The interstellar cascade I: the world's largest turbulent MHD box experiment

turbulence… it's less turbulent than we think

Château de Goutelas

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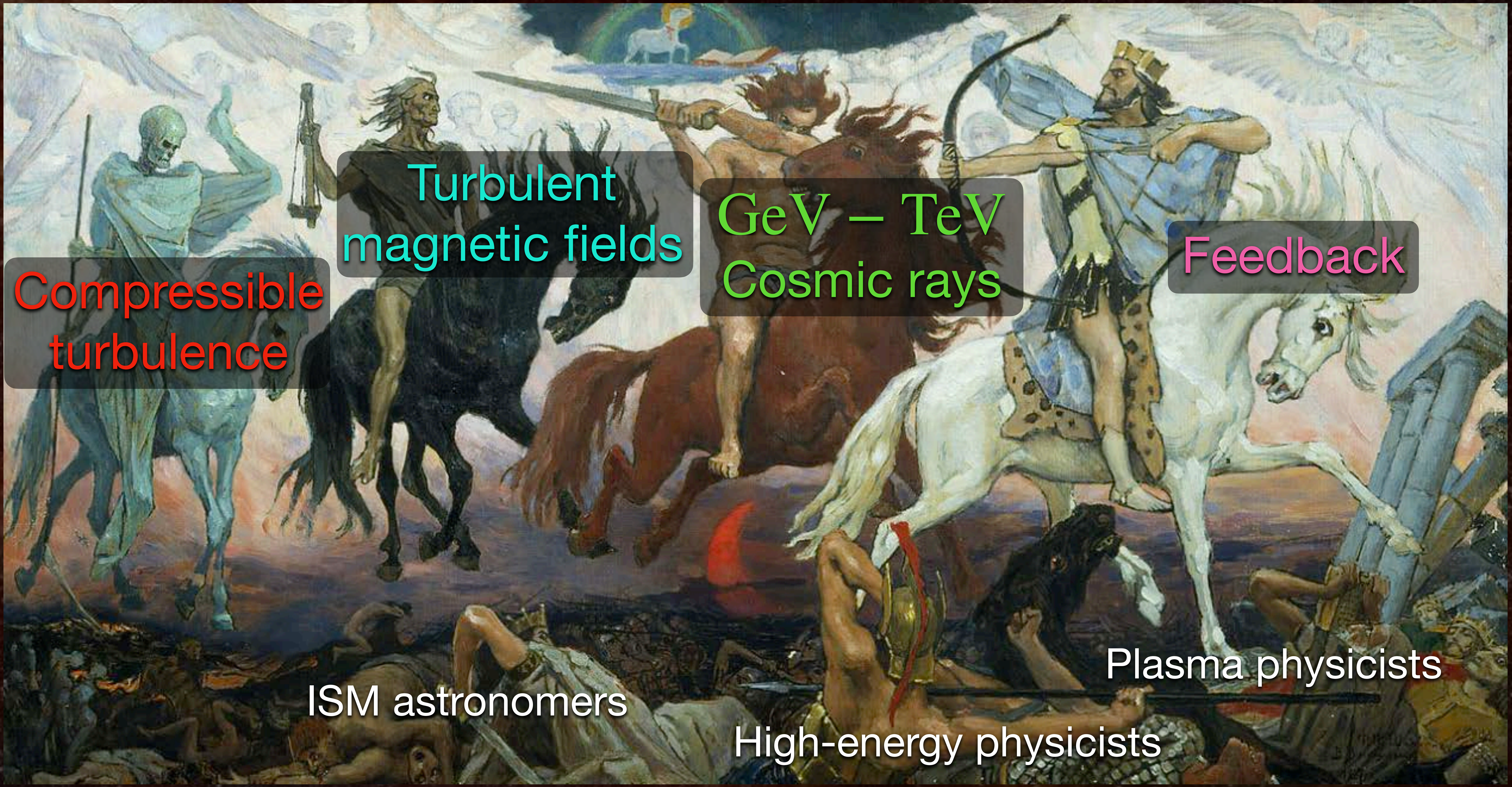
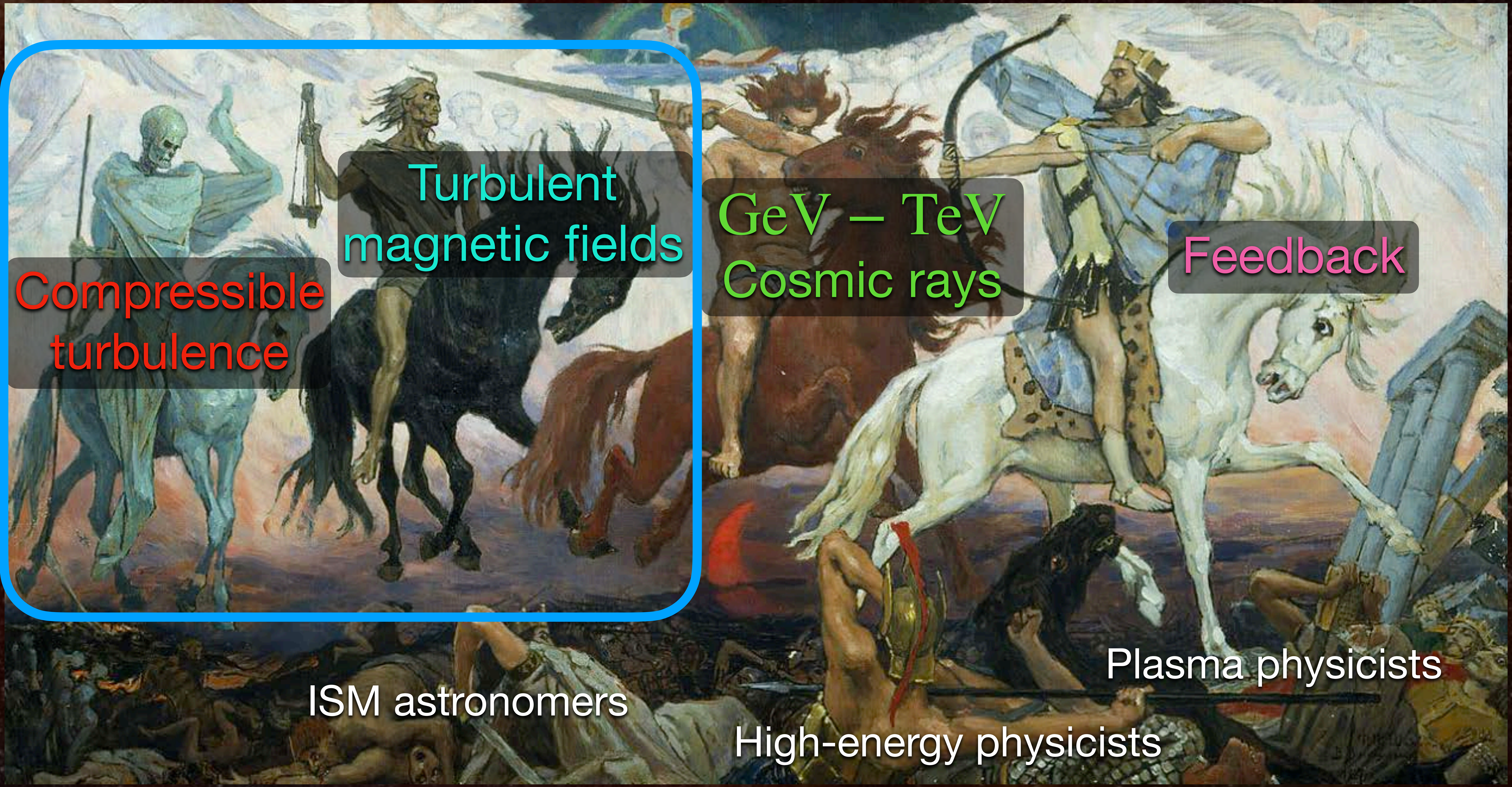
Princeton / CITA

In collaboration: Amitava Bhattacharjee (Princeton),
Christoph Federrath (ANU), Ralf Klessen (UH), Salvatore Cielo (LRZ)

