

# **ASTR 610**

# **Theory of Galaxy Formation**

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**Lecture 19: Elliptical Galaxies**

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# Structure & Formation of Elliptical Galaxies

In this lecture we discuss the structure and formation of elliptical galaxies. After a very brief overview of some of the main observational properties of ellipticals, we discuss two ‘competing’ pictures for the formation of ellipticals.

Topics that will be covered include:

- Dichotomy of elliptical
- Fundamental Plane
- Intrinsic Shapes
- Monolithic collapse picture
- Sizes & the Merger picture
- Formation Scenarios

# Observational Facts

## Surface Brightness Profiles

Elliptical galaxies have surface brightness profiles that are well described by a **Sersic profile**:

$$I(R) = I_0 \exp \left[ -\beta_n \left( \frac{R}{R_e} \right)^{1/n} \right]$$

$$\beta_n \simeq 2n - 0.324$$

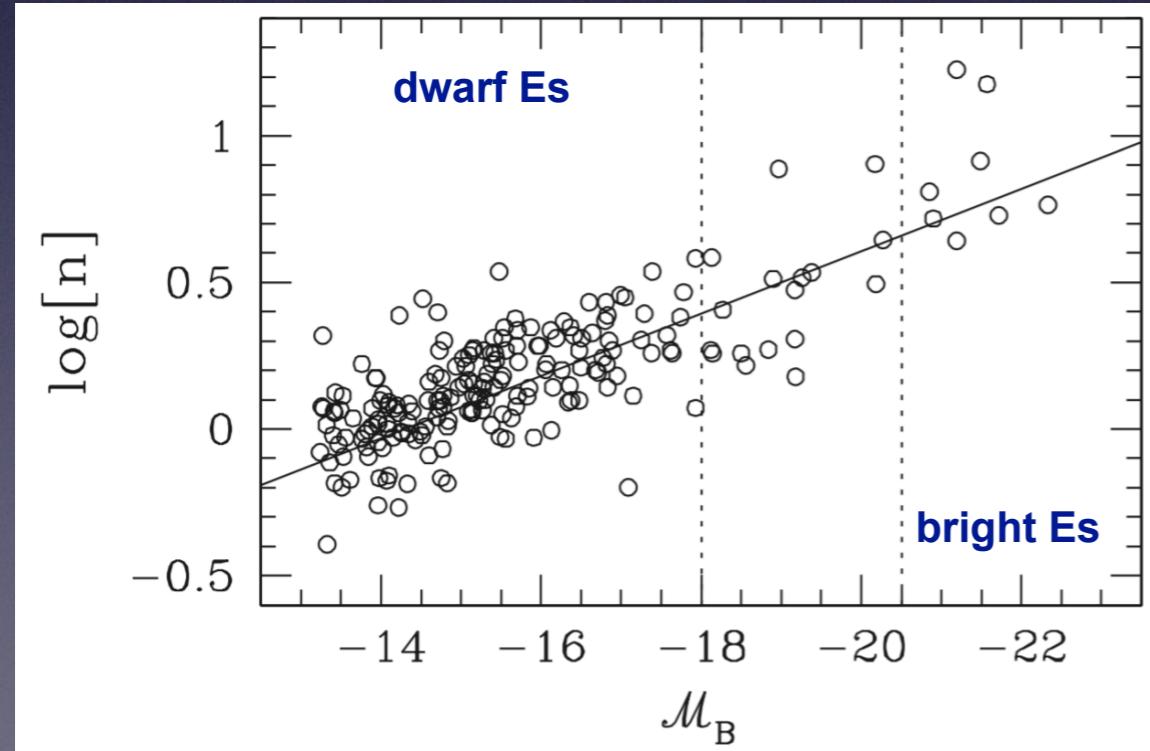
follows from definition of  $R_e$

$$L = 2\pi \int_0^\infty I(R) R dR = \frac{2\pi n \Gamma(2n)}{(\beta_n)^{2n}} I_0 R_e^2$$

$n$ : Sersic index

$R_e$ : effective radius that encloses half of the total light

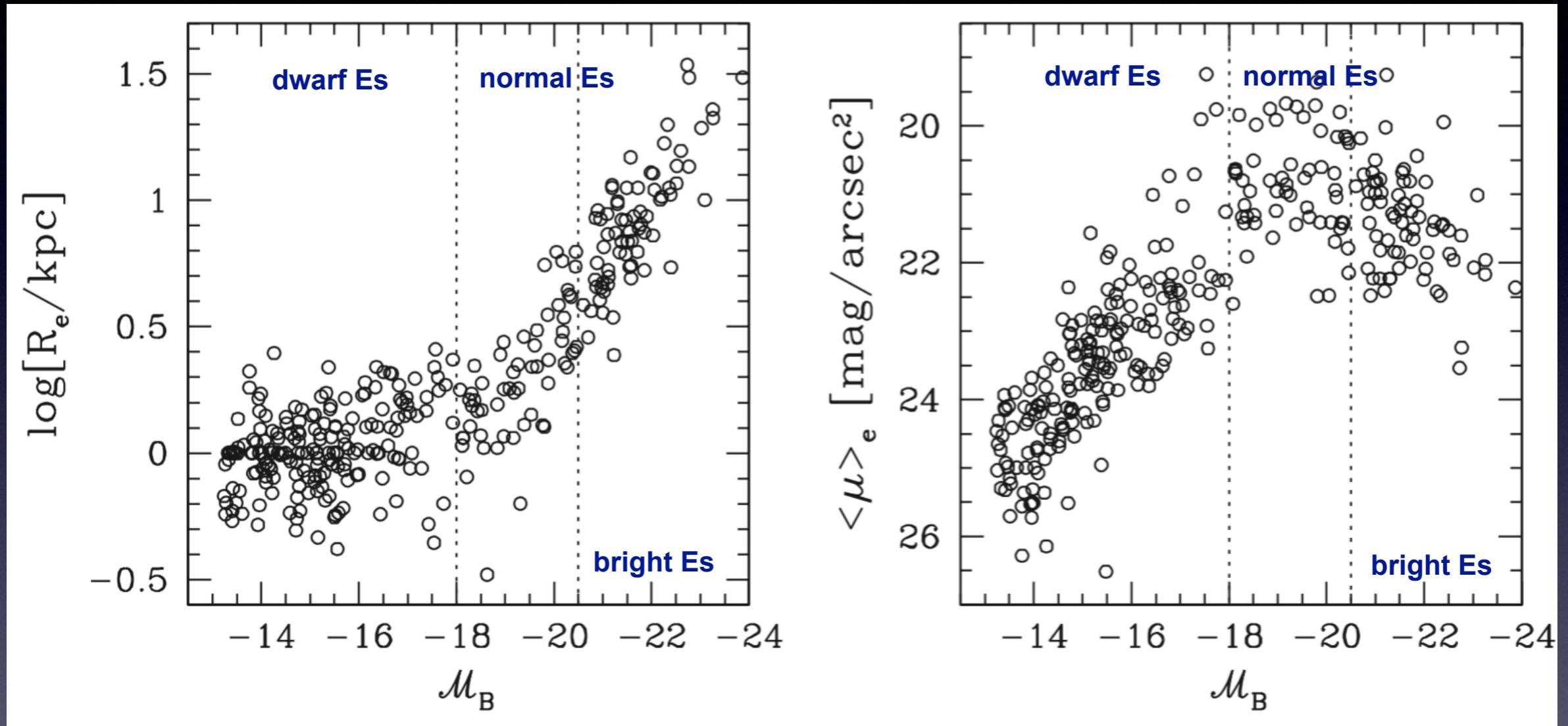
For  $n=4$  this is known as the 'de Vaucouleurs' profile, while  $n=1$  corresponds to an exponential profile



Typically, brighter ellipticals have a larger **Sersic index**. Faintest ellipticals (dwarf spheroidals) have  $n \sim 1$ , corresponding to an exponential profile.

# Observational Facts

## Surface Brightness Profiles

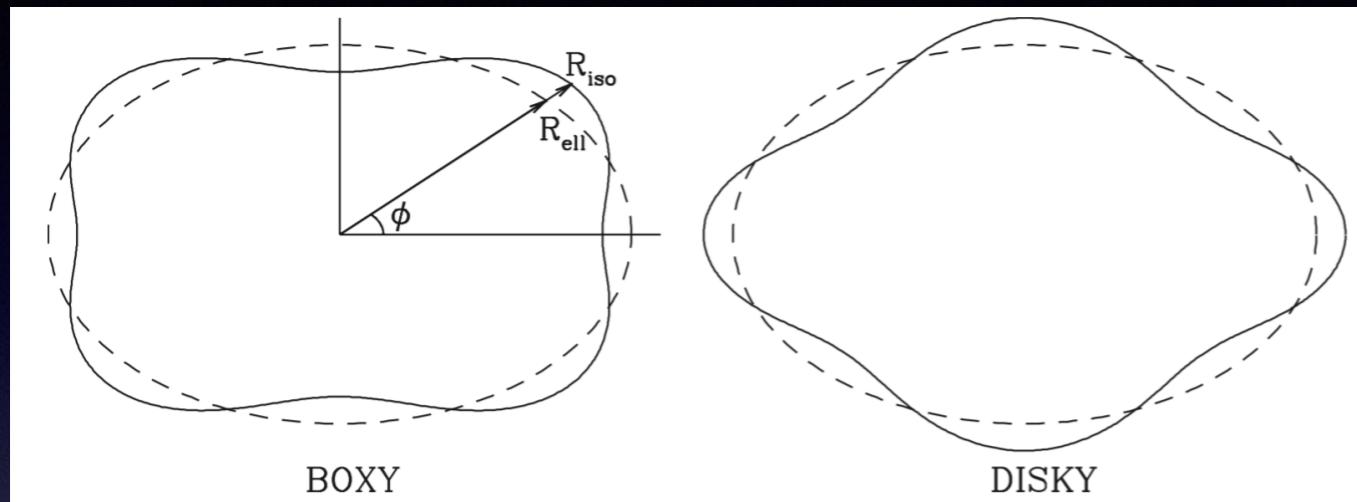


MBW, Fig. 2.14

Brighter ellipticals are larger and have higher surface brightness  
But, trends are not continuous: for  $M_B < -20.5$  surface brightness decreases with increasing luminosity, while there is little to no magnitude-size trend for dwarfs...

# Isophotal Shapes

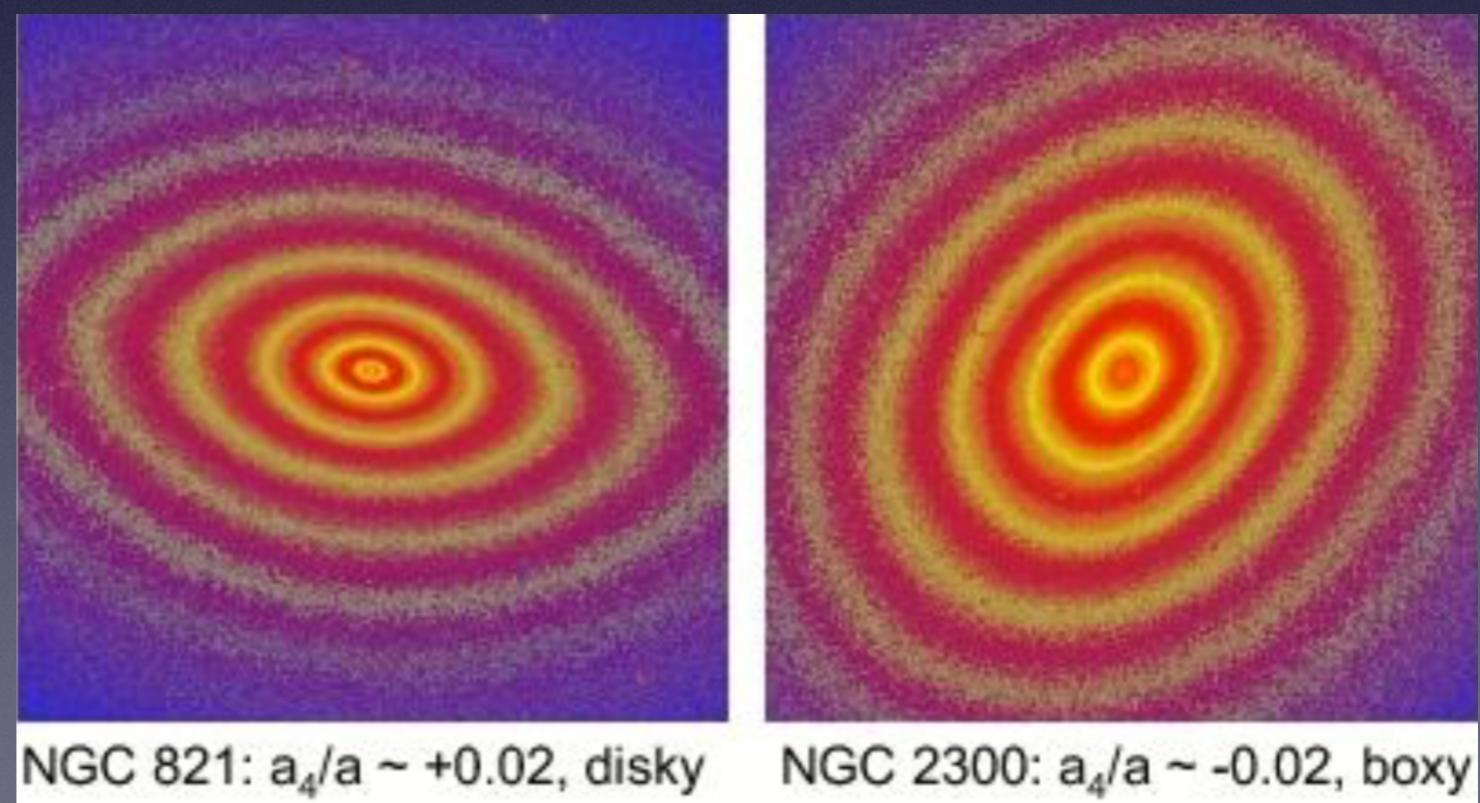
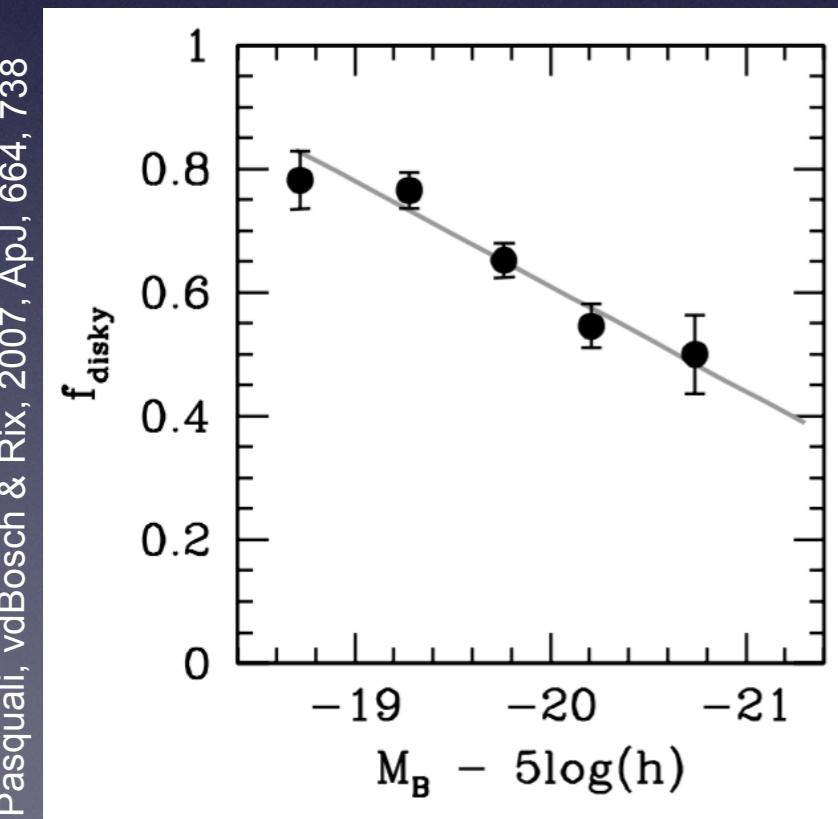
The **isophotes** of elliptical galaxies are elliptical, but not perfectly so. They show deviations that are typically either '**disky**' or '**boxy**'



