FREmu

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With Prof. Jun-Qing Xia

Power Spectrum Emulator for f(R) Gravity

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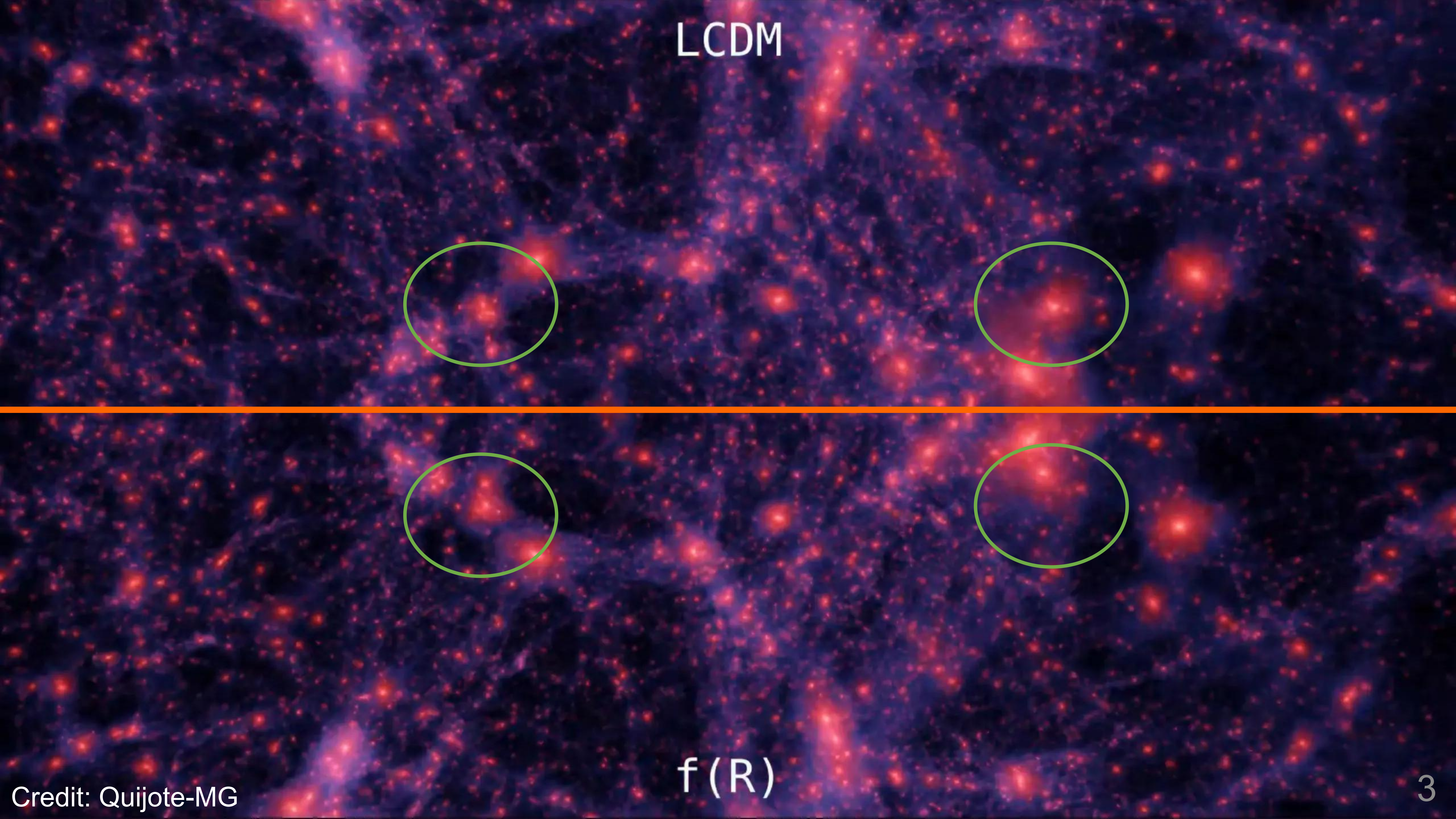
F(R) Gravity & Large-scale Structure of the Universe

Action: $$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{R + f(R)}{16\pi G} + \mathcal{L}_\mathrm{m} \right]$$

$$f_{R_0}$$

Poisson's Equation: $$\nabla^2 \Phi = \frac{16\pi G}{3}\delta\rho - \frac{1}{6}\delta R$$



Credit: Quijote-MG

F(R) Gravity & Large-scale Structure of the Universe

$$P(\mathbf{k}) \equiv V\langle|\delta_{\mathbf{k}}|^2\rangle = \int d^3r\, e^{-i\mathbf{k}\cdot\mathbf{r}}\xi(\mathbf{r})$$ depends on cosmology

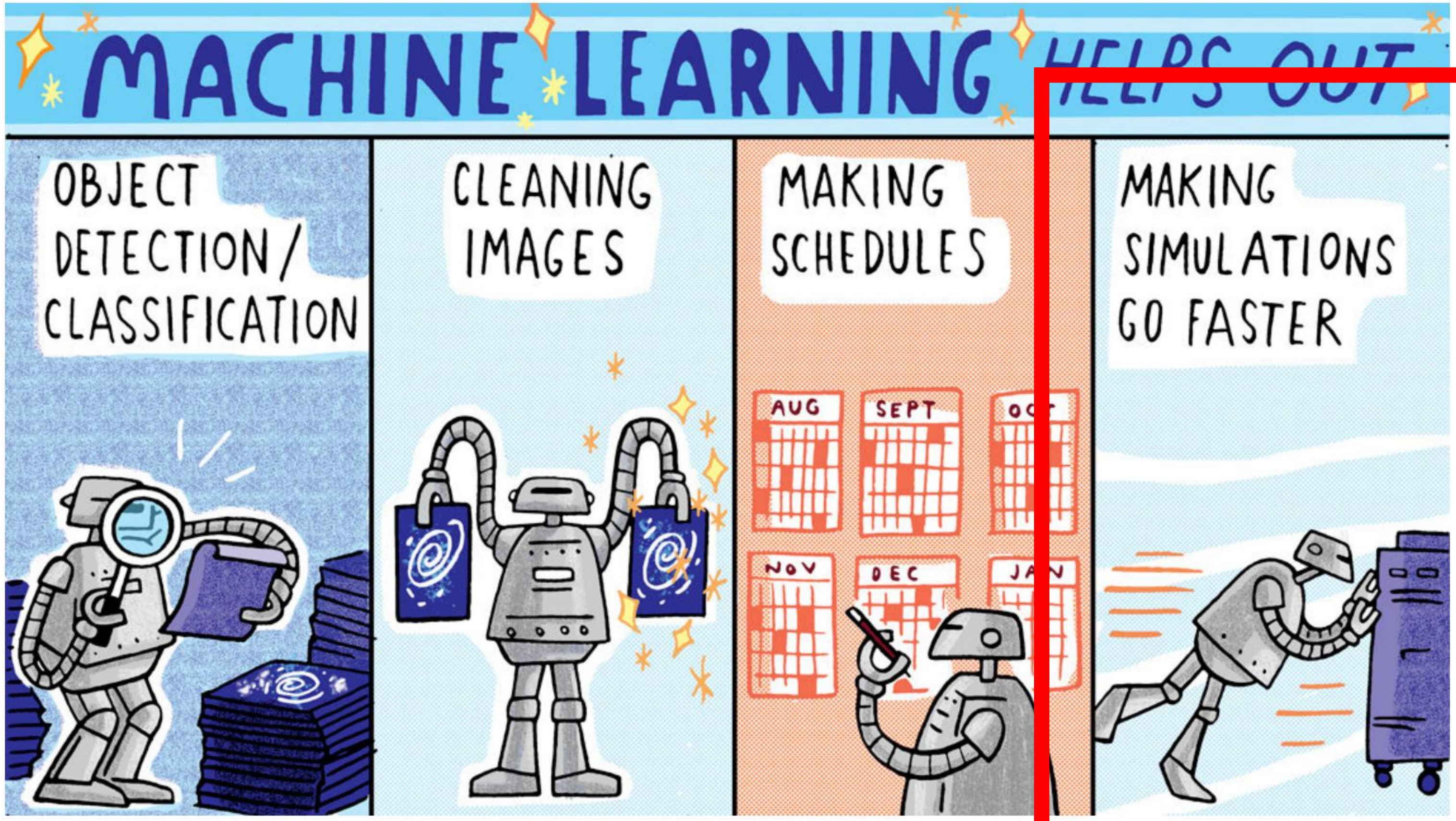
We can now predict linear P(k) precisely, but non-linear effects cannot be ignored.

N-body simulations => Non-linear P(k)

$\boxed{\text{MCMC?}}$

MONEY & TIME (Months) IMPOSSIBLE!!!

