Markov Chain Monte Carlo

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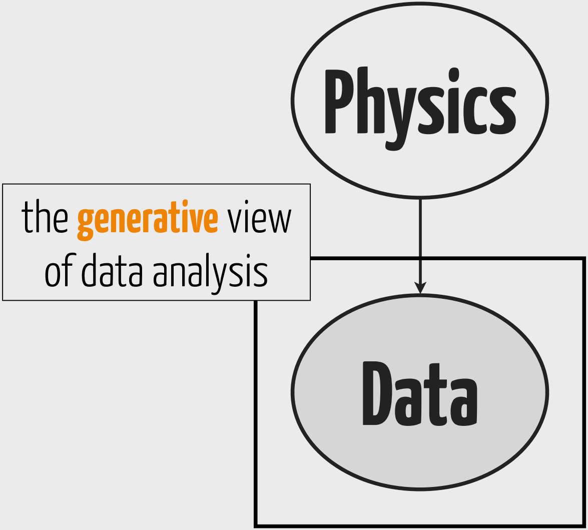
Borrowing heavily from Dan Foreman-Mackey's slides
https://speakerdeck.com/dfm/data-analysis-with-mcmc

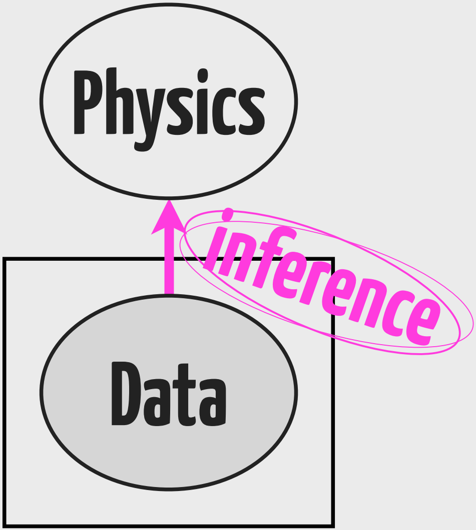
data analysis with

Markov chain Monte Carlo

Dan Foreman-Mackey
CCPP@NYU



The a sketch of graphical model of my research.



a sketch of
The graphical model of my research.

$$p(\mathrm{data} \,|\, \mathrm{physics})$$

likelihood function/generative model

$$\downarrow$$

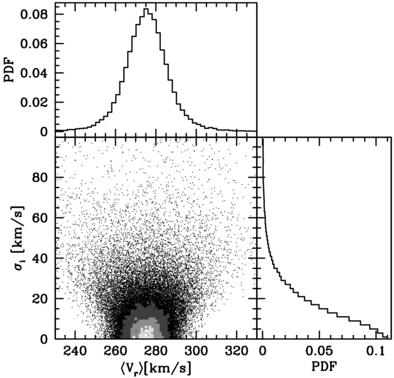
$$p(\mathrm{physics} \,|\, \mathrm{data}) \propto p(\mathrm{physics})\, p(\mathrm{data} \,|\, \mathrm{physics})$$

posterior probability

Why we often need MCMC

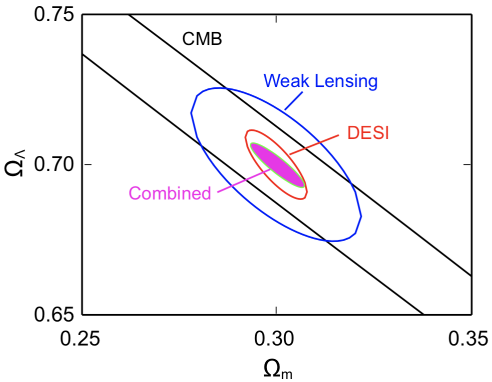
- We want to put constraints on parameters of a physical model based on observations
- constraints = posterior $= p(\mathrm{parameters}|\mathrm{data})$
  ("The stellar mass of the Andromeda galaxy is $10 \pm 2 \times 10^{10}\mathrm{M}_\odot$")
- $\propto \mathrm{prior} \times \mathrm{likelihood}$
- $\propto p(\mathrm{parameters}) \times p(\mathrm{data}|\mathrm{parameters})$
- Real-life models and likelihoods are often complex
- . . . so the resulting constraints have complicated distributions (not Gaussians!)
- . . . but we can represent them with samplings

