Problem B of the 9th China Trajectory Optimization Competition

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1 Background

Introducing Low Earth Orbit (LEO) satellite for navigation enhancement can shorten the positioning time, improve the positioning accuracy and anti-interference ability. In recent years, with the improvement of orbit determination accuracy of LEO satellites and small satellite techniques, navigation enhancement by LEO satellites has become an research hotspot. Problem B of the 9th CTOC (China Trajectory Optimization Competition) focuses on the **design and deployment of a locally enhanced navigation constellation**.

2 Mission Description

The mission is to design a constellation offering navigation enhancement service to several key cities in China. The constellation is deployed in two ways which are **launching**

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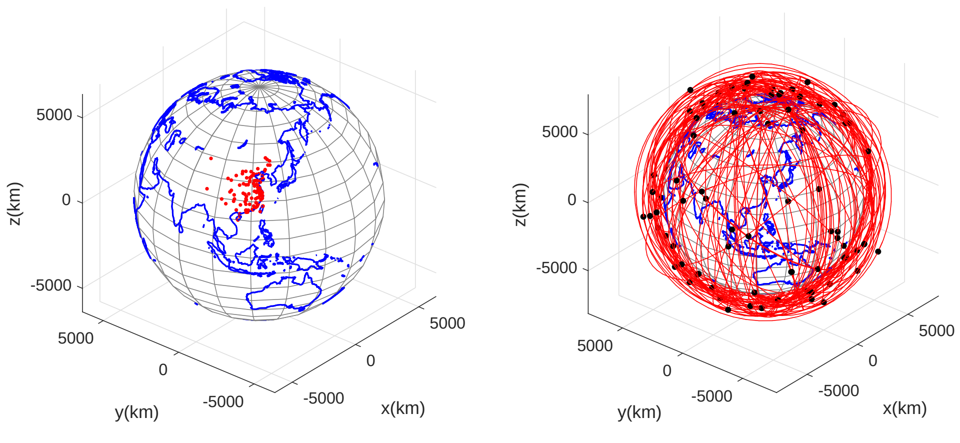
new rockets and **utilizing piggyback missions** respectively. It is expected to design a suitable constellation configuration and optimize the navigation performance of key cities under the conditions of limited construction cost.

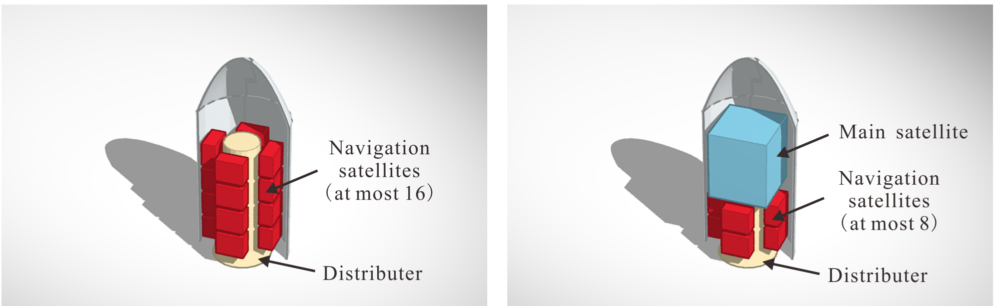
Fig.1(a) shows the positions of 108 key cities in China, denote $[\lambda_k, \phi_k], k = 1, 2, ..., K$ as the latitude and longitude of the k-th city, the data of all cities can be found in ‘city.txt’. Fig.1(b) illustrates 100 piggyback missions under Earth-Centered-Inertial (ECI) frame, the data of all piggyback missions can be found in ‘carry.txt’.