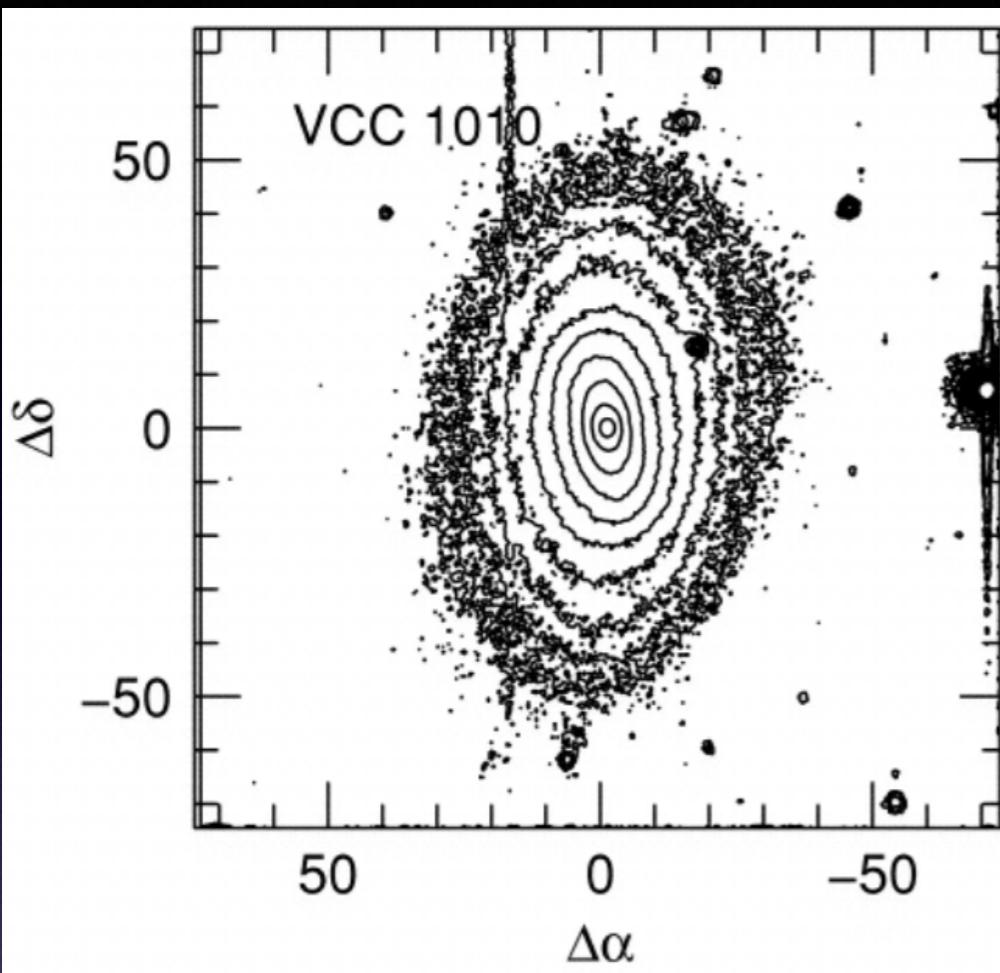
$$\Delta\phi \equiv R_{\text{iso}}(\phi) - R_{\text{ell}}(\phi)$$
$$= \sum_{n=3}^{\infty} [a_n \cos(n\phi) + b_n \sin(n\phi)]$$

disky and boxy correspond to  $a_4 > 0$  and  $a_4 < 0$ , respectively

Brighter ellipticals are more likely boxy (disky fraction decreases with luminosity)



# Isophotal Twist



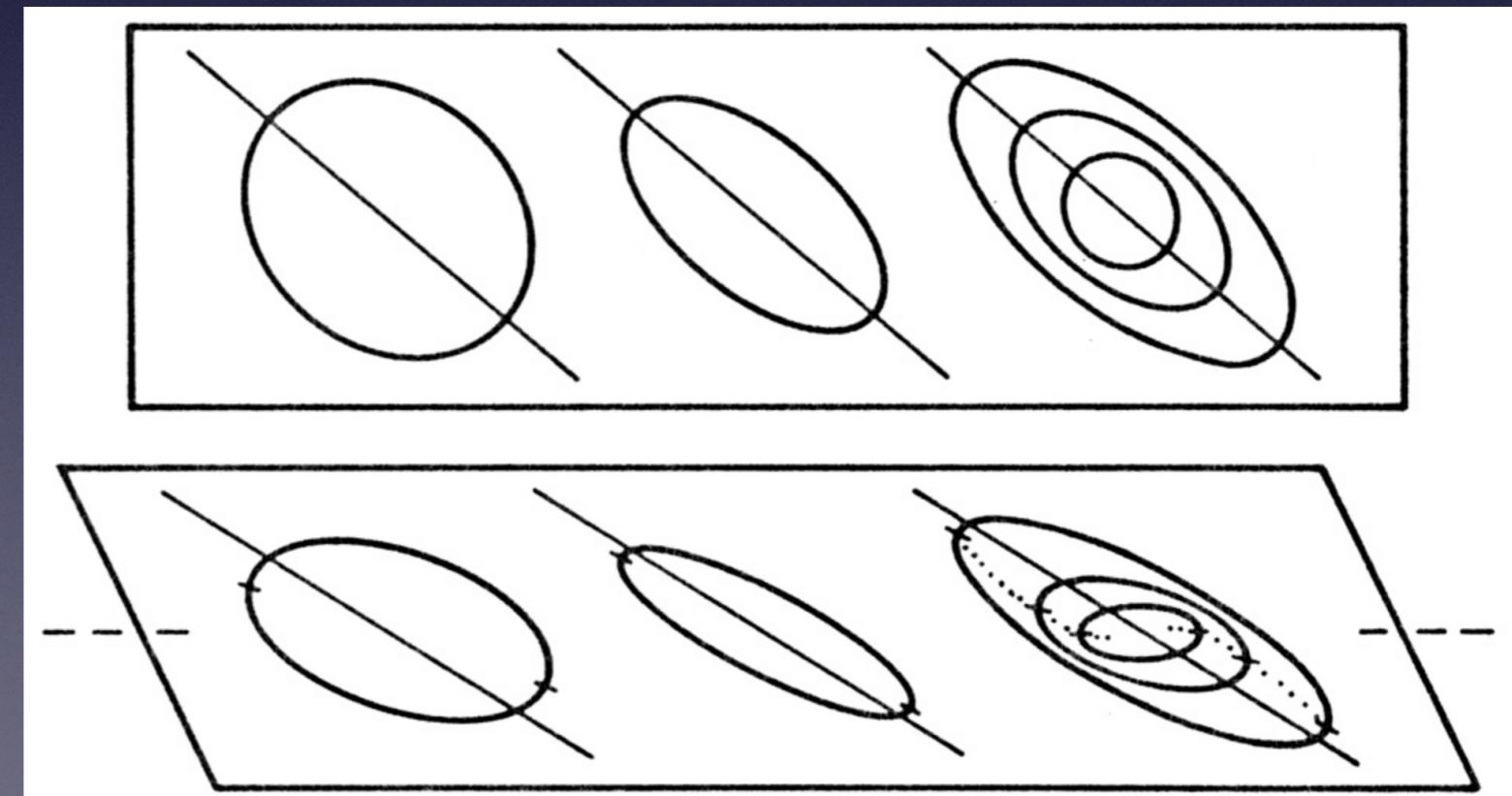
The presence of isophote twists among (bright/boxy) ellipticals is often taken as evidence that, as a class, they must be **triaxial**.

Some ellipticals reveal **isophotal twists**, with direction of major axis of isophote changing with isophotal level

Most of these ellipticals are **boxy**

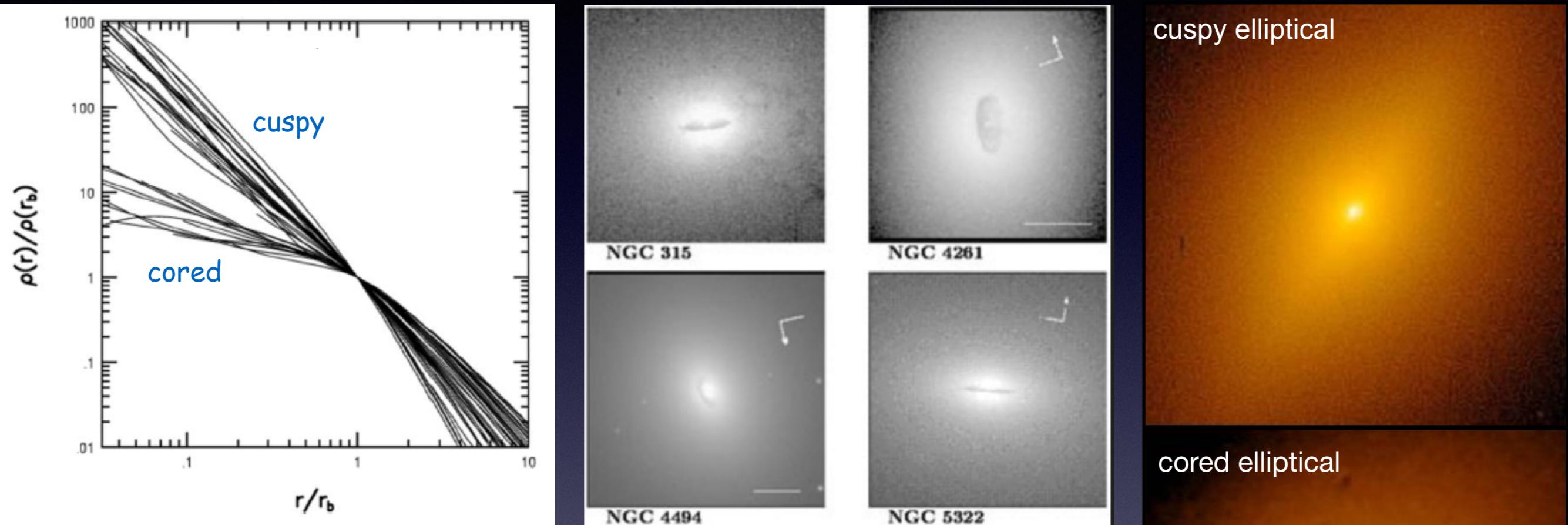
The simplest explanation is that (these) elliptical galaxies are **triaxial** (rather than oblate/prolate), and have their intrinsic axis ratios change with radius.

Such a system in **projection** will reveal **isophote twist**



# The Nuclei of Elliptical Galaxies

High resolution imaging with the HST revealed that the central regions of ellipticals reveal a dichotomy in their central surface brightness profile; ‘cusps’ vs ‘cores’

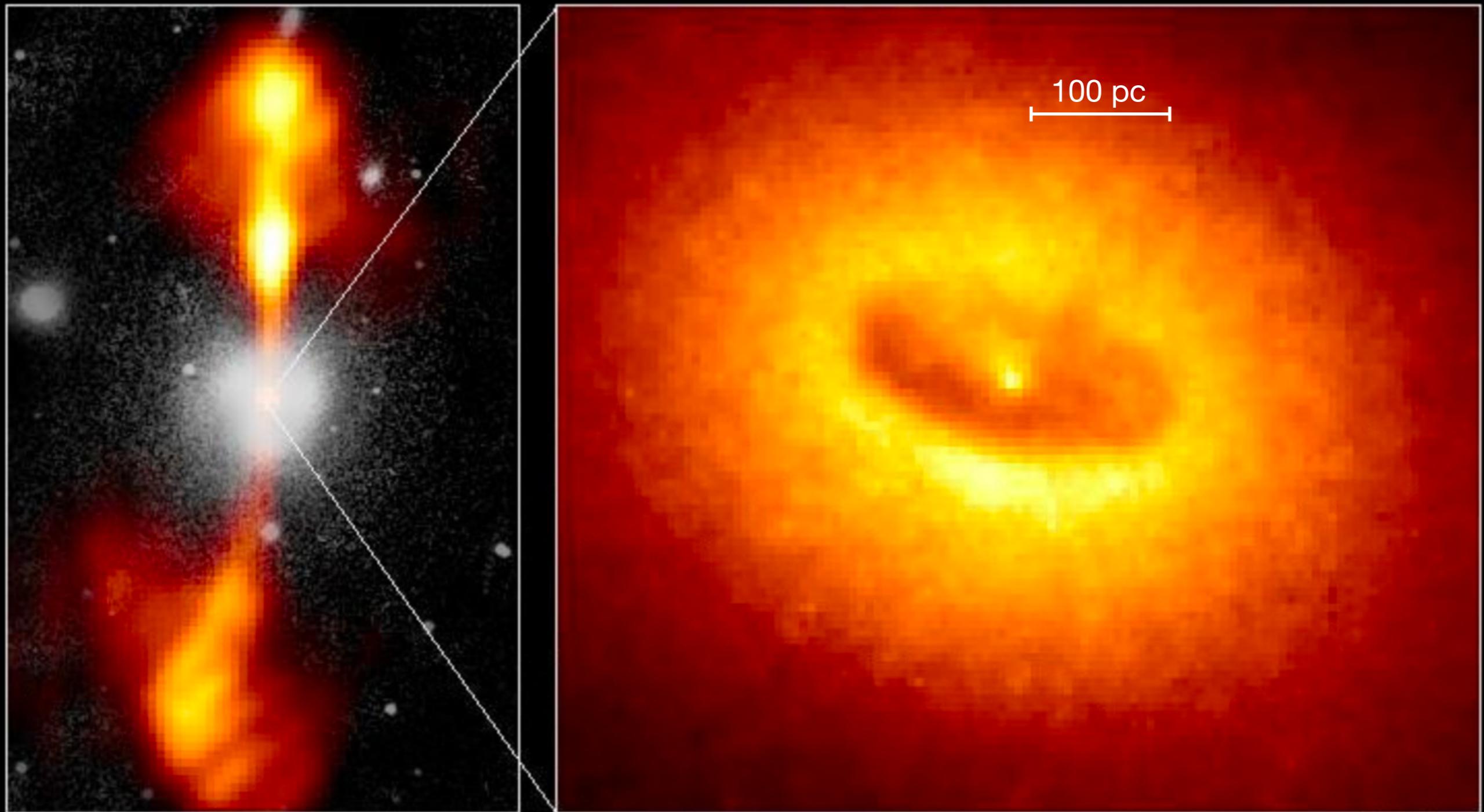


Typically cored ellipticals are bright ( $M_B \lesssim -20.5$ ) and boxy, while cuspy ellipticals are fainter and disky.