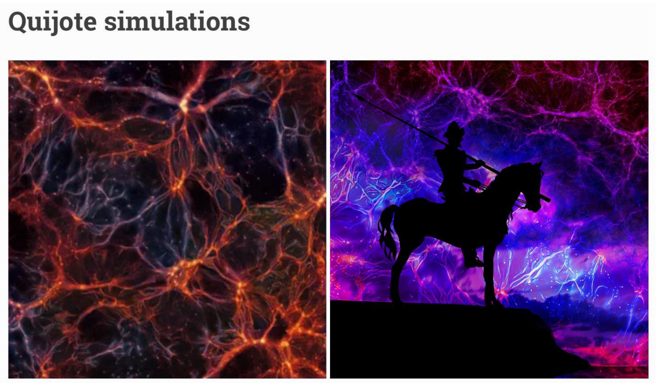
Simulation & Emulation



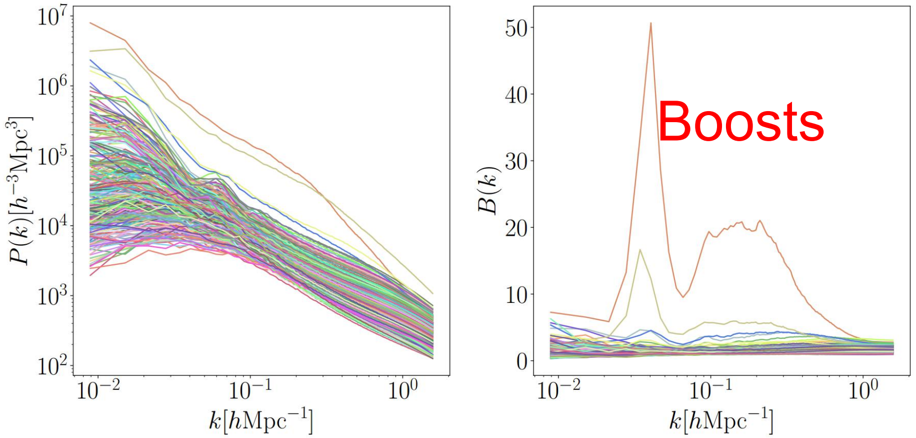
Emulation

Directly predict P(k) in only few seconds!