Constellation design process can be divided into two phases which are construction and service respectively. The construction phase starts from Modified Julian Day 2000 (MJD2000) 7305 to 7396. During this phase, the navigation constellation shall be constructed by launching new rockets or utilizing piggyback missions. Fig.2 displays the payload layout of the two methods. See Fig.2(a) for the payload layout of launching new rocket. In the beginning, the distributer carries at most 16 navigation satellites and located at a circular parking orbit with altitude equal to 900km. After that, the satellites separate from distributer and transfer to target orbits with chemical propulsion respectively. See Fig.2(b) for the payload layout of piggyback mission. In the beginning, the distributer carries at most 8 navigation satellites and stays in an orbit of piggyback missions. Before that the main satellite inserted into its orbit. Since MJD2000 = 7396, the constellation construction was completed and the service phase begins. The navigation performance of the constellation for all cities was obtained at some sampling points with fixed interval $\Delta t = 120$s. Fig.3 shows the flow chart of a single task.



(a) Chinese key cities　　(b) Piggyback missions (MJD2000 = 7305)

Figure 1: Chinese key cities and piggyback missions (ECI frame)



(a) New rocket (b) Piggyback mission

Figure 2: Payload layout for two deployment methods

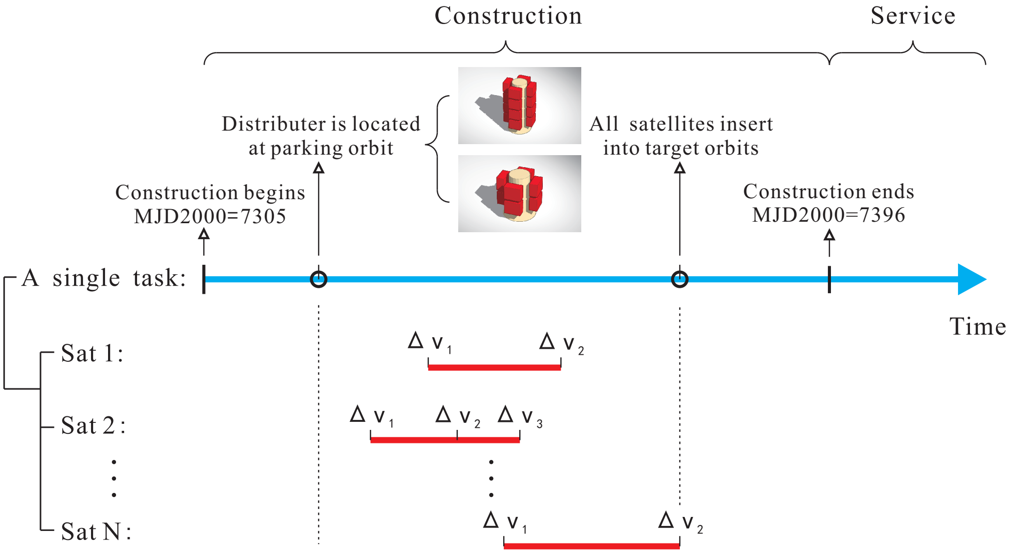


Figure 3: The flow chart of a single task

3 Evaluation Index

The **first index** is to maximize the total navigation revenue of all key cities,

$$\begin{aligned} \mathrm{Obj}_1 &= \max\left(\textstyle\sum_{k=1}^{K}\delta_k w_k\right) \\ \delta_k &= \begin{cases} 1, \text{ if } \max\left(\mathrm{GDOP}_{k,l}\right) \le 10 \\ 0, \text{ if } \max\left(\mathrm{GDOP}_{k,l}\right) > 10 \end{cases}, l = 1, 2, ..., L \end{aligned} \tag{1}$$

where, δ_k is 0-1 variable, and it is used to measure whether the k-th city can meet the navigation accuracy requirements, w_k is the weight of the k-th city (see 'city.txt'), $\mathrm{GDOP}_{k,l}(k = 1, 2, ..., K, l = 1, 2, ..., L)$ is the Geometric Dilution of Precision (GDOP) of the k-th city at the l-th sampling point (see Appendix 8.2 for the calculation of GDOP).

