

(c) Calculate average over all directions of \vec{r} :

$\langle (\vec{r} \cdot \vec{a}) \vec{r} \rangle$; $|\vec{r}| = R$ & \vec{a} is a fixed vector

$$\text{Ans } \langle (\vec{r} \cdot \vec{a}) \vec{r} \rangle = \langle r_i a_i r_j \rangle = a_i \langle r_i r_j \rangle \\ = a_i R^2 \left\langle \frac{r_i}{R} \frac{r_j}{R} \right\rangle = a_i R^2 \langle n_i n_j \rangle$$

$$= \frac{a_i R^2}{3} \delta_{ij}$$

$$= \frac{a_i R^2}{3}$$

$$\Rightarrow \langle (\vec{r} \cdot \vec{a}) \vec{r} \rangle = \frac{R^2}{3} \vec{a}$$
