Finite Elements Method Project

Derivation of elastic deformation formulas by Piotr Rusak

Problem

$$-\frac{d}{dx}\left(E(x)\frac{d}{dx}u(x)\right)=0$$

$$u(2)=0,\ \frac{d}{dx}u(0)+u(0)=10$$

$$E=\begin{cases}3, & x\in[0,1]\\ 5, & x\in(1,2]\end{cases}$$

$$[0,2]\ni x\ \rightarrow u(x)\in R$$

Solution

$$-\frac{d}{dx}E(x)\frac{d}{dx}u(x)-E(x)\frac{d^2}{dx^2}u(x)=0$$

$$-E(x)\frac{d^2}{dx^2}u(x)v(x)=0$$

$$-\int_0^2 E(x)u''(x)v(x)dx=0$$

$$-5u'(2)v(2)+3u'(0)v(0)+\int_0^2 E(x)\,u'(x)v'(x)dx=0$$

$$30v(0)-3u(0)v(0)+\int_0^2 E(x)u'(x)v'(x)dx=0$$

$$\int_0^2 E(x)u'(x)v'(x)dx-3u(0)v(0)=-30v(0)$$

$$B(u,v)=\int_0^2 E(x)u'(x)v'(x)dx-3u(0)v(0)\qquad L(v)=-30v(0)$$

Rest of the solution in R Source File