Whether this is a true ‘dichotomy’ or not is still debated...

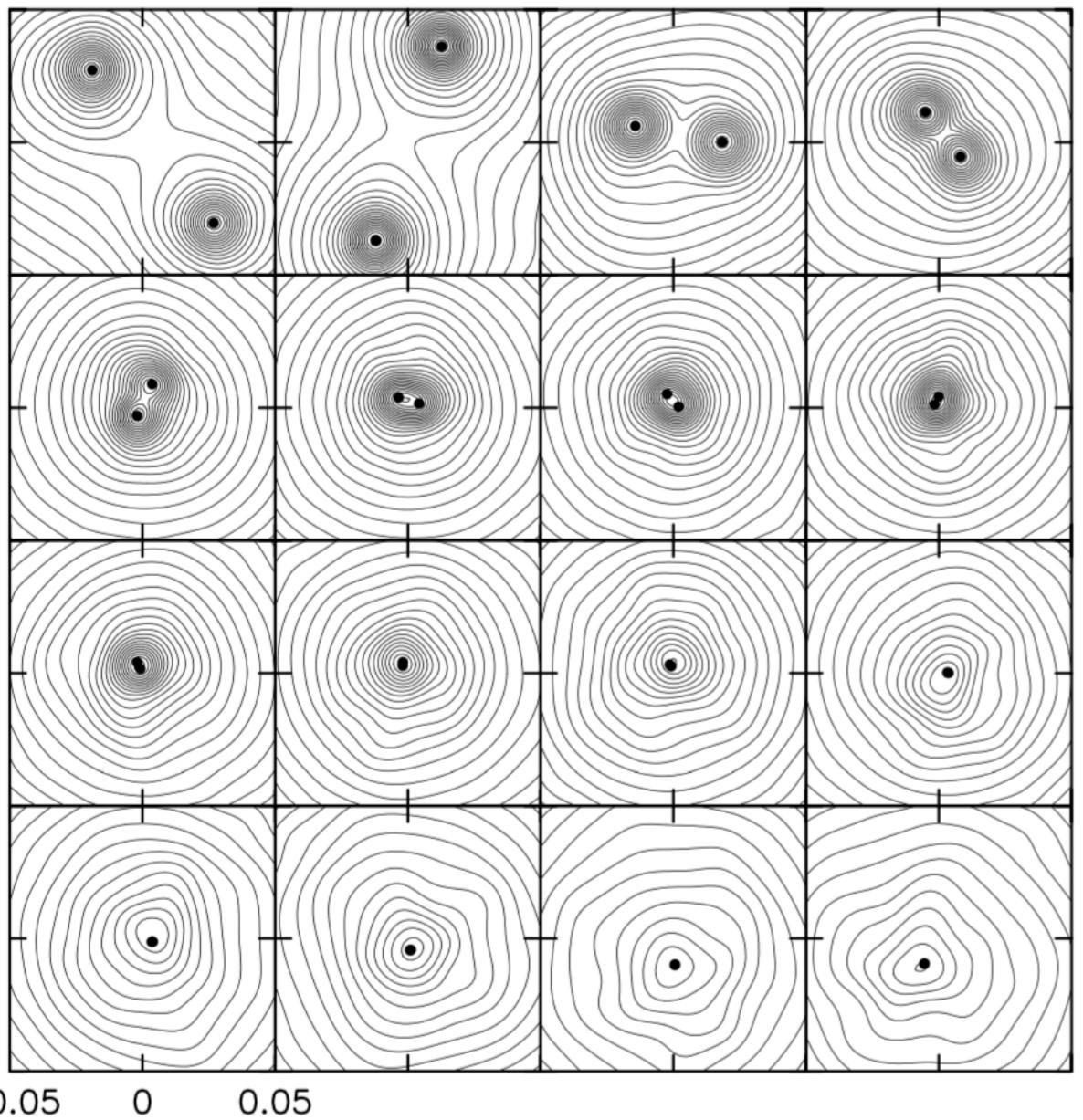
Nuclei of elliptical galaxies also often harbour small (few 100pc) disks of gas/dust and/or stars

# The Nuclear Dust Disk of NGC 4261



Dust disk  $\sim$ 100 pc size, oriented perpendicular to radio jets; is this the material that feeds the accretion disk surrounding the SMBH at the center?

# Creating Cores with Scouring SMBH Binaries



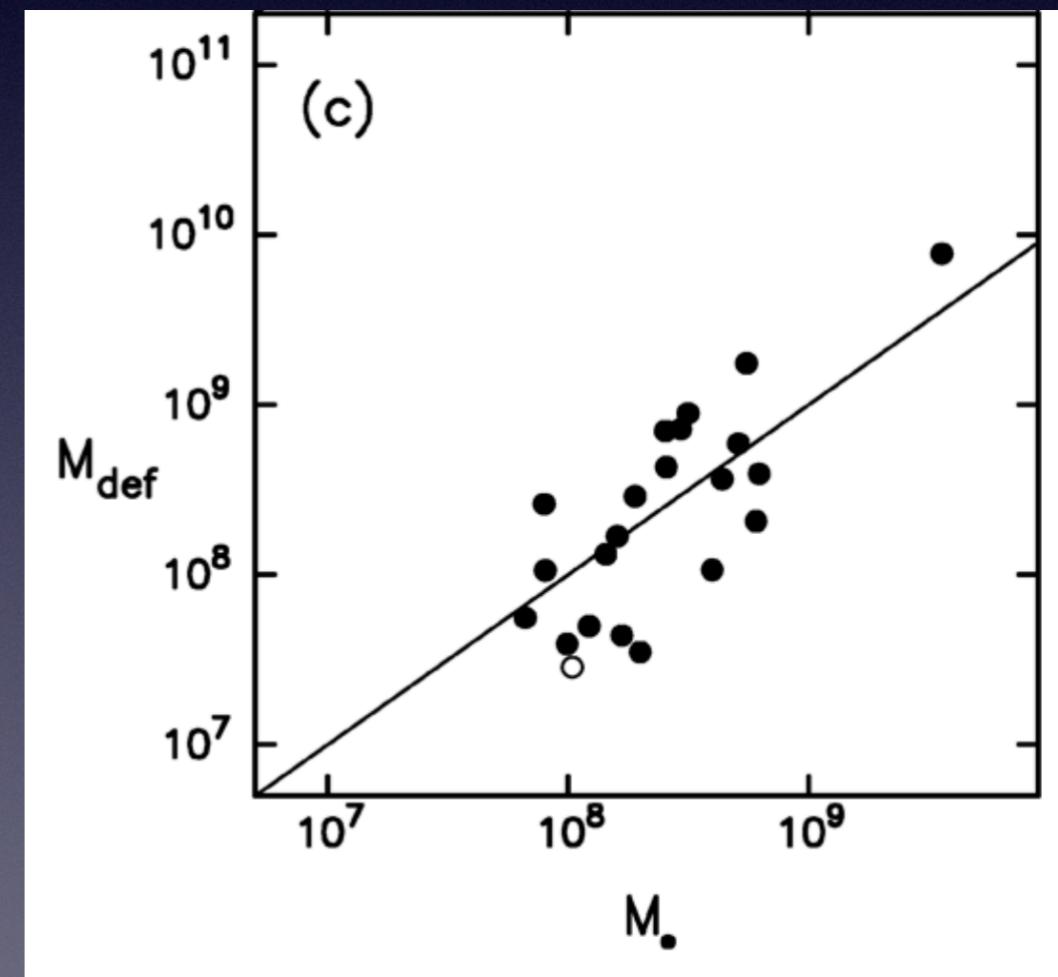
Milosavljevic & Merritt, 2001, ApJ, 563, 34

Rule of thumb  $M_{ej} \sim 0.5 (M_{\bullet,1} + M_{\bullet,2}) \ln(a_h/a_{gr})$

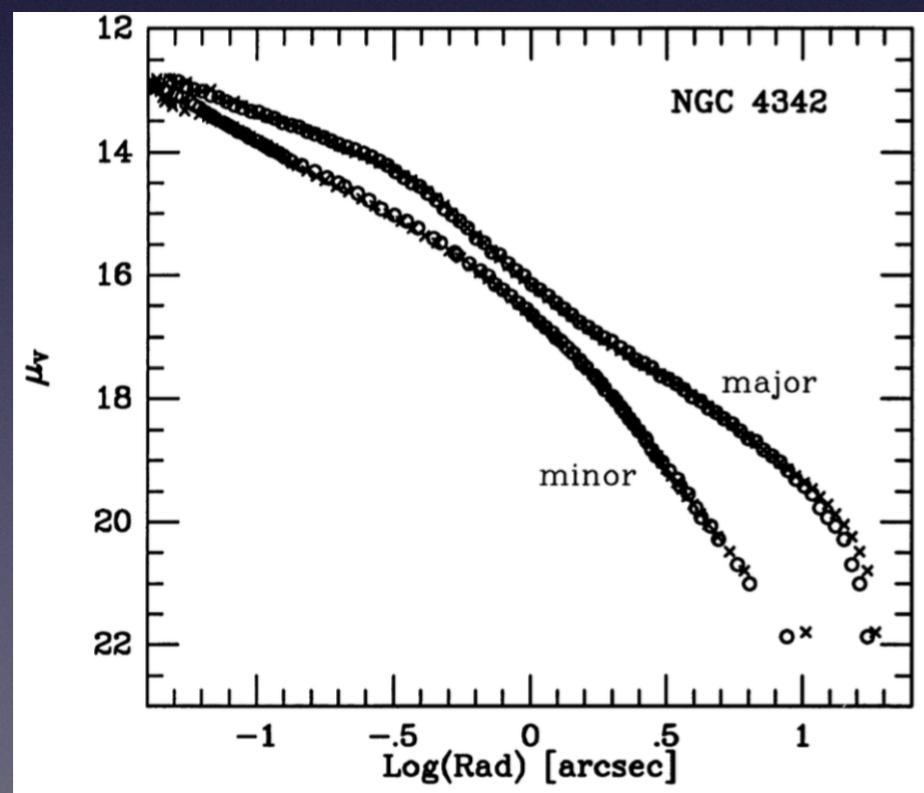
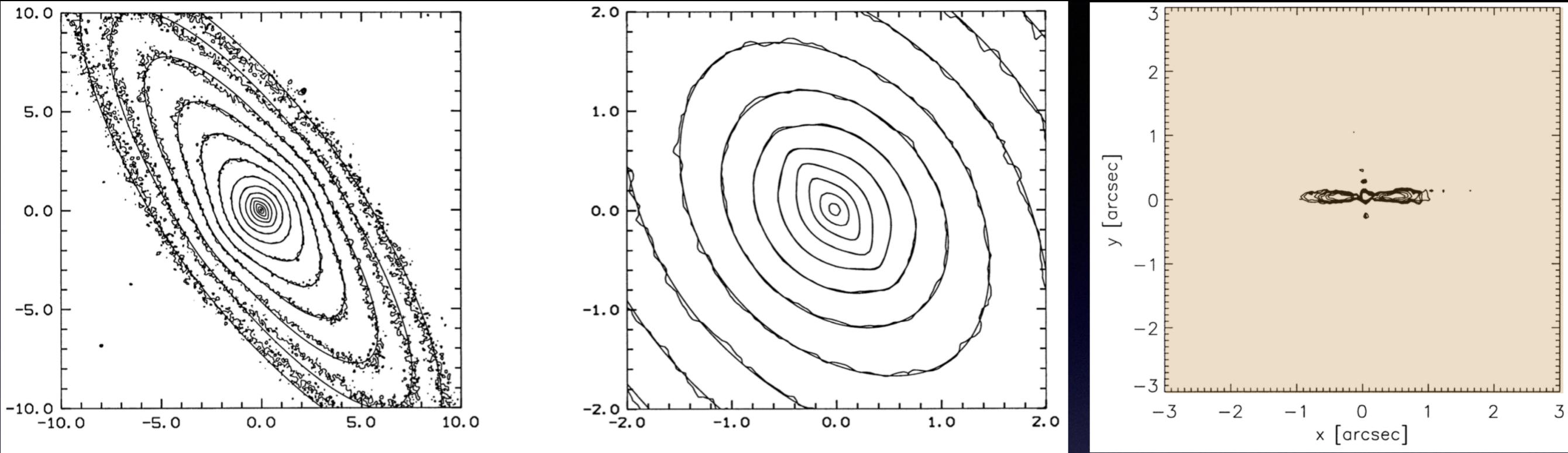
$a_h$  = semi-major axis of SMBH binary when binary first becomes hard

