Introduction

This is the first lab not based on the book *Getting Started with MicroPython*. In this lab you are going to explore some of the common usages of capacitors in circuits. One use is to use an RC circuit to time events or set the oscillation time of a circuit. A third usage is to filter signals in frequency, , and a third usage is to *decouple* DC, AC, and noise signals from each other. This covers a lot of territory, so let's get started!

1. Capacitors for Timing

Capacitors store charge and the basic formula is

$$V = \frac{Q}{C} \qquad \text{Eq. 1}$$

where V is the voltage across a capacitor in Volts (abbreviated V), Q is the charge on the capacitor in Coulombs (abbreviated C), and C is the capacitance value in Farads (abbreviated F.) It turns out 1 F is a very large capacitor. The capacitors we will use are in the ranges of µF, nF, or pF. When a current runs into a capacitor, charge accumulates in it and the voltage across it increases. Consider the following circuit

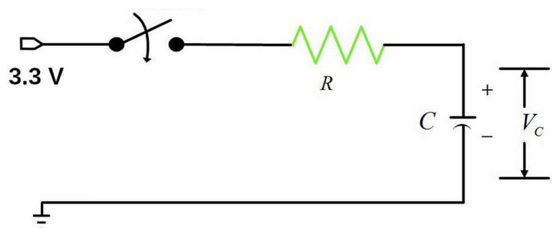

Figure 1: A charging RC circuit. If Vc starts at zero when the switch is closed, the capacitor will charge and Vc will increase. ***Note*** *3.3V is also called Vcc.*

Challenge:

- If R = 100 kΩ and C = 100 µF, what will the initial current be?
- What do you think the maximum Vc will be when the capacitor is fully charged?

The Charge – Discharge Function

You should have seen the formula for a capacitor charging of discharging through a resistor

$$V(t) = V_0 \pm V_1 e^{-t/\tau} \qquad \text{Eq. 2}$$

where $\tau = RC$.

Challenge:

- Find the value of τ in seconds for the Challenges above.

- Find the values for V_0 and V_1 for the circuit in Figure 1 when the switch is closed.

You are going to make an oscillator out the circuit below by driving the circuit with a digital output, and reading V_C with an ADC input. You need to write a program where it turns on the digital output. When V_C = ⅔V_{CC}, set the digital output to 0, when V_C = ⅓V_{CC}, turn the digital output to 1. Figure 2 shows what the waveform should look like, but ton and toff will be equal.

Super Challenge:

- How long will it take the RC circuit to charge from ⅓V_{CC} to ⅔V_{CC}?
- What will T be in terms of R, C, and V_{CC}?
- For R = 100 kΩ and C = 100 μF, what will T be?

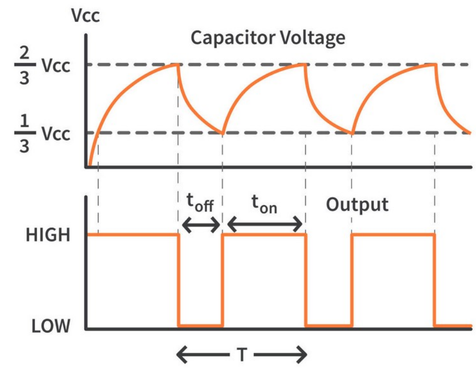

Figure 2: The charge-discharge curve for your oscillator. You will make t_on and t_off the same.

Working with Your Oscillator

1) Build the oscillator circuit with the components above, program the PicoW, and measure its frequency using an oscilloscope. In the space below, analyze your data and compare it to your calculated T. What is the percentage difference?

2) Change the components to make a 1 kHz oscillator. Is the result within the tolerance of the components? Show your work below.

