Galaxy formation and evolution
PAP 318, 5 op, autumn 2020

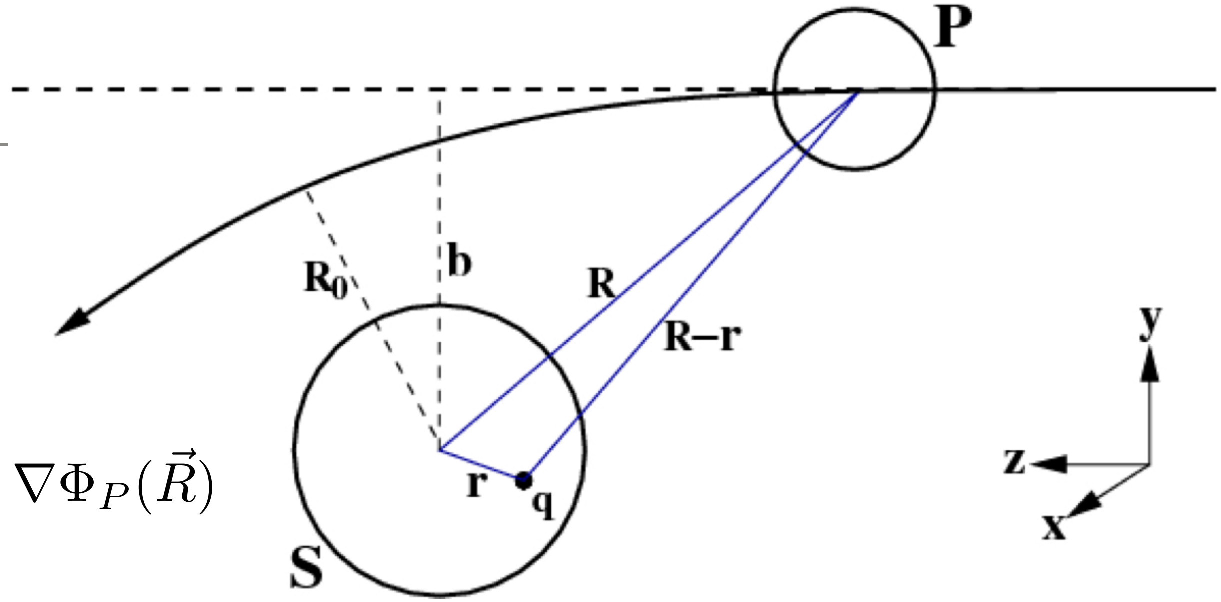
on Zoom

Lecture 11: Galaxy interactions and transformations, 20/11/2020

On this lecture we will discuss

1. Galaxy interactions – Basics and definitions.
2. High-speed encounters and the impulse approximation.
3. Tidal stripping and the formation of tidal arms.
4. Dynamical friction: Intuitive picture and mathematical formulation.
5. Orbital decay: Theory and applications.
6. The lecture notes correspond to: MBW: pages 544-561 (§12.1-12.3)

11.1 Galaxy interactions



$$\vec{F}_{\rm tid}(\vec{r}) = -\nabla\Phi_P(|\vec{R}-\vec{r}|) + \nabla\Phi_P(\vec{R})$$

- Consider a body S of finite size which has an encounter with a perturber P with impact parameter b and initial velocity v_∞.
- Let q be a particle (star) in S, at a distance **r**(t) from the centre of S and let **R**(t) be the position vector of P from S.
- Since the gravitational force due to P is not uniform over the body S, the particle q experiences a tidal force per unit mass.

Interaction timescales I

- The timescale for tidal interaction can be defined as t_{tide}~R_{gal}/σ, where R_{gal} and σ are the size and velocity dispersion of the system that experiences the tides.
- The timescale for the encounter can be defined as t_{enc}~R/V, where the radius R=MAX[R_0,R_S,R_P]. R_0 is the minimum distance of the encounter and R_S and R_P are the characteristic radii of S and P. V is the encounter velocity at the minimum distance, R=R_0.
- If $t_{enc}>>t_{tide}$ we are in the adiabatic limit, i.e. the system has sufficient time to respond to tidal deformations. Deformation during the approach and departure cancel each other and there is no net effect.
- $t_{enc}>>t_{tide}$ would imply that $V<<(R_{max}/R_{gal})\sigma$, which is not possible since $R_{max}\geq R_{gal}$ by construction and $V\gtrsim\sigma$, because the same gravitational field is responsible for both velocities (you cannot have a close approach due to self-gravity, with speed below the velocity dispersion). Thus, the situation $t_{enc}>>t_{tide}$ never occurs for collisionless (i.e. no gas) galactic systems.