My four horsemen of ISM physics



My four horsemen of ISM physics



Turbulence

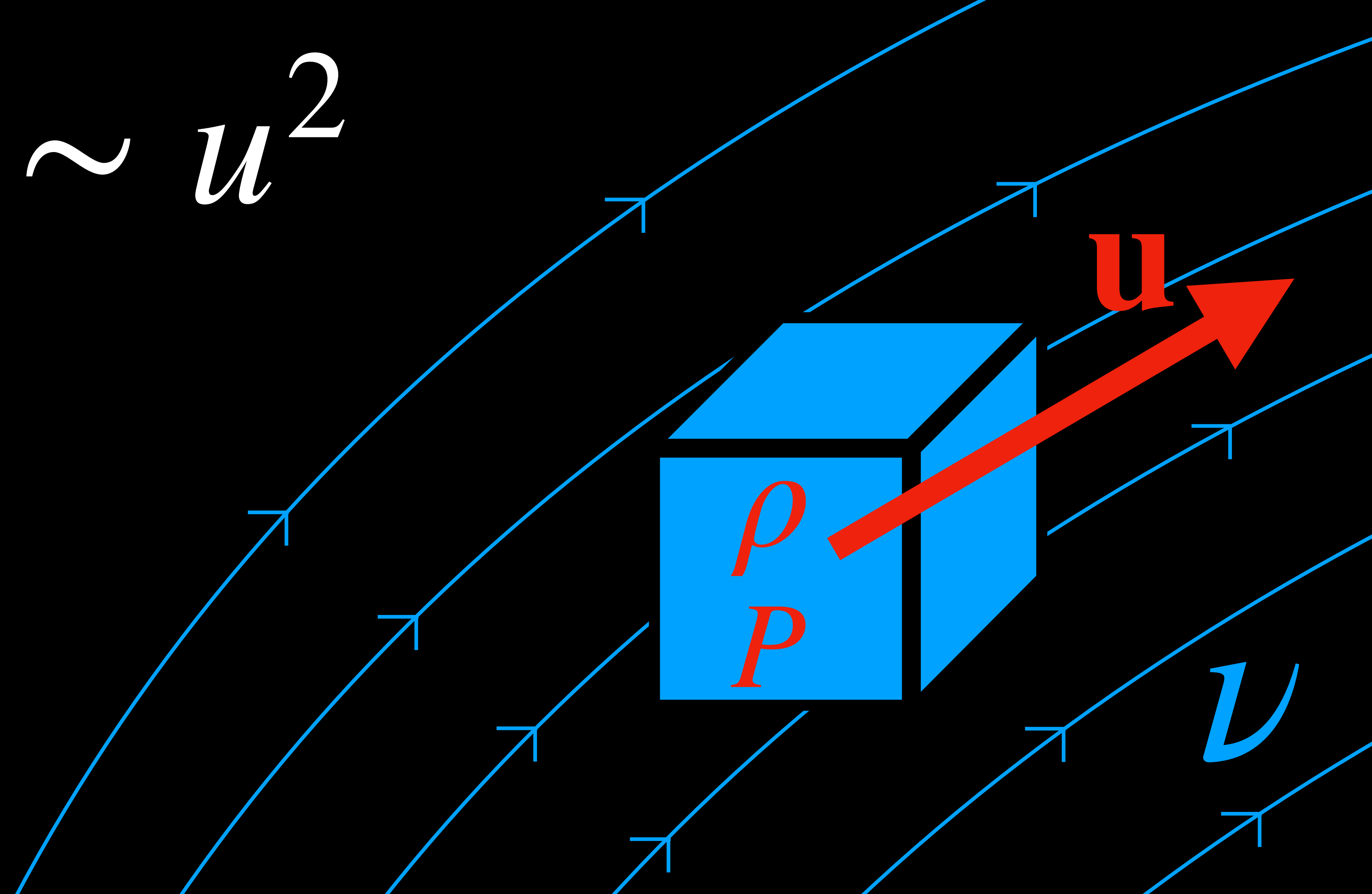
What is it (hand wavey)?

Momentum conservation for a hydrodynamical fluid element

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = - \nabla \cdot P\mathbb{I} + 2\nu \nabla \cdot (\rho \mathcal{S})$$

$$\mathbf{u} \otimes \mathbf{u} = u_i u_j \sim u^2$$

$$\lambda_{\mathrm{mfp}}/L \ll 1$$



Turbulence

What is it (hand wavey)?

Reynolds stress

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla \cdot P\mathbb{I} + 2\nu\nabla \cdot (\rho \mathcal{S})$$

$$\mathbf{u} \otimes \mathbf{u} = u_i u_j \sim u^2$$

Viscous stress

Turbulence

What is it (hand wavey)?

quadratic nonlinearity

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla \cdot P\mathbb{I} + 2\nu \nabla \cdot (\rho \mathcal{S})$$

$$\mathbf{u} \otimes \mathbf{u} = u_i u_j \sim u^2$$

Smooths out nonlinear things in the fluid

Turbulence

What is it (hand wavey)?

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = -\nabla \cdot P\mathbb{I} + 2\nu\nabla \cdot (\rho \mathcal{S})$$

Creating nonlinear things in the fluid

$$\mathrm{Re} = \frac{|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|}{|2\nu\nabla \cdot (\rho \mathbb{S})|} \sim \frac{UL}{\nu}$$

