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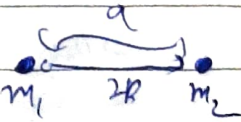
Q 2) Find the change in time of radius of a double star system due to "GRad". Initial dist. b/w star is 2R & masses m_1, m_2.

Ans: Assumption :- Velocities are non-relativistic



The total energy of system is :-

$$E_{orb} = -\frac{m_1 m_2}{2(2a)} = -\frac{m_1 m_2}{4a}$$

So, the total mass is :- $M = m_1 + m_2 - \frac{m_1 m_2}{4a}$

& the orbital freq is :- $\Omega = \frac{2\pi}{T} = \frac{(m_1+m_2)^{1/2}}{(2a)^{3/2}}$

& the Quadrupole moment is :-

$\phi_0 + \Omega t$ → for masses separated at this angle

$$Q_{ij} = \frac{m_1 m_2}{m_1+m_2}(2a)^2 \begin{pmatrix} \cos 2\phi - \frac{1}{3} & \cos\phi \sin\phi & 0 \\ \cos\phi \sin\phi & \sin^2\phi - \frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}$$

$$\Rightarrow \dot{Q}_{ij} = \frac{m_1 m_2}{m_1+m_2} a^2 \begin{pmatrix} -2\cos\phi\sin\phi & \frac{2\cos 2\phi}{2} & 0 \\ \frac{2\cos 2\phi}{2} & 2\sin\phi\cos\phi & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \dddot{Q}_{ij} = \frac{m_1 m_2}{m_1+m_2} a^2 \begin{pmatrix} \sin 2\phi & 4\cos 2\phi & 0 \\ 4\cos 2\phi & -4\sin 2\phi & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

GW wave power

$$\Rightarrow -\langle \dot{E} \rangle = \frac{1}{5}\langle \dddot{Q}_{ij} \dddot{Q}_{ij} \rangle = \frac{32}{5}\left(\frac{m_1 m_2}{m_1+m_2}\right)^2 (2a)^4 \Omega^6$$

$$= \frac{32}{5} \frac{m_1^2 m_2^2 (m_1+m_2)}{(2a)^5} = \frac{32 m_1^2 m_2^2 (m_1+m_2)}{5 \cdot 2^5 a^5}$$

We know that

$$\langle \dot{E} \rangle = 2\dot{a}\left(\frac{m_1+m_2}{2a} - \frac{m_1 m_2}{2a}\right) = \frac{m_1 m_2}{2a^2}\dot{a}$$

$$\Rightarrow \dot{a} = -\frac{64}{5} \frac{m_1 m_2 (m_1+m_2)}{a^3} \Rightarrow \frac{a^4}{4} - \frac{(2R)^4}{4} = -\frac{64}{5} m_1 m_2 (m_1+m_2) t$$

$$\Rightarrow \int da = -\frac{64}{5} m_1 m_2 (m_1+m_2) \int dt$$

$$\Rightarrow \boxed{a^4(t) = 16R^4 - \frac{256}{5} m_1 m_2 (m_1+m_2) t}$$