Interaction timescales II

- In actual cases $t_{enc}<t_{tide}$, the response of the system lags behind the instantaneous tidal force, causing a back reaction on the orbit.
- The net effect is the transfer of orbital energy to internal energy of both S and P, i.e. the velocity dispersions, σ of both S and P increases.
- Under certain conditions, if enough orbital energy is transferred, the two bodies can become gravitationally bound to each other. This process is called gravitational capture.
- If the transfer of orbital energy continues, the capture will ultimately result in the merger of the two objects.
- When the internal energy gain is large, some particles may however become unbound (note that a velocity dispersion, σ implies a distribution of velocities) resulting in mass loss.

11.2 High-speed encounters I

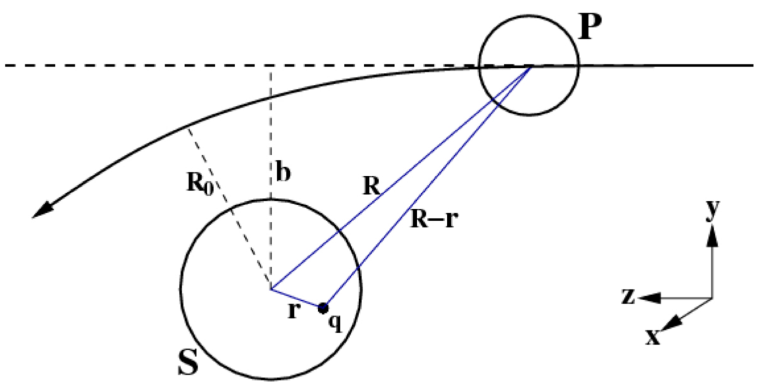
- In general numerical simulations are required to investigate the outcome of a gravitational encounter. However, in the special case of a high-speed encounter ($V>>\sigma$), i.e. the encounter velocity is much larger than the internal velocity dispersion, the change in the internal energy can be obtained analytically using the impulse approximation.
- We can consider the encounter between S and P in the figure on slide 3. In the impulse approximation we may consider the particle q to be stationary with respect to the centre of S. Thus, during the encounter, q only experiences a velocity change Δv, but its potential energy remains unchanged:

$$\Delta E_q = \frac{1}{2}(\vec{v} + \Delta\vec{v})^2 - \frac{1}{2}\vec{v}^2 = \vec{v}\cdot\Delta\vec{v} + \frac{1}{2}|\Delta\vec{v}|^2$$

High-speed encounters II

- We are interested in calculating ΔE_S, obtained by integrating ΔE over the entire system S. Because of symmetry the first term **v**Δ**v** vanishes (the average of **v** is zero):

$$\Delta E_S = \frac{1}{2}\int |\Delta\vec{v}(\vec{r})|^2 \rho(\vec{r}) d^3\vec{r}$$



- In the large velocity limit (v_∞), R_0->b and R(t)=(0,b,v_Pt).
- In the distant encounter approximation b>>MAX[R_S,R_P] and the perturber P may be considered a point mass. The potential due to P at r is:

$$\Phi_P = -\frac{GM_P}{|\vec{r}-\vec{R}|}, \qquad |\vec{r}-\vec{R}| = \sqrt{R^2 - 2rR\cos\phi + r^2}$$

High-speed encounters III

- The angle ϕ is the angle between the vectors **r** and **R**.
- The denominator can be expanded using the Taylor series for:

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{15}{48}x^3 + \ldots$$

- Using this Taylor series and the fact that r<<R we get the potential:

$$\Phi_P = -\frac{GM_P}{R} - \frac{GM_P r}{R^2}\cos\phi - \frac{GM_P r^2}{R^3}\left(\frac{3}{2}\cos^2\phi - \frac{1}{2}\right) + O(r^3/R^3)$$

- The first term is a constant term yielding no tidal force. The second term describes how the centre of mass of S changes under a uniform acceleration. We are interested in the third term that describes the tidal force per unit mass, note that this is proportional to ~R^{-3}.

