

Minkowski metric is:-

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

In cylindrical coordinate system:-

$$x = r \cos \theta \rightarrow dx = dr \cos \theta - r \sin \theta d\theta$$

$$y = r \sin \theta \rightarrow dy = dr \sin \theta + r \cos \theta d\theta$$

$$z = z \rightarrow dz = dz$$

$$\omega = \frac{d\theta}{dt} \rightarrow d\theta = \omega dt$$

$$\begin{aligned} \Rightarrow dx^2 &= (dr \cos \theta - r \sin \theta d\theta)^2 \\ &= dr^2 \cos^2 \theta + \omega^2 r^2 \sin^2 \theta dt^2 - 2\omega r \sin \theta \cos \theta dr dt \\ &= dr^2 \cos^2 \theta + \omega^2 r^2 \sin^2 \theta dt^2 - \omega r \sin 2\theta dr dt \end{aligned}$$

$$\begin{aligned} \Rightarrow dy^2 &= (dr \sin \theta + r \cos \theta d\theta)^2 \\ &= dr^2 \sin^2 \theta + \omega^2 r^2 \cos^2 \theta dt^2 + 2\omega r \sin \theta \cos \theta dr dt \\ &= dr^2 \sin^2 \theta + \omega^2 r^2 \cos^2 \theta dt^2 + \omega r \sin 2\theta dr dt \end{aligned}$$

$$\begin{aligned} \Rightarrow ds^2 &= dt^2 - dr^2 (\cos^2 \theta + \sin^2 \theta) - \omega^2 r^2 (\sin^2 \theta + \cos^2 \theta) dt^2 \\ &\quad + \cancel{\omega r \sin 2\theta dr dt} - \cancel{\omega r \sin 2\theta dr dt} - dz^2 \end{aligned}$$

$$= dt^2 - dr^2 - \omega^2 r^2 dt^2 - dz^2$$

$$= \underline{(1 - \omega^2 r^2) dt^2 - dr^2 - dz^2}$$

Thus, the metric we get is:-

$$ds^2 = (1 - \omega^2 r^2) dt^2 - dr^2 - dz^2$$