Build an Emulator - Overview

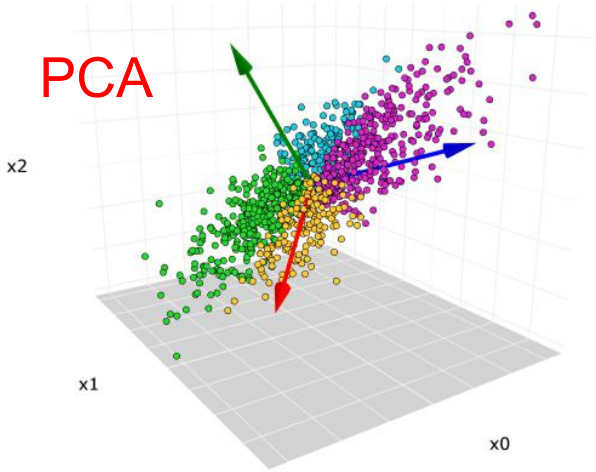


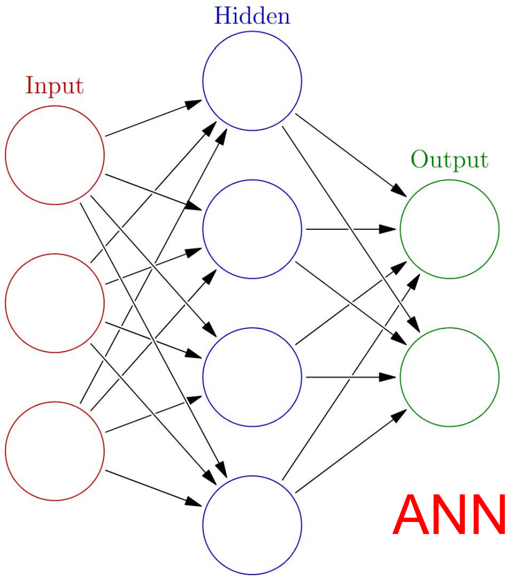
Simulation Data



Boosts

PCA

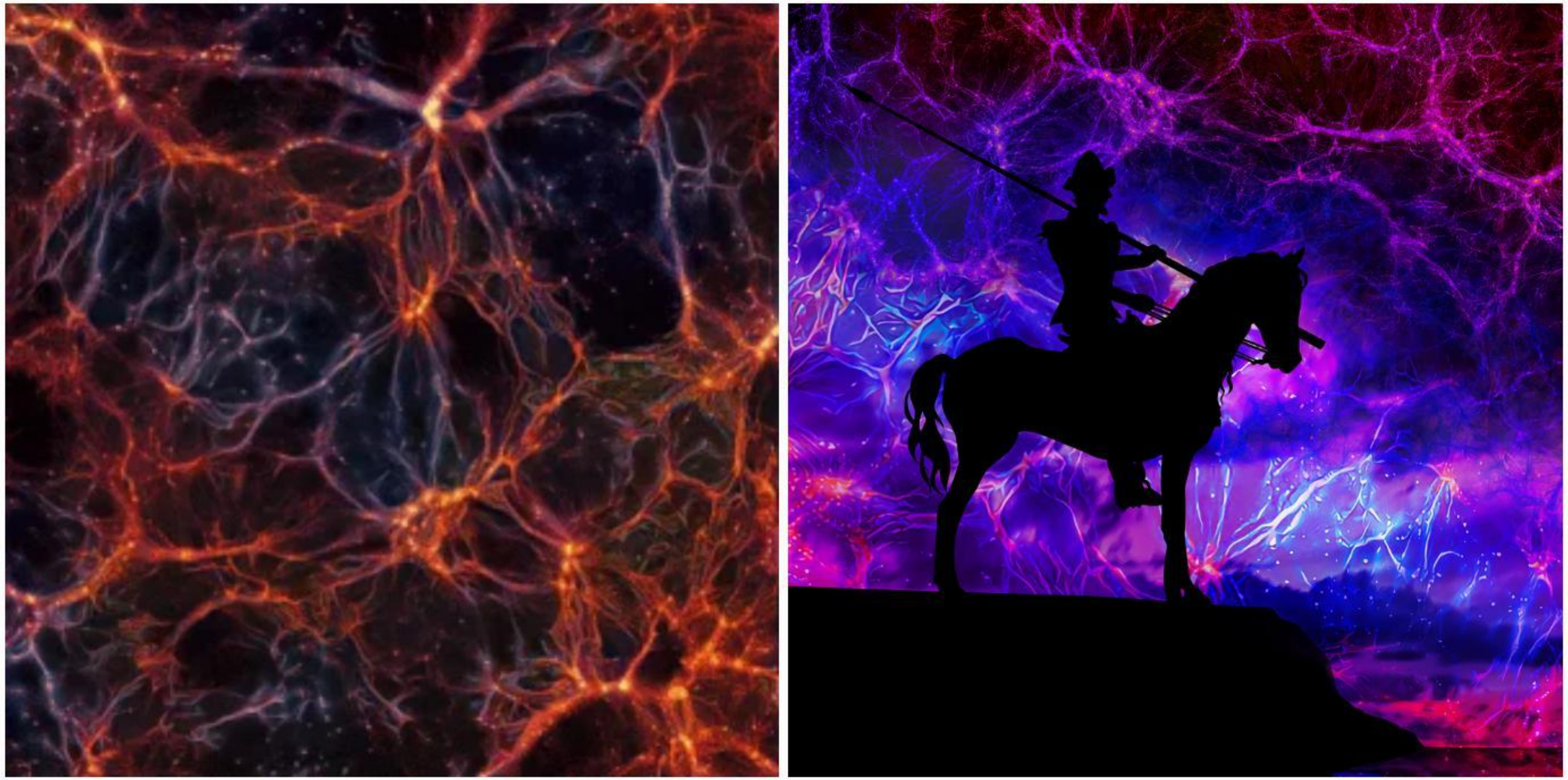




ANN

Build an Emulator - Data

Quijote simulations



Quijote-MG

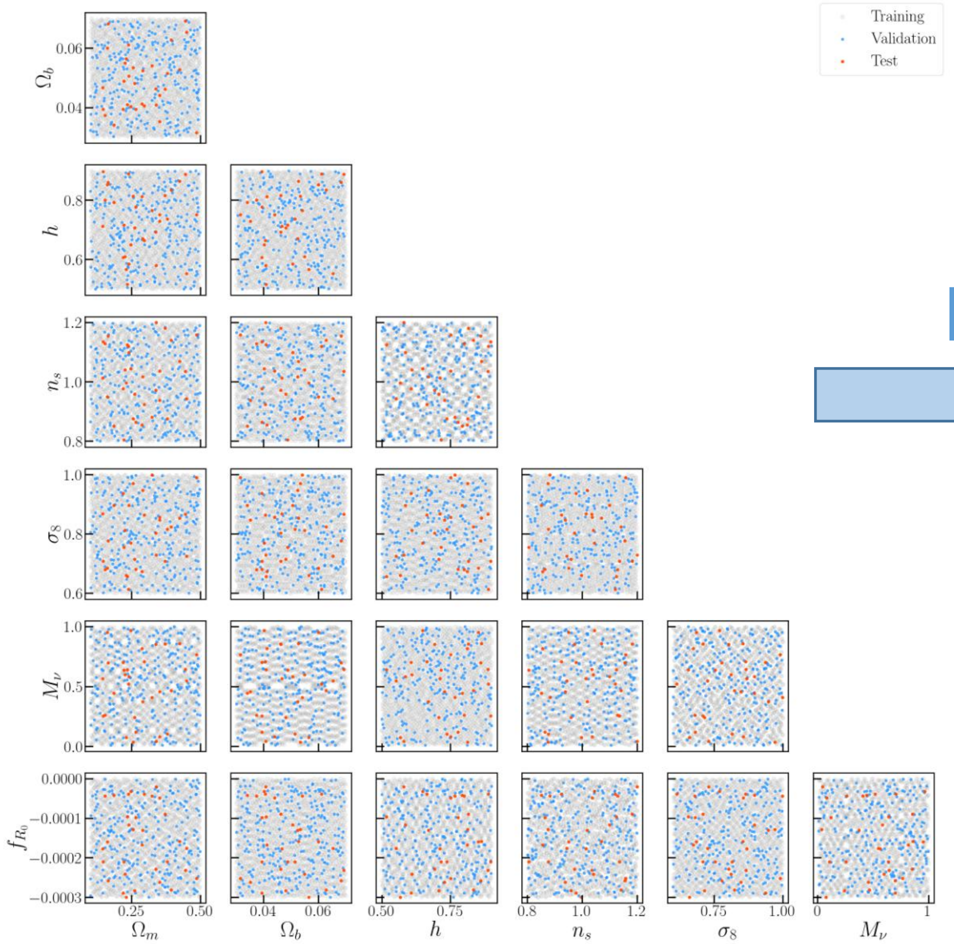
2048 sets

7 params

Ω_m, Ω_b, h, n_s, σ_8, M_ν, and f_{R_0}

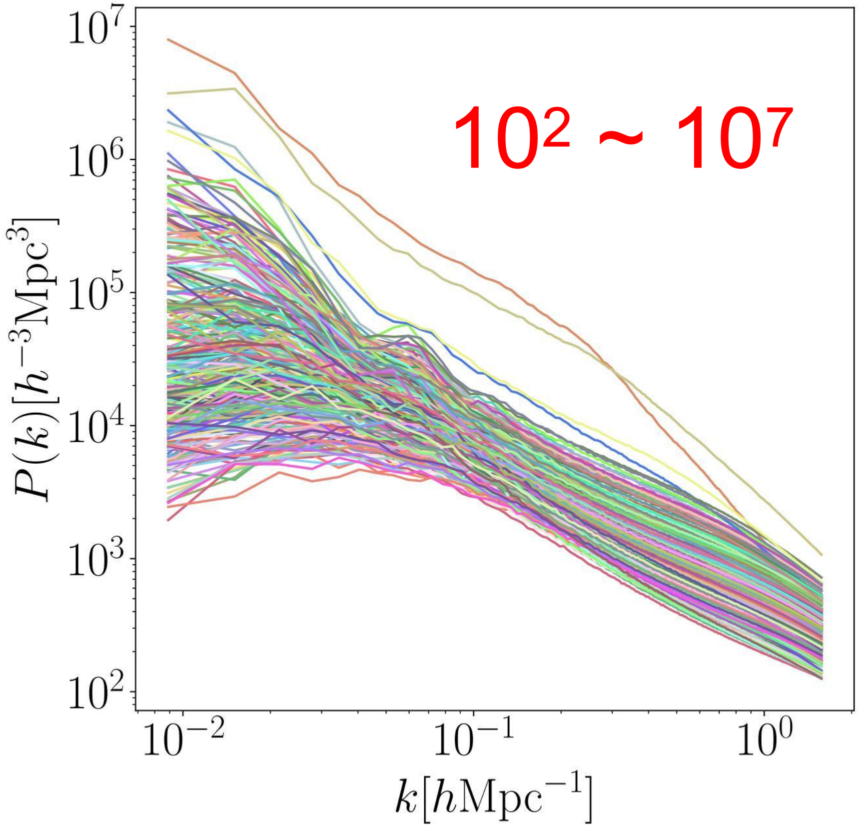
Build an Emulator - Targets

Parameters + Redshift



Map

Power spectrum



Build an Emulator - Boosts

$$B(k) = P^{\mathrm{nonlin}}_{f(R)}(k) / P^{\mathrm{halofit}}_{\Lambda\mathrm{CDM}}(k)$$