Impulse approximation I

- Taking the gradient of the potential and dropping the second term (constant acceleration) gives the tidal force per unit mass:
$$\vec{F}_{\rm tid}(\vec{r}) = \nabla\Phi_P = d\vec{v}/dt$$
- Integrating the force over time gives the cumulative change in the velocity (see MBW page 546 for details):
$$\Delta\vec{v} = \frac{2GM_P}{v_P b^2}(x, 0, -z)$$
- Substituting this expression for the total change in energy of S:
$$\Delta E_S = \frac{1}{2}\int |\Delta\vec{v}(\vec{r})|^2 \rho(r) d^3\vec{r} = \frac{2G^2M_P^2}{v_P^2 b^4}\int \rho(r)(x^2+z^2)d^3\vec{r} = \frac{2G^2M_P^2}{v_P^2 b^4} M_S \langle x^2+z^2\rangle$$
- Assuming spherical symmetry: $\langle x^2+z^2\rangle = 2/3\langle r^2\rangle$
$$\Delta E_S = \frac{4}{3}G^2 M_S\left(\frac{M_P}{v_P}\right)^2\frac{\langle r^2\rangle}{b^4}$$

Impulse approximation II

$$\Delta E_S = \frac{4}{3} G^2 M_S \left(\frac{M_P}{v_P} \right)^2 \frac{\langle r^2 \rangle}{b^4}$$

- The formula of the impulse approximation is surprisingly accurate even for relatively slow encounters with $v_P \sim \sigma_S$ as long as the impact parameter $b \geq 5\, \mathrm{MAX}(R_P, R_S)$. For smaller impact parameters one needs to account for the detailed internal mass distribution of P (see MBW page 547 for details).
- The change in energy scales as $\Delta E_S \propto b^{-4}$, closer encounters have a much larger impact.
- If ΔE_S exceeds the binding energy of S, the system S will be tidally disrupted.

Impulsive heating

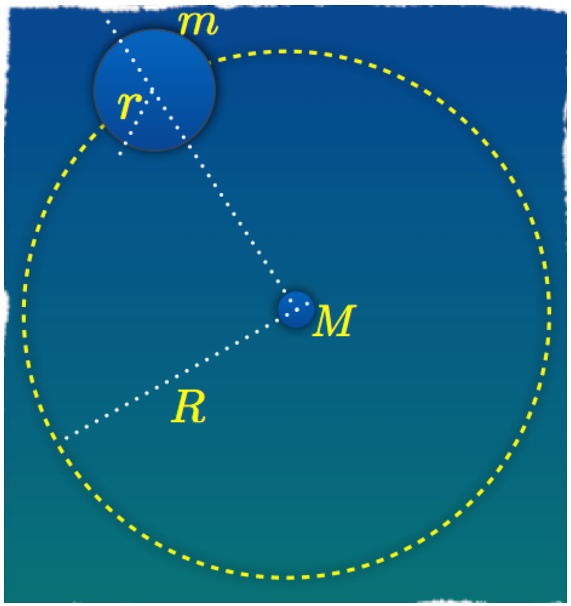
- In the impulse approximation the encounter only changes the kinetic energy of S and leaves the potential energy constant.
- After the encounter S is no longer in virial equilibrium and consequently S must undergo relaxation to re-establish virial equilibrium. Let K_S be the original pre-encounter kinetic energy, then:

1. Virial equilibrium: $E_S=-K_S$.
2. After encounter: $E_S \rightarrow E_S+\Delta E_S$.
3. All new energy is kinetic: $K_S \rightarrow K_S+\Delta E_S$.
4. After relaxation: $K_S=-(E_S+\Delta E_S)=-E_S-\Delta E_S$

- Relaxation decreases the kinetic energy by $2\Delta E_S$ and this energy is transferred to the potential energy, which becomes less negative. Hence, tidal shocks cause the system to expand and make it less bound.

11.3 Tidal stripping I

- Even in the general non-impulsive case, tidal forces can strip matter (=tidal stripping).
- Let us consider a mass m, with radius r, orbiting a point mass M on a circular orbit of radius R.
- Calculating the tidal gravitational acceleration at the edge of m closest to the central mass M:



$$\vec{g}_{\rm tid}(r) = \frac{GM}{R^2} - \frac{GM}{(R-r)^2} \simeq \frac{2GMr}{R^3} \qquad (r << R)$$

- If the tidal acceleration exceeds the binding force per unit mass (Gm/r^2), the material at distance r from the centre of m will be stripped. This defines the tidal radius:

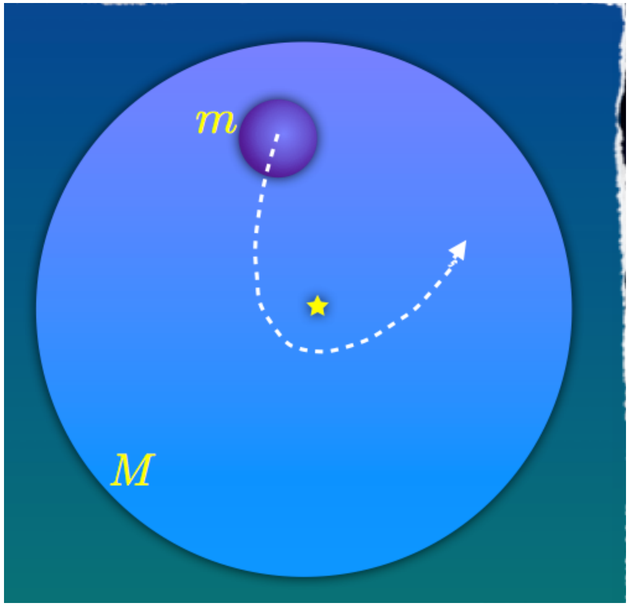
$$r_t = \left(\frac{m}{2M}\right)^{1/3} R$$