Smoothing out nonlinear things in the fluid

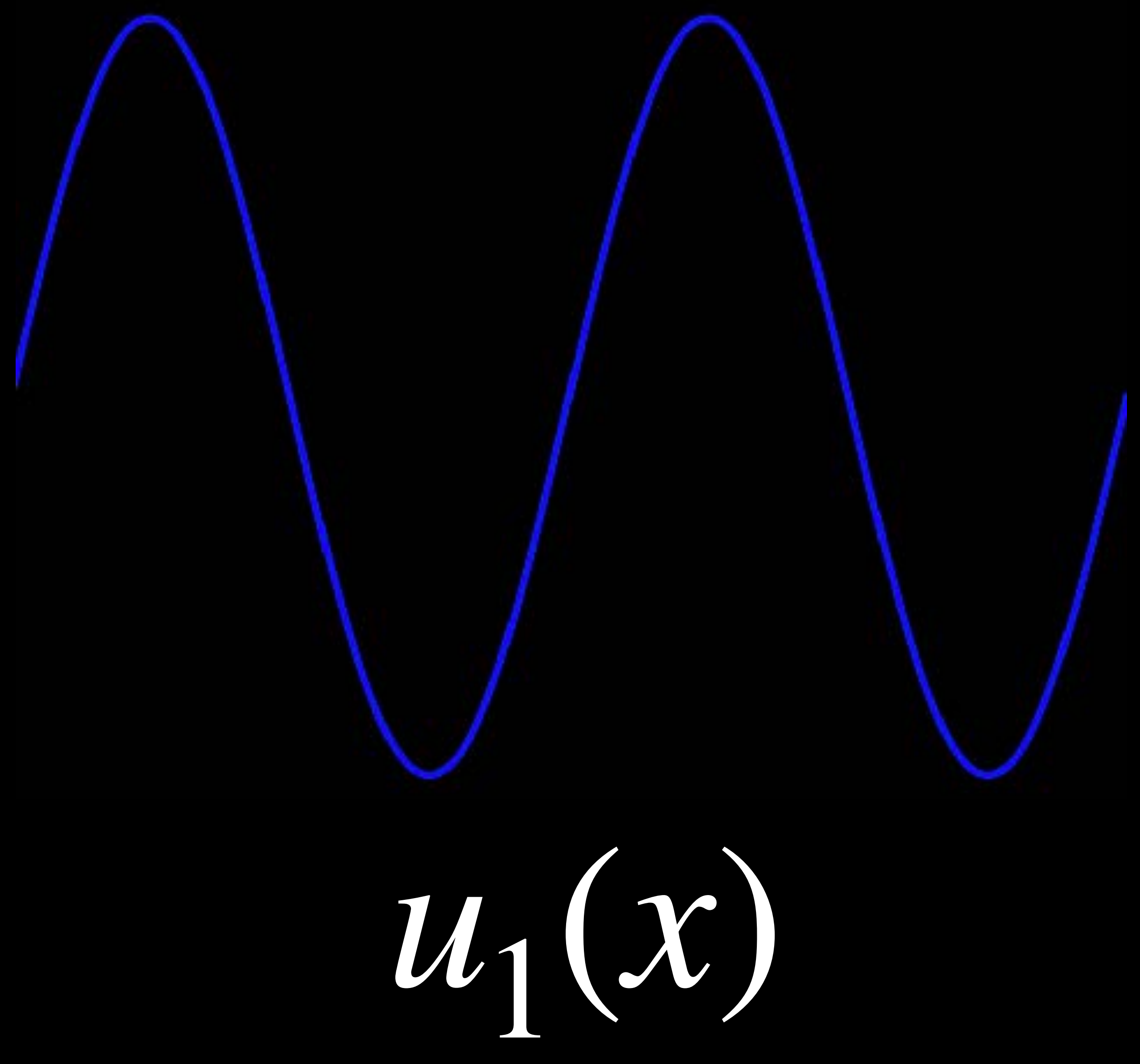
Turbulence

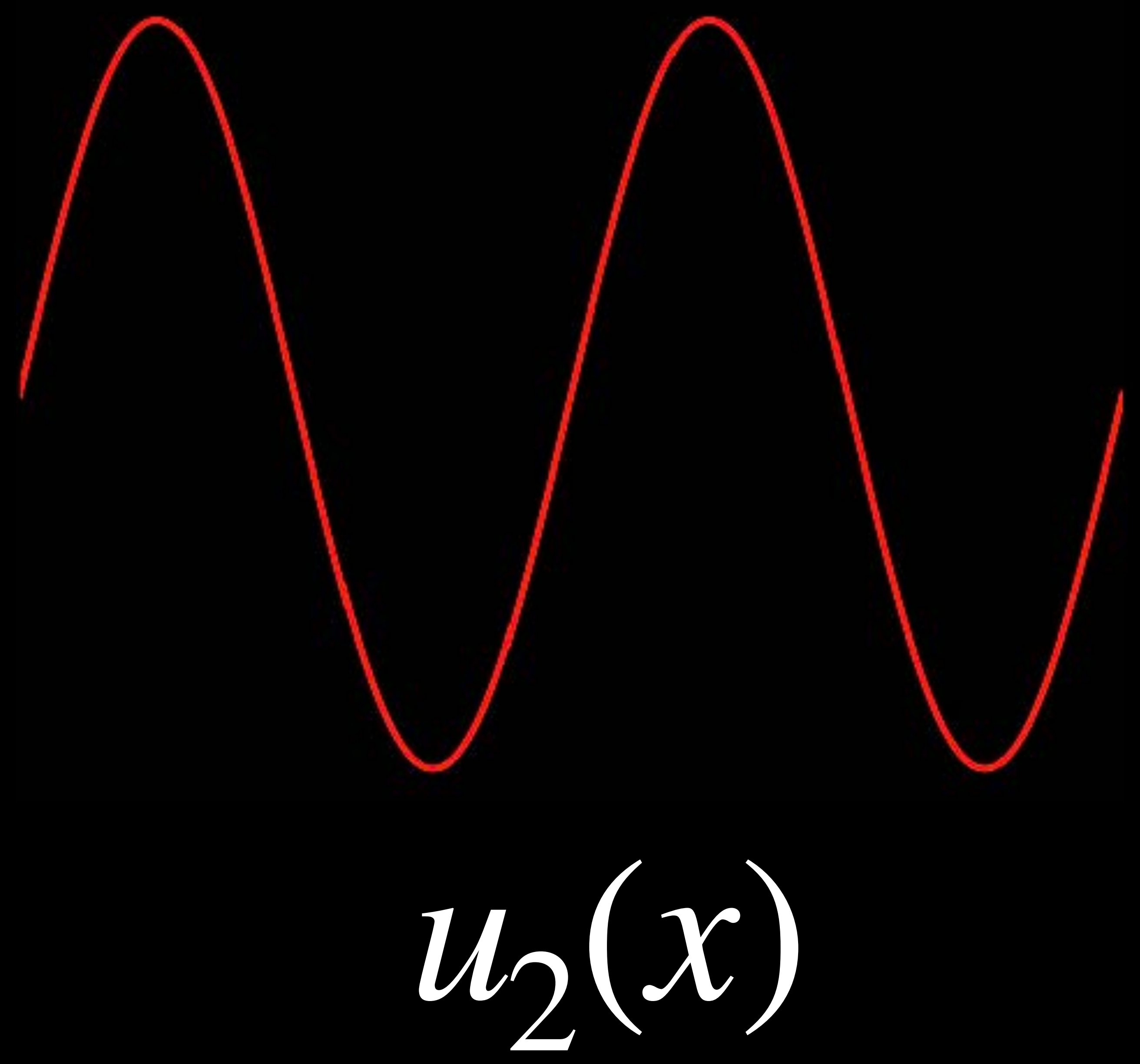
What does $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$ want to do?

$$k_1 = k_2$$

Consider two waves

$$u_1(x) = \sin(k_1 x)$$
$$u_2(x) = \sin(k_2 x)$$





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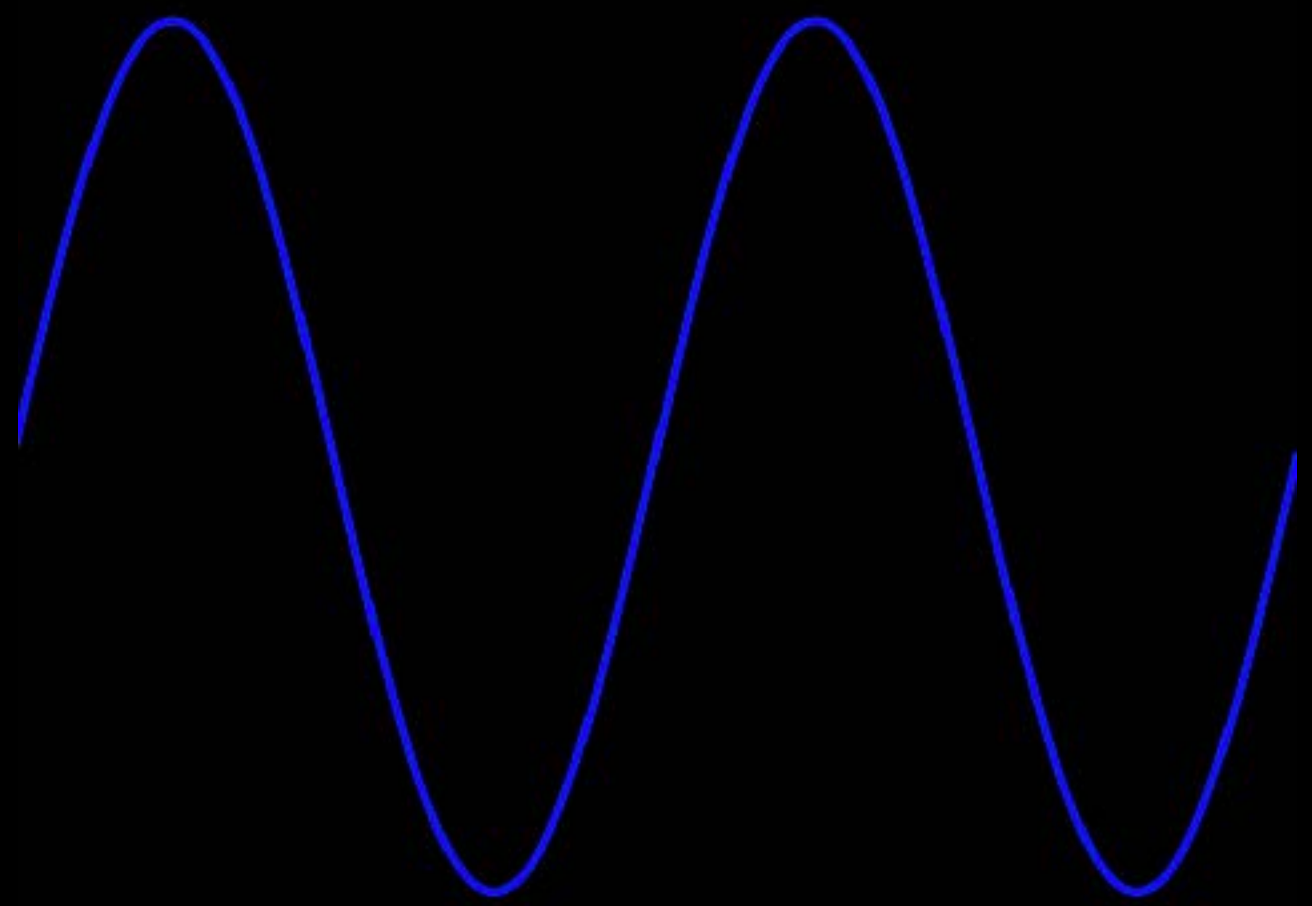
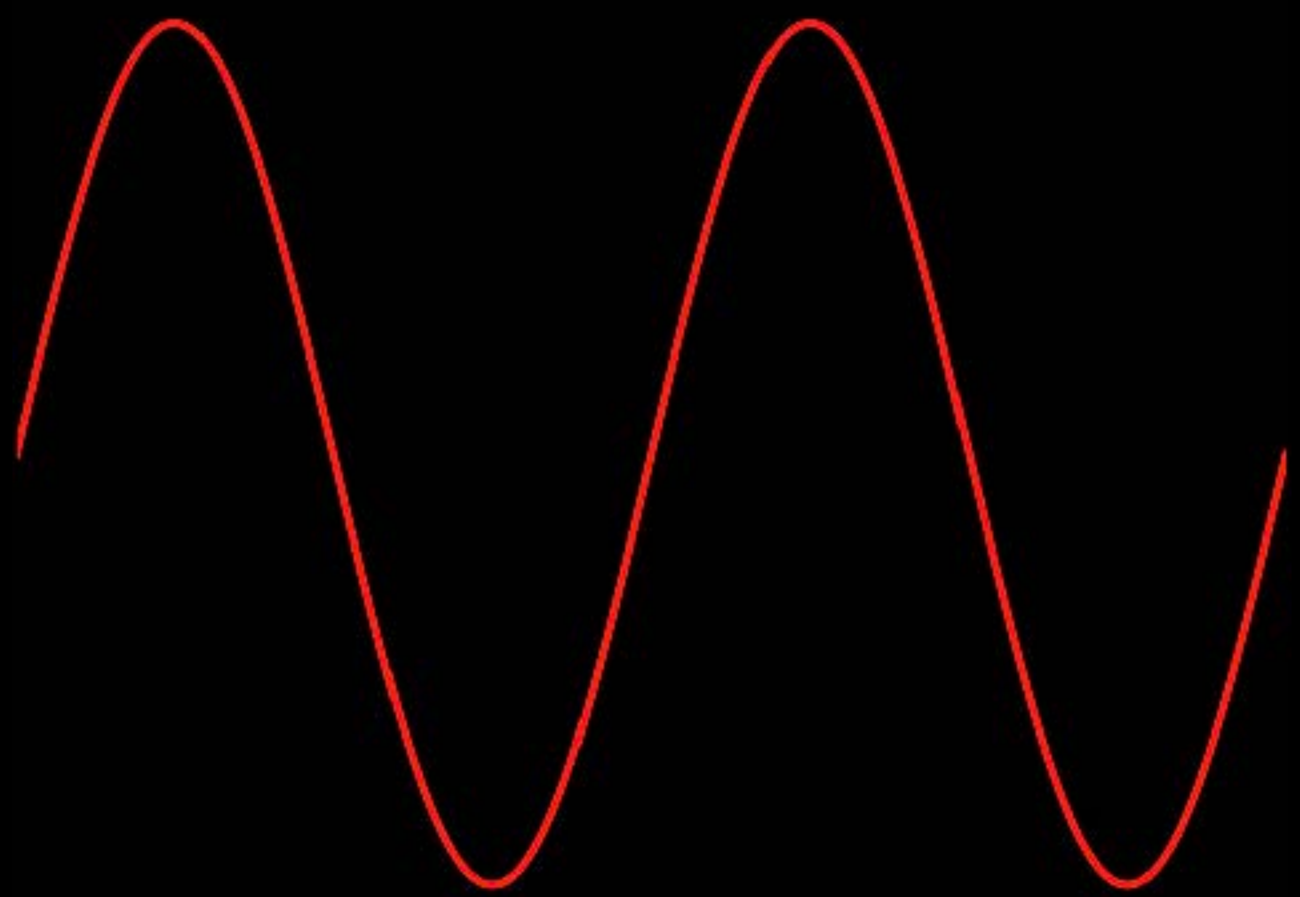