Samplings to represent constraints - examples



- From https://arxiv.org/abs/1910.04899
- With a sampling: Marginalize over a parameter by projecting it out

Samplings to represent constraints - examples



- From https://arxiv.org/abs/1611.00036

MCMC

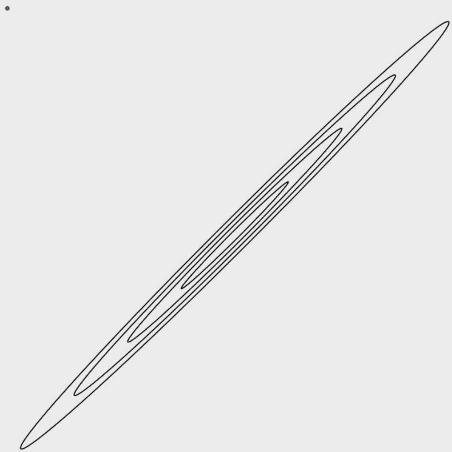
draws samples from a probability function

and all you need to be able to do is

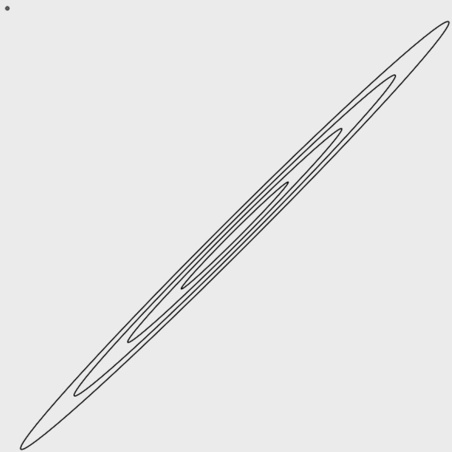
evaluate

the function

(up to a constant)

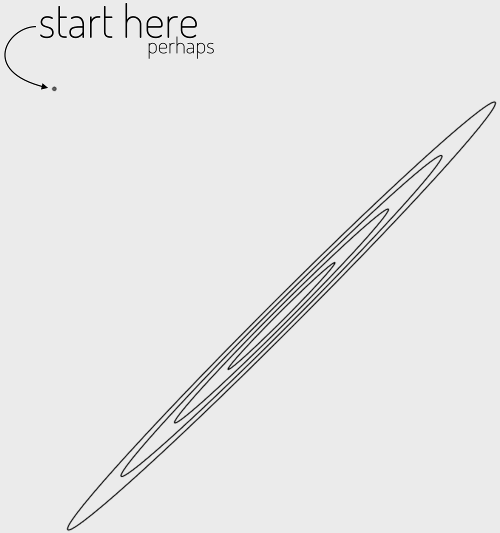


Metropolis–Hastings



Metropolis–Hastings

in an ideal world



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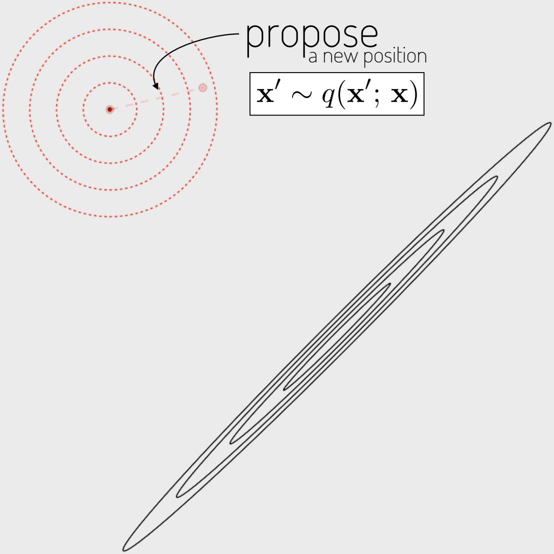