Tidal stripping II

- Taking into account the centrifugal force associated with the circular motion results in a more accurate tidal radius (MBW p. 548):

$$r_t = \left(\frac{m/M}{3+m/M}\right)^{1/3} R$$

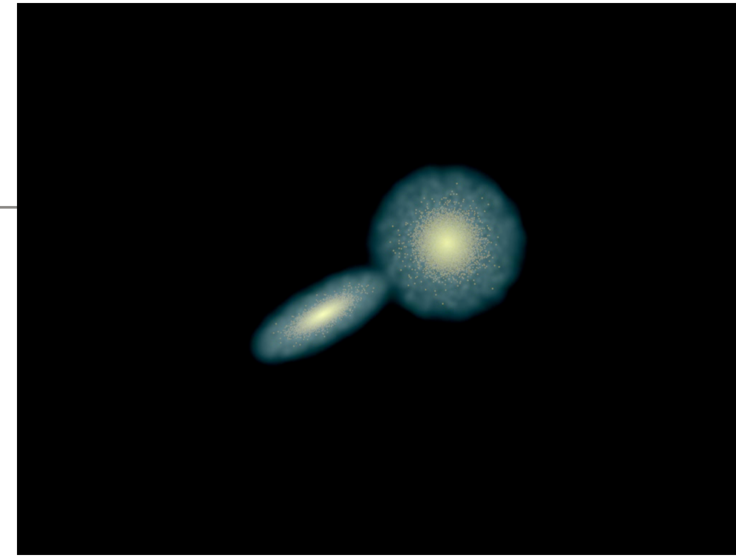
- In a more realistic case object m is on an eccentric orbit within an extended mass M.
- The tidal radius, which can be derived in this case should be taken with a grain of salt as the concept of tidal radius is now poorly defined:

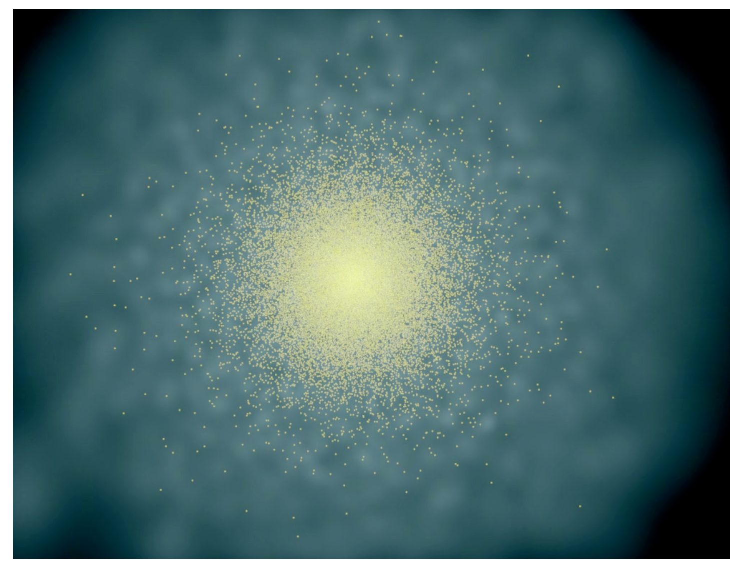


$$r_t = \left[\frac{m(r_t)/M(R_0)}{2 + \frac{\Omega^2 R_0^3}{GM(R_0)} - \frac{d\ln M}{d\ln R}\Big|_{R_0}}\right]^{1/3} R_0$$

Tidal arms

- During the mergers of galaxies tidal relics such as tidal tails are often formed.
- The tails are narrow, because they originate from dynamically cold disks. Mergers between dynamically hot spheroids do not produce narrow tidal tails.
- Prograde mergers in which the orbital angular momentum is aligned with the spin vectors of the initial disk galaxies result in the most prominent tidal tails.