The nonlinear term creates a new wave

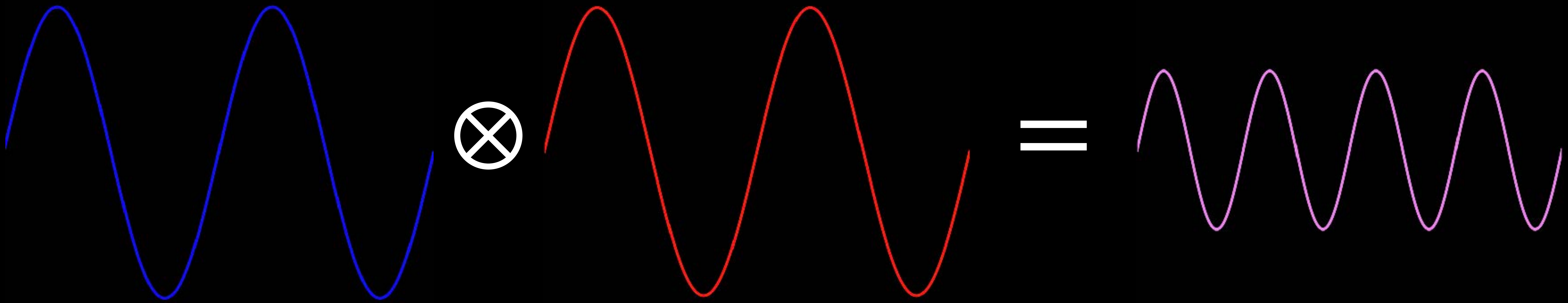
$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) \sim u_1 \partial_x u_2 = k_2 \sin(k_1 x) \cos(k_2 x) \propto \sin(k_3 x)$$

$$k_1 + k_2 = k_3$$ (momentum conservation)

Turbulence

What does $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$ want to do — cascade!!!

$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$$



$$k_1 + k_2 = k_3$$

Turbulence

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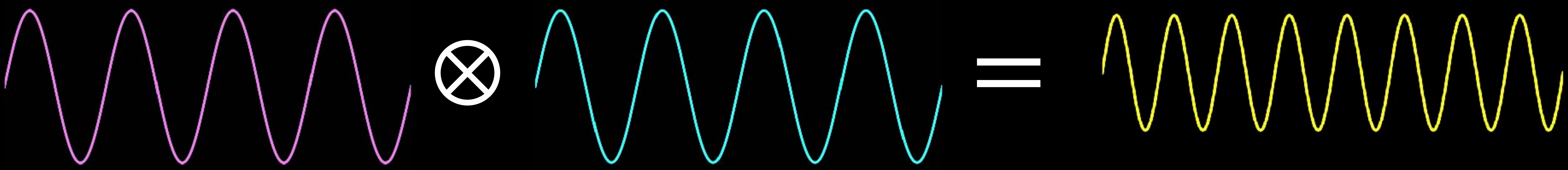
$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$$

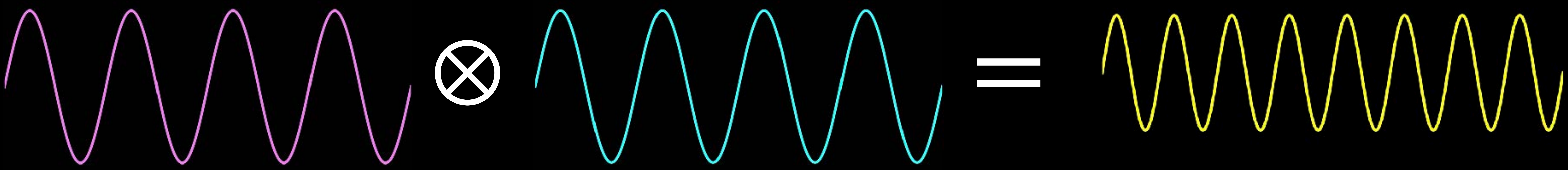


$$k_3 + k_4 = k_5$$

Turbulence

What does $\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$ want to do — cascade!!!

$$\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})$$



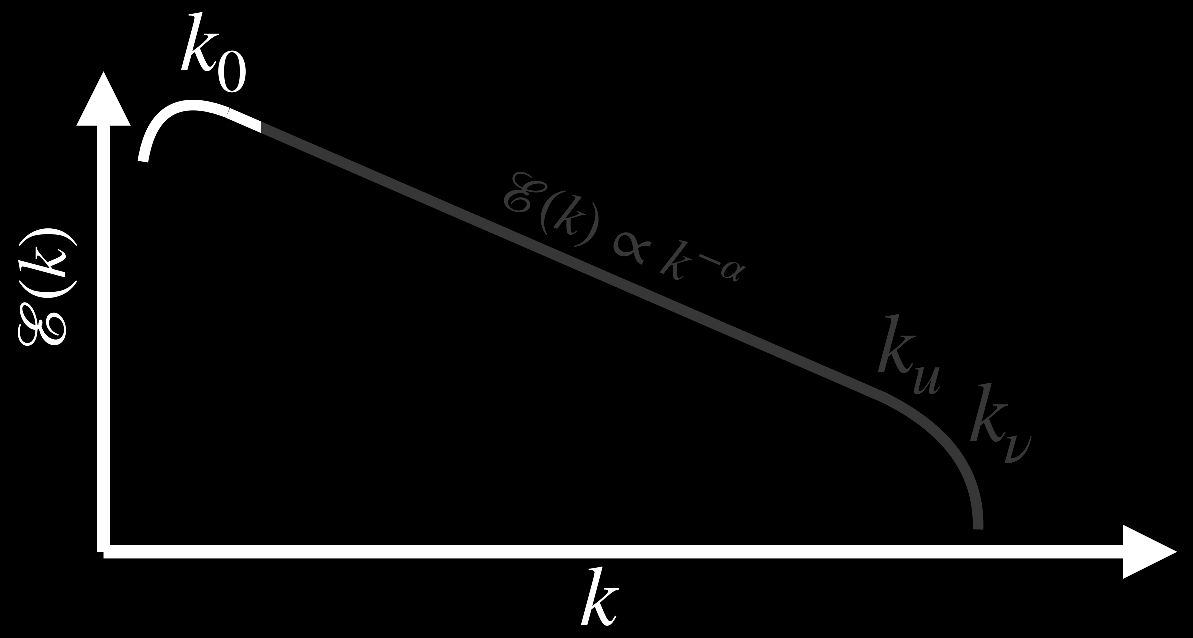
$$k_3 + k_4 = k_5$$

New waves modes created on nonlinear timescales:

$$t_{\rm nl} \sim [\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})]^{-1/2} \sim \ell / u$$

