Prove that for Kerr sol^n, the normal vector to surface $r = r_+$ is light

$$r_+ = K M + \sqrt{(KM)^2 - a^2} = r.$$

light like means $n_\mu n^\mu = 0$.

for $r = r_+$, ~~see~~ n_μ :-

∵ surface is given by :- $r = r_+$.

So, normal at any pt. is given by :-

$$n_\mu = (0, 1, 0, 0).$$

$$\Rightarrow g_{\mu\nu} n^\mu = n_\nu \Rightarrow n^\mu = g^{\mu\nu} n_\nu$$

$$g_{\mu\nu} = \begin{pmatrix} \frac{-a^2 \sin^2\theta}{\rho^2} & 0 & 0 & \frac{2KMa r_+ \sin^2\theta}{\rho^2} \\ 0 & -\frac{\rho^2}{\Delta} & 0 & 0 \\ 0 & 0 & -\rho^2 & 0 \\ \frac{2KMa r_+ \sin^2\theta}{\rho^2} & 0 & 0 & -\frac{A \sin^2\theta}{\rho^2} \end{pmatrix}$$

$\frac{\Delta}{\rho^2} = 0$ for $r = r_+$

$$n_\mu n^\mu = n_1 n^1 \quad \text{(as others} = 0\text{)}$$

$$= n^1 = g^{1\nu} n_\nu = g^{11} n_1 = g^{11} = \frac{\Delta}{\rho^2} = 0$$