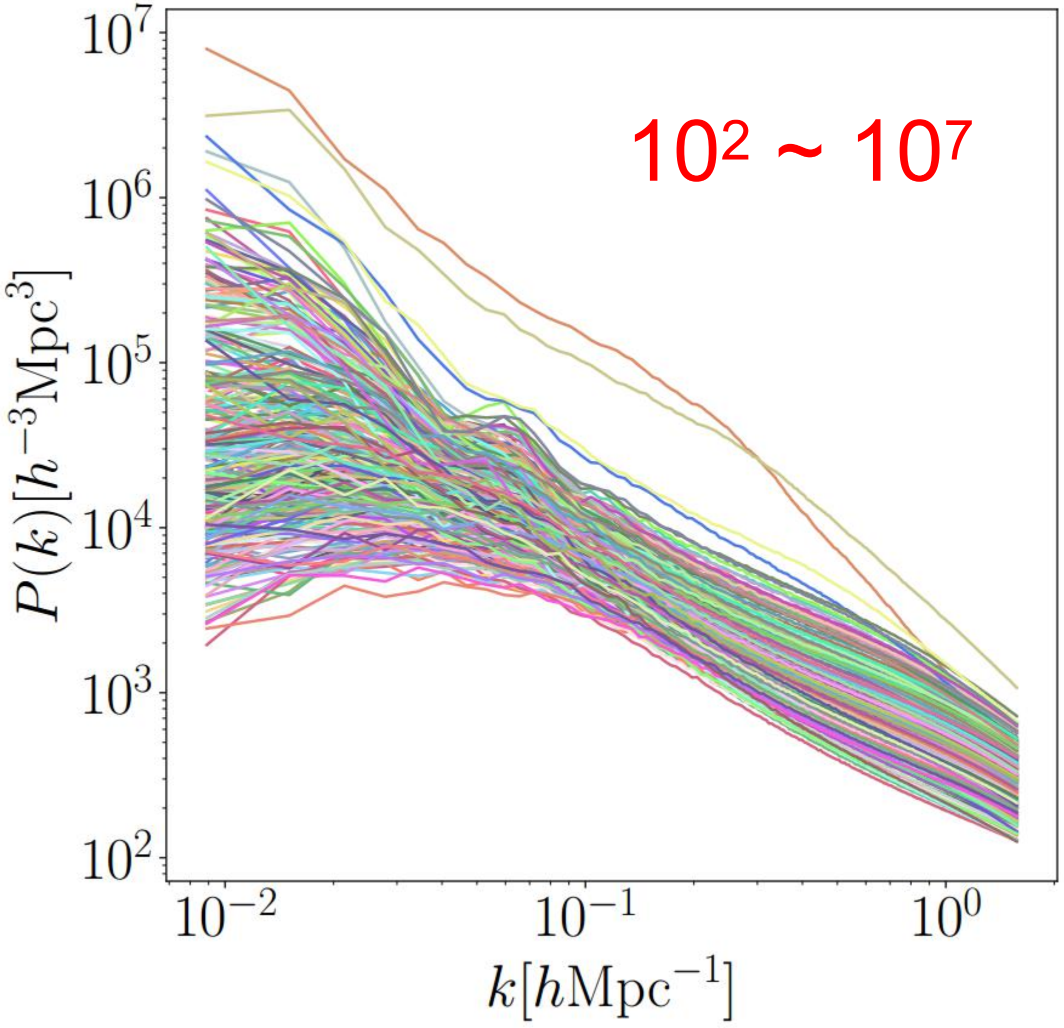
Halofit: Mead2020
computed with CAMB

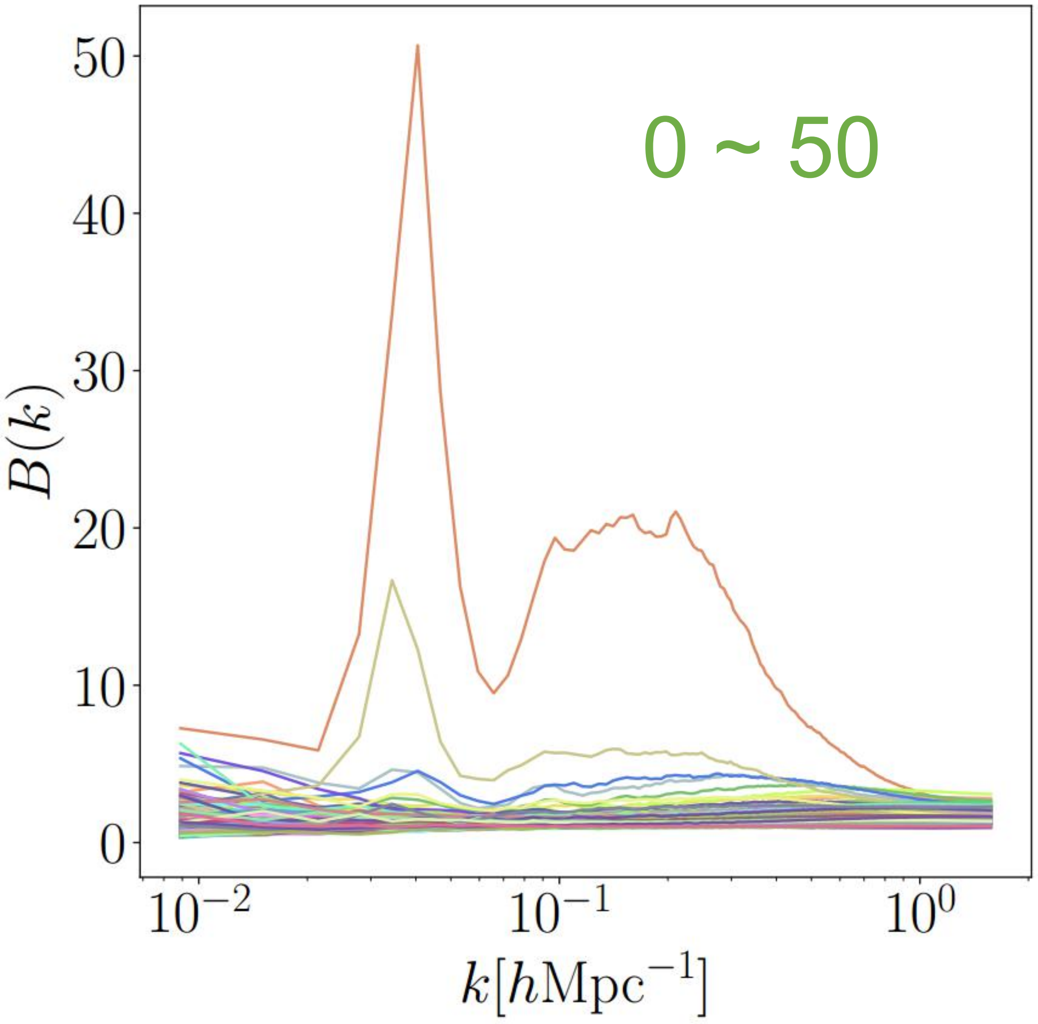
$$\mathrm{P}_{\mathrm{f(R)}}(\mathrm{k}) = \mathrm{P}_{\mathrm{halofit}}(\mathrm{k}) \times \mathrm{B}(\mathrm{k})$$

Fast Fast

Focus on f(R) & M_ν features

Build an Emulator - Boosts





Build an Emulator - PCA

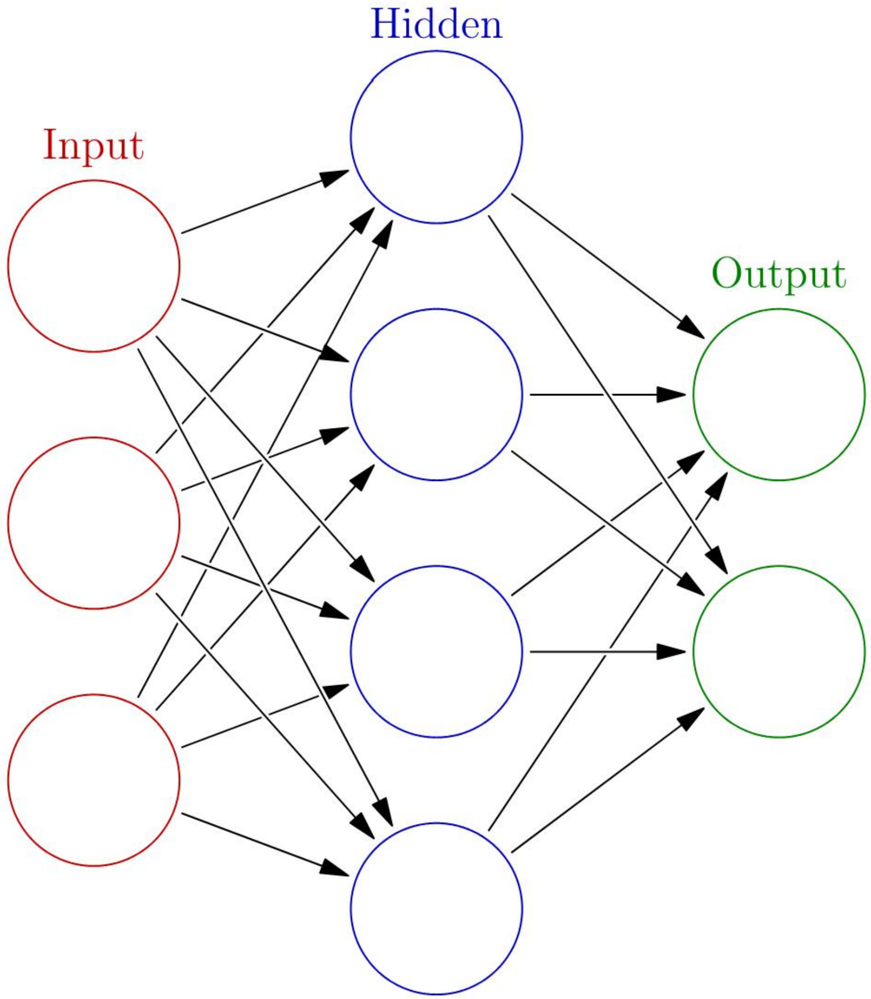
7 parameters → (Map) → Hundreds of points

7 parameters → (Map) → 24 PCA coefficients

PCA

$$B(k, z; \theta) = \mu_B(k, z) + \sum_{i=1}^{N_{pc}} \phi_i(k, z) w_i(\theta) + \epsilon$$

