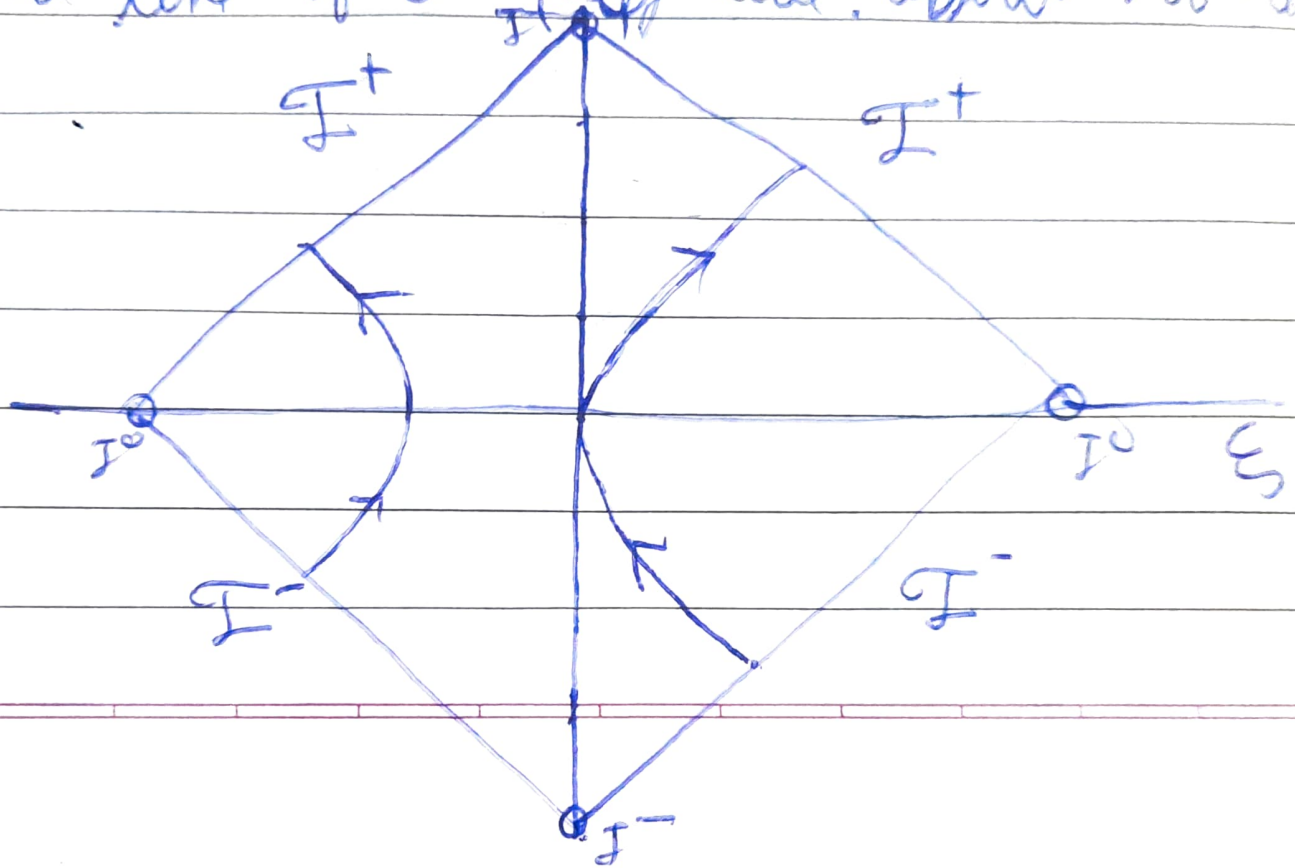
Q11 Show world line of eternally accel. observer of Minkowskian S.T.

Ans.



→ For an accelerating observer, the equation of motion is:-

$$\left(x + \frac{1}{a}\right)^2 - t^2 = \frac{1}{a^2}$$

& the coordinate transformation for Penrose-Carter diagrams is:-

$$t \pm x = \tan\left(\frac{\psi \pm \xi}{2}\right)$$

$$\Rightarrow \left(x + t + \frac{1}{a}\right)\left(-\frac{1}{a} - (t - x)\right) = \frac{1}{a^2}$$

Substituting:-

$$\left(\frac{1}{a} + \tan\left(\frac{\psi+\xi}{2}\right)\right)\left(\frac{1}{a} - \tan\left(\frac{\psi-\xi}{2}\right)\right) = \frac{1}{a^2}$$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{a}\left(\tan\left(\frac{\psi+\xi}{2}\right) - \tan\left(\frac{\psi-\xi}{2}\right)\right) - \tan\left(\frac{\psi+\xi}{2}\right)\tan\left(\frac{\psi-\xi}{2}\right) = \frac{1}{a^2}$$

$$\Rightarrow \boxed{a \tan\left(\frac{\psi+\xi}{2}\right)\tan\left(\frac{\psi-\xi}{2}\right) = \tan\left(\frac{\psi+\xi}{2}\right) - \tan\left(\frac{\psi-\xi}{2}\right)}$$

Which is what we have plotted.