Unmasking the role of inter correlation among nuclear mater properties on neutron star observables: Machine Learning approach

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Background– The nuclear matter parameters (NMPs) which characterize the properties of infinite nuclear matter are usually extracted from their correlations with the specific finite nuclei observables. These correlations are determined either by considering a diverse set of mean field models or more recently by applying a co-variance approach to a single mean field model. The NMPs so obtained have been employed to constrain the properties of neutron stars such as the radius and tidal deformability. In this process, the correlation among various NMPs as imposed by the finite nuclei observable are ignored [1]. It is important to investigate the role of inter correlations among NMPs on considering neutron star properties.

The compact stars such as neutron stars (NSs), observed as pulsars, are believed to contain matter upto few times nuclear saturation density in its core. The NSs present one of the densest forms of matter in the observable universe. They are the ideal cosmic laboratories to shed light directly or indirectly on different theories of physics as well as on the physics beyond the standard scenario. To explain and understand the extreme properties of such stars, one needs to connect different branches of physics including low energy nuclear physics, QCD under extreme conditions, general theory of relativity (GR) etc. The internal structure of the neutron star (NS) and its properties, such as mass, radius, quadrupole deformation and moment of inertia, are depends on the hydrostatic equilibrium between the inward gravitational pull of matter and the outward neutron degeneracy pressure. If we assumed the correctness of GR, to understand the internal structure of NS predominantly, one needs the theory of the behavior of matter at extreme conditions, i.e., the theory of infinite nuclear matter equation of state (EOS). The EOS is conventionally defined as energy (or pressure) as a function of density, over a wide range of densities.

The energy per nucleon at a given density $\rho = \rho_n + \rho_p$ with ρ_n and ρ_p the neutron and proton densities, respectively, and asymmetry $\delta = (\rho_n - \rho_p)/\rho$, can be decomposed, to a good approximation, into the EoS for symmetric nuclear matter $e(\rho, 0)$, and the density dependent symmetry energy coefficient $S(\rho)$:

$$e(\rho, \delta) \simeq e(\rho, 0) + S(\rho)\delta^2. \tag{1}$$

Expanding the isoscalar contribution until fourth order and the isovector until third order we obtain for the isoscalar part $e(\rho, 0)$:

$$e(\rho, 0) = e(\rho_0) + \frac{K_0}{2}x^2 + \frac{Q_0}{6}x^3 + \mathcal{O}(x^4) \tag{2}$$

and for the isovector part $S(\rho)$:

$$S(\rho) = J_0 + L_0 x + \frac{K_{\mathrm{sym},0}}{2}x^2 + \mathcal{O}(x^3). \tag{3}$$

where $x = \frac{\rho-\rho_0}{3\rho_0}$ and $J_0 = S(\rho_0)$ is the symmetry energy at the saturation density. The incompressibility K_0, the skewness coefficient Q_0, the symmetry energy slope L_0, and its curvature $K_{\mathrm{sym},0}$ evaluated at saturation density are defined in, e.g., Ref. [2]. The key nuclear matter parameters (NMPs) of the EOS are: K_0, Q_0, J_0, L_0 and $K_{\mathrm{sym},0}$.

We can construct large number of EOS database as a point in the seven dimensional space of NMPs using multivariate Gaussian distribution (MVGD) [3], the parameters being $e_0, \rho_0, K_0, Q_0, J_0, L_0$, and $K_{\mathrm{sym},0}$. Symbolically, the 'i$^{\mathrm{th}}$' EOS is written as

$$\begin{aligned}\mathrm{EOS}_i = \ & \{e_0, \rho_0, K_0, Q_0, J_0, L_0 \text{ and } K_{\mathrm{sym},0}\}_i \\ & \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})\end{aligned}$$

where $\boldsymbol{\mu}$ designates the mean value of the parameters and $\boldsymbol{\Sigma}$ is the co-variance matrix. The diagonal elements of $\boldsymbol{\Sigma}$ represent the variance or the squared error for the parameter set $\boldsymbol{p}$. The off-diagonal elements of $\boldsymbol{\Sigma}$ are the covariance between different parameters and measure the correlations among them. Once the values of all the seven NMPs are given, the 'i$^{\mathrm{th}}$' EOS can be calculated either in Taylor expansion mode (as discussed previously) or in mean field formalism. This EOS can be employed to calculate the various properties of neutron stars such as its maximum mass, radius and tidal deformability by solving the NS structure equations [4]. Hence we able to calculate correlations of NMPs with neutron star properties [5].

