The Physics Teacher

PAPERS | JANUARY 01 2023

Determining Charge Stored on a Capacitor Using Numerical Integration with a Spreadsheet

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Phys. Teach. 61, 31–33 (2023)
https://doi.org/10.1119/5.0077442

Determining Charge Stored on a Capacitor Using Numerical Integration with a Spreadsheet

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Experiments involving RC circuits are an integral part of introductory physics courses. Previous articles using RC circuits describe determining the charge on a Van de Graaff generator,[1] measuring currents and voltages in a charging RC circuit using a multimeter[2] or an Arduino,[3,4] measuring the decay constants of complex RC circuits,[5] and using the Pasco ScienceWorkshop 750 Interface to examine various physical properties of capacitors.[6] In this paper, I will show how to determine the capacitance of a capacitor charged with a battery of known voltage using an RC circuit and numerical integration performed on a spreadsheet.

To determine capacitance, the charge stored on the capacitor must be found. Charge can be calculated by integrating the current flow over time. This paper will provide specific suggestions on how to determine the charge by numerically integrating the measured current using a spreadsheet. Students will explicitly be shown how to perform numerical integration on current measurements using a spreadsheet to obtain the stored charge. Teaching students how to use numerical integration in this manner supports efforts to teach computational thinking, which is now recognized as a fundamental goal in science education.[7] The charge found through numerical integration can be used along with the initial voltage used to charge the capacitor to experimentally find the capacitance. This method will typically determine the capacitance to within 10%.

Theory

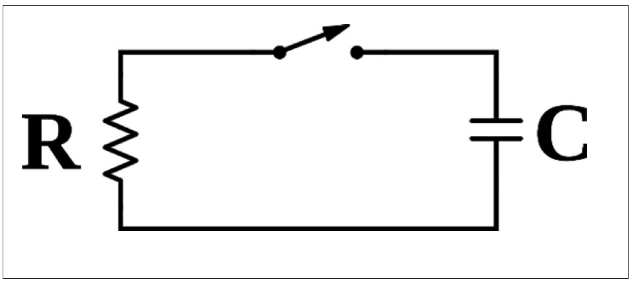

Fig. 1. Discharging RC circuit.

For this experiment, a charged capacitor is connected to a switch and a resistor (see Fig. 1). When the switch is closed, the current I in the circuit is given by[8]

$$I(t) = I_0 e^{-\left(\frac{t}{RC}\right)}, \tag{1}$$

where I_0 is the initial current in the circuit after the switch is closed, R is the resistance in the circuit, C is the capacitance in the circuit, and t is the time.

The total charge stored in the capacitor Q can be found by integrating the current $I(t)$, which is equivalent to finding the area under the $I(t)$ curve[9]:

$$Q = \int_0^\infty I(t)\,dt. \tag{2}$$

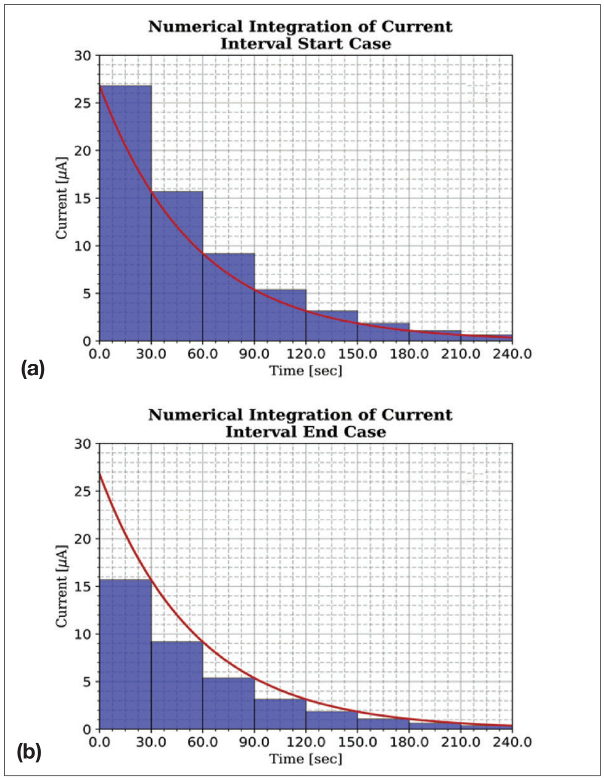

Fig. 2. Numerical integration of current. (a) Overestimation of charge. (b) Underestimation of charge.