Turbulence

Cascade



WIM: $\mathrm{Re} \sim 10^7$ $\lambda_{\mathrm{mfp}} \sim 5\ \mathrm{R}_\oplus$

WNM: $\mathrm{Re} \sim 10^7$ $\lambda_{\mathrm{mfp}} \sim 1.3\ \mathrm{AU}$

CNM: $\mathrm{Re} \sim 10^{10}$ $\lambda_{\mathrm{mfp}} \sim 0.3\ \mathrm{pc}$

Ferrière, 2020; Plasma Physics and Controlled Fusion

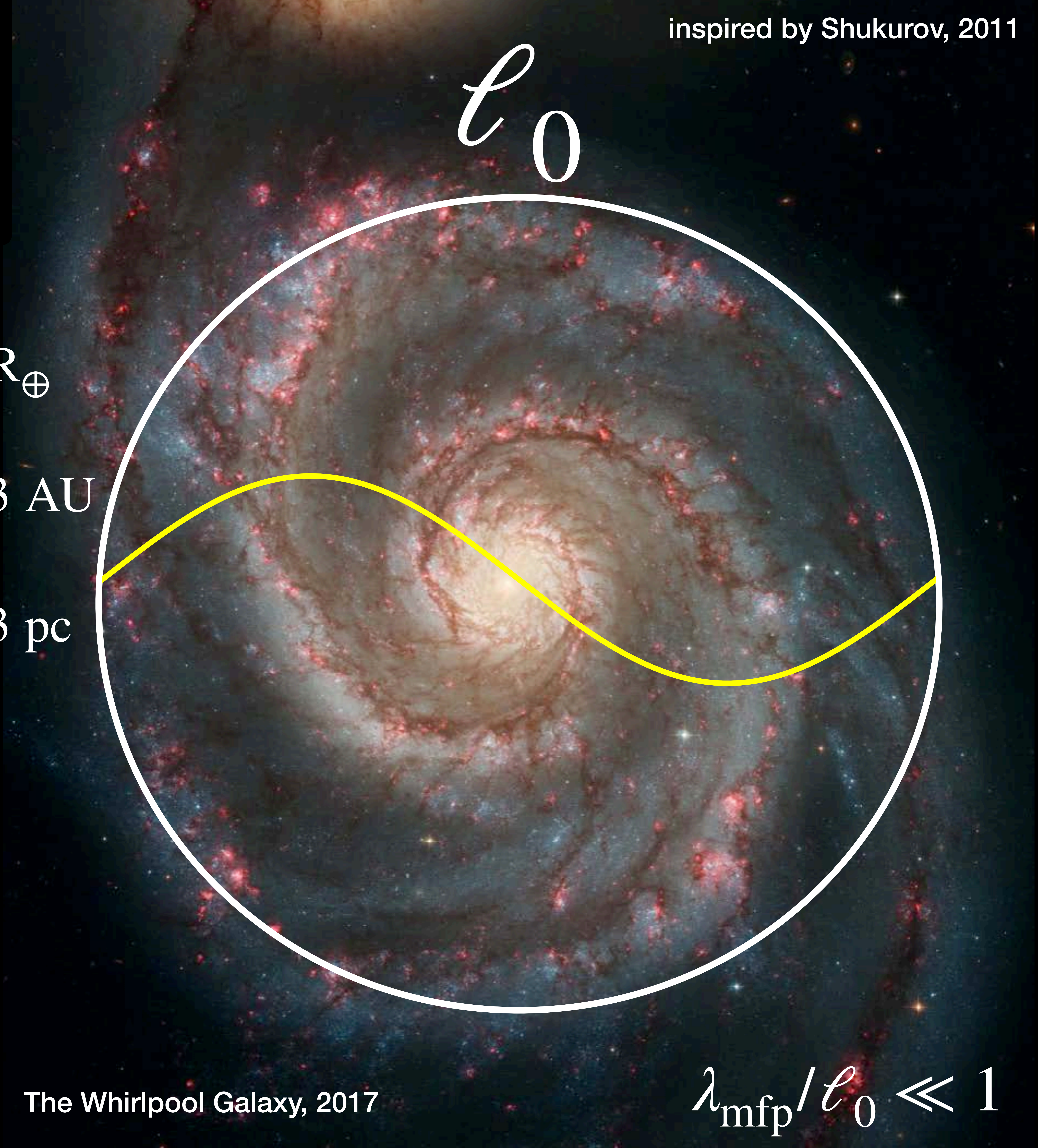
The quadratic nonlinear term

$$|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|$$

dominates on large scales, ℓ

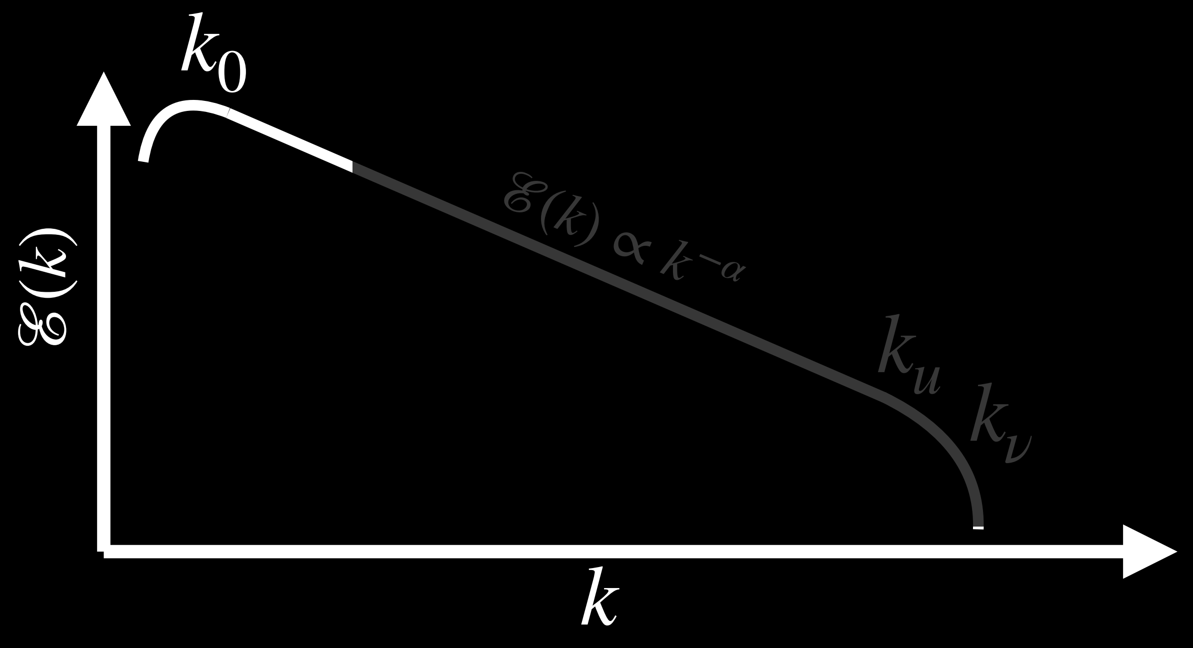
Credit: NASA, ESA, S. Beckwith (STScI) and the Hubble Heritage Team (STScI/AURA)