$a_h$  = semi-major axis of SMBH binary when gravitational radiation starts to dominate

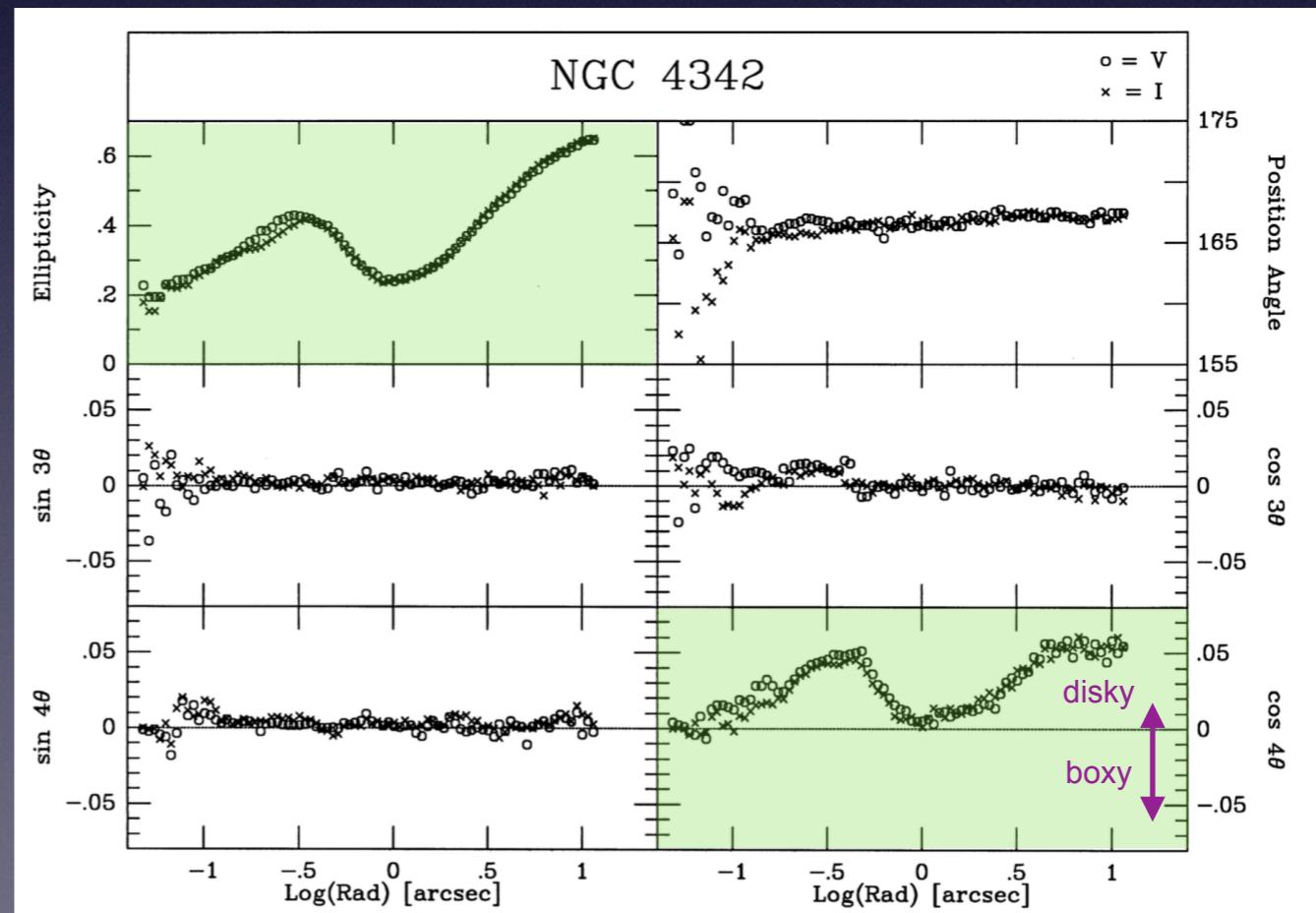
Cores can be created due to scouring by a SMBH binary. Dynamical friction acting on the SMBHs tightens the binary, and transfers momentum to the cusp stores, thereby creating a core. This process becomes inefficient once gravitational wave radiation becomes important..



# The Nuclear Stellar Disk of NGC 4342



SOURCE: vdBosch, Jaffe & vdMarel, 1998, MNRAS, 293, 343



# Kinematics

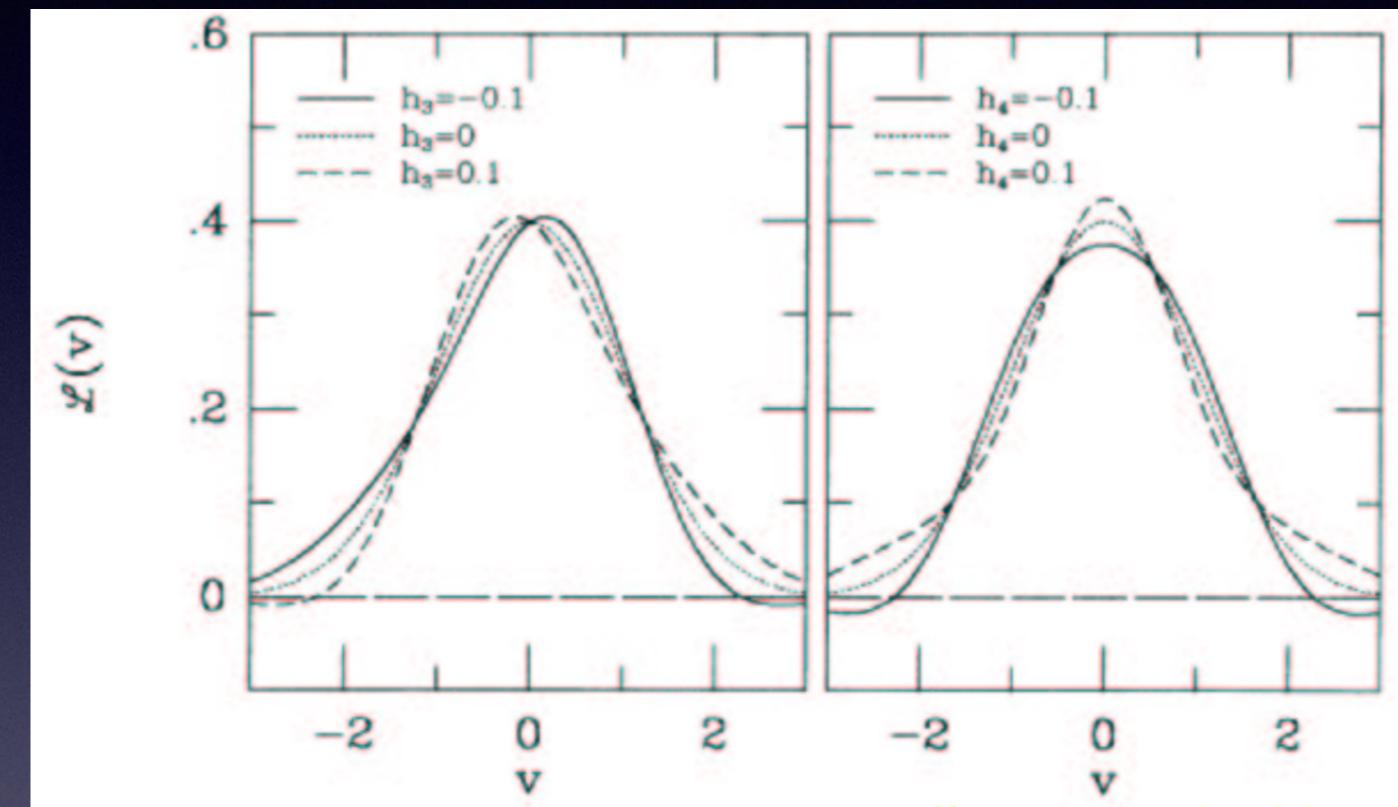
The observed spectrum of an elliptical galaxy is a convolution of the template spectrum, which is the luminosity weighted spectrum of all the various stars along the line-of-sight (LOS) and a broadening function, which is a combination of an instrumental broadening function and the line-of-sight velocity distribution (LOSVD)

A typical functional form for the LOSVD is a simple Gaussian.

$$\mathcal{L}(v) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}w^2} \quad w = (v - V)/\sigma$$

However, the LOSVD is generally not Gaussian and it has become standard practice to adopt a Gauss-Hermite series

$$\mathcal{L}(v) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}w^2} \left[ 1 + \sum_{j=3}^N h_j H_j(w) \right]$$

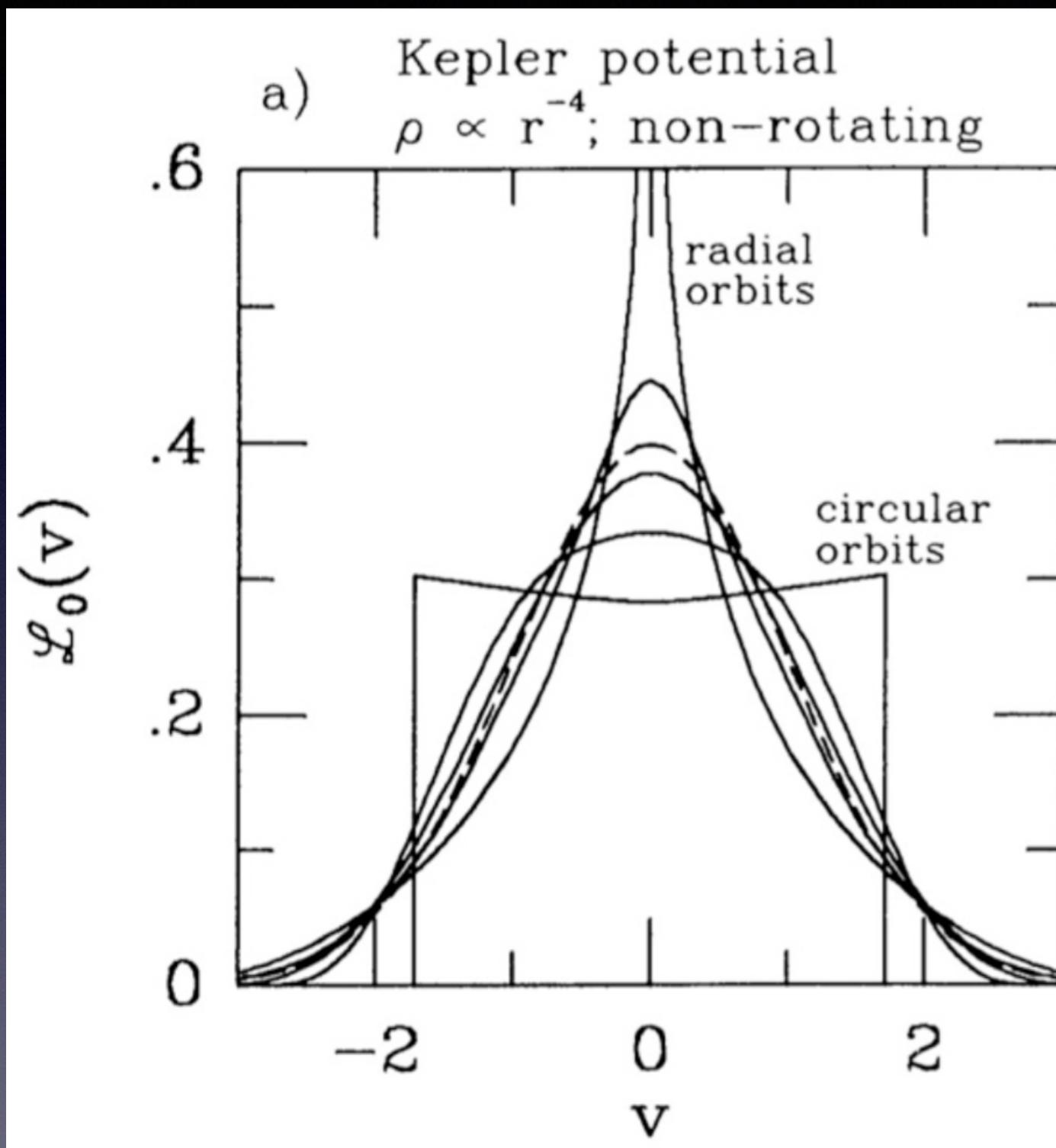


Typically one truncates the series at  $N=4$ , such that LOSVD is described by four parameters:  $V, \sigma, h_3, h_4$

related to skewness

related to kurtosis

# Kinematics

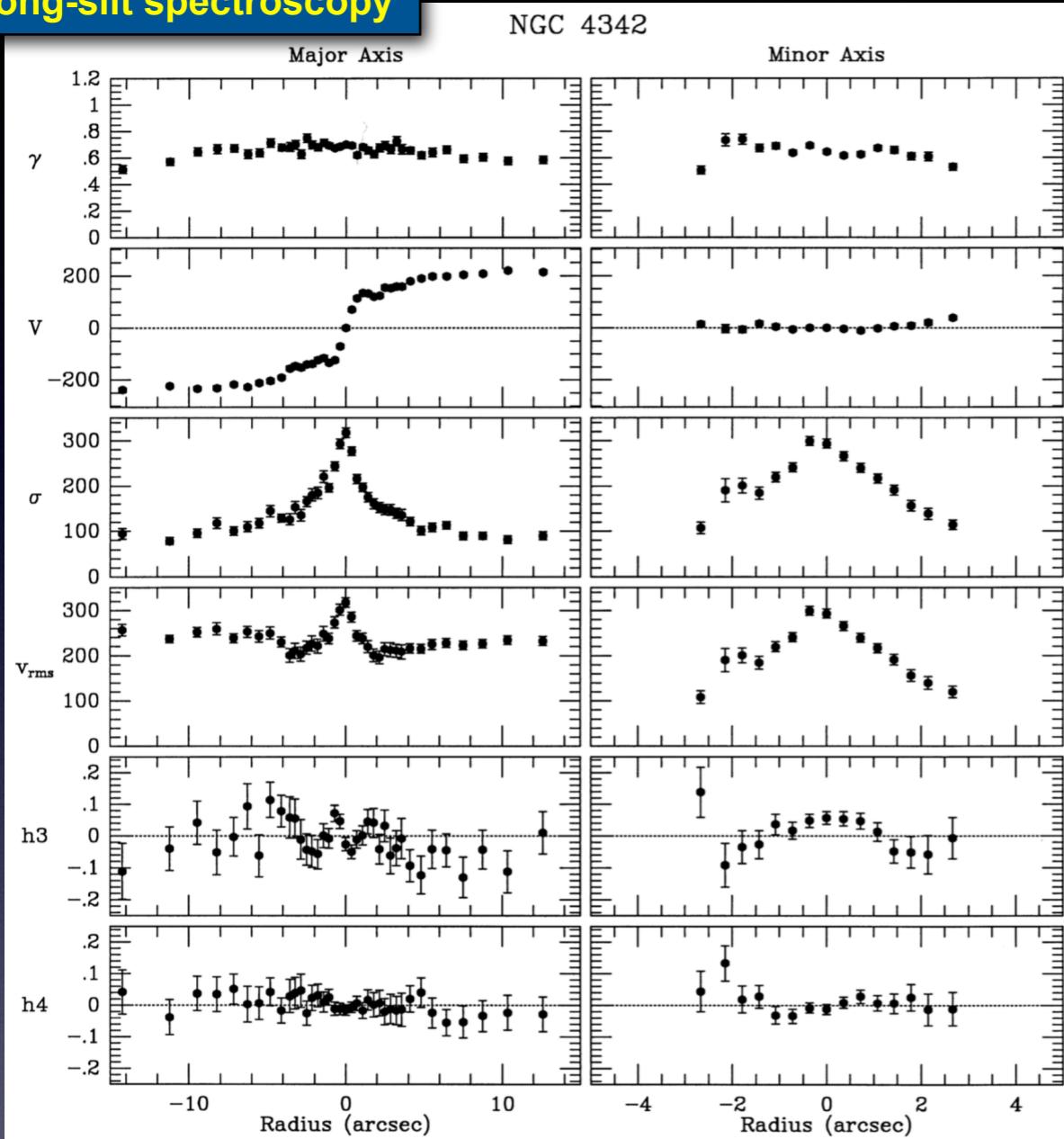


van der Marel & Franx 1993

The  $h_4$  Gauss-Hermite moment is especially powerful as it is sensitive to the orbital distribution of the galaxy, and can therefore be used to break the mass-anisotropy degeneracy that hampers kinematic models.