This integral (and the area under the curve) can be approximated using a sum over N finite time increments of length δt[10]:

$$Q \approx \sum_{i=1}^{N} I(t_i)\delta t = I(t_1)\delta t + I(t_2)\delta t + \cdots + I(t_N)\delta t. \tag{3}$$

There are two simple ways to calculate this sum. The first method uses the current at the start of each time interval (i.e., $t_1 = 0$, $t_2 = \delta t$, $t_3 = 2\delta t \ldots$). As seen in Fig. 2(a), this method overestimates the area under the curve and the charge stored on the capacitor. The second method uses the current at the end of each time interval (i.e., $t_1 = \delta t$, $t_2 = 2\delta t$, $t_3 = 3\delta t \ldots$). Figure 2(b) shows that this method underestimates the charge.

A good balance between the two methods is to use the average current over each interval:

$$Q \approx \sum_{i=1}^{N}\left(\frac{I(t_{i-1}) + I(t_i)}{2}\right)\delta t = \left(\frac{I(0) + I(t_1)}{2}\right)\delta t + \left(\frac{I(t_1) + I(t_2)}{2}\right)\delta t + \cdots. \tag{4}$$

Figure 3(a) shows that this sum over the averages has parts that overestimate and other parts that underestimate the area over each time interval. This method is equivalent to calculating the area of each segment using trapezoids rather than rectangles (i.e., the trapezoidal method).[11] Figure 3(b) shows that the trapezoidal approach is effective in estimating the

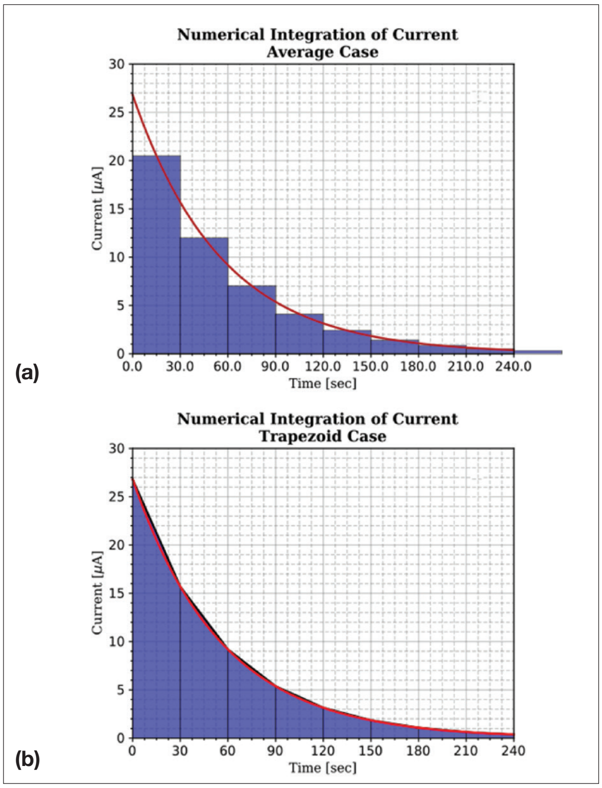

Fig. 3. Charge calculation using (a) average (midpoint) over interval and (b) trapezoidal method.

area under the curve. This average-over-interval method is used to determine the charge stored on a capacitor.

When introducing this experiment to students, the graphs in Fig. 2 could be shown to students. After showing these graphs, the instructor could, by asking guiding questions, lead the students to the idea of using the average across each interval rather than the values at the beginning or end of the interval. The graphs in Fig. 3 could then be shown to the students to show how this "average over interval" works better.

Methodology

The components used in this experiment are shown in Table I. These components are connected using the circuit diagram shown in Fig. 4(a). Figure 4(b) shows how the physical elements are connected.

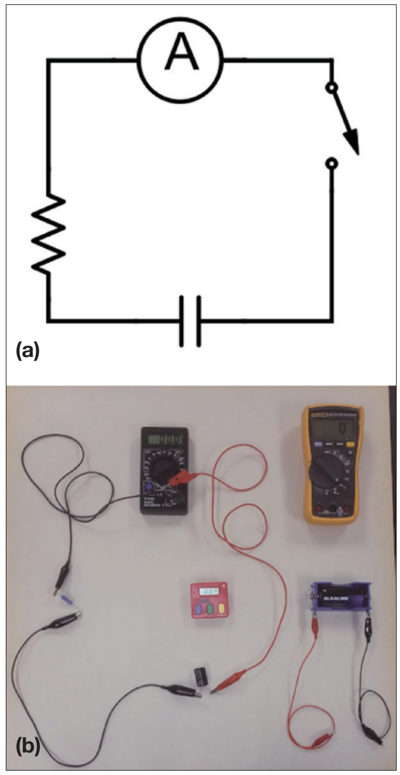

Fig. 4. Experimental setup. (a) Circuit diagram. (b) Picture showing connections.