inspired by Shukurov, 2011



The Whirlpool Galaxy, 2017

Turbulence
Cascade



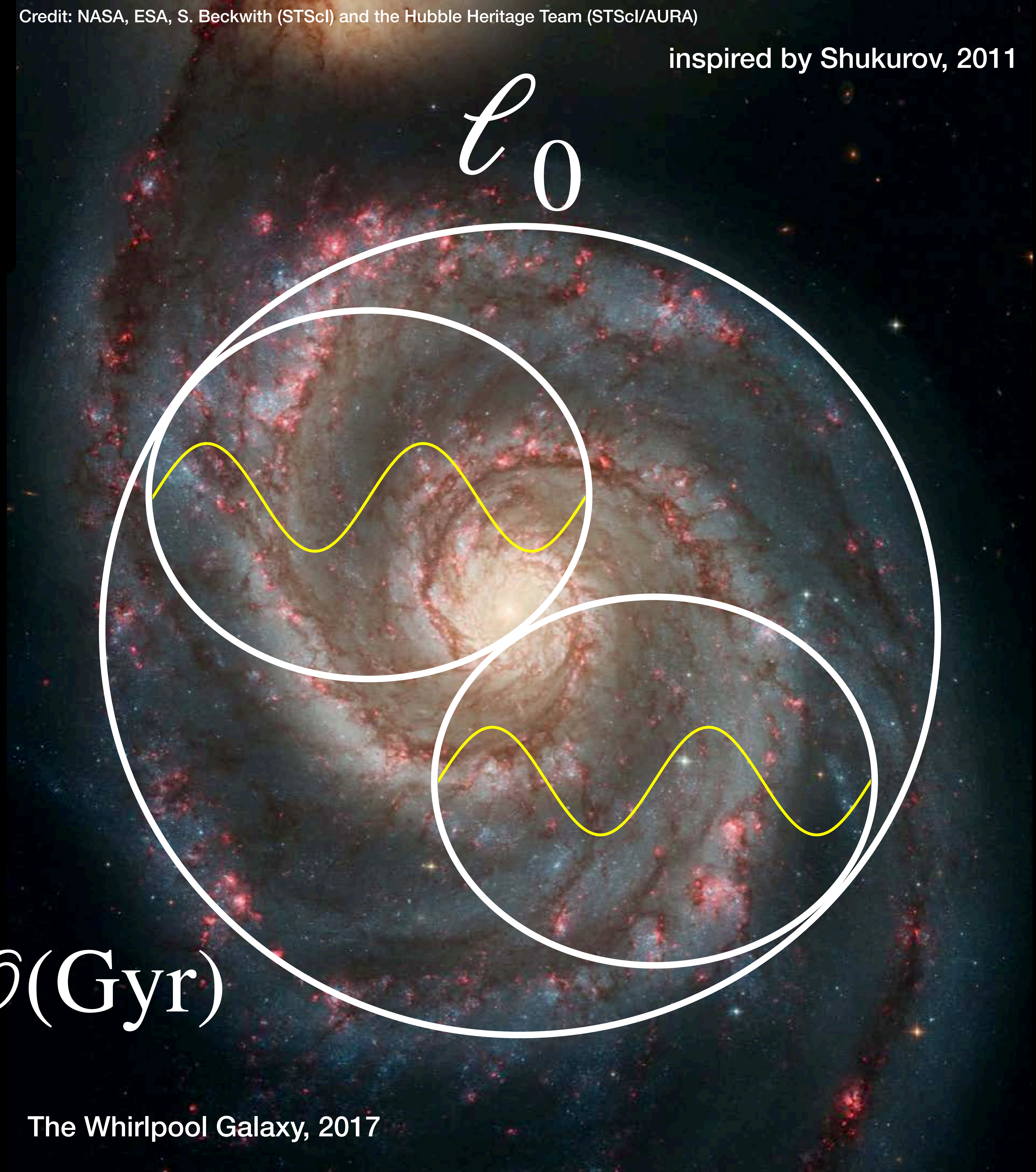
The quadratic nonlinear term

$$|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|$$

dominates on large scales, ℓ

Creates new modes on

$$t_{\rm nl} \sim [\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})]^{-1/2} \sim \mathcal{O}(\mathrm{Gyr})$$



The Whirlpool Galaxy, 2017

Turbulence

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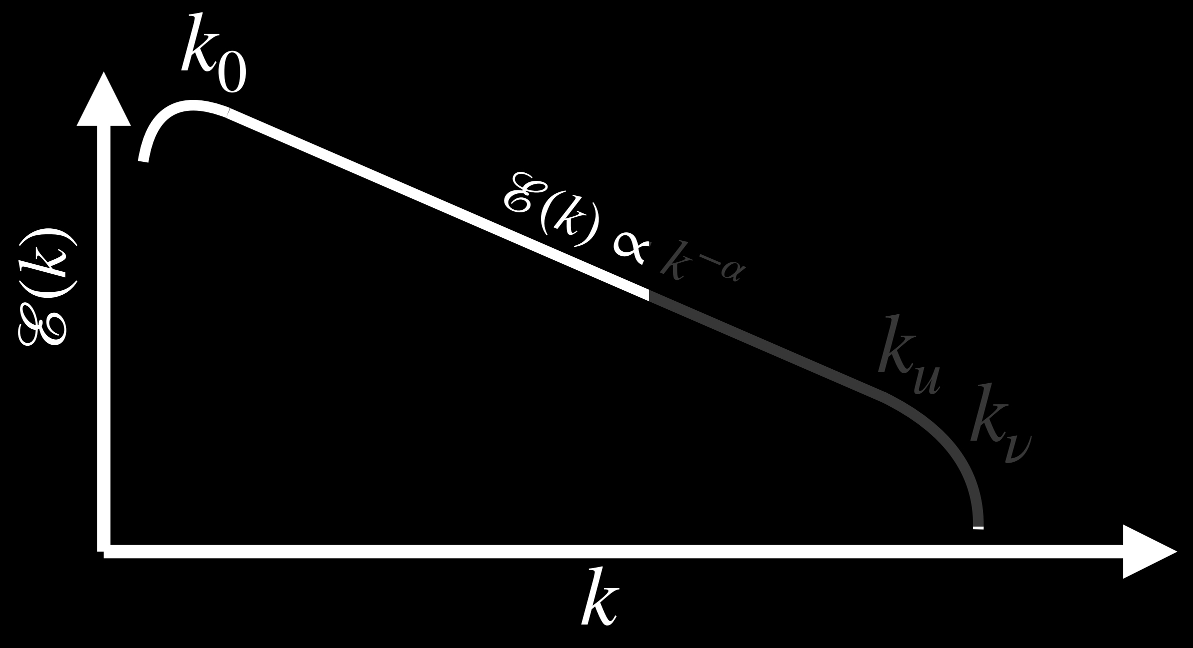
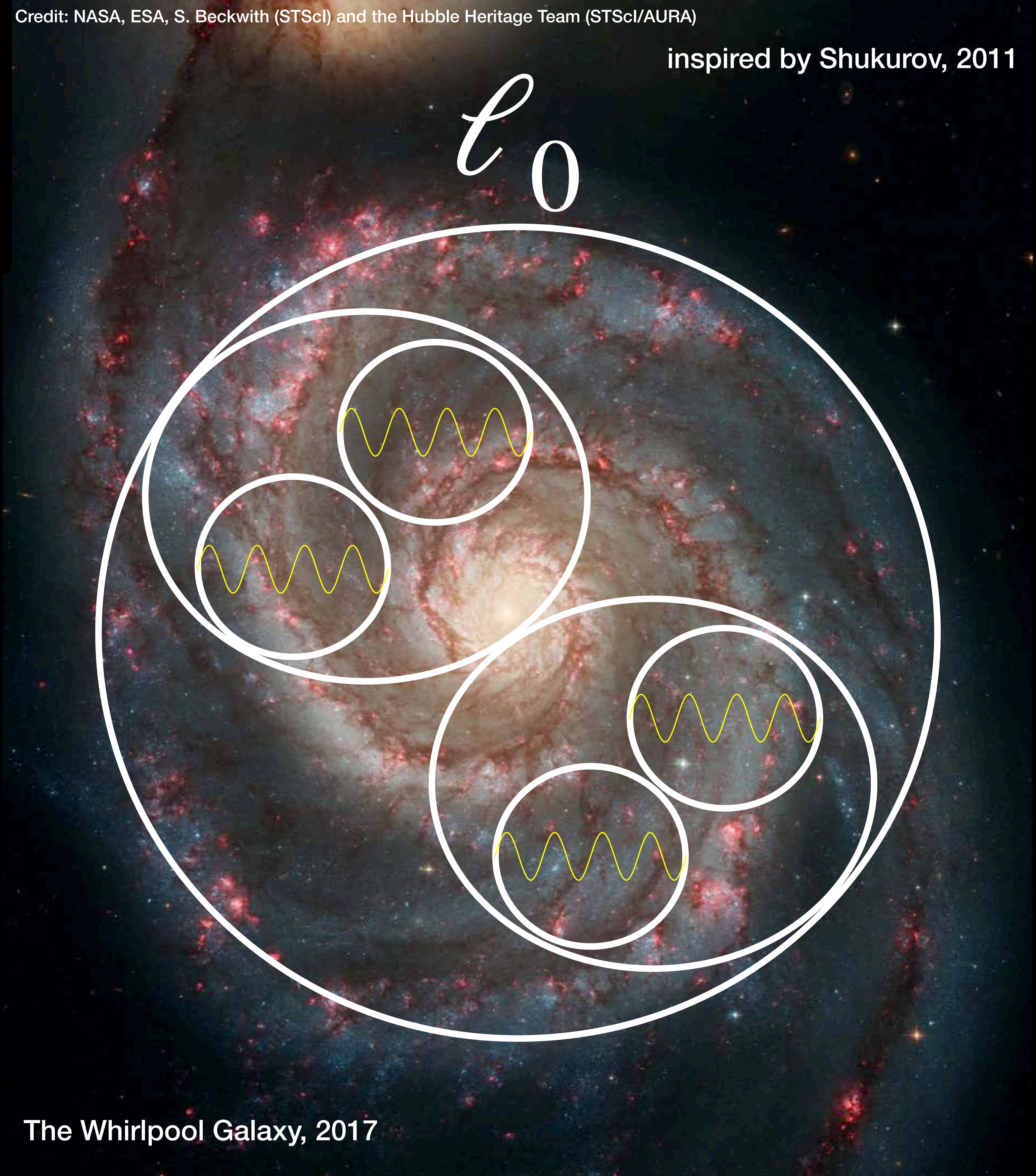