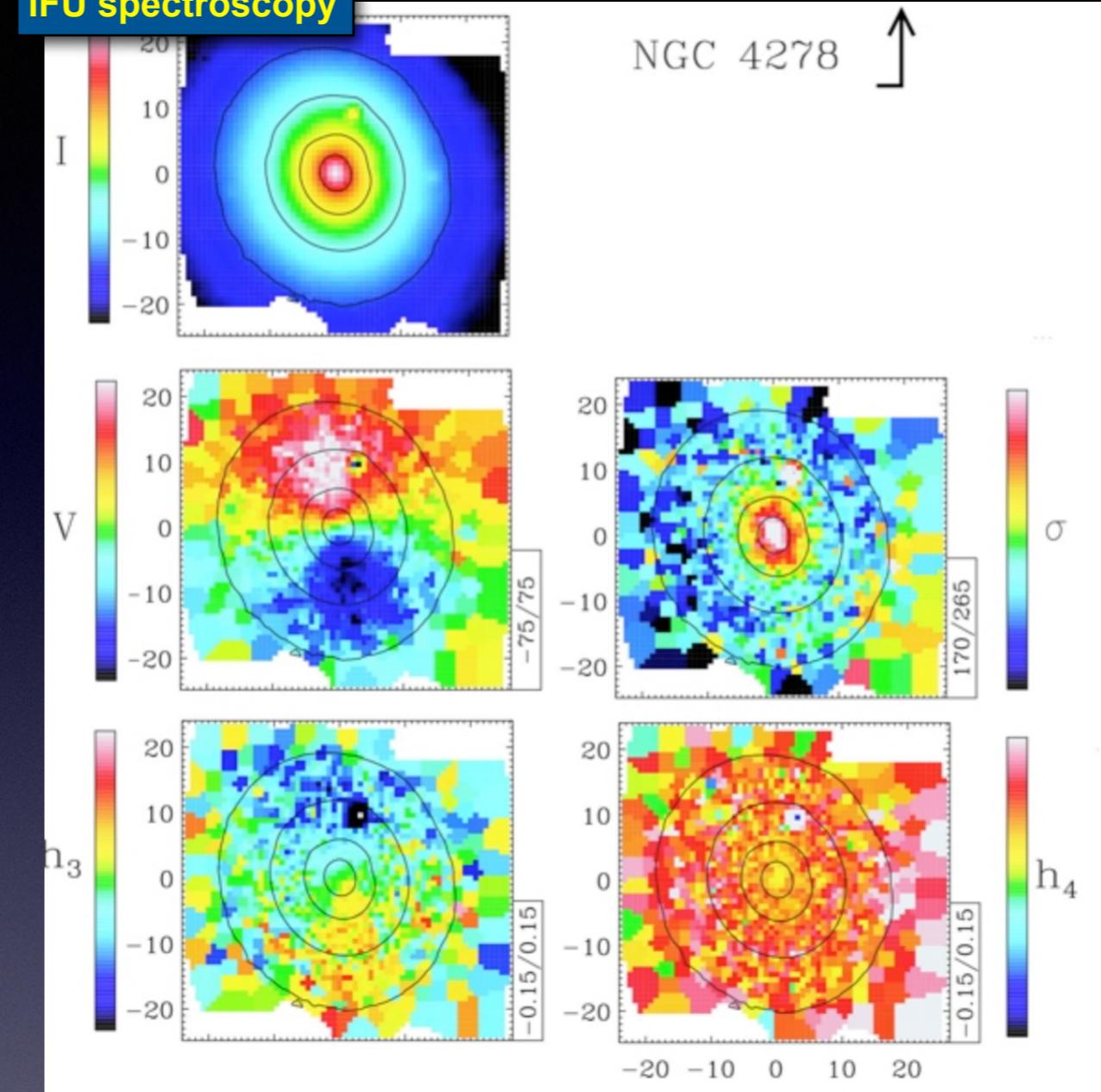
# Kinematics

long-slit spectroscopy



vdBosch, Jaffe & vdMarel, 1998, MNRAS, 293, 343

IFU spectroscopy



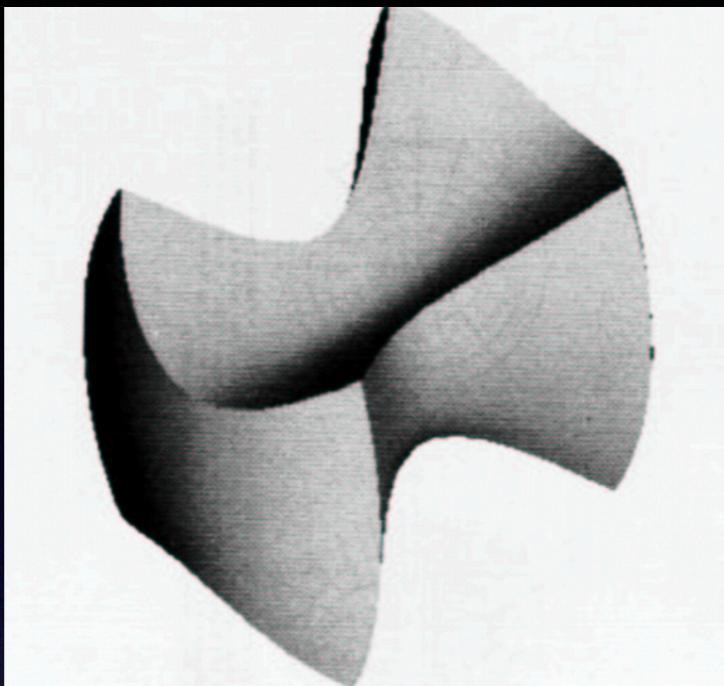
Emsellem et al. 2004

Disky ellipticals typically reveal strong rotation along major axis, consistent with them being ‘oblate rotators’ (oblate in shape, with flattening due to rotation)

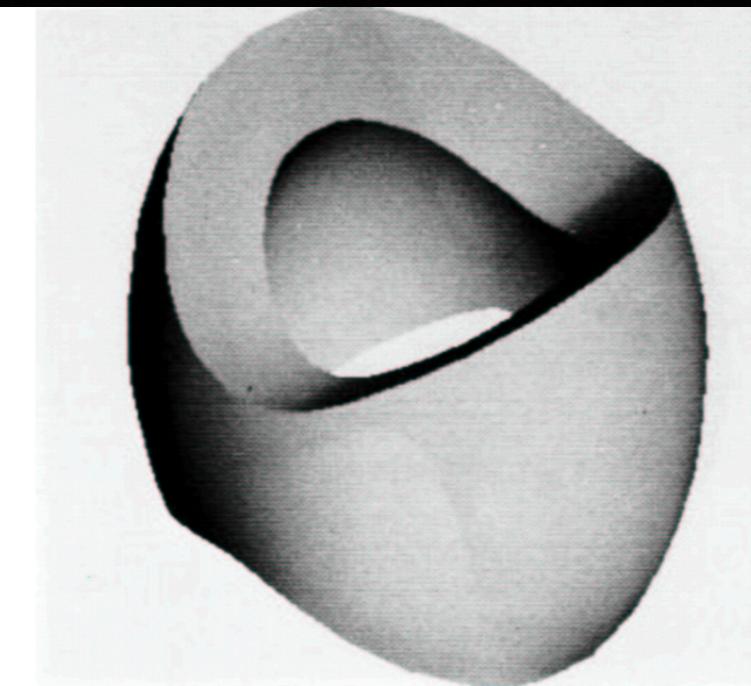
Boxy ellipticals reveal little rotation, and occasionally rotation along the minor axis. Latter is a clear sign that (boxy) ellipticals are triaxial

# Orbital Families in Triaxial Potentials

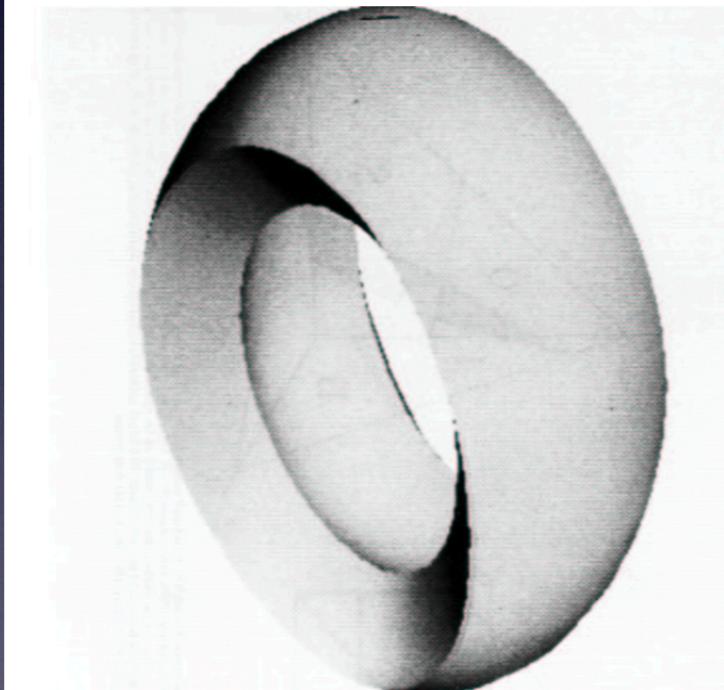
box orbit



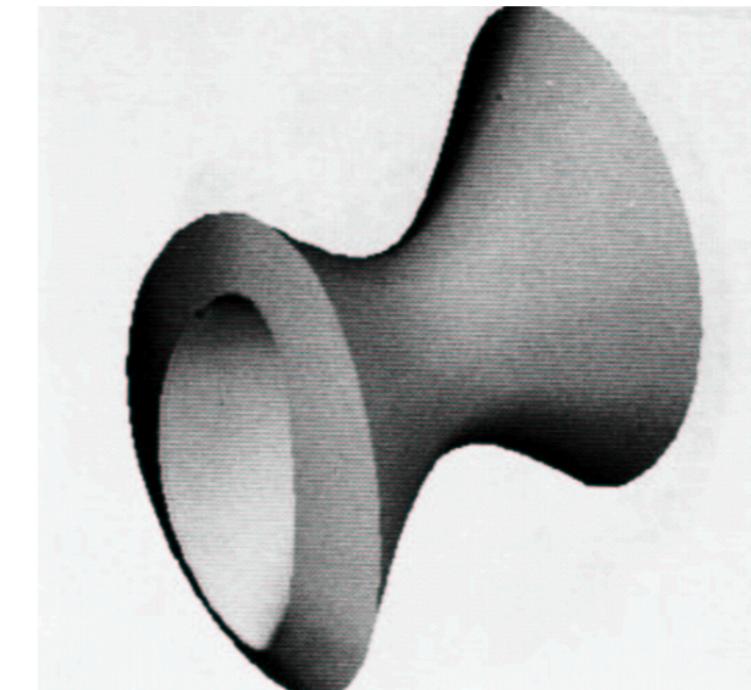
short-axis tube orbit



outer long-axis tube orbit



inner long-axis tube orbit



In triaxial potentials, there are four families of regular orbits.

Box orbit have no net angular momentum; the orbit comes arbitrarily close to the centre. Tube orbits, on the other hand, have an angular momentum barrier

# The Tensor Virial Theorem

We now examine how the structure of elliptical galaxies relates to their kinematics.