Build an Emulator - ANN



7 Input

1020 Hidden

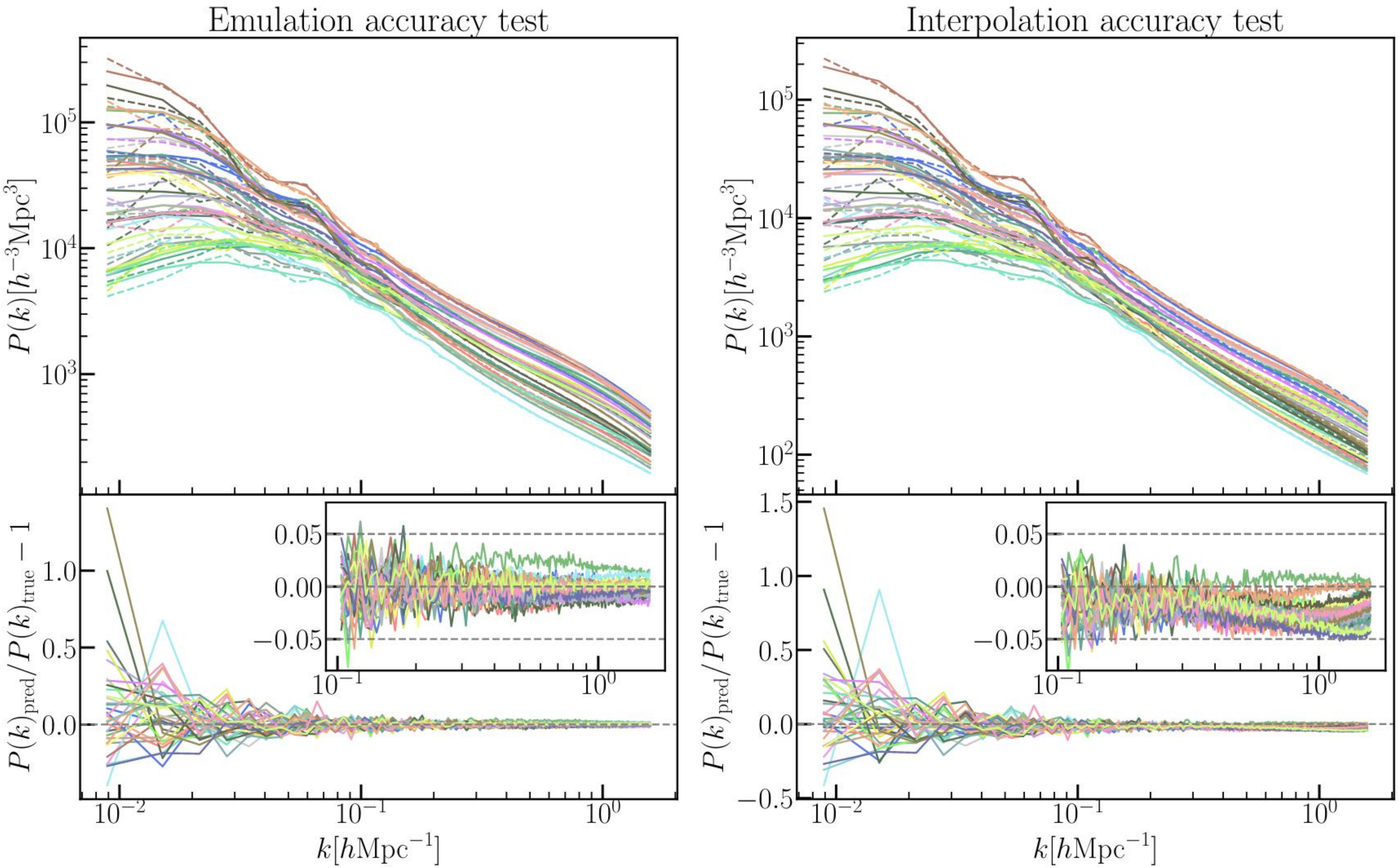
24 Output

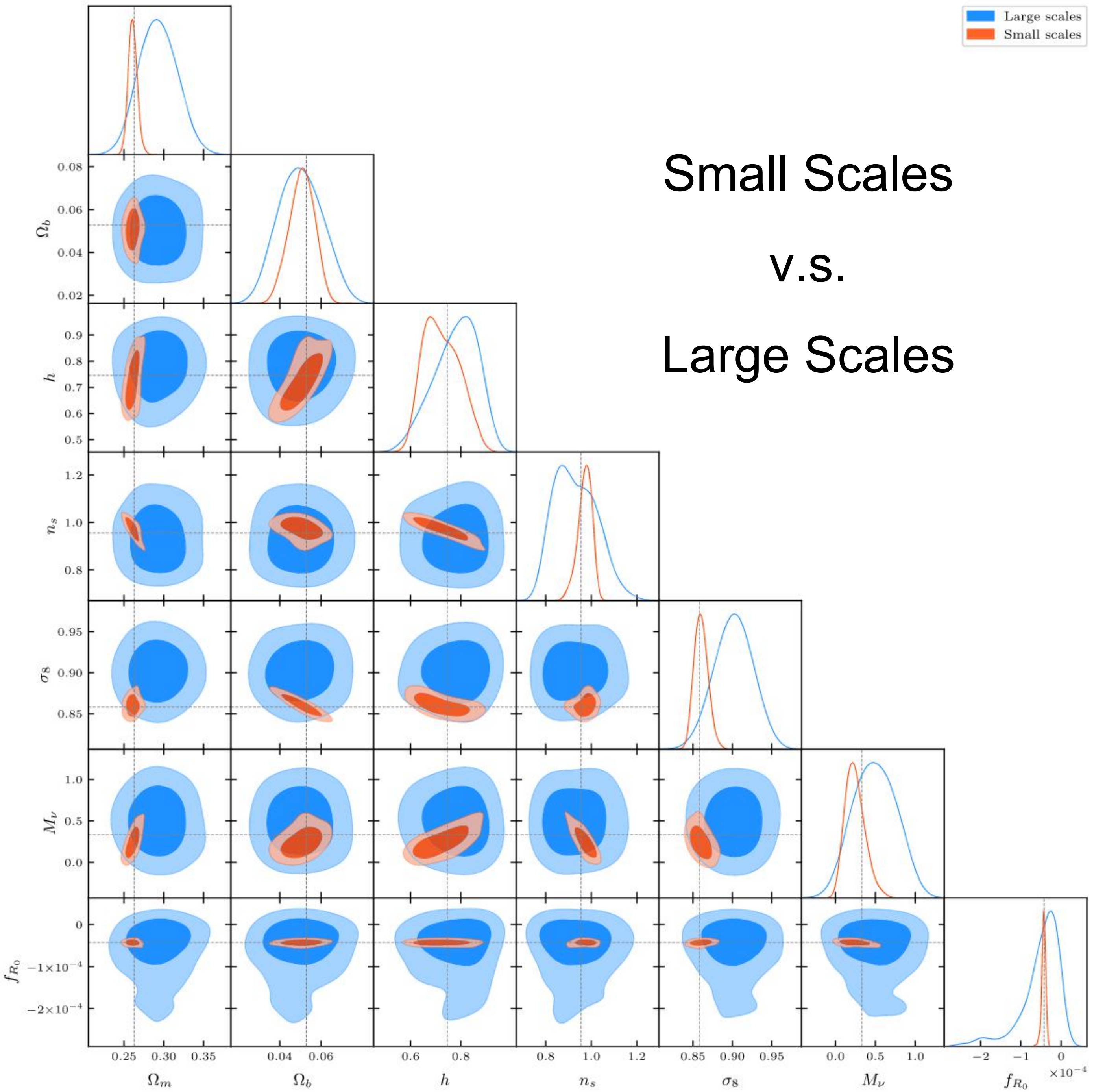
Linear - Sigmoid - Linear

Results

Accuracy

Application (Mock & Real Data)


Emulation accuracy test
Interpolation accuracy test
$P(k)[h^{-3}\mathrm{Mpc}^3]$
$P(k)_{\mathrm{pred}}/P(k)_{\mathrm{true}} - 1$
$k[h\mathrm{Mpc}^{-1}]$



Application with Mock Data

Small Scales

v.s.

Large Scales

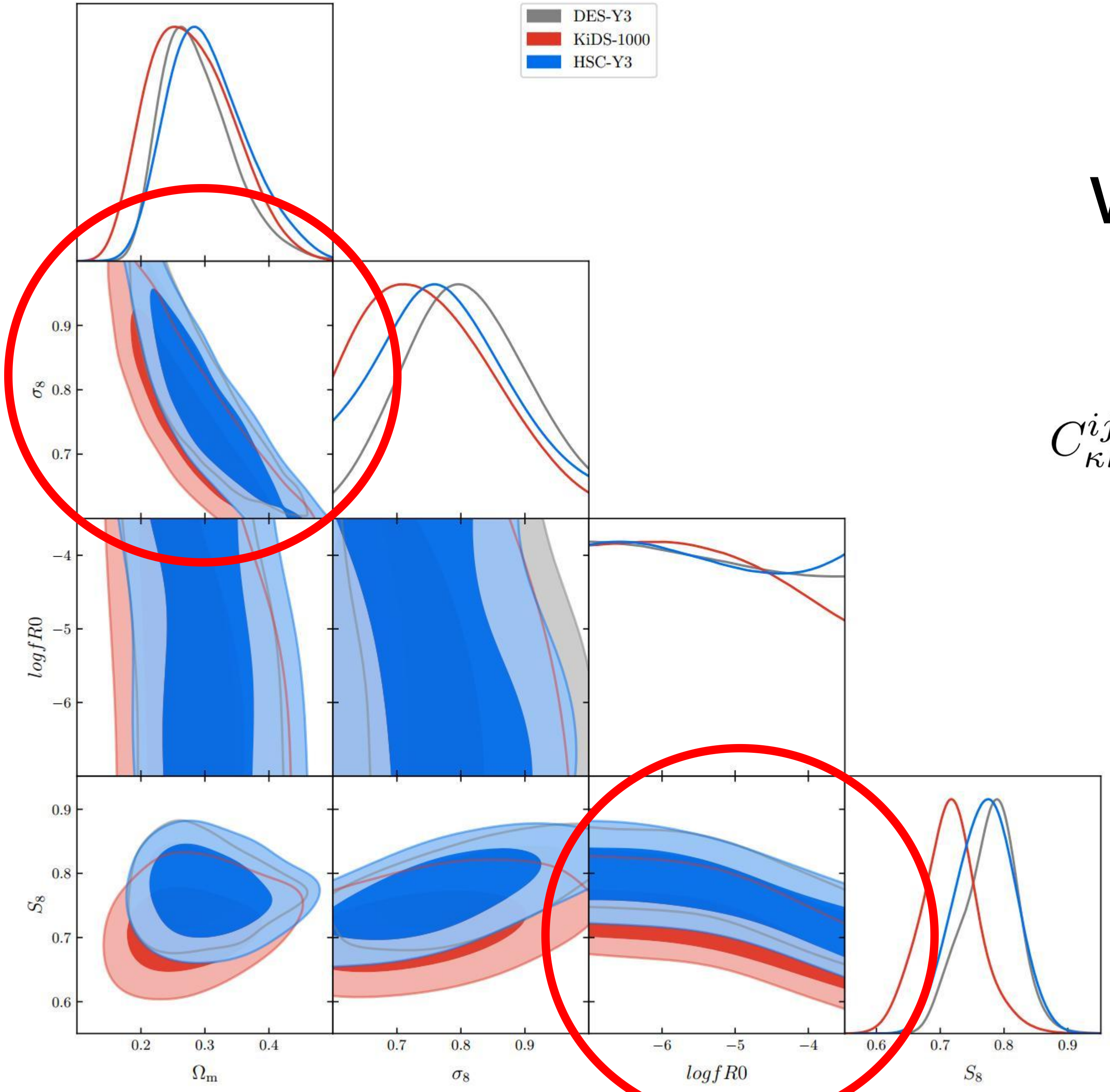
$$\Sigma\left(k_1, k_2\right)=\left[\frac{2}{N_{k_1}}+\sigma_{\mathrm{sys}}^2\right] P^2\left(k_1\right) \delta_{k_1 k_2}$$

$$\mathcal{L}(\theta \mid D)=\exp \left(-\frac{1}{2}(\mathbf{O}(D)-\mathbf{M}(\theta))^T \Sigma^{-1}(\mathbf{O}(D)-\mathbf{M}(\theta))\right)$$