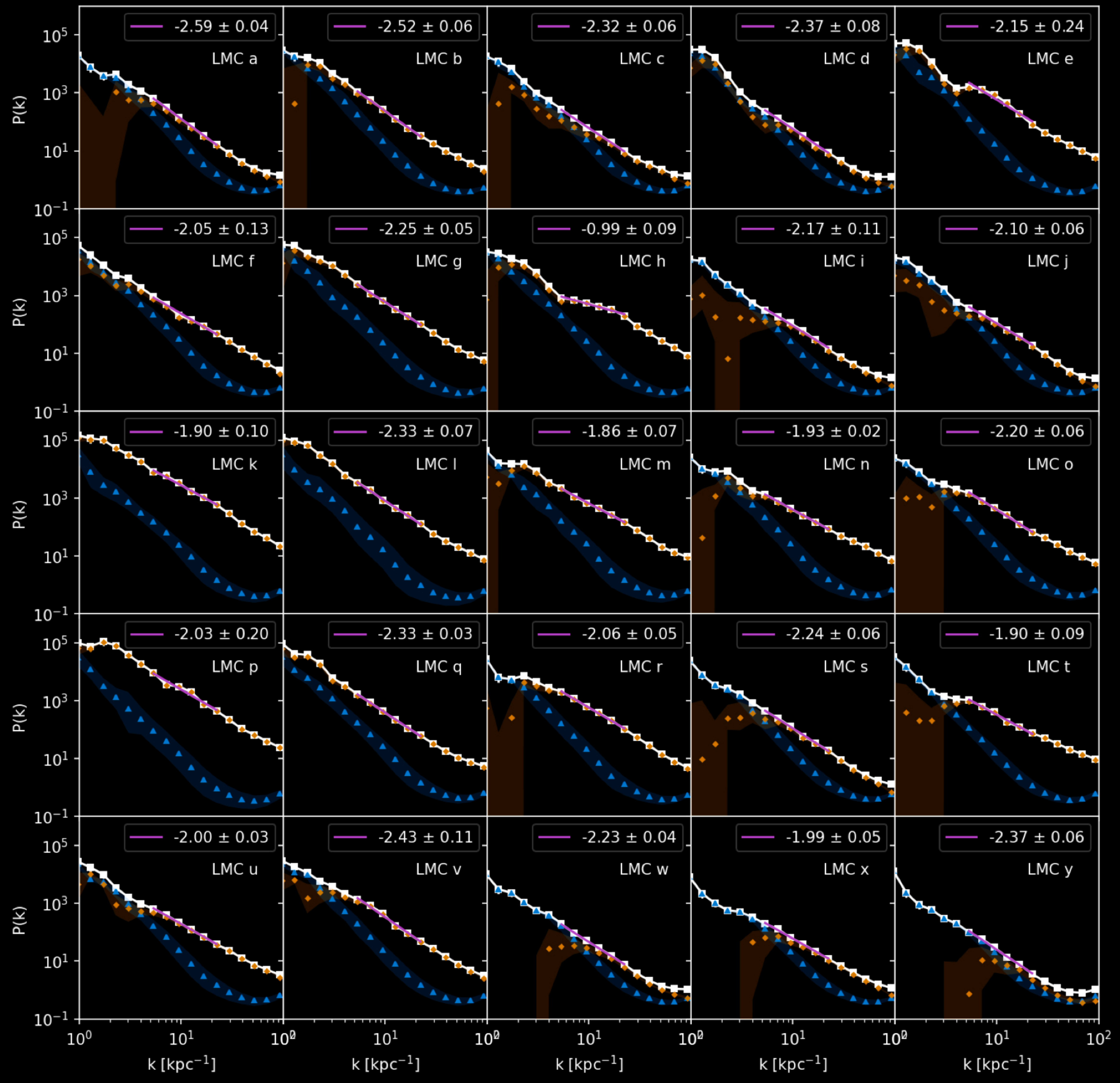
inside of the cascade

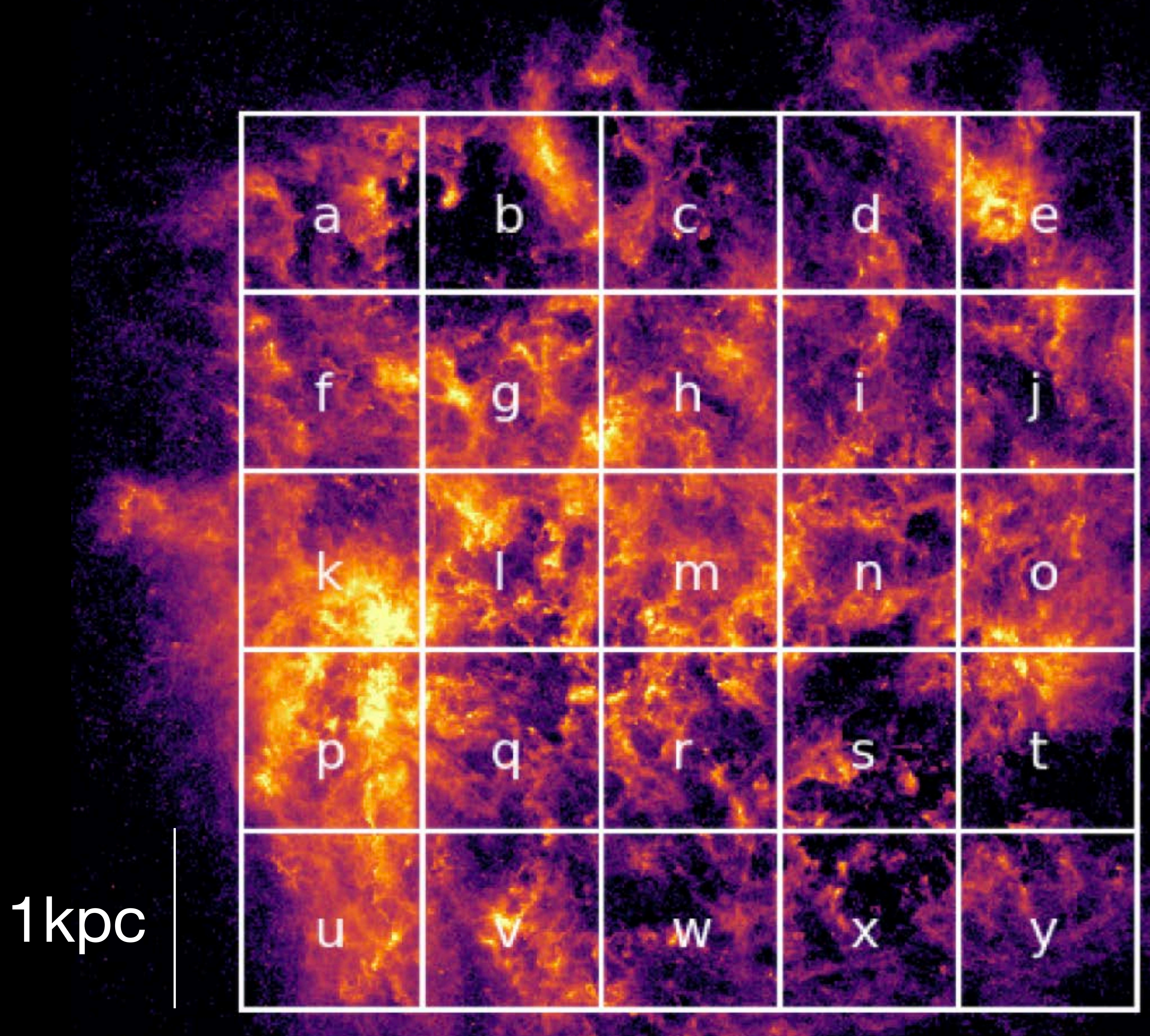


The Whirlpool Galaxy, 2017

Turbulence

LMC: 500μm, *Herschel* (processing Gordon et al. 2014)