Metropolis–Hastings
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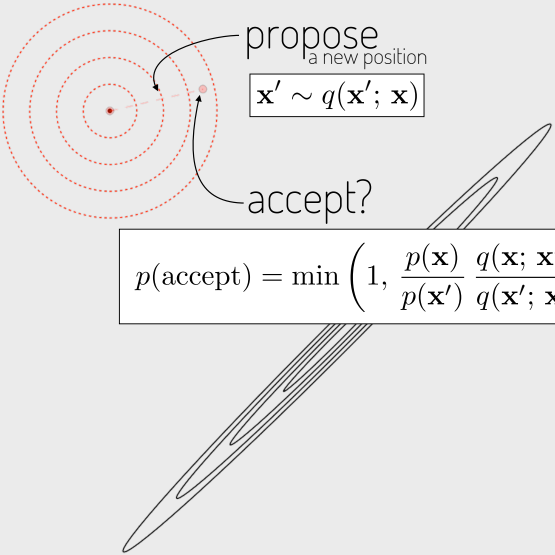
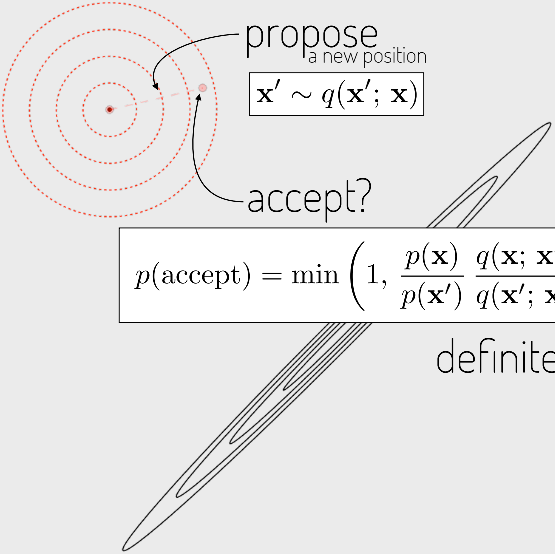


$$\boxed{\mathbf{x}' \sim q(\mathbf{x}';\, \mathbf{x})}$$

$$\boxed{p(\text{accept}) = \min\left(1,\ \frac{p(\mathbf{x})}{p(\mathbf{x}')}\,\frac{q(\mathbf{x};\, \mathbf{x}')}{q(\mathbf{x}';\, \mathbf{x})}\right)}$$

Metropolis–Hastings

in an ideal world



propose
a new position

$$\boxed{\mathbf{x}' \sim q(\mathbf{x}';\, \mathbf{x})}$$

accept?

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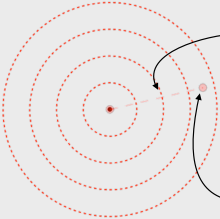
definitely.

Metropolis–Hastings

in an ideal world



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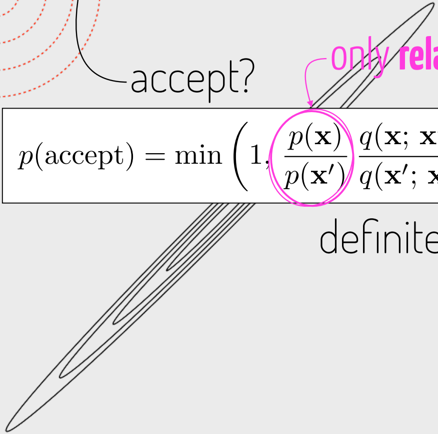

only **relative** probabilities

$$\boxed{p(\text{accept}) = \min\left(1, \frac{p(\mathbf{x})}{p(\mathbf{x}')}\frac{q(\mathbf{x};\, \mathbf{x}')}{q(\mathbf{x}';\, \mathbf{x})}\right)}$$