2. Frequency Filters with Resistors and Capacitors.

Simple circuits, like Figure 1, have a much more important usage – they frequency filter signals. Very often when you are working with a signal, it is mostly near some frequency. If you have noise present, it usually has many frequencies present. Electronically you can eliminate noise from your measurement by using a frequency filter.

a. R-C Circuit

Impedance extends resistance to AC (Alternating Current) frequencies and to capacitors and inductors. The impedance of a capacitor of value *C* is

$$Z_C = \frac{1}{j2\pi f C} \qquad \text{Eq. 3}$$

where Z_C is the impedance, *j* is the pure complex number $\sqrt{-1}$, *f* is the frequency in Hz, and *C* is the capacitance in Farads. Notice two important differences from resistance. First, the impedance is frequency dependant, and second, it is complex. The impedance is useful because it can be used *just like resistance* in familiar series and parallel formula. The circuit in Figure 1 is a voltage divider. For resistors the output voltage would be

$$V_{out} = V_{\text{in}} \frac{R_2}{R_1 + R_2} \qquad \text{Eq. 4}$$

where R_1 is the upper resistor and R_2 the lower. For the circuit above, replace R_1 with *R* and R_2 with $\frac{1}{j2\pi f C}$.

$$V_{out} = V_{\text{in}} \frac{\frac{1}{j2\pi f C}}{R + \frac{1}{j2\pi f C}} \qquad \text{Eq. 5}$$

It takes a bit of algebra to make the denominator real, but the result is

$$V_{out} = V_{\text{in}} \frac{1 - j2\pi R C}{1 + (2\pi f R C)^2} \qquad \text{Eq. 6}$$

The *transfer function* is defined as

$$H(f) = \frac{V_{out}}{V_{\text{in}}} = \frac{1 - j2\pi R C}{1 + (2\pi f RC)^2} \qquad \text{Eq. 7}$$

This function is usually described by the *magnitude* and the *phase*. The magnitude is $|H(f)| = \sqrt{H(f) H_f^{\div}}$ where $^{\div}$ means complex conjugate. Again after some algebra, you get the final important answer sthat the amplitude of the transfer function

$$|H(F)| = \frac{1}{\sqrt{1 + (2\pi f RC)^2}} \qquad \text{Eq. 8}$$

and the phase is

$$\phi_{out} = \arctan\left(\frac{\text{Im}(H(f))}{\text{Re}(H(f))}\right) = \arctan(-2\pi f R C) \qquad \text{Eq. 9}$$

Challenges:

- What is the limiting value of the magnitude as $f \to 0$? As $f \to \infty$? When $f = 1 / 2\pi RC$?

- What is the limiting value of the phase as $f \to 0$? As $f \to \infty$? When $f = 1 / 2\pi RC$?

- What is the instructor's quick way of reasoning through what a capacitor does in an AC circuit as $f \to 0$? and $f \to \infty$?

- Why is this called a *low-pass* filter?

Last observation: Note that the combination $2\pi f\, RC$ occurs several times in the formulas and it has to be a unit less quantity. The combination $2\pi f\, RC$, then has to have units of 1 / *Hz*. The quantity $f_0 = \frac{1}{2\pi RC}$ is the *characteristic frequency* of the filter. Eq. 7 & 8 can then be written

$$|H(f)| = \frac{1}{\sqrt{1+(f/f_0)^2}} \qquad \text{Eq. 10}$$

The phase is

$$\phi_{out} = \arctan(-f/f_0) \qquad \text{Eq. 11}$$

Now it is easy to see when $f = f_0$, the amplitude is ½ and the phase is -45º.

b. C-R Circuit

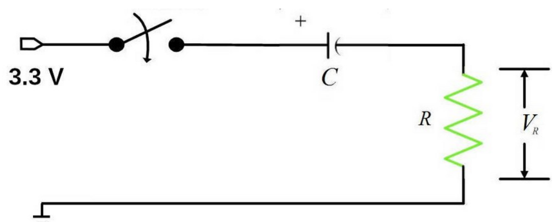

Figure 3: A "C-R" circuit. This can be analyzed as a voltage divider, but using the impedance for the capacitor.

