

$$\text{Ans } ds^2 = \eta_{AB} dx^A dx^B.$$

$$= (dx^0)^2 - \sum_{i=1}^3 (dx^i)^2 - (dx^D)^2$$

$$u x^0 = \sinh(u \tau_+) + \frac{(u \vec{x}_+)^2}{2} e^{u \tau_+}$$

$$\Rightarrow u dx^0 = u \cosh(u \tau_+) d\tau_+ + (u \vec{x}_+) d\vec{x}_+ e^{u \tau_+} + \frac{(u \vec{x}_+)^2}{2} u e^{u \tau_+} d\tau_+$$

$$\Rightarrow dx^0 = \left[ \cosh(u \tau_+) + \frac{(u \vec{x}_+)^2}{2} e^{u \tau_+} \right] d\tau_+ + e^{u \tau_+} d\vec{x}_+$$

$$\Rightarrow (dx^0)^2 = \left[ \cosh^2(u \tau_+) + \frac{(u \vec{x}_+)^2}{2} e^{u \tau_+} + \frac{(u \vec{x}_+)^2}{2} \cosh(u \tau_+) e^{u \tau_+} \right] d\tau_+^2 \\ + e^{2u \tau_+} (\vec{x}_+)^2 (d\vec{x}_+)^2 + 2 \left[ \cosh(u \tau_+) + \frac{(u \vec{x}_+)^2}{2} e^{u \tau_+} \right] e^{u \tau_+} d\tau_+ d\vec{x}_+$$

$$u x^i = u x_+^i e^{u \tau_+}$$

$$\Rightarrow u dx^i = u e^{u \tau_+} dx_+^i + u x_+^i e^{u \tau_+} d\tau_+$$

$$\Rightarrow (dx^i)^2 = e^{2u \tau_+} dx_+^{i2} + u^2 x_+^{i2} e^{2u \tau_+} d\tau_+^2 + 2u e^{2u \tau_+} x_+^i dx_+^i d\tau_+ \\ = e^{2u \tau_+} \left[ dx_+^{i2} + u^2 x_+^{i2} d\tau_+^2 + 2x_+^i dx_+^i d\tau_+ \right]$$

$$\Rightarrow \sum_{i=1}^3 (dx^i)^2 = e^{2u \tau_+} \left[ d\vec{x}_+^2 + u^2 \vec{x}_+^2 d\tau_+^2 + 2\vec{x}_+ d\vec{x}_+ d\tau_+ \right]$$

$$u x^D = -\left[ \cosh(u \tau_+) + \frac{(u \vec{x}_+)^2}{2} \right] e^{u \tau_+}$$

$$\Rightarrow dx^D = -\sinh(u \tau_+) d\tau_+ + \vec{x}_+ d\vec{x}_+ e^{u \tau_+} + \frac{(u \vec{x}_+)^2}{2} u e^{u \tau_+} d\tau_+$$

$$\Rightarrow dx^D = \left[ -\sinh(u \tau_+) + \frac{(u \vec{x}_+)^2}{2} e^{u \tau_+} \right] d\tau_+ + \vec{x}_+ d\vec{x}_+ e^{u \tau_+}$$

$$\Rightarrow (dx^D)^2 = \left( \sinh^2(u \tau_+) + \frac{(u \vec{x}_+)^2}{2} e^{2u \tau_+} - 2\sinh(u \tau_+) \right) d\tau_+^2 \\ + (\vec{x}_+)^2 d\vec{x}_+^2 e^{2u \tau_+} + 2 ( ) ( )$$

$$\Rightarrow ds^2 = \left( \cosh^2(u \tau_+) - \sinh^2(u \tau_+) \right) d\tau_+^2 + \text{rest terms cancel} =$$

$$\Rightarrow ds^2 = d\tau_+^2 - e^{2u \tau_+} d\vec{x}_+^2$$