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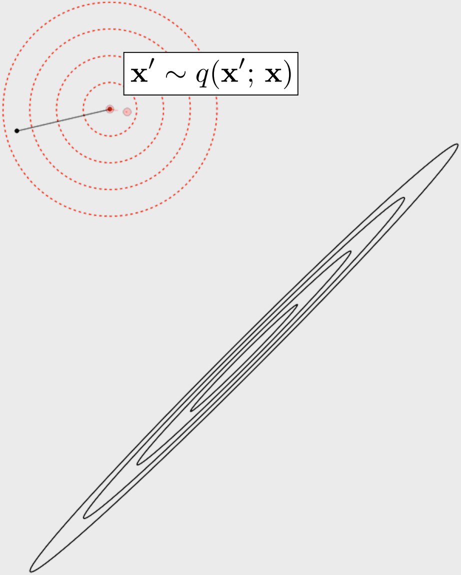
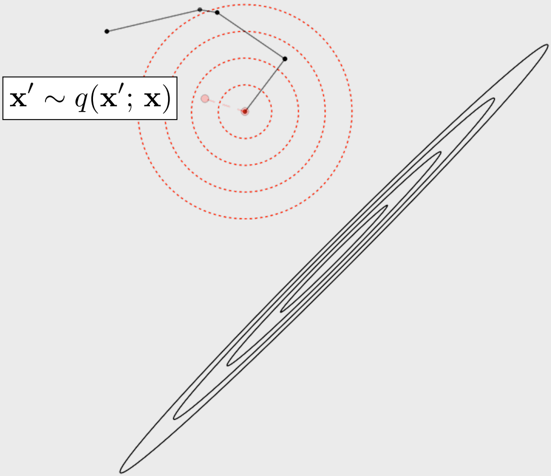
Metropolis–Hastings

in an ideal world



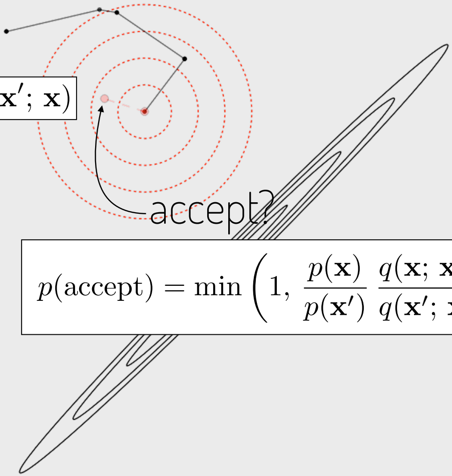
Metropolis–Hastings

in an ideal world



Metropolis–Hastings

in an ideal world



$$\mathbf{x}' \sim q(\mathbf{x}'; \mathbf{x})$$

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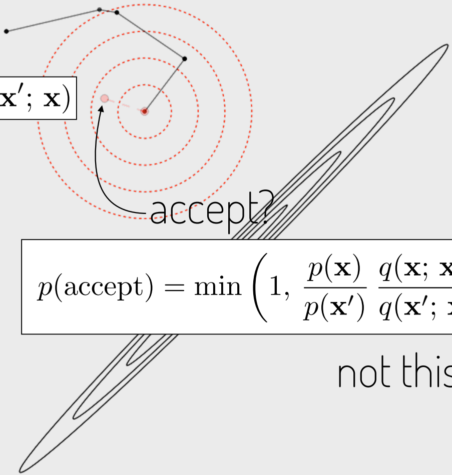
Metropolis–Hastings

in an ideal world



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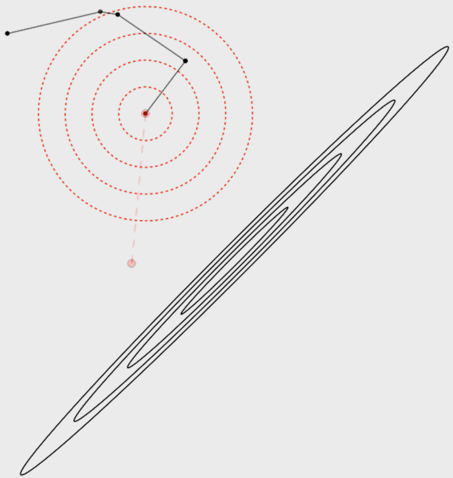
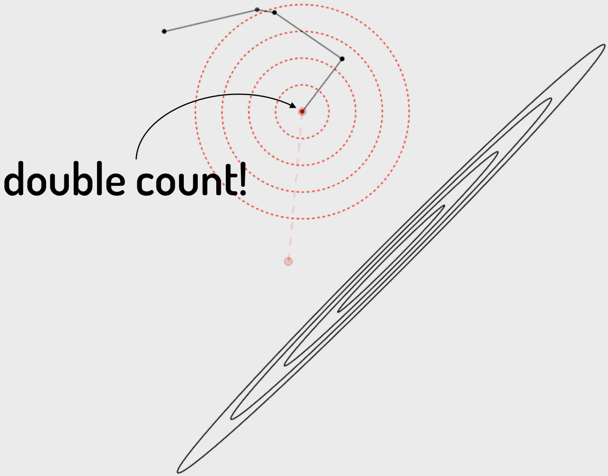
not this time.