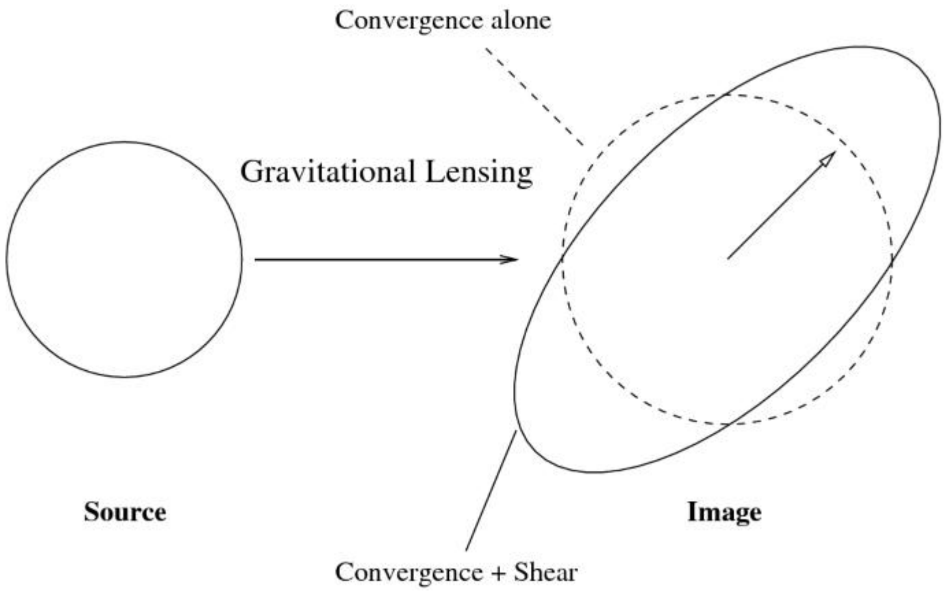
True Values of Parameters Alongside Constraint Results

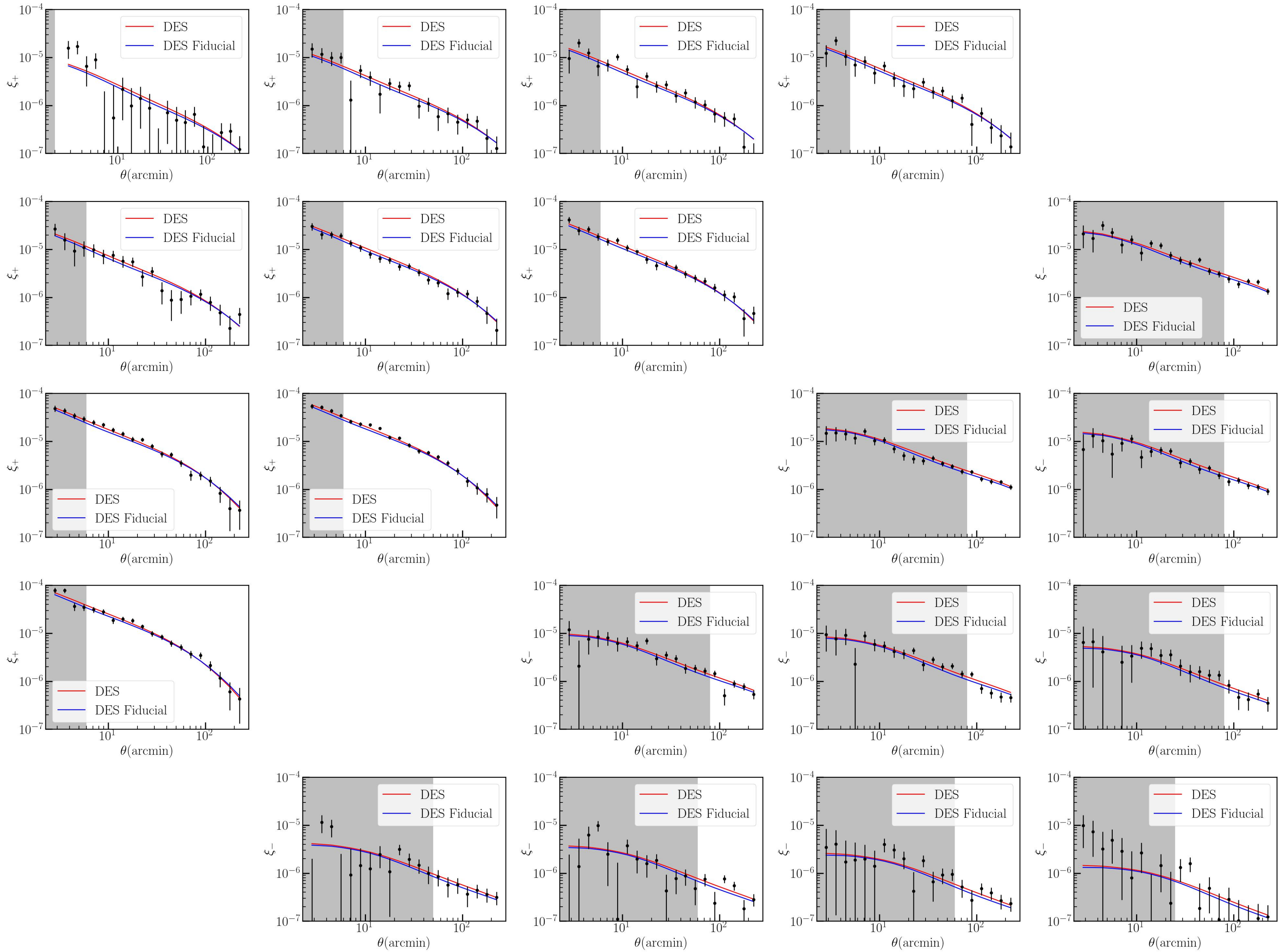
Parameter	True	Small-scale	Large-scale
$\boldsymbol{\Omega_m}$	0.263	$0.261^{+0.012}_{-0.011}$	$0.293^{+0.045}_{-0.042}$
$\boldsymbol{\Omega_b}$	0.053	$0.051^{+0.012}_{-0.012}$	$0.050^{+0.018}_{-0.018}$
$\boldsymbol{h}$	0.746	$0.72^{+0.14}_{-0.12}$	$0.78^{+0.13}_{-0.17}$
$\boldsymbol{n_s}$	0.955	$0.973^{+0.057}_{-0.066}$	$0.93^{+0.17}_{-0.14}$
$\boldsymbol{\sigma_8}$	0.858	$0.860^{+0.018}_{-0.017}$	$0.902^{+0.044}_{-0.046}$
$\boldsymbol{M_\nu}$	0.332	$0.24^{+0.26}_{-0.23}$	$0.50^{+0.48}_{-0.45}$
$\boldsymbol{10^5 f_{R_0}}$	-4.24	$-4.3^{+1.0}_{-1.0}$	$-4.9^{+5.7}_{-10}$

Application with Weak Lensing Data



$$C^{ij}_{\kappa\kappa}(\ell) = \int_0^{\chi_H} d\chi \frac{q^i_\kappa(\chi) q^j_\kappa(\chi)}{\chi^2} P_m\left[\frac{\ell + 1/2}{\chi}, z(\chi)\right]$$




DES
DES Fiducial
ξ_+
ξ_-
θ(arcmin)
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{1}
10^{2}

Conclusions

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- FREmu: Boost + PCA + ANN = 95% + 1s Emulation
- FRCCL (PyCCL+FREmu): Theory code for 2pt, 3x2pt...
- FRCobaya: A Full Pipeline for real data analyses and MCMC
- Future Works: More Features & More Data

FREmu

Thanks!

Backups

The fundamental principle commonly employed to elucidate natural phenomena is the principle of least action, which also governs gravity. In $f(R)$ gravity, this principle is encapsulated by the Einstein–Hilbert action augmented with an extra $f(R)$ term, expressed as

$$S = \int d^4x\sqrt{-g}\left[\frac{R + f(R)}{16\pi G} + \mathcal{L}_\mathrm{m}\right]. \quad (1)$$

Here, g signifies the determinant of the metric tensor $g_{\mu\nu}$, R represents the Ricci scalar, $f(R)$ denotes a function of the Ricci scalar, G stands for the gravitational constant, and $\mathcal{L}_\mathrm{m}$ denotes the Lagrangian density of matter fields.

