{x}) - \mathcal{D}[u]_\Gamma(\mathbf{x}) \quad \forall \mathbf{x} \in \Omega_i \cup \Omega_e, \quad \mathbf{x} \notin \Gamma$$

Continuation with 0 such that $[u]_\Gamma = \gamma_0^i u$ and $\left[\frac{\partial u}{\partial n}\right]_\Gamma = \gamma_1^i u$ (ends with equation for natural traces)

Continuation by continuity: $[u]_\Gamma = 0$, we note $q = \left[\frac{\partial u}{\partial n}\right]_\Gamma(\mathbf{x})$

Representation: $\quad u(\mathbf{x}) = \mathcal{S}q \quad \forall \mathbf{x} \in \Omega_i \cup \Omega_e, \quad \mathbf{x} \notin \Gamma$

Equation: $\quad u_n = (\frac{I}{2} + D')q$

Neumann Interior problem with a continuation

u is extended in Ω_e such that it still satisfies Helmholtz equation in Ω_e

$$u(\mathbf{x}) = \mathcal{S}\left[\frac{\partial u}{\partial n}\right]_\Gamma(\mathbf{x}) - \mathcal{D}[u]_\Gamma(\mathbf{x}) \quad \forall \mathbf{x} \in \Omega_i \cup \Omega_e, \quad \mathbf{x} \notin \Gamma$$

Continuation with 0 such that $[u]_\Gamma = \gamma_0^i u$ and $\left[\frac{\partial u}{\partial n}\right]_\Gamma = \gamma_1^i u$ (ends with equation for natural traces)

Continuation by continuity: $[u]_\Gamma = 0$, we note $q = \left[\frac{\partial u}{\partial n}\right]_\Gamma(\mathbf{x})$

Representation: $u(\mathbf{x}) = \mathcal{S}q \quad \forall \mathbf{x} \in \Omega_i \cup \Omega_e, \quad \mathbf{x} \notin \Gamma$

Equation: $u_n = (\frac{I}{2} + D')q$

Continuation by continuity of normal deriv. $\left[\frac{\partial u}{\partial n}\right]_\Gamma = 0, \varphi = [u]_\Gamma$

Repres.: $u(\mathbf{x}) = -\mathcal{D}\varphi(\mathbf{x}) \quad \forall \mathbf{x} \in \Omega_i \cup \Omega_e, \quad \mathbf{x} \notin \Gamma$

Equation: $u_n = -N\varphi$

TP1: Représentation intégrale pour domaine extérieur

D'après le théorème de représentation intégrale, on a:

$$u(\mathbf{x}) = \mathcal{S}\left[\frac{\partial u}{\partial n}\right]_\Gamma(\mathbf{x}) - \mathcal{D}[u]_\Gamma(\mathbf{x}) \quad \forall \mathbf{x} \in \Omega_i \cup \Omega_e$$

On note $u^+(\mathbf{x}) = u(\mathbf{x})$ si $\mathbf{x} \in \Omega_e$ et on choisit un prolongement par continuité $[u]_\Gamma = 0$ et $p = \left[\frac{\partial u}{\partial n}\right]_\Gamma = -\partial_\mathbf{n} u^+ - \partial_\mathbf{n} u^{inc}$

On obtient bien que:

Le champ diffracté dans le cas de conditions à la frontière de type Dirichlet est donné par la représentation intégrale

$$u^+(\mathbf{x}) = \int_\Gamma G(\mathbf{x}, \mathbf{y}) p(\mathbf{y}) d\Gamma(\mathbf{y})$$

$p = -\partial_\mathbf{n} u^+ - \partial_\mathbf{n} u^{inc}$, $\mathbf{n}$ normale ext, $G(\mathbf{x}, \mathbf{y}) = \frac{i}{4} H_0^{(1)}(k||\mathbf{x} - \mathbf{y}||)$.

TP2: Résolution de l'équation intégrale de frontière

Lors du TP1, p a été obtenu analytiquement. Dans le TP2, nous allons le déterminer numériquement en résolvant l'équation intégrale.

On part de la représentation intégrale

$$u(\mathbf{x}) = \mathcal{S}p(\mathbf{x}), \quad \forall \mathbf{x} \in \Omega_e, \quad \mathbf{x} \notin \Gamma$$

On prend la trace de Dirichlet et on utilise la condition de Dirichlet $[u]_\Gamma = 0$.

La densité p est alors donnée par

$$\text{Trouver } p \in H^{-1/2}(\Gamma) \text{ tel que} \int_\Gamma G(\mathbf{x},\mathbf{y})p(\mathbf{y})d\Gamma(\mathbf{y}) = -u^{inc}(\mathbf{x})$$

Le but de ce TP2 est de (i) vérifier que la solution numérique correspond bien à la solution analytique et (ii) supprimer l'utilisation de la solution analytique pour calculer le champ diffracté dans Ω^+.