Metropolis–Hastings

in an ideal world



Metropolis–Hastings
in an ideal world



Metropolis–Hastings
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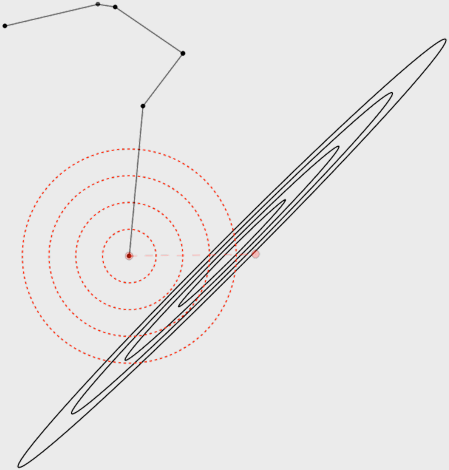

Metropolis–Hastings

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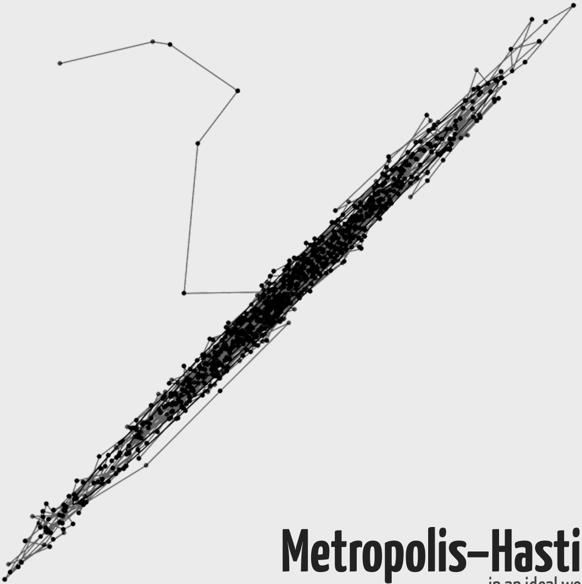
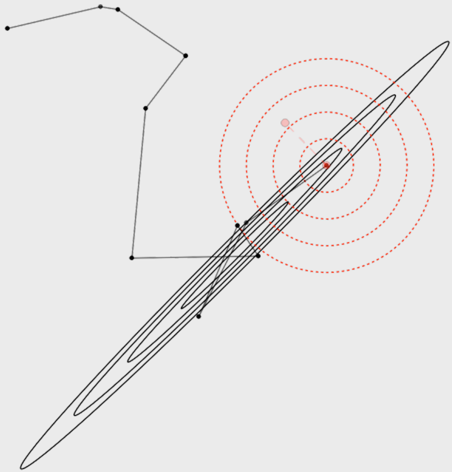
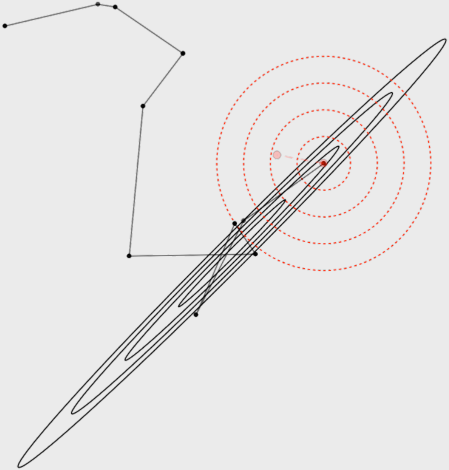