I. THE EFFECT OF INTER CORRELATIONS AMONG NMPS ON THE CORRELATION OF NEUTRON STAR PROPERTIES WITH NMPS

Three different distributions of the NMPs has been generated, namely, Case-I, Case-II and Case-III and obtain the corresponding sets of Skyrme the EOSs [6, 7]. The central values and the uncertainties on every parameters are exactly the same for the Cases I and II, listed in Table I. The Case I corresponds to the independent distribution of NMPs, i.e., the correlation among different NMPs are assumed to be zero. In the Case II the $L_0 - K_{\mathrm{sym},0}$ correlation is switched on and the correlation coefficient is assumed to be 0.8. The Case III is similar to Case II but the values of e_0, ρ_0, K_0 and Q_0 are kept fixed to their central values. For Case I,II and III,

	MVGD	
	$\overline{P}_i$	σ_{P_i}
e_0	-16.0	0.25
ρ_0	0.16	0.005
K_0	230.0	20
Q_0	-300	100
J_0	32.0	3
L_0	60.0	20
$K_{\mathrm{sym},0}$	-100.0	100

Table I. The mean $\overline{P}_i$ and standard deviation σ_{P_i} of the multivariate Gaussian distribution, where $\sigma^2_{P_i}$ is the variance of the parameter P_i. The P_i considered are the binding energy per nucleon e_0, saturation density ρ_0, the nuclear matter incompressibility K_0, the skewness Q_0, symmetry energy J_0, slope of symmetry energy L_0 and the curvature parameters $K_{\mathrm{sym},0}$. All the quantities are in the unit of MeV except for ρ_0 is in unit of fm^{-3}. Our EOSs are sampled for three different cases using three different distribution for P_i. The EOSs for the Case I are sampled assuming that there are no correlations among the parameters and for Case II we allow the correlation between $L_0 - K_{\mathrm{sym},0}$ to be 0.8. The Case III is obtained from Case II by freezing the values of e_0, ρ_0, K_0 and Q_0 to their central values.

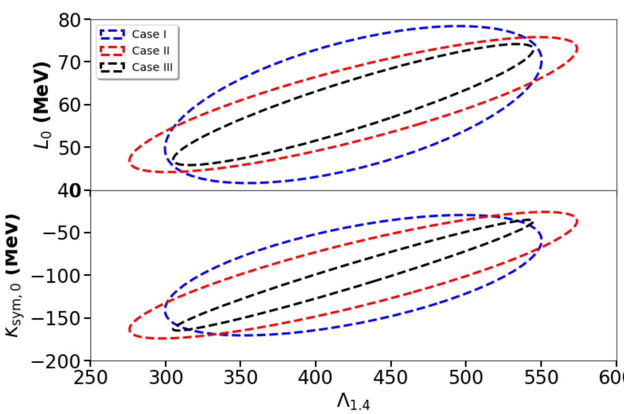

Figure 1. The 1σ confidence ellipses in the plans of $\Lambda_{1.4} - L_0$ (top) and $\Lambda_{1.4} - K_{\mathrm{sym},0}$ (bottom) with $\Lambda_{1.4}$ being the tidal deformability corresponding to the neutron star with the canonical mass for Cases I,II and III.

the distributions of NMPs are filtered out such that the EOSs satisfy the casualty condition and yield the maximum mass of NS above 1.8 $M_\odot$. The central value for the maximum NS mass for each of the distributions is $\sim 2.01\ M_\odot$. The number of filtered EOSs for each of the distributions is about 3000.

In Figure 1, we show the confidence ellipse within 1σ for the tidal deformability of neutron star with canonical mass $\Lambda_{1.4}$ verses the slope L_0 (upper) and its curvature $K_{\mathrm{sym},0}$ (lower) of the symmetry energy for the Case I, II and III. The narrowing down of the ellipses indicate the increase in the correlations for the Case II and Case III compared to Case I. The overall improvement in the correlations from Case I to case II is mainly due to the inclusion of $L_0 - K_{\mathrm{sym},0}$ correlation. These correlations further improves for the Case III. Since, the iso-scalar parameters are fixed in Case III, the interference of these parameters in the values of the tidal deformability is absent. The properties of low mass NS is predominately governed by the symmetry energy and its density derivatives. As the mass of the NS increases, their properties become sensitive also to the iso-scalar NMPs such as K_0 and Q_0. That is why, the correlations for the Case II are marginally weaker than those for Case III.

II. WORK TO BE DONE!!

We have theory to calculate the EOS for a given NMPs and further to calculate neutron star properties with that EOS. But to do that for a large number of NMPs is very time consuming. Whenever we will demand different correlations among NMPs again this time consuming process has to repeat. We need to develop a Machine Learning model for that. Once our model is trained then it should predict the neutron star properties for a given NMPs (more clearly input will be NMPs MVGD and outputs are the distributions of neutron star properties, such as mass, radius and tidal deformability).

- To design the final database of EOS for training the network.
- To investigate should we take MVGD distribution or uniform distribution of NMPs for the training set.
- To design a suitable ML framework for this particular problem we are interested.
- Systematic study for the effect by the of-diagonal terms in the co-variance matrix of NMPs one by one on the neutron star properties.

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