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) Show that $\hbar^2 \Box e^{-i\frac{\psi}{\hbar}} = 0 \longrightarrow g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi = 0$ as $\hbar \to 0$

Ans $\Box = \frac{1}{\sqrt{|g|}} \partial_\mu g^{\mu\nu} \sqrt{|g|}\, \partial_\nu$

$= g^{\mu\nu} \partial_\nu \partial_\mu$.

$\Rightarrow \hbar^2 \Box e^{-i\frac{\psi}{\hbar}} = g^{\mu\nu} \hbar^2 \partial_\nu \partial_\mu e^{-i\frac{\psi}{\hbar}} = 0$

$= g^{\mu\nu} \hbar^{\cancel{2}} \partial_{\nu} \left(-\frac{i}{\cancel{\hbar}} \partial_\mu \psi \, e^{-i\frac{\psi}{\hbar}} \right) = 0$

$= (-i) g^{\mu\nu} \hbar (\partial_\nu \partial_\mu \psi) e^{-i\frac{\psi}{\hbar}}$
$+ (-i) g^{\mu\nu} \cancel{\hbar} (\partial_\mu \psi) \left(-\frac{i}{\cancel{\hbar}}\right) \partial_\nu \psi e^{-i\frac{\psi}{\hbar}} = 0$

$= (-i) g^{\mu\nu} \hbar (\partial_\nu \partial_\mu \psi) e^{-i\frac{\psi}{\hbar}} - g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi e^{-i\frac{\psi}{\hbar}} = 0$

as $\hbar \to 0$, first term $\to 0$

$\Rightarrow -(g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi) e^{-i\frac{\psi}{\hbar}} = 0$

$\Rightarrow \underline{g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi = 0}$