The dynamics of (elliptical) galaxies are governed by the CBE:  $df/dt = 0$

Multiplying the CBE with velocity, and integrating over velocity-space yields the Jeans equations (which are momentum equations)

$$\frac{\partial(\rho\langle v_j \rangle)}{\partial t} + \frac{\partial(\rho\langle v_i v_j \rangle)}{\partial x_i} + \rho \frac{\partial \Phi}{\partial x_j} = 0$$

Multiplying all terms with  $x_k$  and integrating over all of configuration space yields

$$\frac{\partial}{\partial t} \int \rho x_k \langle v_j \rangle d^3 \vec{x} = - \int x_k \frac{\partial(\rho\langle v_i v_j \rangle)}{\partial x_i} d^3 \vec{x} - \int \rho x_k \frac{\partial \Phi}{\partial x_j} d^3 \vec{x}$$

Using integration by parts, the first terms on the rhs can be written as

$$\int x_k \frac{\partial(\rho\langle v_i v_j \rangle)}{\partial x_i} d^3 \vec{x} = - \int \rho \langle v_k v_j \rangle d^3 \vec{x} \equiv -2\mathcal{K}_{kj}$$

where we have defined the kinetic energy tensor,  $\mathcal{K}_{kj}$

# Centrifugal Support vs. Pressure Support

We split kinetic energy tensor into contributions from ordered and random motions:

$$\mathcal{K}_{ij} \equiv \mathcal{T}_{ij} + \frac{1}{2}\Pi_{ij}$$

$$\mathcal{T}_{ij} \equiv \frac{1}{2} \int \rho \langle v_i \rangle \langle v_j \rangle d^3\vec{x}$$

$$\Pi_{ij} \equiv \int \rho \sigma_{ij}^2 d^3\vec{x}$$

In addition to the kinetic energy tensor, we also define the potential energy tensor:

$$\mathcal{W}_{ij} \equiv - \int \rho x_i \frac{\partial \Phi}{\partial x_j} d^3\vec{x}$$

Combining the above, and using that both  $\mathcal{K}$  and  $\mathcal{W}$  are symmetric, we have that

$$\frac{1}{2} \frac{d}{dt} \int \rho [x_k \langle v_j \rangle + x_j \langle v_k \rangle] d^3\vec{x} = 2\mathcal{K}_{jk} + \mathcal{W}_{jk}$$

# The Tensor Virial Theorem

Finally, we define the moment of inertia tensor

$$\mathcal{I}_{ij} \equiv \int \rho x_i x_j d^3 \vec{x}$$

Differentiating wrt time and using the continuity equation (i.e., zeroth moment of CBE):

$$\frac{d\mathcal{I}_{jk}}{dt} = \int \frac{\partial \rho}{\partial t} x_j x_k d^3 \vec{x} = - \int \frac{\partial \rho \langle v_i \rangle}{\partial x_i} x_j x_k d^3 \vec{x} = \int \rho [x_j \langle v_k \rangle + x_k \langle v_j \rangle] d^3 \vec{x}$$

students: try this at home

so that

$$\frac{1}{2} \frac{d}{dt} \int \rho [x_j \langle v_k \rangle + x_k \langle v_j \rangle] d^3 \vec{x} = \frac{1}{2} \frac{d^2 \mathcal{I}_{jk}}{dt^2}$$

which allows us to write the **Tensor Virial Theorem** as

$$\frac{1}{2} \frac{d^2 \mathcal{I}_{jk}}{dt^2} = 2\mathcal{T}_{jk} + \Pi_{jk} + \mathcal{W}_{jk}$$

which relates the gross kinematic and structural properties of gravitational systems

# The Tensor Virial Theorem

If the system is in a steady state, the moment of inertia tensor is stationary, and the **tensor virial theorem** reduces to

$$2\mathcal{K}_{jk} + \mathcal{W}_{jk} = 0$$

The common (scalar) virial theorem ( $2K+W=0$ ) is simply the trace of this tensor equation.

We now use this **tensor virial theorem**, to relate the flattening of an elliptical to its kinematics. Consider an **oblate system** with it's symmetry axis along the z-direction. Because of symmetry considerations we have that

$$\langle v_R \rangle = \langle v_z \rangle = 0 \quad \langle v_R v_\phi \rangle = \langle v_z v_\phi \rangle = 0$$

If we write  $\langle v_x \rangle = \langle v_\phi \rangle \sin \phi$  and  $\langle v_y \rangle = \langle v_\phi \rangle \cos \phi$  then we obtain

$$\mathcal{T}_{xy} = \frac{1}{2} \int \rho \langle v_x \rangle \langle v_y \rangle d^3\vec{x} = \frac{1}{2} \int_0^{2\pi} d\phi \sin \phi \cos \phi \int_0^\infty dR \int_{-\infty}^{+\infty} dz \rho(R, z) \langle v_\phi \rangle^2(R, z) = 0$$

A similar analysis shows that all other non-diagonal elements of  $\mathcal{T}$ ,  $\Pi$  and  $\mathcal{W}$  have to be zero. In addition, because of symmetry considerations we must also have that  $\mathcal{T}_{xx} = \mathcal{T}_{yy}$ , and similar for  $\Pi$  and  $\mathcal{W}$ .

# The Tensor Virial Theorem

Given these symmetries, the only independent, non-trivial virial equations are

$$2\mathcal{T}_{xx} + \Pi_{xx} + \mathcal{W}_{xx} = 0$$

$$2\mathcal{T}_{zz} + \Pi_{zz} + \mathcal{W}_{zz} = 0$$

If the only streaming motion is rotation about the z-axis, then  $\mathcal{T}_{zz} = 0$  and

$$2\mathcal{T}_{xx} = \frac{1}{2} \int \rho \langle v_\phi \rangle^2 d^3 \vec{x} = \frac{1}{2} M v_0^2$$

where  $v_0$  is the mass-weighted rotation velocity. Similarly we can write

$$\Pi_{xx} = M \sigma_0^2; \quad \Pi_{zz} = (1 - \delta) M \sigma_0^2$$

where  $\sigma_0^2 \equiv (1/M) \int \rho \sigma_{xx}^2 d^3 \vec{x}$  is the mass-weighted velocity dispersion along los, and  $\delta \equiv 1 - \Pi_{zz}/\Pi_{xx} < 1$  is a measure of the anisotropy of the velocity dispersion

Taking the ratio between the two non-trivial virial equations above then yields

$$\frac{\mathcal{W}_{xx}}{\mathcal{W}_{zz}} = \frac{1}{1 - \delta} \left( 1 + \frac{1}{2} \frac{v_0^2}{\sigma_0^2} \right)$$

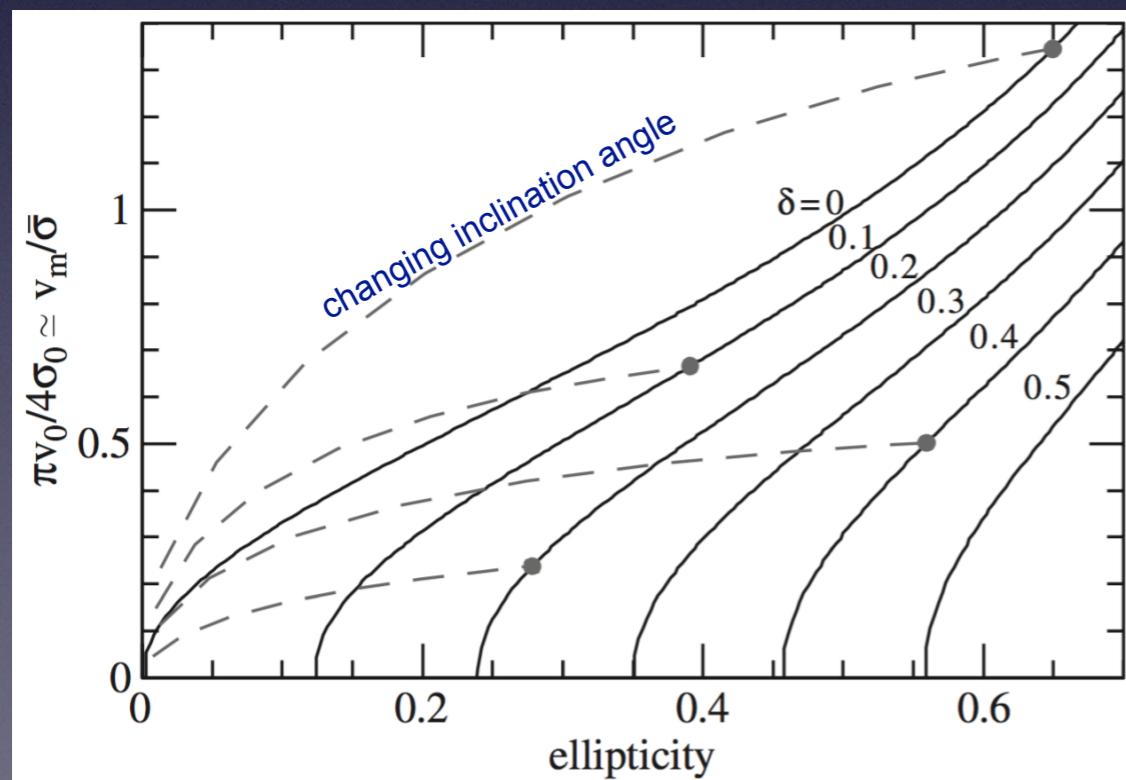
# The Tensor Virial Theorem

As shown by Roberts (1962), for systems stratified on similar coaxial oblate ellipsoids, the ratio  $\mathcal{W}_{xx}/\mathcal{W}_{zz}$  depends only on the ellipticity  $\varepsilon$

$$\frac{\mathcal{W}_{xx}}{\mathcal{W}_{zz}} = \frac{1}{1 - \delta} \left( 1 + \frac{1}{2} \frac{v_0^2}{\sigma_0^2} \right)$$

The above expression therefore makes it clear that a stellar system can be flattened either by rotation, or by anisotropic velocity dispersion (i.e.,  $\delta > 0$ )

It is customary to identify  $\sigma_0$  with  $\bar{\sigma}$ , the mean velocity dispersion interior to half the effective radius, and  $v_0$  with  $4v_m/\pi$ , with  $v_m$  the maximum rotation velocity (Binney 2005)

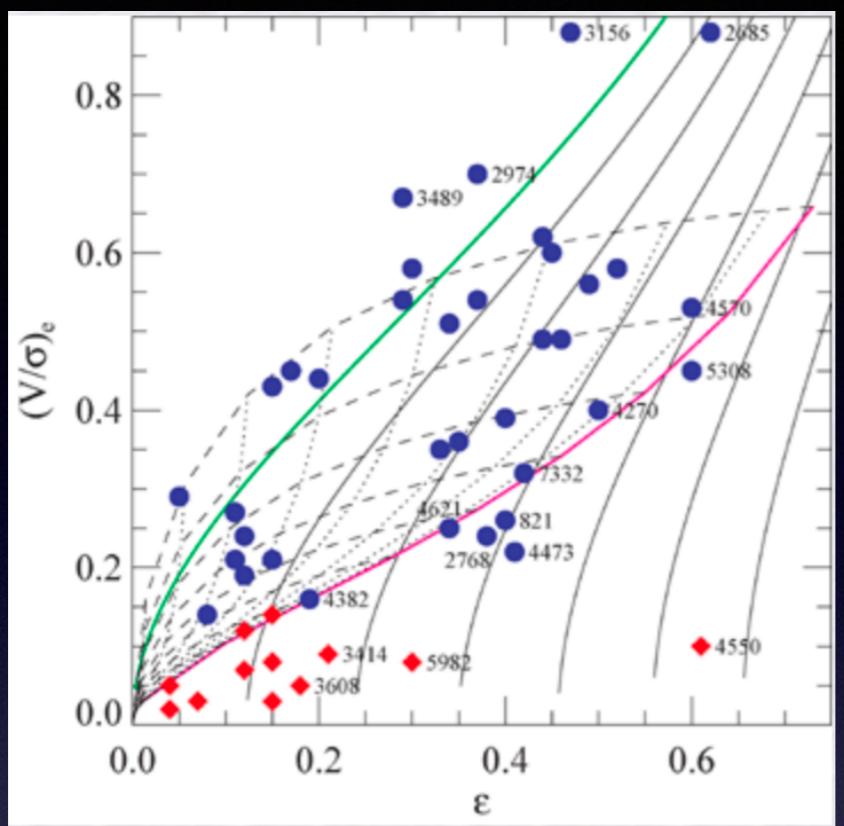


Solid lines are for edge-on system; dashed lines show impact of projection for decreasing inclination angle.

For isotropic case, to good approximation we have

$$\frac{v_m}{\bar{\sigma}} \approx \sqrt{\frac{\varepsilon}{1 - \varepsilon}}$$

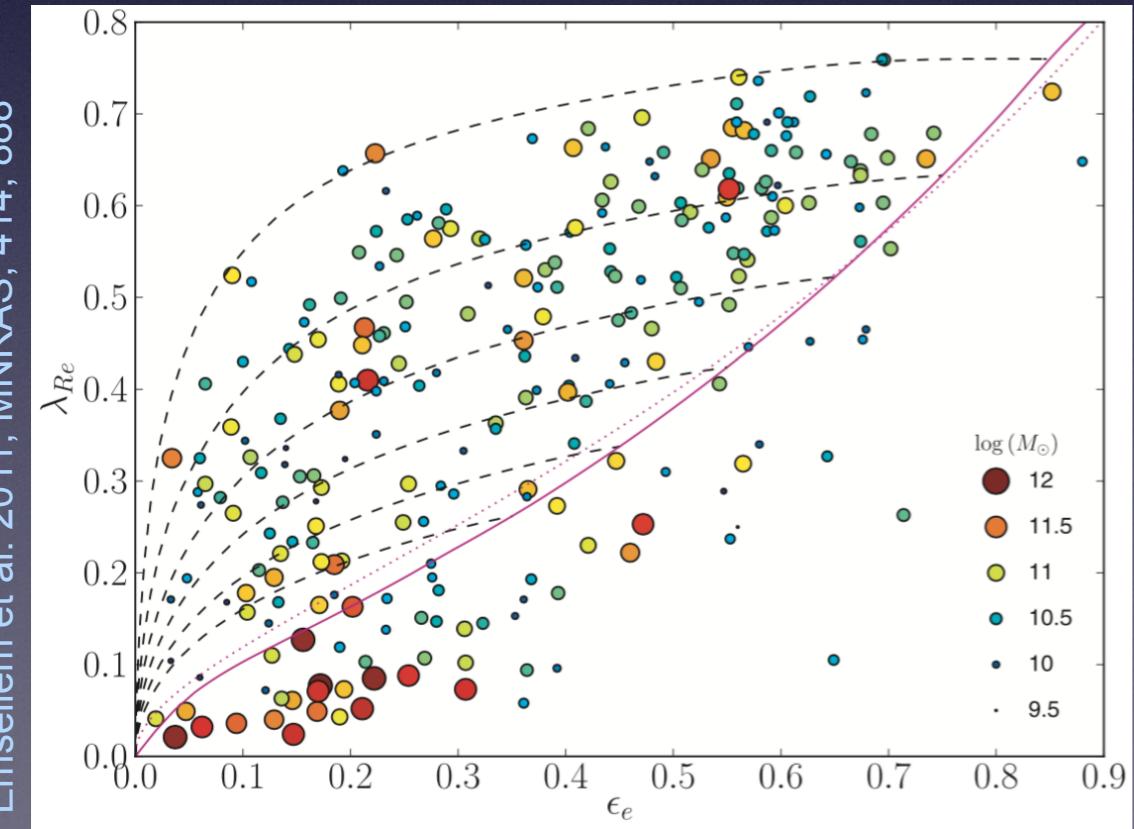
# Fast vs. Slow Rotators



One can split ellipticals in two kinematic classes:

Fast rotators; kinematics consistent with oblate rotators,  
shape is flattened by rotation

Slow rotators; very little rotation; shape is due to  
anisotropic pressure support

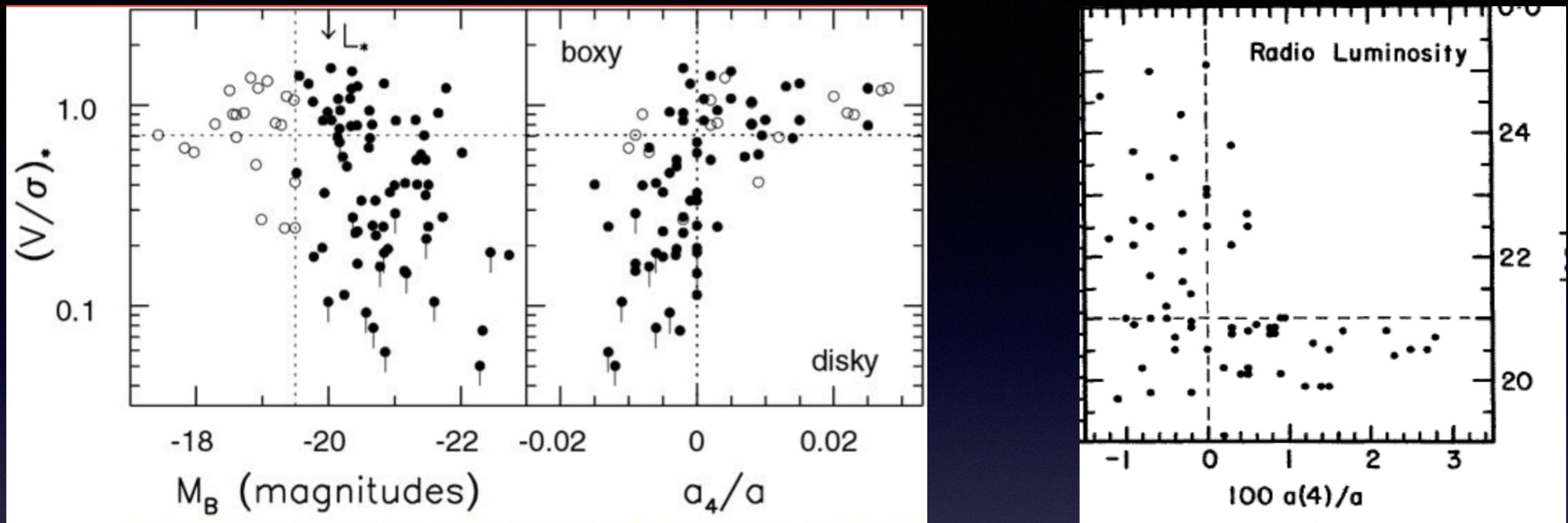


Typically, slow rotators are more massive, and are often  
boxy. Fast rotators are disk-like ellipticals or S0s, and are  
often less luminous.

$$\lambda_R = \frac{\langle R | V | \rangle}{\langle R \sqrt{V^2 + \sigma^2} \rangle}$$

a modern replacement for  $v_m/\bar{\sigma}$

# The Dichotomy among Ellipticals



## Faint ellipticals ( $M_B \gtrsim -20.5$ )

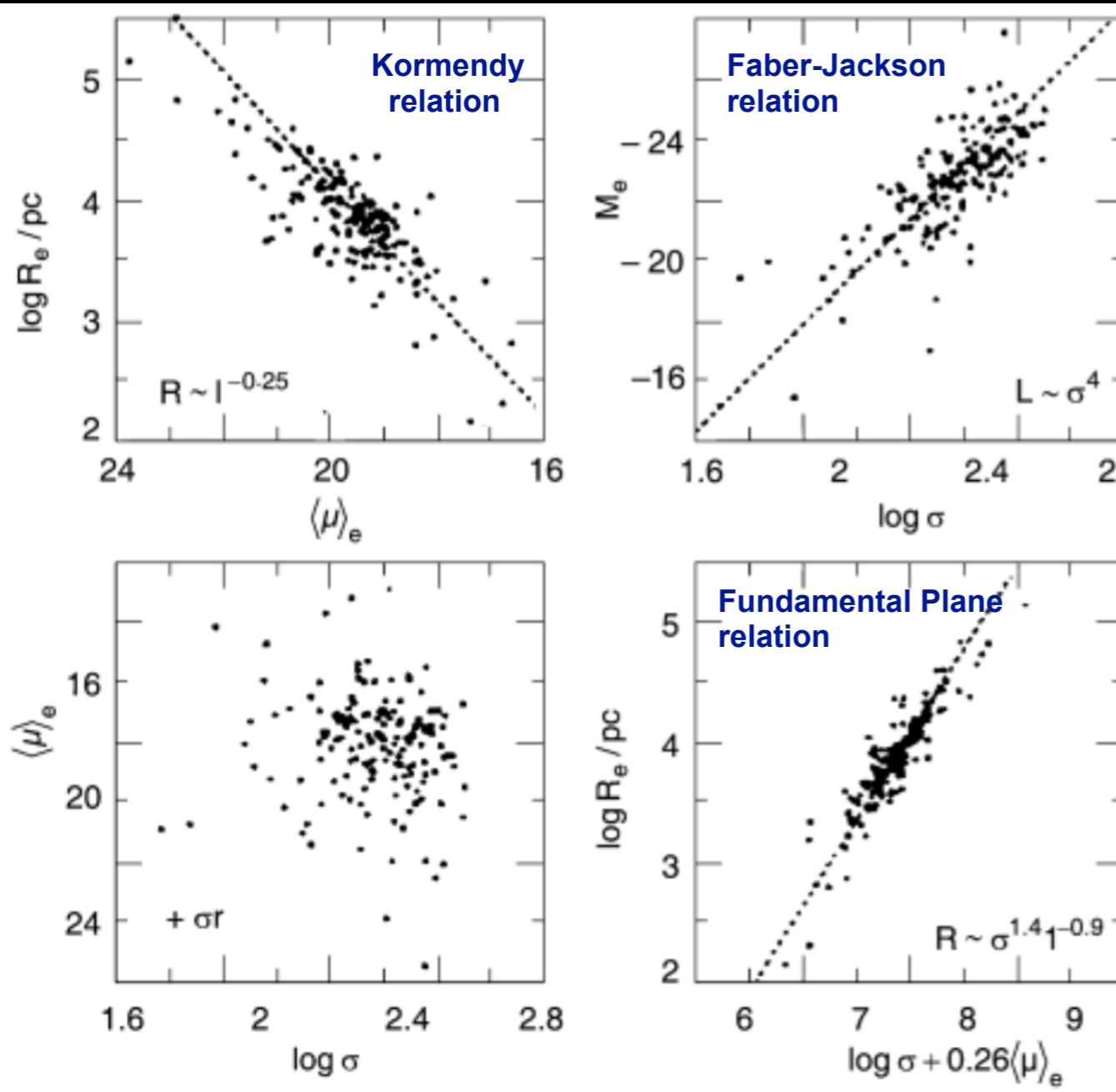
disky isophotes  
cuspy SB profile  
fast rotator  
weak in radio/X-ray  
isophotal twists rare

## Bright ellipticals ( $M_B \lesssim -20.5$ )

boxy isophotes  
cored SB profile  
slow rotator  
often strong radio/X-ray emitter  
isophotal twists common

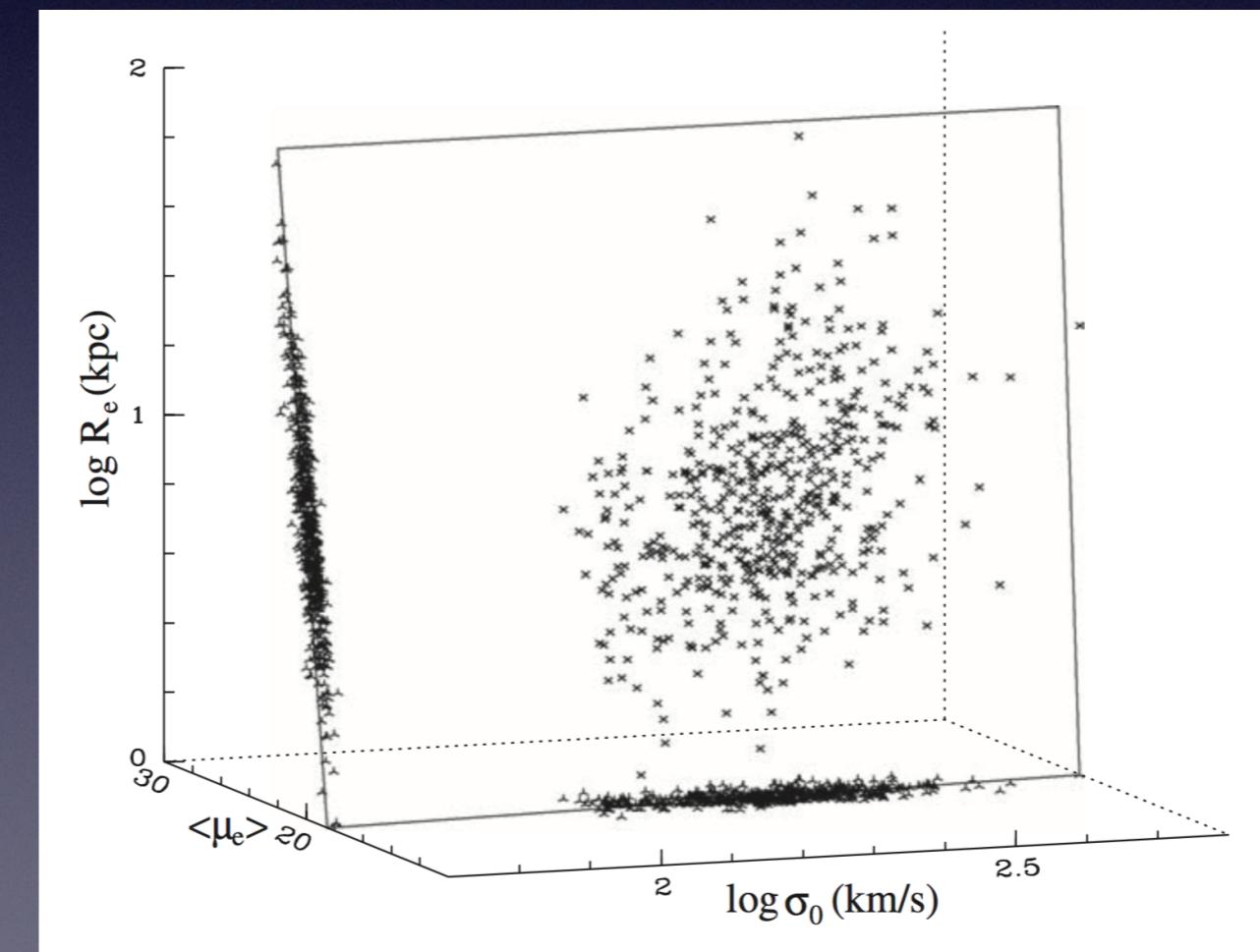
Disky ellipticals are consistent with being more bulge-dominated versions of S0 galaxies.

# The Fundamental Plane



Similar to the TF-relation for disk galaxies, ellipticals reveal a scaling relation between luminosity and velocity dispersion, known as the Faber-Jackson (FJ) relation:

However, unlike the TF, the scatter in FJ \*is\* correlated with size, giving rise to a three-parameter Fundamental Plane relation.



# The Fundamental Plane

The FP-relation is generally written in the form

$$\log R_e = a \log \sigma_0 + b \log \langle I \rangle_e + \text{cst}$$

Best-fit parameters are  $a \sim 1.2$  to  $1.5$  (depending on waveband), and  $b \sim -0.8$

The FP-relation is usually interpreted in terms of the Virial Theorem

$$\frac{G M}{\langle R \rangle} = \langle v^2 \rangle$$

$\langle R \rangle$  = average radius, such that lhs is abs. value of mean potential energy per unit mass

$\langle v^2 \rangle$  = average rms velocity, such that half that value is mean kinetic energy per unit mass

Let  $R_e = \kappa_R \langle R \rangle$   
 $\sigma_0 = \kappa_V \sqrt{\langle v^2 \rangle}$



$$R_e = \frac{1}{2\pi G \kappa_R \kappa_V^2} \sigma_0^2 \langle I \rangle_e^{-1} (M/L)^{-1}$$

If ellipticals are homologous (i.e.  $\kappa_R$  and  $\kappa_V$  constant), and the mass-to-light ratio is constant, then the Virial Theorem predicts a FP-relation with  $a=2$  and  $b=-1$

The deviation from this prediction is called the ‘tilt’ of the fundamental plane, and reflects that ellipticals as a class are not homologous, and/or that  $(M/L) \propto L^\alpha \langle I \rangle_e^\beta$  with  $(\alpha, \beta) \neq 0$ .

although still debated, the latter option seems to explain most of the tilt.