The **second index** is to minimize the total cost of constellation construction.

$$\mathrm{Obj}_2 = \min\left(N_{\mathrm{launch}}C_{\mathrm{launch}} + N_{\mathrm{carry}}C_{\mathrm{carry}} + N_{\mathrm{naviSat}}C_{\mathrm{naviSat}}\right) \tag{2}$$

where, $N_{\mathrm{launch}}, N_{\mathrm{carry}}$ and N_{naviSat} are the number of new launches, the number of piggy-back missions and the number of navigation satellites of the constellation. $C_{\mathrm{launch}}, C_{\mathrm{carry}}$

and C_{naviSat} are single launch cost, single piggyback cost and cost of single navigation satellite. See Tab.1 for relevant prices (the price unit is virtual currency unit).

After submission of results by each team, the design indexes were listed from higher to lower, as Obj_1 and Obj_2. If the relative deviation of the results from the two teams were within 1e-3, it is believed that the two results are the same. For this case, the sequence is determined by the time order emailed to the official mailbox (ctoc2017@163.com).

4 Orbital Dynamics

During the construction and service phases, only earth's central gravitational field and the perturbation of J_2 are considered. The Cartesian coordinate description is implemented during the orbit transfer, and the dynamic model is shown below [1]

$$\begin{cases} \ddot{x} &= -\frac{\mu x}{r^3}\left(1+\frac{3}{2}J_2(\frac{R_\text{e}}{r})^2(1-5\frac{z^2}{r^2})\right) \\ \ddot{y} &= -\frac{\mu y}{r^3}\left(1+\frac{3}{2}J_2(\frac{R_\text{e}}{r})^2(1-5\frac{z^2}{r^2})\right) \\ \ddot{z} &= -\frac{\mu z}{r^3}\left(1+\frac{3}{2}J_2(\frac{R_\text{e}}{r})^2(3-5\frac{z^2}{r^2})\right) \end{cases} \tag{3}$$

where, μ is gravitational constant of the earth, x, y, z are position components of the satellite in the ECI coordinate system, $r = \sqrt{x^2+y^2+z^2}$ is geocentric distance, R_e is equatorial radius, J_2 is earth oblateness gravity coefficient. The navigation satellite was propelled by chemical engine, the magnitude and direction of each impulse shall be optimized, but no limit was imposed on the number of impulses. Residual mass m_f after an impulse is given by

$$m_f = m_i \exp\left(\frac{-\Delta v}{I_{\text{sp}} g_0}\right) \tag{4}$$

where, m_i is satellite mass before the impulse, Δv is magnitude of velocity impulse, I_{sp} is specific impulse, g_0 is gravitational acceleration at the sea level.

Note that, **the orbit propagation with mean orbital elements is implemented before the first impulse and after the orbit injection (including the service phase) due to its efficiency. (i.e., Cartesian description is only used in orbit transfer).** The dynamic model of mean orbital elements under J_2 perturbation is given by

$$\begin{cases} \dot{a} &= 0 \\ \dot{e} &= 0 \\ \dot{i} &= 0 \\ \dot{\Omega} &= -\frac{3A_n J_2}{2(1-e^2)^2}\left(\frac{R_\text{e}}{a}\right)^2 \cos i \\ \dot{\omega} &= \frac{3A_n J_2}{4(1-e^2)^2}\left(\frac{R_\text{e}}{a}\right)^2 (5\cos^2 i - 1) \\ \dot{M} &= \frac{3A_n J_2}{4(1-e^2)^{3/2}}\left(\frac{R_\text{e}}{a}\right)^2 (3\cos^2 i - 1) \end{cases} \tag{5}$$

where, a is semi-major axis, e is eccentricity, i is inclination, Ω is right ascension of ascending node, ω is argument of perigee, M is mean anomaly. $A_n = \sqrt{\mu/a^3}$ is the mean motion of the orbit. Once Ω_0, ω_0 and M_0 at t_0 are given, the elements at time t can be evaluated by

$$\begin{cases} \Omega = & \Omega_0 + \dot{\Omega}(t - t_0) \\ \omega = & \omega_0 + \dot{\omega}(t - t_0) \\ M = & M_0 + (A_n + \dot{M})(t - t_0) \end{cases} \tag{6}$$

Position and velocity can be obtained by orbital elements of orbit (see Appendix 8.4 for the conversion).