Before the circuit is constructed, the battery voltage V_0 and resistance of the resistor R should be measured using the multimeter. The physical circuit does not include a switch. Instead, the ammeter remains unconnected until after the capacitor is charged. Charging the capacitor with a 1.5-V D battery and using an 82-kΩ resistor should produce a maximum current of about 18 μA. With this in mind, the multimeter should be able to measure currents in the microamp range. With these values, the ammeter should be set to the 200-μA range. Using a higher voltage to charge the capacitor tends to make the experiment less accurate.

After the capacitor is charged with the 1.5-V battery, the ammeter is connected, and the current in the circuit is measured every 30 s. Students must carefully watch the ammeter to record the initial maximum discharge current $I(0)$.

With the values stated above, the current will fall below the level detectable by the multimeter on the 200-μA scale within about 8 min. Measuring the current every 30 s leads to about 16 data points. As shown in Fig. 5, the data can be quickly analyzed using a spreadsheet.

The experimental value of the capacitance C_{exp} is found by dividing total charge Q_{total} by the initial voltage V_0 used to charge the capacitor:

$$C_{exp} = \frac{Q_{total}}{V_0}. \qquad (5)$$

In principle, the initial voltage V_0 should be equal to the initial current $I(0)$ divided by the resistance R [$V_0 = I(0)/R$]. However, the ammeter has a finite response time to measure the current in the circuit. This finite response time means that the initial current measured by the ammeter is smaller than the instantaneous maximum current in the circuit. For this reason, $I(0)/R$ should not be used to find the initial voltage across the capacitor V_0. Instead, the voltage across the battery should be measured

Table I. Materials list (prices as of fall 2022).

Item	Manufacturer/ Retailer	Model Number	Price
Digital (non-rescaling) multimeter	Amazon[a]	DT-830B	$8.99
D battery	Digikey[b]	547-813BULKJ-ND	$1.78
82-kΩ resistor	Digikey[b]	BC4539CT-ND	$0.64
1000-μF bipolar electrolytic capacitor	Digikey[b]	493-12662-1-ND	$0.85
1 alligator-to-alligator connecting lead	Digikey[b] (pack of 10)	1927-1085-ND	$2.25
2 alligator-to-banana leads	Digikey[b]	290-1974-ND	$5.94
Capacitance meter	Digikey[b]	FLUKE-115	$248.99
Timer	Ward's Science[c]	470149-406	$9.50
D battery holder	Ward's Science[c]	470004-252	$4.85

[a] https://www.amazon.com; [b] https://www.digikey.com; [c] https://wardsci.com/store/.

Time [sec]	I [uA]	Iave [uA]	dQ [uC]
0		-----------	--------
30		$=\frac{I(0)+I(30)}{2}$	$=I_{ave}(30)*30$
60		$=\frac{I(30)+I(60)}{2}$	$=I_{ave}(60)*30$
⋮	⋮	⋮	⋮
600		$=\frac{I(570)+I(600)}{2}$	$=I_{ave}(600)*30$
		$Q_{Total}=$	$=\Sigma dQ_i$

Fig. 5. Total charge calculation using a spreadsheet.

using the multimeter and used for the initial voltage V_0.

After the experimental value of the capacitance is found, the actual value can be found using a capacitance meter. As stated by manufacturers, typical uncertainties for electrolytic capacitors are ±20%. For best results, a capacitance meter should be used to determine the actual capacitance of the capacitor. With the values given in this paper, typical students will be able to determine the capacitance found using the capacitance meter to within 10%.

Conclusion

Since the average-over-interval method shown here is equivalent to using the trapezoidal method, the procedure could be simplified by applying the trapezoidal method[11]:

$$Q_{\text{total}} \approx \frac{1}{2}I(0)+I(t_1)+I(t_2)+\cdots+I(t_{N-1})+\frac{1}{2}I(t_N). \quad (6)$$

However, this simplification obscures the role of using average values over the interval. Using an average-over-interval approach is consistent with other experiments with parameters that change over time. An average-over-interval approach also makes this experiment understandable to non-calculus-based (i.e., trigonometry-based) physics students.

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