# The Fundamental Plane

As originally proposed by Bender+92, it is useful to use an orthogonal combination of the three observables that enter the FP-relation, which facilitates interpretation.

$$\kappa_1 \equiv (\log \sigma_0^2 + \log R_e)/\sqrt{2}$$

$$\kappa_2 \equiv (\log \sigma_0^2 + 2 \log \langle I \rangle_e - \log R_e)/\sqrt{6}$$

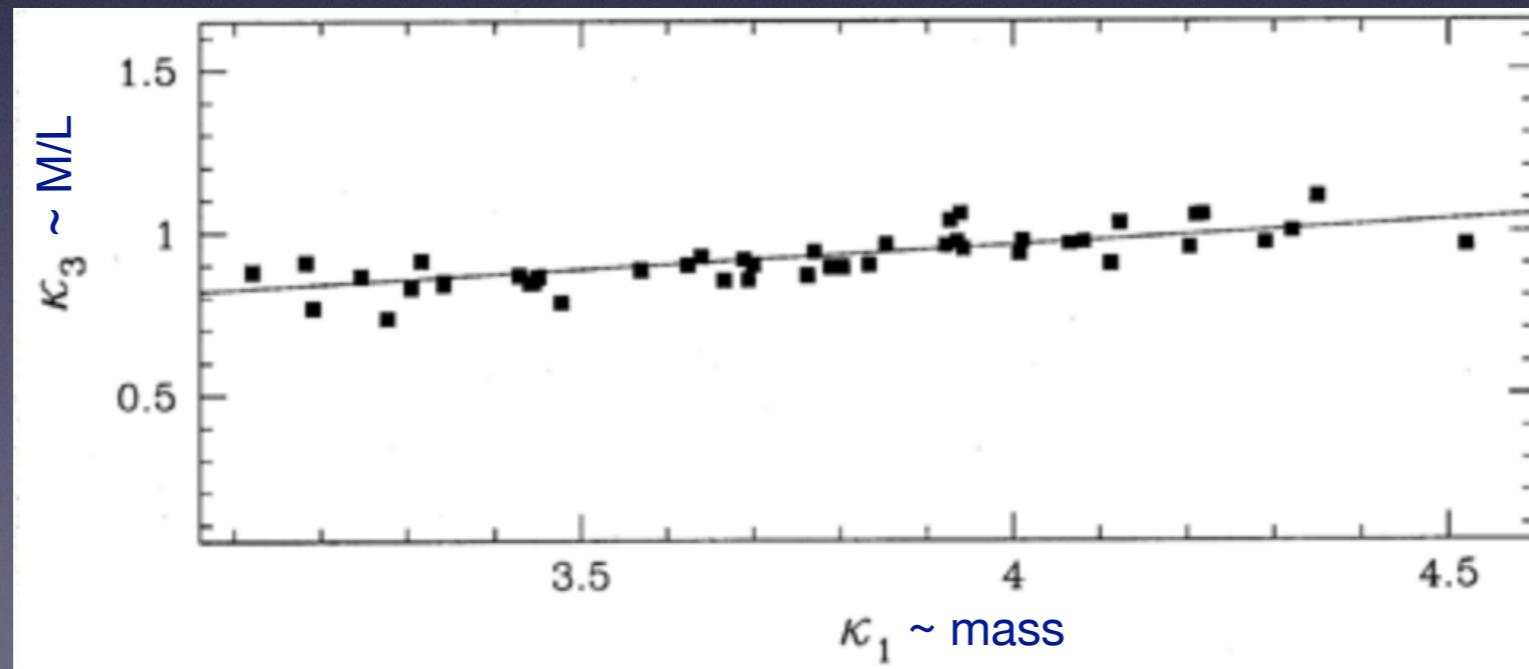
$$\kappa_3 \equiv (\log \sigma_0^2 - \log \langle I \rangle_e - \log R_e)/\sqrt{3}$$



$$\kappa_1 \propto \log(\sigma_0^2 R_e) \propto \log M$$

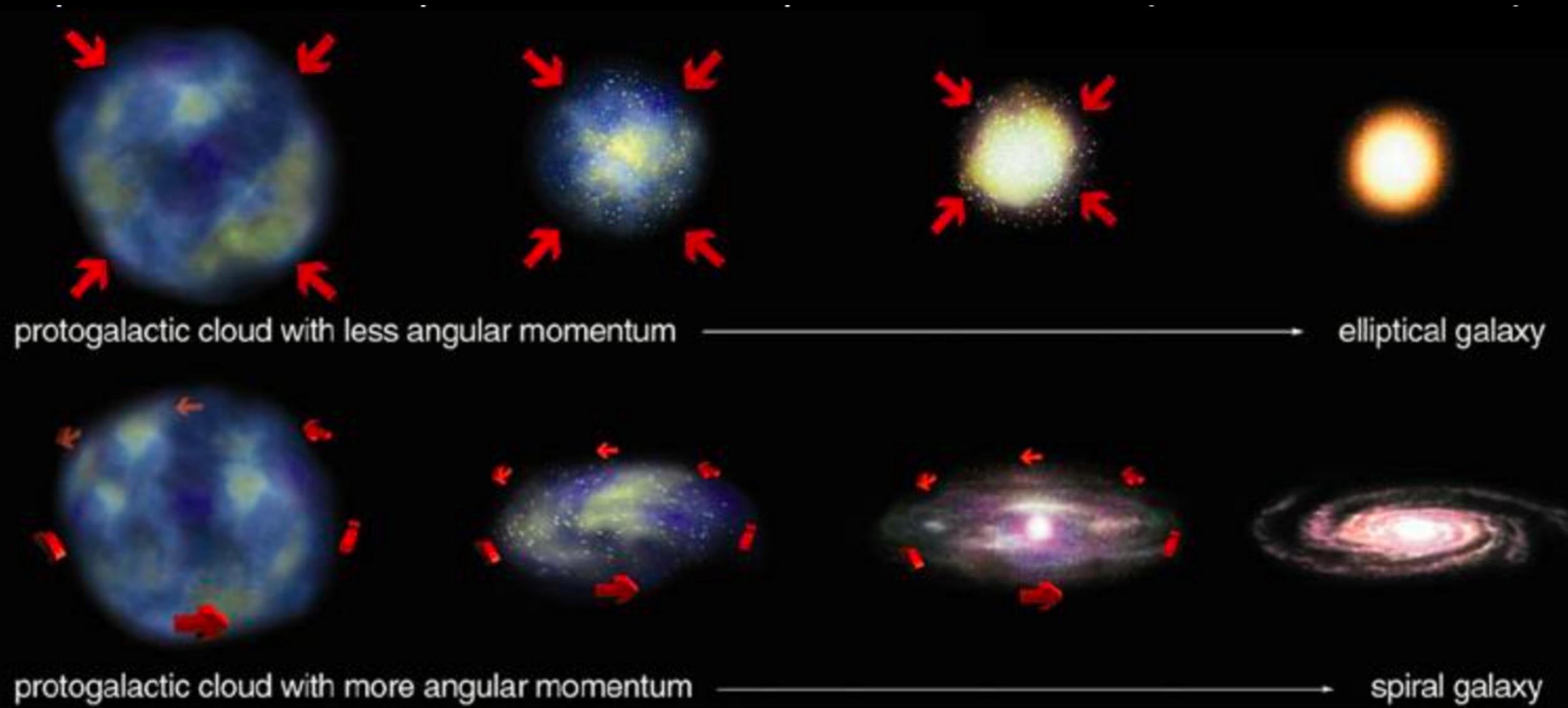
$$\kappa_3 \propto \log(\sigma_0^2 R_e / \langle I \rangle_e R_e^2) \propto \log(M/L)$$

In this ‘ $\kappa$ -space’, the  $\kappa_1$ - $\kappa_2$  projection is very close to a face-on projection of the FP, while the  $\kappa_1$ - $\kappa_3$  projection shows the FP nearly edge-on. In fact, if ellipticals are homologous, and  $(M/L) \propto M^\gamma$ , the virial theorem implies that  $\kappa_3 = \sqrt{2/3}\gamma\kappa_1 + \text{cst}$



# Formation of Elliptical Galaxies

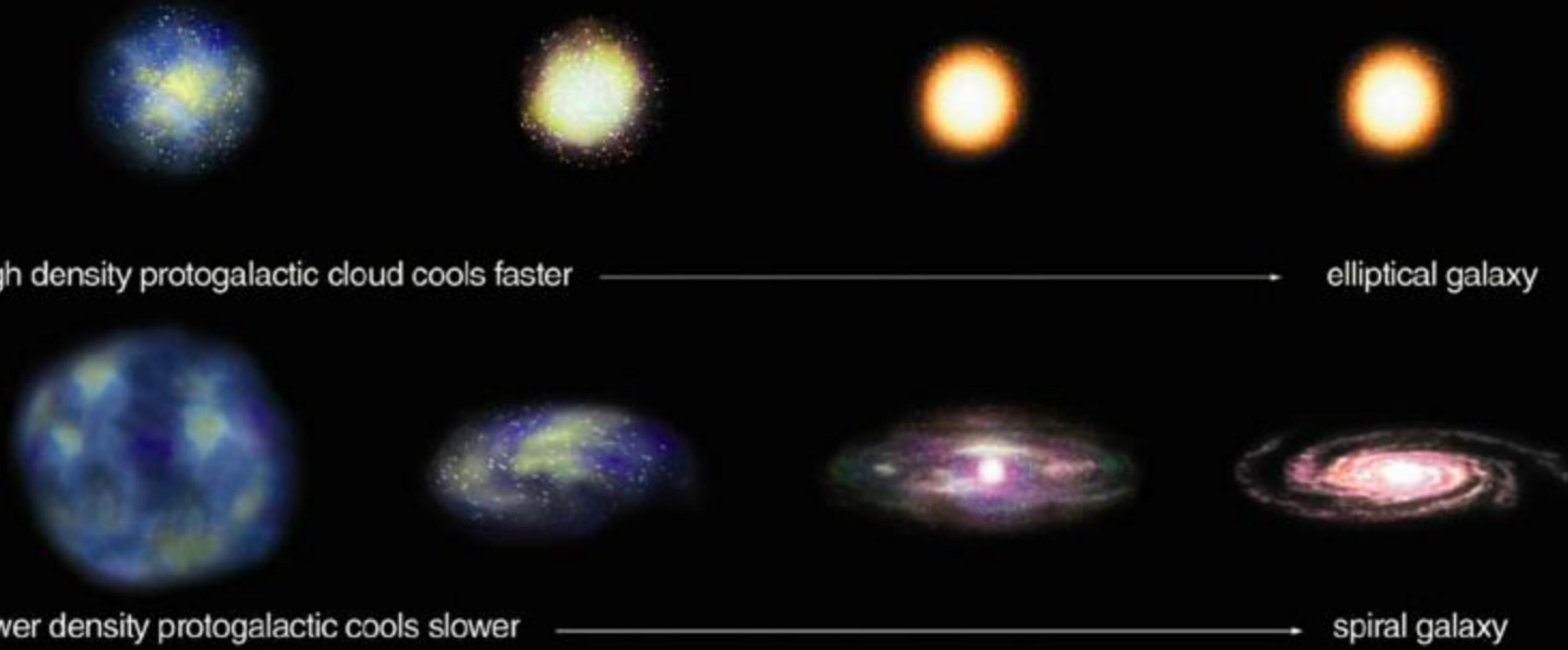
One ‘obvious’ scenario for why some galaxies are ellipticals and others are spirals is to assume this is governed by angular momentum....



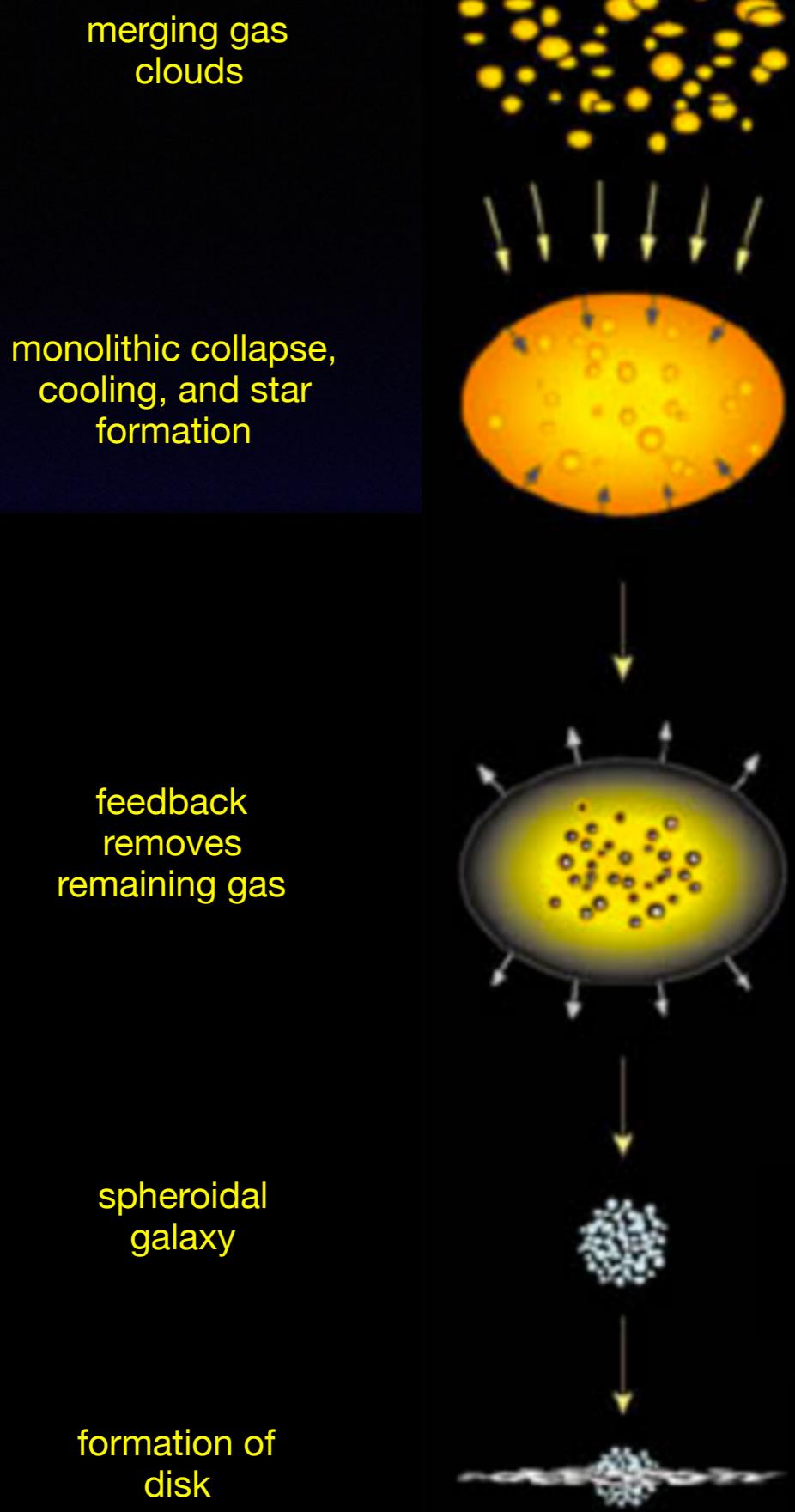
# Formation of Elliptical Galaxies

an alternative is to assume that the difference relates to the **density** of the proto-galaxy

Fig. credit: 2004, Pearson Education, publishing as Addison-Wesley



## monolithic collapse



## hierarchical merger scenario