5 Constraints

- **Constraints on orbit altitude**
  At any time during construction and service phases, the orbit altitude of all satellites shall be no less than 500km.

- **Time constraints on construction phase**
  The earliest start time for the construction phase is MJD2000 = 7305, and the latest completion time is MJD2000 = 7396.

- **The initial state constraints on launching new rockets**
  In the beginning of a task, the distributer is located at a circular parking orbit with altitude of 900km. The eccentricity of the orbit is $e = 0$, and other elements can be set freely. The distributer can carry up to 16 navigation satellites.

- **The initial state constraints on piggyback mission**
  At the beginning of a task, position and velocity of the distributer are the same as that of a piggyback mission. The states of all piggyback missions can be obtained by propagating equation Eq.(5) and Eq.(6) analytically. The distributer can carry up to 8 navigation satellites.

- **Cost constraints on constellation construction**
  The total cost of constellation construction must satisfy the constraint $C_{\text{all}} \leq 10$ (currency unit), where $C_{\text{all}} = N_{\text{launch}} C_{\text{launch}} + N_{\text{carry}} C_{\text{carry}} + N_{\text{naviSat}} C_{\text{naviSat}}$

- **Mass and specific impulse constraints**
  As to the two deployments, the initial mass of the navigation satellite m_{all} can be expressed as

$$m_{\text{all}} = m_{\text{dry}} + m_{\text{fuel}} \tag{7}$$

where, dry mass m_{dry} equal to 80kg, fuel mass m_{fuel} equal to 20kg. The satellite is equipped with chemical propulsion system and its specific impulse $I_{\text{sp}} = 300\text{s}$.

- **State constraints at the injection point**
  The teams have to provide the mean orbital elements of navigation constellation on MJD2000 = 7396, see Section 6 for more details. After several impulses of orbit transfer, the states of satellite at the injection point shall be identical with that obtained by a reversed propagation with mean orbital elements of the corresponding satellite as listed in 'constellation.txt'. The precision of position for injection shall be less than 1e-3km/s, the velocity precision shall be less than 1e-6km/s and the mass precision shall be under 1e-4kg.

- **Constraints on visibility of city and satellite**
  The precondition for visibility of city and satellite is a ground elevation of no less than 10 degrees (see Appendix 8.4 for visibility between city and satellite).

- **Inclination constraints on elliptical orbit**
  If elliptical orbit is used, critical inclination orbit shall be adopted due to its fixed apsidal line (see Annex 8.3 for calculation of critical inclination).

6 Requirements for Submitting the Results

Please compress the following three files and then send them to ctoc2017@163.com, naming the compressed folders like '[2017MMDD]_B_TeamName.zip.

1. Provide a description document in word or pdf format and name it as 'result'. Major design results shall be included in this document: the two design indexes $(\text{Obj}_1, \text{Obj}_2)$, number of new launches, number of piggyback missions, number of navigation satellites, and a figure of constellation under ECI frame (only single revolution for each satellite), in addition, an introduction of design process is also needed.

2. Provide a text file of mean orbital elements of the constellation at MJD2000 = 7396 and the file shall be named as 'constellation.txt'. Please find format requirements in 'constellationFormat.txt'. All data must be given in double precision.

3. Provide a text file of orbit transfer data during MJD2000 = 7305 to 7396 and the file shall be named as 'transfer.txt'. Please find format requirements in 'transferFormat.txt'. All data must be given in double precision.

7 Descriptions of Verification Program

Before submitting the results, run ‘check.m’ to check ‘constellation.txt’ and ‘transfer.txt’, and output design indexes. The procedures of the verifying program are shown below

1. Call ‘constellation.txt’ to obtain the orbital elements of the navigation constellation at MJD2000 = 7396. Propagate orbits on the 1st, 7th and 30th day (i.e., MJD2000 = 7396 to 7397, MJD2000 = 7402 to 7403 and MJD2000 = 7425 to 7426) with Eq.(5) and Eq.(6), the time step $\Delta t = 120$s. The navigation performance for each city will be evaluated and the design index Obj_1 is obtained.

