Galaxy formation and evolution
PAP 318, 5 op, autumn 2020

on Zoom

Lecture 6: Correlation functions and the spectrum of the initial fluctuations – Additional notes, 09/10/2020

Lecture 6 additional notes I

- Page 4: Overdensity delta definition:

$$\Delta = \frac{\delta\rho}{\rho_0}, \quad \rho = \rho_0[1 + \Delta(\bar{x})] = \rho_0\left[1 + \frac{\delta\rho}{\rho_0}\right] = \rho_0 + \delta\rho$$

- Page 6: Fourier transform and inverse Fourier transform:

$$\Delta(\bar{r}) = \frac{V}{(2\pi)^3}\int \Delta_{\bar{k}} e^{-i\bar{k}\cdot\bar{r}} d^3k$$

$$\Delta_{\vec{k}} = \frac{1}{V}\int \Delta(\vec{r}) e^{i\bar{k}\cdot\bar{r}} d^3x$$

Lecture 6 additional notes II

- Page 6: Definition and short-hand notation:

$$\langle \Delta^2 \rangle = \frac{1}{V} \int \Delta^2(\bar{r}) d^3x$$

- Page 7: Basics of complex numbers: Δ^*=complex conjugate

$$|\Delta(\bar{r})|^2 = |\Delta(\bar{r})\Delta^*(\bar{r})|$$

$$e^{i\bar{k}\cdot\bar{r}} = \cos(\vec{k}\cdot\vec{r}) + i\sin(\vec{k}\cdot\vec{r})$$

Lecture 6 additional notes III

- Page 8: Two-point correlation function integration:

$$\xi(r) = \frac{V}{(2\pi)^3} \int |\Delta_k|^2 \cos(kr\cos\theta) \frac{1}{2} \sin\theta d\theta k^2 dk$$

$$\xi(r) = \frac{V}{(2\pi)^3} \int |\Delta_k|^2 \frac{\sin kr}{kr} k^2 dk$$

- Page 9: Initial Power spectrum: sin (kr) Taylor expansion

$$\sin kr \approx kr - \frac{(kr)^3}{6} + \frac{(kr)^5}{120} + \ldots$$

Lecture 6 additional notes III

- Page 9: Initial Power spectrum:

$$\xi(r) \propto \int_0^{k_{\max}} k^{n+2} dk \propto k^{n+3}$$

$$k_{\max} \approx 1/r, \quad \xi \propto r^{-(n+3)}, \quad \xi(M) \propto M^{-(n+3)/3}$$

- Page 10: Relation between the density fluctuation and the 2-point correlation function:

$$\langle \Delta^2 \rangle^{1/2} \propto \xi^{1/2}$$

Lecture 6 additional notes IV

- Page 11: Horizon mass in the radiation dominated era

$$M_{\rm H} \propto \rho_{\rm d}(ct)^3 \propto a^{-3}(a^2)^3 = a^3, \quad a \propto M_{\rm H}^{1/3}$$

- Page 18: Bias can also be defined in terms of the σ_8 parameter, which is defined to be root-mean-square density fluctuation when smoothed with a top-hat filter of radius $8h^{-1}$ Mpc.

$$b^2 = \frac{\sigma_8^2(\rm gal)}{\sigma_8^2(\rm mass)}$$