Metropolis–Hastings
in an ideal world



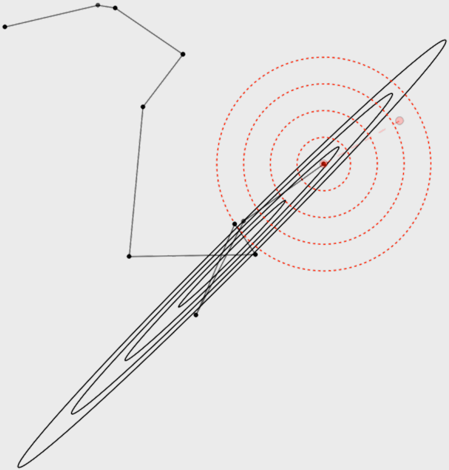
Metropolis–Hastings

in the real world

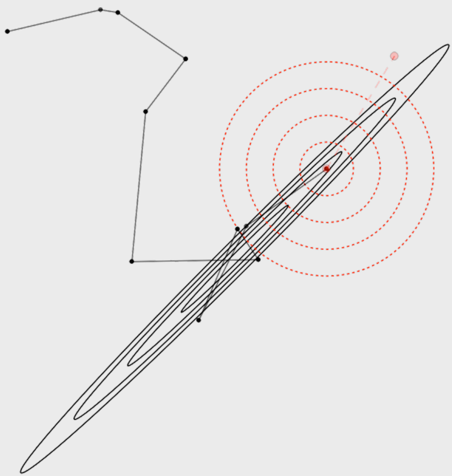


Metropolis–Hastings

in the real world



Metropolis–Hastings
in the real world



Metropolis–Hastings

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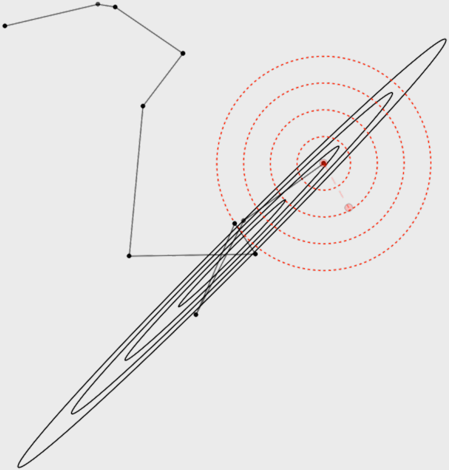
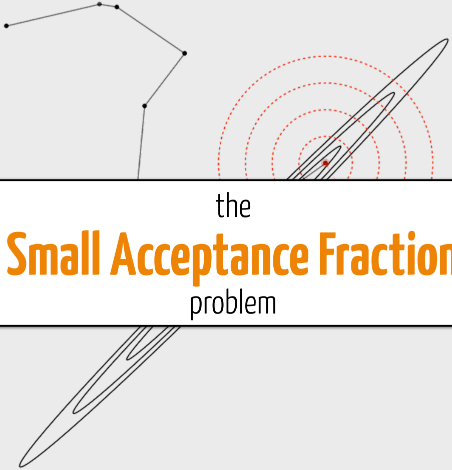


Metropolis–Hastings

in the real world

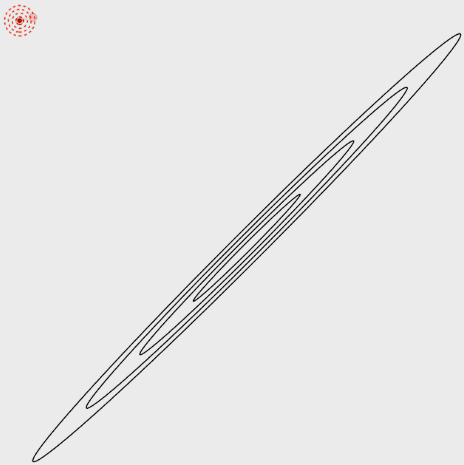


the

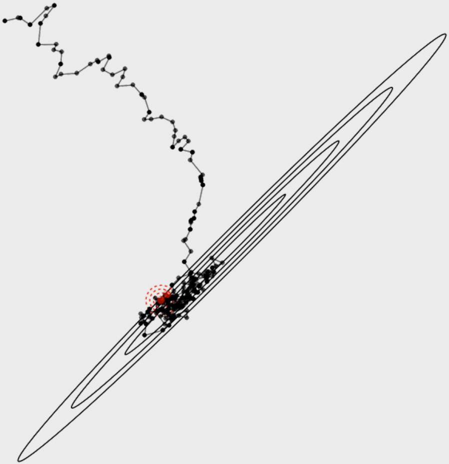
Small Acceptance Fraction

problem

Metropolis–Hastings
in the real world



Metropolis–Hastings
in the real world



Metropolis–Hastings

in the real world

the

Huge Acceptance Fraction

problem