11.4 Dynamical friction

- When an object of mass M_S moves through a large collisionless system whose constituent particles ("field particles) has a mass $m \ll M_S$, it experiences a drag force, called dynamical friction.
- Dynamical friction transfers the orbital energy of orbiting satellite galaxies and dark matter subhaloes to the dark matter particles and stars that make up the host halo, causing the satellite (subhalo) to "sink" to the centre of the potential well, where it can ultimately merge with the central galaxy (galactic cannibalism).
- The physics behind dynamical friction can be understood using the following two intuitive pictures: 1) Equipartition and 2) The formation of a gravitational wake.

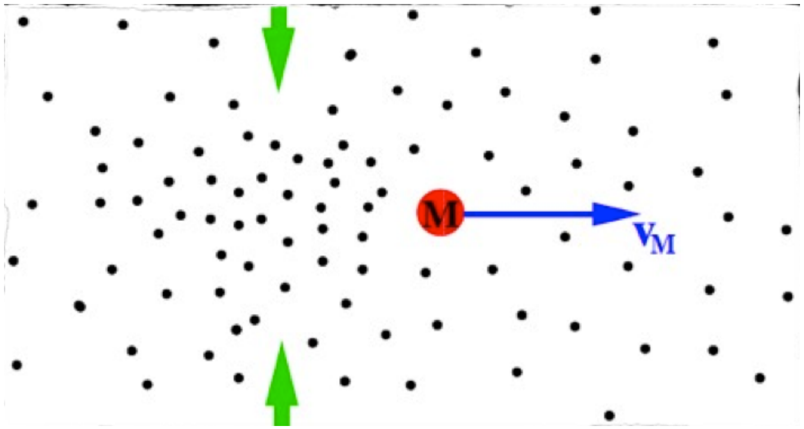
Dynamical friction: Intuitive pictures

1. Two-body interactions move the system towards equipartition.

$$m_1\langle v_1^2\rangle = m_2\langle v_2^2\rangle = m_3\langle v_3^2\rangle = \mathrm{etc}$$

Since initially $v_S \sim \sigma_{field}$ and $M_S >> m_{field}$, the subject mass will on average loose energy and slow down.

2. Gravitational wake: The moving subject mass perturbs the distribution of the field particles creating a trailing density enhancement ("wake"). The gravitational force of this wake on M_S slows it down.



Gravitational wake formed by the action of dynamical friction.

Dynamical friction: Mathematical formulation I

- Chandrasekhar (1943) derived an analytical expression for the dynamical friction force (see MBW §12.3) and the Galactic Dynamics course (Spring 2021):

$$\vec{F}_{\rm df} = M_S \frac{d\vec{v}_S}{dt} = -4\pi \left(\frac{GM_S}{v_S}\right)^2 \ln\Lambda \rho(< v_S) \frac{\vec{v}_S}{v_S}$$

- Here $\rho(<v_S)$ is the local density of the field particles with speeds less than v_S, and $\ln\Lambda$ is the Coulomb logarithm, which can be approximated as:

$$\ln\Lambda \approx \ln\left(\frac{b_{\rm max}}{b_{90}}\right) \quad b_{\rm max} \sim R \quad b_{90} \sim \frac{G(M_S + m)}{v_\infty^2}$$

- The dynamical friction force is proportional to M_S^2 and independent of the individual masses of the field particles. For small velocities v_S, $F_{\rm df} \propto v_s$ and for large velocities v_S, $F_{\rm df} \propto v_s^{-2}$, unlike hydrodynamical friction which always increases with the velocity.

Dynamical friction: Mathematical formulation II

- Chandrasekhar's expression for the dynamical friction is based on the following three assumptions:

1. The subject mass and the field particles are point masses.
2. The self-gravity of the field particles can be ignored.
3. The distribution of the field particles is infinite, homogenous and isotropic.

- Chandrasekhar's dynamical friction is considered as the sum of uncorrelated two-body interactions between a field particle and the subject mass. However, this ignores the collective effects due to self-gravity of the field particles.
- In reality dynamical friction is a global phenomenon , which is evident from the fact that the subject mass experiences dynamical friction even if it orbits beyond the outer edge of the finite host system. The proper way to treat dynamical friction would be instead using linear response theory.