The field equations are derived by varying the action with respect to the metric tensor, resulting in modified gravitational field equations that couple the Einstein tensor $G_{\mu\nu}$ with terms involving derivatives of $f(R)$ with respect to R. These equations are represented as

$$G_{\mu\nu} + f_R R_{\mu\nu} - \left(\frac{f}{2} - \Box f_R\right)g_{\mu\nu} - \nabla_\mu\nabla_\nu f_R = \kappa^2 T_{\mu\nu}. \quad (2)$$

Here, $R_{\mu\nu}$ denotes the Ricci tensor, $G_{\mu\nu}$ represents the Einstein tensor, $T_{\mu\nu}$ signifies the energy-momentum tensor of matter, ∇_μ denotes the covariant derivative compatible with the metric, $\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$, and f_R is the derivative of $f(R)$ with respect to R, referred to as the scalar degree of freedom (SDOF).

In the quasi-static regime, these modified field equations can be approximated to yield the modified Poisson equation and the equation of motion for the SDOF:

$$\boldsymbol{\nabla}^2\Phi = \frac{16\pi G}{3}\delta\rho - \frac{1}{6}\delta R \quad (3)$$

$$\boldsymbol{\nabla}^2 f_R = \frac{1}{3}(\delta R - 8\pi G\delta\rho), \quad (4)$$

where Φ denotes the Newtonian potential, $\delta\rho$ and δR represent perturbations to the matter density and Ricci scalar, respectively, and $\boldsymbol{\nabla}$ is the three-dimensional gradient operator.

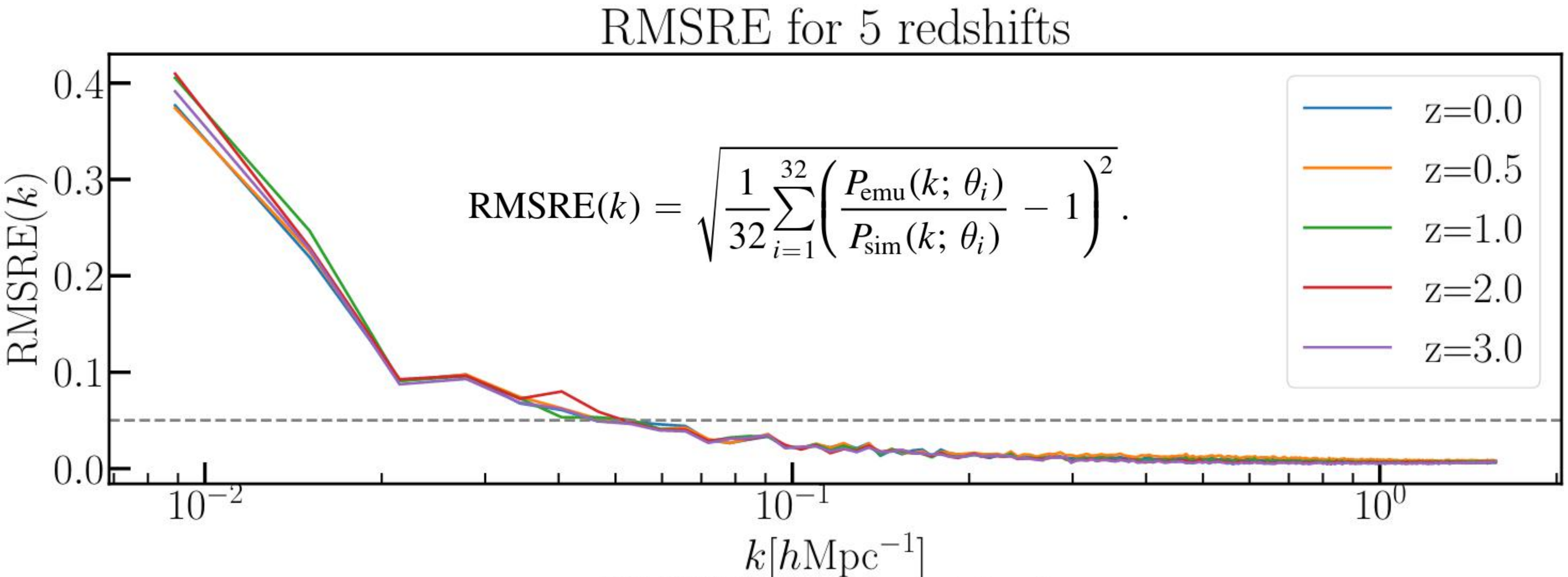
To parameterize the function $f(R)$, a commonly used form, proposed by Hu & Sawicki (2007), is

$$f(R) = -m^2\frac{c_1(R/m^2)^n}{c_2(R/m^2)^n + 1}, \quad (5)$$

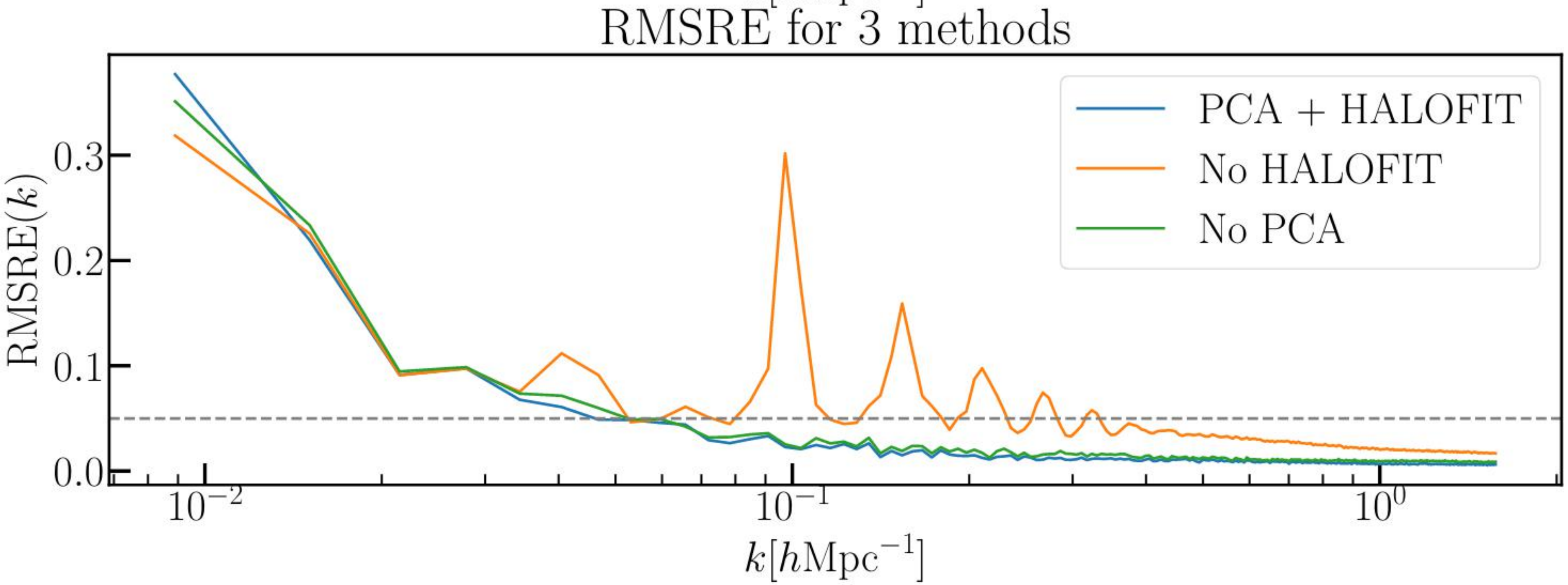
where c_1, c_2, and n are dimensionless parameters, and m is a curvature scale defined as

$$m^2 \equiv \frac{\Omega_m H_0^2}{c^2}. \quad (6)$$

Here, Ω_m represents the current fractional matter density, and H_0 is the Hubble constant. By appropriately choosing the parameters c_1 and c_2, one can ensure that the background evolution mimics that of the ΛCDM model.