Consider the circuit in Figure 3. In this case

$$V_{out} = V_{\text{in}} \frac{R}{R + \frac{1}{j2\pi f C}} \qquad \text{Eq. 12}$$

The algebra is still a mess, but the result is

$$|V_{out}| = |V_{\text{in}}| \frac{(f/f_0)}{\sqrt{1+(f/f_0)^2}} \qquad \text{Eq. 13}$$

The phase is

$$\phi_{out} = \arctan\left(f_0/f\right) \qquad \text{Eq. 14}$$

Note: the fraction is reversed in the phase.

Challenge (his should be easy now!)

- What is the limiting value of the magnitude as $f \to 0$? As $f \to \infty$? When $f = f_0$?
- What is the limiting value of the phase as $f \to 0$? As $f \to \infty$? When $f = f_0$?
- Why is this called a *high-pass* filter?

Exercise: Measuring Filters

The goal of this exercise is to build these two filters and test them using the PicoW. You have to extend the DAC work you did in the lab on *Interfaces* to step through a series of frequencies, and for each frequency, measure the amplitude of the transfer function, $H(f)$.

1. Modify the Code to Step Frequency.

You are going to build a filter with f_0 about 1 kHz. To the the formulas it is good to make measurements from $f_0/10$ to 10 f_0 (100 – 10,000 Hz). A general principle of experimental design, is *whenever your experimental variable varys by more a factor of three, use logarithmic spacing*. So the *logarithms* of your frequency should be equally spaced. A code fragment showing how to do this is below. Copy and paste this into Thonny and run it. You will see each frequency is about 10% larger than the previous one, and the significant digits repeat in each decade. You will have to adapt it to your own program.

```
freq_start =   100.0
freq_stop  = 10000.0
n_freq = 21

freq_steps = np.linspace(np.log(freq_start), np.log(freq_stop), n_freq)
for freq in freq_steps:
    print(freq, " Hz")
```

2. Wire the Circuit

Wire the DAC output to the low pass RC Circuit in Figure 1. Use f_0 = 1 kHz, and R = 100 kΩ to calculate the capacitor value. From the assortment of capacitor, choose the closes value. Calculate the expected f_0 below:

Expected f_0:

3. Write Program to Process and Print the Data

Like the program from Lab on Interfaces, you need to program PicoW to output a sine wave, then read the value with the DAC. This time however, you want to read V_{out} from the RC filter.

This time, however, instead of printing the DAC data, you want to calculate and print the amplitude of the filter's transfer function.

For an AC signal of amplitude V_0 , the AC *RMS amplitude* is defined as

$$V_{RMS} = \sqrt{\frac{1}{T} \int_{t=0}^{T} \left(V(t) - V_{avg}\right)^2} \qquad \text{Eq. 15}$$

For discrete data, you can calculate this using the formula below.

$$V_{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(V_i - V_{avg}\right)^2} \qquad \text{Eq. 15}$$

and V_{avg} is the average of the sine wave.

Programming Trick

It is common to calculate the RMS value of data. You can do this by keeping a running sum of the data values and the squares of the data values. Here is a code fragment that does this. You will have to adapt it to your particular case:

```
# The number of data points to take is n_points

data_sum = 0.0
data_square = 0.0
for i_point in range(n_points):
    data = get_data()
    data_sum += data
    data_square += data**2
data_average = data_sum / n_points
data_rms = math.sqrt(data_square / n_points - data_average**2)
```

You want your program to:

- Step through frequencies.
- At each frequency take data and calculate the average and RMS.
- Print the frequency, average, and RMS.
- You should save this data in a CSV file.

Analysis

Every time you take data you need to plot and analyze it. Refer to the *Python Skills* assignment to plot your data. You should plot the RMS versus frequency on both a linear and a log frequency scale. Also, each plot should include a model line of $|H(\mathrm{f})|$ using Equation 8.

Repeat with an RC High Pass Filter