11.5 Orbital decay I

- Let us consider a subject mass on a circular orbit in a spherical, singular isothermal host halo with constant circular velocity V_c and the density distribution: $\rho(r)=V_C^2/(4\pi G r^2)$
- Assuming that the velocity distribution of field particles is a Maxwell-Boltzmann distribution with velocity dispersion $\sigma=V_C/\sqrt{2}$ the dynamical friction force can be expressed as:

$$F_{\rm df} = -0.428 \frac{GM_S^2}{r^2} \ln\Lambda \frac{\vec{v}_S}{v_S}$$

- For an object on a circular orbit, the rate at which the subject mass loses its orbital angular momentum $L_S=rv_S$ is given by:

$$\frac{dL_S}{dt} = r\frac{dv_S}{dt} = r\frac{F_{\rm df}}{M_S} = -0.428 \frac{GM_S}{r} \ln\Lambda$$

Orbital decay II

- In the isothermal halo model adopted here the circular speed is independent of the radius, thus the subject mass continues to orbit with a speed v_S as it spiral inwards and the orbit radius changes:

$$v_S \frac{dr}{dt} = -0.428 \frac{GM_S}{r} \ln\Lambda \Rightarrow r\frac{dr}{dt} = -0.428 \frac{GM_S}{V_c} \ln\Lambda$$

- Now we can solve for the dynamical friction time, i.e. how long it takes for the orbit to decay from some initial radius r_i to r=0:

$$t_{\mathrm{df}} = \frac{1.17}{\ln\Lambda} \frac{r_i^2 V_c}{GM_S} \rightarrow t_{\mathrm{df}} = \frac{1.17}{\ln\Lambda} \left(\frac{r_i}{r_h}\right)^2 \left(\frac{M_h}{M_S}\right) \frac{r_h}{V_c}$$

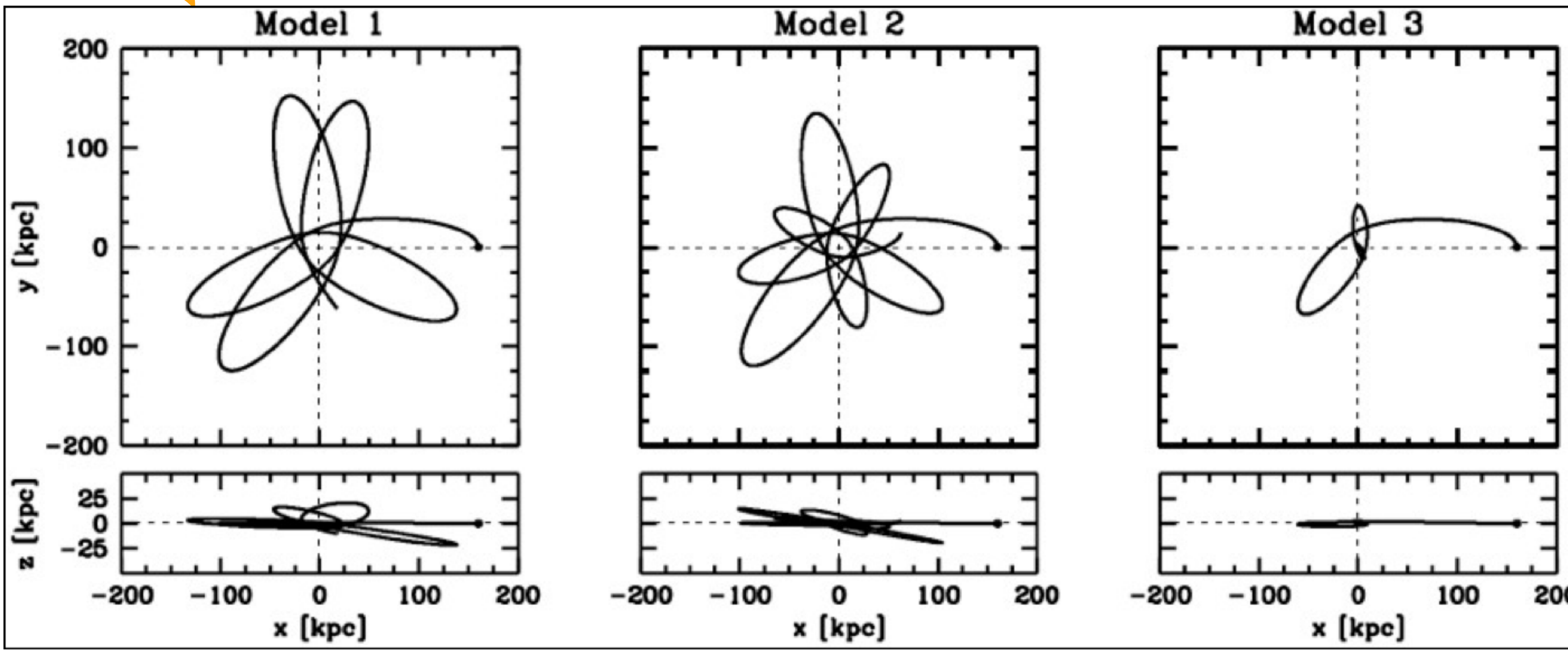
Orbital decay III

- Finally, using the following approximations $r_H/V_C \sim 1/[10H(z)]=0.1t_H$ and that $\ln \Lambda \sim \ln(M_h/M_S)$ (MBW p. 557) results in:

$$t_{\rm df} \simeq 0.117 \frac{(M_h/M_s)}{\ln(M_h/M_s)} t_H$$

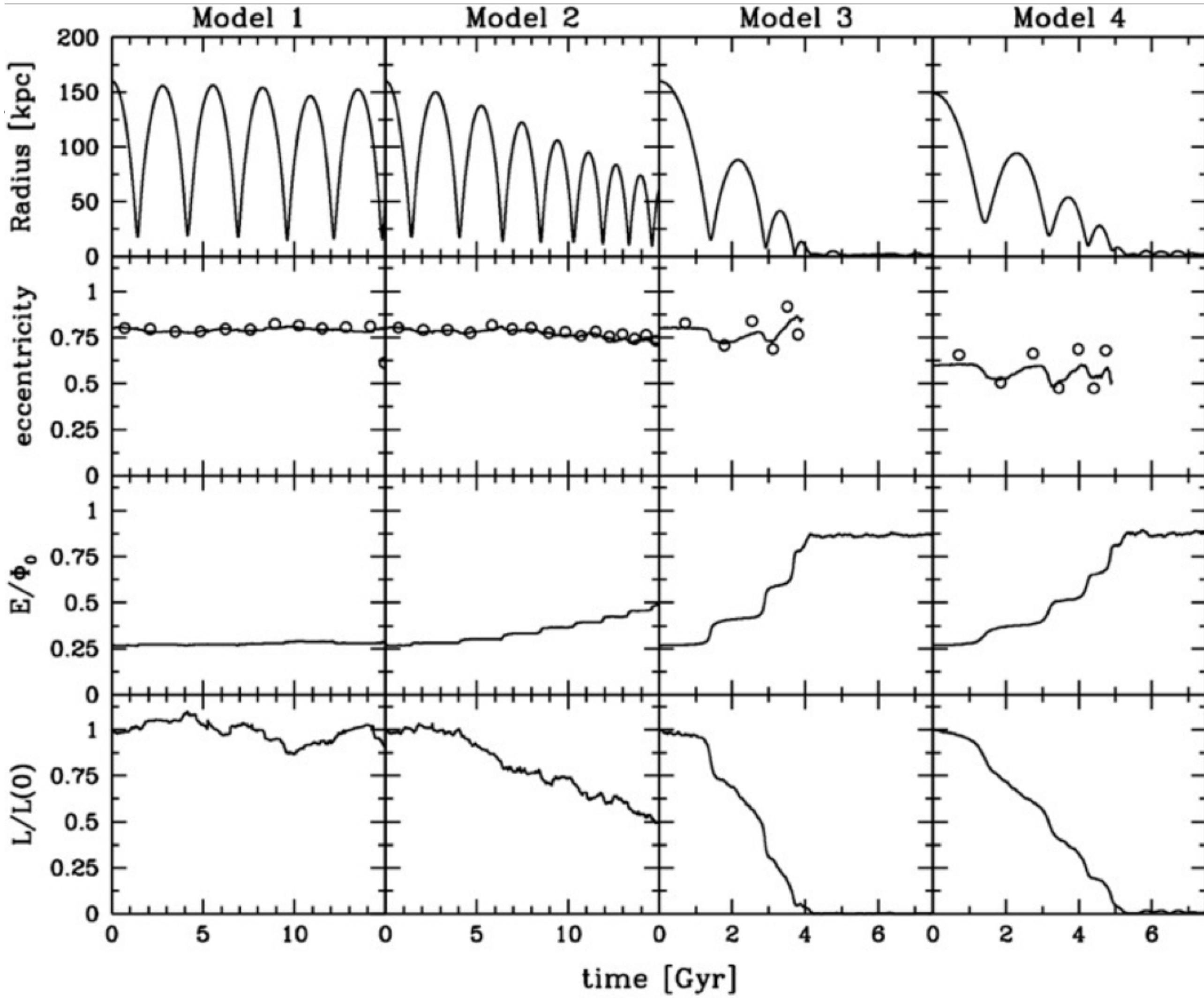
- This means that only systems with $M_S/M_h>0.03$ experience significant mass segregation during the age of the Universe.
- The above formulas are based on questionable assumptions, which in general are not true. Haloes are not singular isothermal spheres, orbits are not circular (although DF can circularise them) and tidal stripping implies mass loss.
- Also when the orbits are eccentric, dynamical friction may cause the orbit's eccentricity to evolve. The net result is that more eccentric orbits decay faster, see MBW pages 557-558 for details.