RMSRE for 5 redshifts
$$\mathrm{RMSRE}(k) = \sqrt{\frac{1}{32}\sum_{i=1}^{32}\left(\frac{P_{\mathrm{emu}}(k;\,\theta_i)}{P_{\mathrm{sim}}(k;\,\theta_i)} - 1\right)^2}.$$
z=0.0
z=0.5
z=1.0
z=2.0
z=3.0
RMSRE(k)
$k[h\mathrm{Mpc}^{-1}]$



RMSRE for 3 methods
PCA + HALOFIT
No HALOFIT
No PCA
RMSRE(k)
$k[h\mathrm{Mpc}^{-1}]$

$$F(k,z)=\frac{P_{\mathrm{BCM}}}{P_{\mathrm{DMO}}}=G\left(k \mid M_c,\eta_b,z\right) S\left(k \mid k_s\right), \tag{13}$$

where G describes the suppression due to the gas and S the small-scale increase due to the central galaxy stars. The gas suppression is best captured by the function

$$G\left(k \mid M_c,\eta_b,z\right)=\frac{B(z)}{1+\left(k/k_g\right)^3}+[1-B(z)] \tag{14}$$

where

$$B(z)=B_0\left[1+\left(\frac{z}{z_c}\right)^{2.5}\right]^{-1}, \quad k_g(z)=\frac{0.7[1-B(z)]^4\eta_b^{-1.6}}{\mathrm{h/Mpc}}, \tag{15}$$

with $z_c=2.3$ and

$$B_0=0.105\log_{10}\left[\frac{M_c}{\mathrm{Mpc/h}}\right]-1.27, \quad \left(\text{for } M_c\geq 10^{12}\mathrm{M}_{\odot}/\mathrm{h}\right) \tag{16}$$

The amount of total suppression is governed by the factor B (and therefore M_c) while the scale of suppression is controlled by k_g which mainly depends on the ejection radius (via the parameter η_b). The stellar component S is well captured by the simple power law, i.e.

$$S\left(k \mid k_s\right)=1+\left(k/k_s\right)^2 \quad k_s=55[\,\mathrm{h/Mpc}]. \tag{17}$$

$$C_{\kappa\kappa}^{ij}(\ell) = \int_0^{\chi_{\rm H}} d\chi \frac{q_\kappa^i(\chi) q_\kappa^j(\chi)}{\chi^2} P_{\rm m}\left[\frac{\ell+1/2}{\chi}, z(\chi)\right] \quad (9)$$

where the Latin indices (i/j) label the tomographic redshift bins, ℓ is the angular multipole, χ denotes the comoving radial distance, $\chi_{\rm H}$ represents the comoving horizon distance, while z and k are redshift and modulus of the wave vector respectively. The lensing kernel $q_\kappa^i(\chi)$ can be expressed as:

$$q_\kappa^i(\chi) = \frac{3H_0^2\Omega_m}{2c^2} \frac{\chi}{a(\chi)} \int_\chi^{\chi_{\rm H}} {\rm d}\chi' \frac{{\rm d}z}{{\rm d}\chi'} \frac{n^i\left[z\left(\chi'\right)\right]}{\bar{n}^i} \frac{\chi'-\chi}{\chi'} \quad (10)$$

In the above, $n^i(z)$ denotes the redshift distribution of galaxies in the i-th tomographic bin, a is the scale factor, $\bar{n}^i$ represents the mean number density of source galaxies. H_0 is the Hubble constant, Ω_m is the matter fraction of the Universe and c is the speed of light.

The two-point correlation functions (2PCFs) for two redshift bins can be expressed in terms of the convergence power spectrum $C_{\kappa\kappa}^{ij}(\ell)$ as:

$$\xi_\pm^{ij}(\theta) = \sum_\ell \frac{2\ell+1}{2\pi\ell^2(\ell+1)^2}\left[G_{\ell,2}^+(\cos\theta) \pm G_{\ell,2}^-(\cos\theta)\right] C_{\kappa\kappa}^{ij}(\ell) \quad (11)$$

where $G_{\ell,2}^\pm(\cos\theta)$ are computed from Legendre polynomials.

$$\xi_\pm^{\rm obs} = \xi_\pm^{\rm true} + \xi_\pm^{II} + \xi_\pm^{GI} + \xi_\pm^{IG} \quad (12)$$

These effects can be involved by adding an additional term to the radial kernel as:

$$q_{\rm IA}^i(\chi) = -A_{\rm IA}(z) n^i(z) \frac{dz}{d\chi} \quad (13)$$

where the IA amplitude can be parametrized as:

$$A_{\rm IA}(z) = A_{\rm IA}\left(\frac{1+z}{1+z_0}\right)^{\eta_{\rm IA}} \frac{0.0139\Omega_{\rm m}}{D(z)} \quad (14)$$

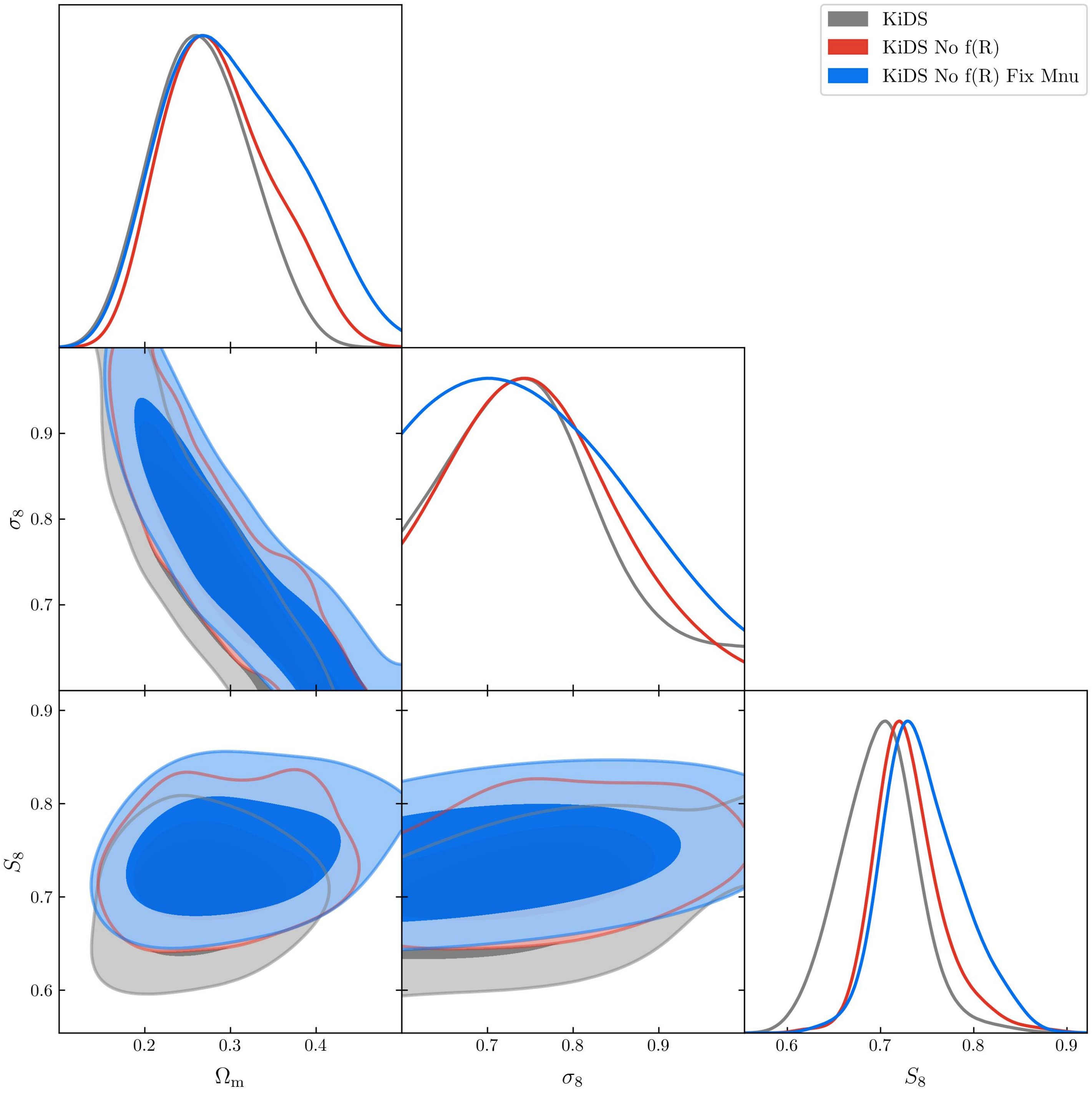
where $D(z)$ represents the linear growth factor, z_0 denotes the pivot redshift, while $A_{\rm IA}$ and $\eta_{\rm IA}$ are free parameters that need to be included in cosmic shear modeling.

The cosmic shear signal can be sensitive to the measurements of redshift distributions, so the uncertainty of these can be parameterized by adding nuisance parameters Δz^i, denoting a displacement of redshift bins:

$$n^i(z) \rightarrow n^i\left(z - \Delta z^i\right) \quad (15)$$

Owing to the uncertainty in the multiplicative and additive biases in cosmic shear measurements, we include nuisance parameters m^i for each independent redshift bin, thus the observed cosmic shear signal can be modified as:

$$\xi_{\rm obs}^{ij} = \left(1+m^i\right)\left(1+m^j\right)\xi_{\rm true}^{ij} \quad (16)$$


KiDS
KiDS No f(R)
KiDS No f(R) Fix Mnu
σ8
S8
Ωm
σ8
S8
0.2
0.3
0.4
0.7
0.8
0.9
0.6
0.7
0.8
0.9