2. Call ‘transfer.txt’ to obtain the orbit transfer data of all navigation satellites. As mean orbital elements of navigation satellite at MJD2000 = 7396 was given in ‘constellation.txt’, the satellite states at injection point shall be identical with that obtained by a reversed propagation of the corresponding satellite listed in ‘constellation.txt’. The orbit transfer for each segment will be checked and the design index Obj_2 is obtained.

8 Appendix

8.1 The visibility condition between city and satellite

Denote $[\lambda_k, \phi_k], (k = 1, 2, ...K)$ as the latitude and longitude of the k-th city, then the position vector under Earth-Centered-Fixed (ECF) frame can be given by

$$\boldsymbol{r}_k^{\mathrm{city}} = R_{\mathrm{e}} \begin{bmatrix} \cos\lambda_k \cos\phi_k \\ \sin\lambda_k \cos\phi_k \\ \sin\phi_k \end{bmatrix}, (k = 1, 2, ..., K) \tag{8}$$

Denote $\boldsymbol{R}_n^{\mathrm{sat}}$ as the position vector of the n-th satellite under ECI frame, then the position vector $\boldsymbol{r}_n^{\mathrm{sat}}$ under ECF frame can be expressed as

$$\boldsymbol{r}_n^{\mathrm{sat}} = \begin{bmatrix} \cos\theta_{\mathrm{g}} & \sin\theta_{\mathrm{g}} & 0 \\ -\sin\theta_{\mathrm{g}} & \cos\theta_{\mathrm{g}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \boldsymbol{R}_n^{\mathrm{sat}} \tag{9}$$

where, θ_{g} is Greenwich sidereal time and it can be obtained by

$$\theta_{\mathrm{g}} = \theta_{\mathrm{g0}} + \omega_{\mathrm{e}} \Delta t \tag{10}$$

where, θ_{g0} is the Greenwich sidereal time at t_0, ω_{e} is earth rotation rate and Δt is the time difference between any time t and the initial time. The attached function ‘jday.m’ is used to evaluate Julian Day (JD) according to yr/mon/day and hr/min/sec. The attached function 'gstime.m' is used to calculate Greenwich sidereal time according to JD. Define ξ as the minimum elevation angle, and the vector from city to satellite is defined as $\boldsymbol{r}_{k,n} = \boldsymbol{r}_n^{\text{sat}} - \boldsymbol{r}_k^{\text{city}}$, then the visibility condition between the n-th satellite and the k-th city is expressed as

$$\arccos\left(\frac{\boldsymbol{r}_k^{\text{city}} \cdot \boldsymbol{r}_{k,n}}{|\boldsymbol{r}_k^{\text{city}}||\boldsymbol{r}_{k,n}|}\right) \leq \frac{\pi}{2} - \xi \tag{11}$$

8.2 GDOP

Geometric Dilution Precision (GDOP) is an important factor to measure navigation precision and it is determined by the geometrical relationship between city and satellites. Lower GDOP value indicates higher precision (see function 'gdop.m' for calculation procedures). Suppose there are N visible satellites for the k-th city at the l-th sampling point, denote a unit vector from k-th city to n-th satellite as $\bar{\boldsymbol{r}}_{k,n} = [(x_n - x)/R_n, (y_n - y)/R_n, (z_n - z)/R_n]$, $n = 1, 2, ..., N$, where $R_n = \sqrt{(x_n - x)^2 + (y_n - y)^2 + (z_n - z)^2}$, $[x_n, y_n, z_n]$ is position components of the n-th satellite, and $[x, y, z]$ is the position components of the k-th city, create the following matrix $\boldsymbol{H}$

$$\boldsymbol{H} = \begin{bmatrix} \frac{(x_1-x)}{R_1} & \frac{(y_1-y)}{R_1} & \frac{(z_1-x)}{R_1} & 1 \\ \frac{(x_2-x)}{R_2} & \frac{(y_2-y)}{R_2} & \frac{(z_2-x)}{R_2} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{(x_N-x)}{R_N} & \frac{(y_N-y)}{R_N} & \frac{(z_N-x)}{R_N} & 1 \end{bmatrix}_{N\times 4} \tag{12}$$