Orbital decay application I



- N-body simulations of dynamical friction in high resolution simulations. Model 1: $M_S/M_h=2x10^{-4}$,Model 2: $M_S/M_h=2x10^{-3}$,Model 3: $M_{S/}M_h=2x10^{-2}$.
- All models have eccentricity e=0.8. More massive objects decay faster.

Orbital decay application II



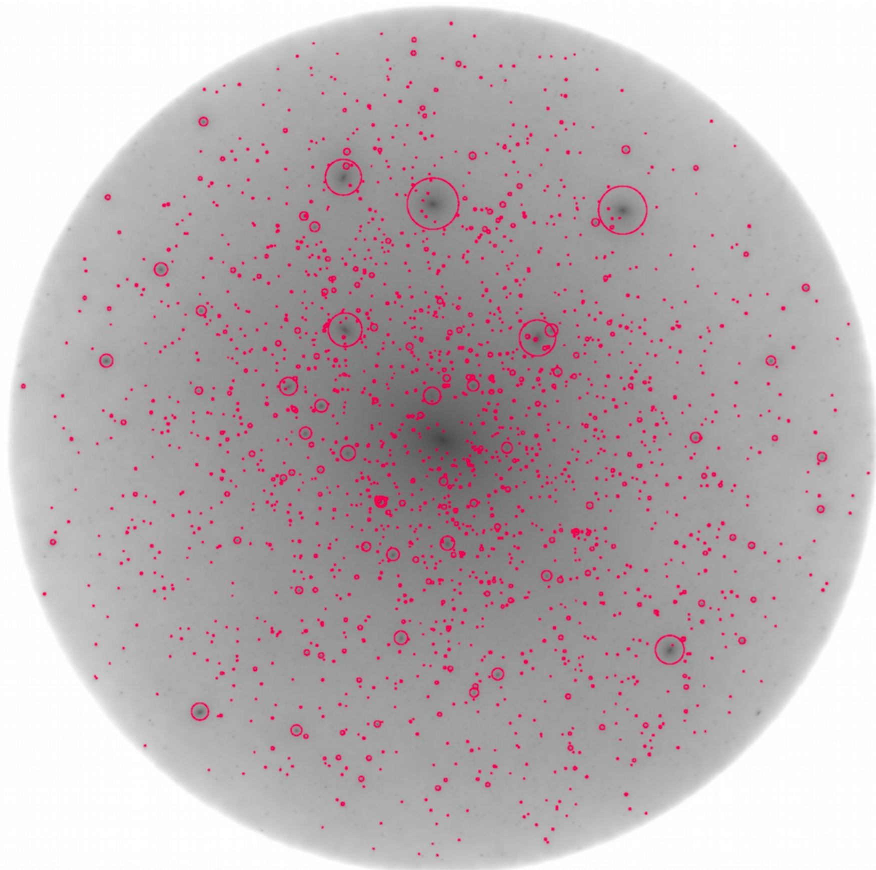
Dynamical friction and mass loss

- When the subject masses are not solid objects, but N-body systems, they can experience mass loss due to tidal stripping and tidal heating.
- Rough analytical estimates (see MBW pages 558-559) indicate that mass loss causes the average dynamical friction time to increase by a factor of ~2.8 when mass loss is accounted for.
- This analytic estimate is in good agreement with the results from accurate numerical simulations.
- Accurate modelling of tidal stripping, tidal heating and dynamical friction is important for predicting the disruption and merger rates of satellite galaxies.

Destruction of dark matter substructues

Sawala et al. (2017)

- Even though halos form hierarchically through mergers, at any given time, most of the matter in a halo is “smooth”, i.e. phase-mixed; only a small fraction is in self-bound substructures.
- Accreted substructures are destroyed via tidal stripping, and their orbits decay via dynamical friction.



What have we learned?

1. In actual interactions the encounter timescale is always shorter than the tidal timescale: The response of the system lags behind the instantaneous tidal force, causing a back reaction on the orbit.
2. In high-speed encounters the impulse approximation can be used as the particle only experiences a velocity change Δv, but its potential energy remains unchanged.
3. Tidal forces are proportional to R^{-3} and if the tidal acceleration exceeds the binding force per unit mass (Gm/r^2), the material at distance r from the centre of m will be stripped.
4. The dynamical friction force is proportional to M^2 and is most effective for slowly moving objects ($\propto v$), for large velocities the effect weakens as ($\propto v^{-2}$).