Define $\boldsymbol{Q} = (\boldsymbol{H}^\top \boldsymbol{H})^{-1}$, and its principal diagonal elements are $Q_{11}, Q_{22}, Q_{33}, Q_{44}$ respectively, then the GDOP can be evaluated by $\text{GDOP}_{k,l} = \sqrt{Q_{11} + Q_{22} + Q_{33} + Q_{44}}$, note that at least 4 visible satellites are required to evaluate GDOP.

8.3 Critical inclination

In the perturbation equations of mean orbital elements under the influence of J_2, a factor $5\cos i^2 - 1$ is involved in Eq.(5). The inclination makes the factor value be 0 is called the critical inclination. And the apsidal line will be fixed when the critical inclination is applied. The critical inclination $\tilde{i}$ is evaluated as below

$$\tilde{i} = \begin{cases} \arccos(\sqrt{1/5}), & \text{prograde orbit} \\ \pi - \arccos(\sqrt{1/5}), & \text{retrograde orbit} \end{cases} \tag{13}$$

8.4 Orbital elements to position and velocity

When the orbital elements $[a, e, i, \Omega, \omega, M]$ is given. First, evaluate eccentric anomaly E with Eq.(14)

$$M = \quad E - e \sin E \tag{14}$$

Then vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ are obtained by Eq.(15).

$$\begin{aligned} \boldsymbol{P} = & \begin{bmatrix} \cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i \\ \sin\Omega\cos\omega + \cos\Omega\sin\omega\cos i \\ \sin\omega\sin i \end{bmatrix} \\ \boldsymbol{Q} = & \begin{bmatrix} -\cos\Omega\sin\omega - \sin\Omega\cos\omega\cos i \\ -\sin\Omega\sin\omega + \cos\Omega\cos\omega\cos i \\ \cos\omega\sin i \end{bmatrix} \end{aligned} \tag{15}$$

At last, the position and velocity vectors $[\boldsymbol{r}, \dot{\boldsymbol{r}}]$ are obtained by

$$\begin{cases} \boldsymbol{r} = & a(\cos E - e)\boldsymbol{P} + a\sqrt{1-e^2}\sin E\boldsymbol{Q} \\ \dot{\boldsymbol{r}} = & \frac{\sqrt{\mu a}}{r}\left[-\sin E\boldsymbol{P} + \sqrt{1-e^2}\cos E\boldsymbol{Q}\right] \end{cases} \tag{16}$$

8.5 Parameters

The simulation parameters are listed in Tab.1

Parameters	Value	Unit	Meaning
μ	398600	km^3/s^2	Gravitational constant of the earth
R_e	6378	km	Equatorial radius
J_2	0.0010826	–	Earth oblateness gravity coefficient
ω_e	7.29211585530e-5	rad/s	Earth rotation rate
I_{sp}	300	s	Specific impulse
g_0	9.80665	m/s^2	Gravity acceleration at sea level
m_{all}	100	kg	Initial mass
m_{dry}	80	kg	Dry mass
m_{fuel}	20	kg	Fule mass
ξ	10	deg	Minimum visibility elevation
C_{launch}	1.2	Currency unit	Single rocket cost
C_{carry}	0.2	Currency unit	Single carry cost
$C_{naviSat}$	0.05	Currency unit	Single satellite cost

Table 1: Simulation parameters

In addition, JD is Julian day, MJD is modified Julian day, MJD2000 is modified Julian day 2000, the relation is given below

$$\begin{aligned} \mathrm{JD} = & \quad \mathrm{MJD2000} + 2451544.5 \\ \mathrm{MJD} = & \quad \mathrm{MJD2000} + 51544 \end{aligned} \tag{17}$$

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