Colman, et al., 2022, MNRAS

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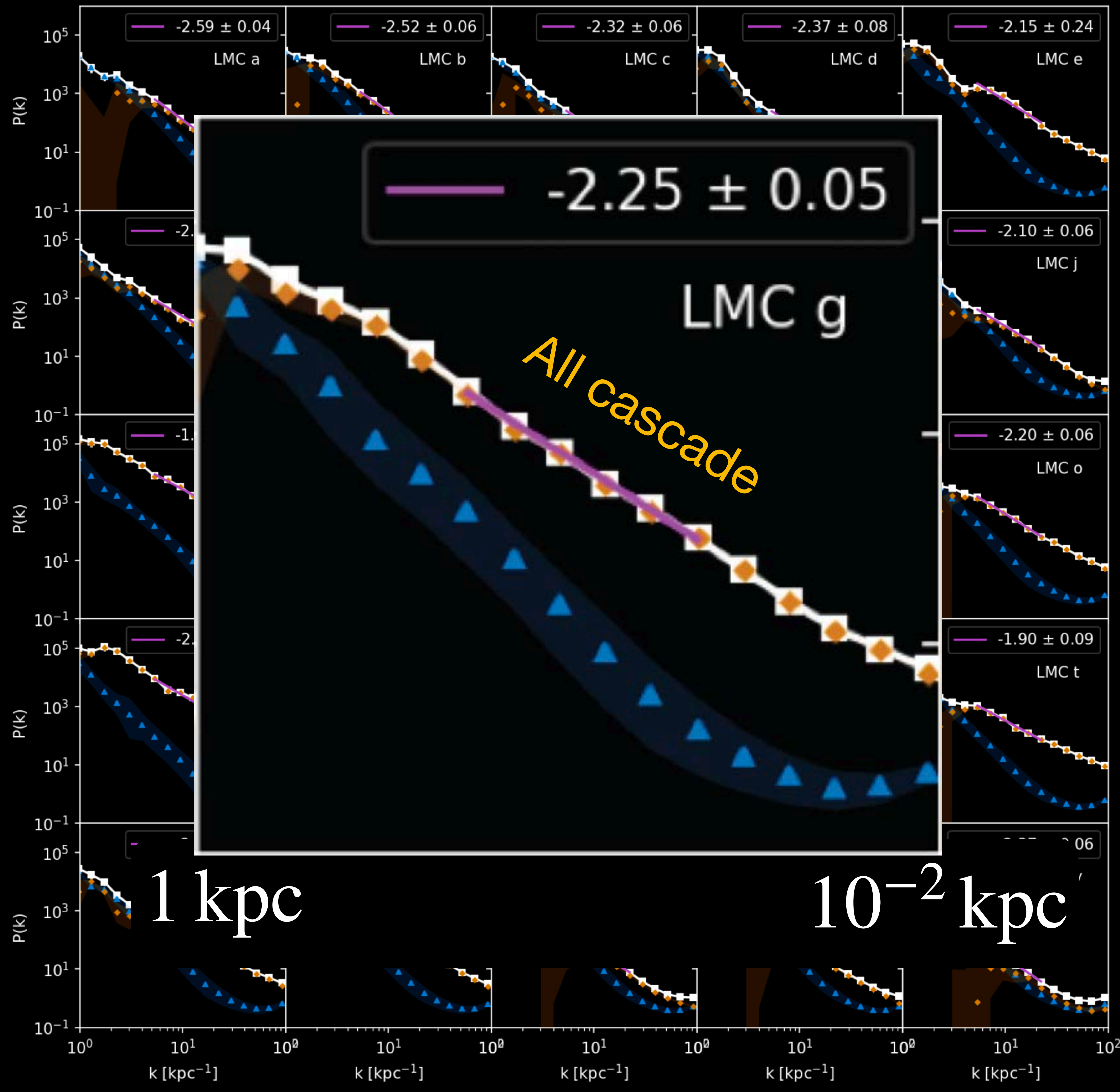

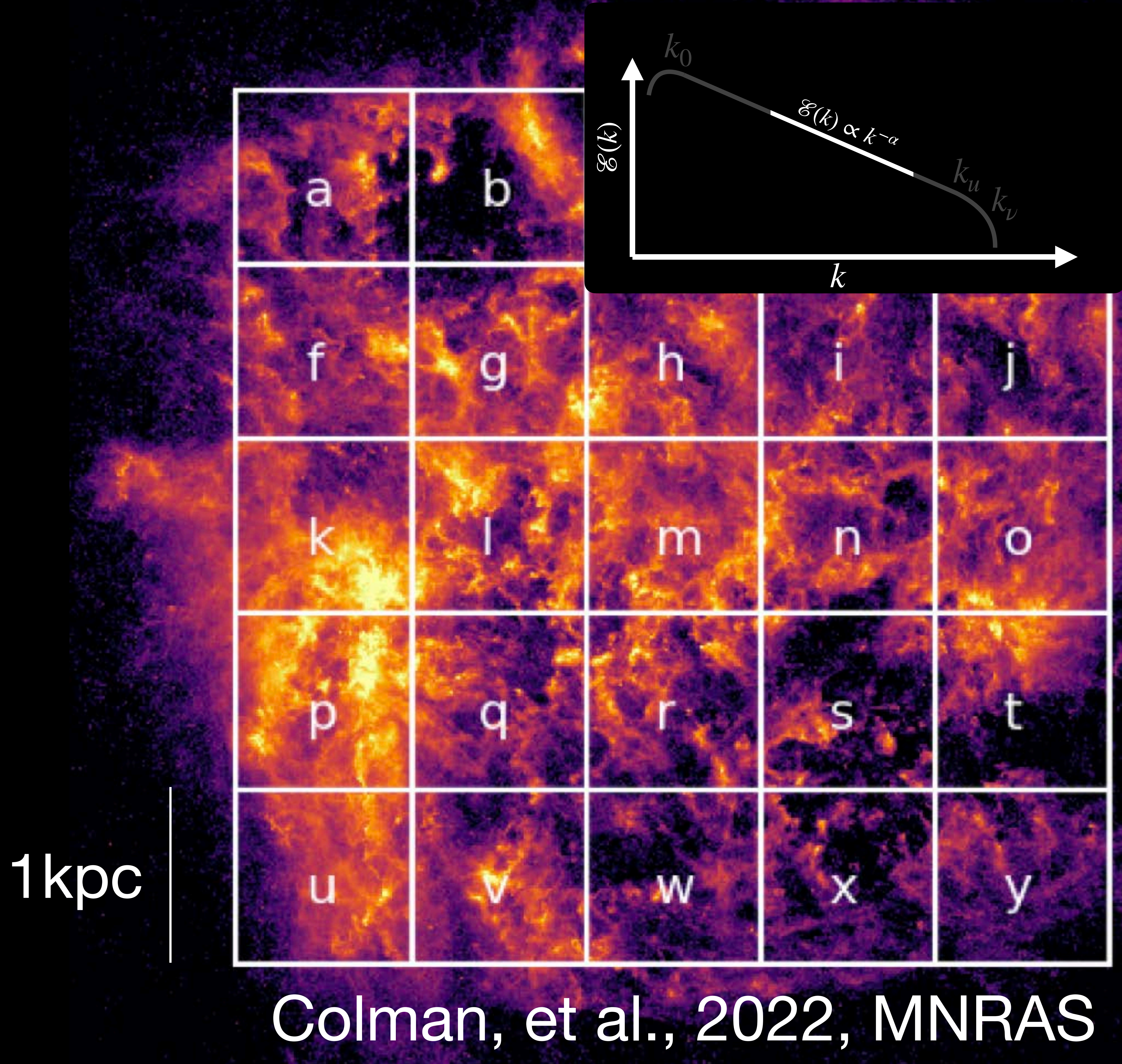

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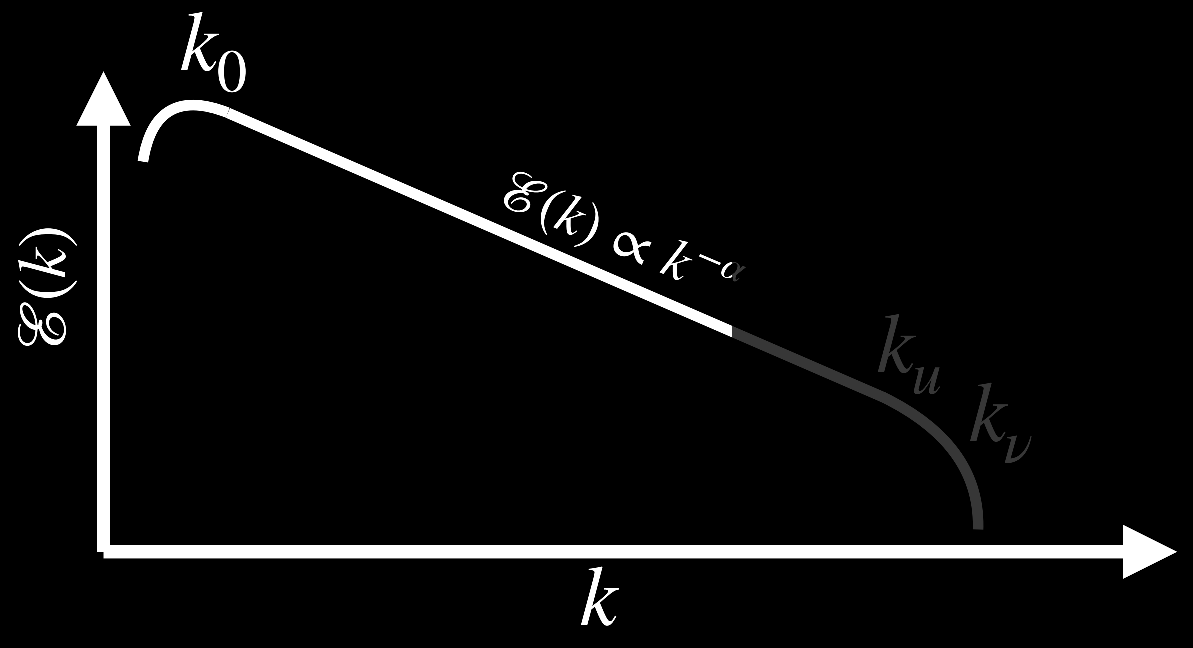
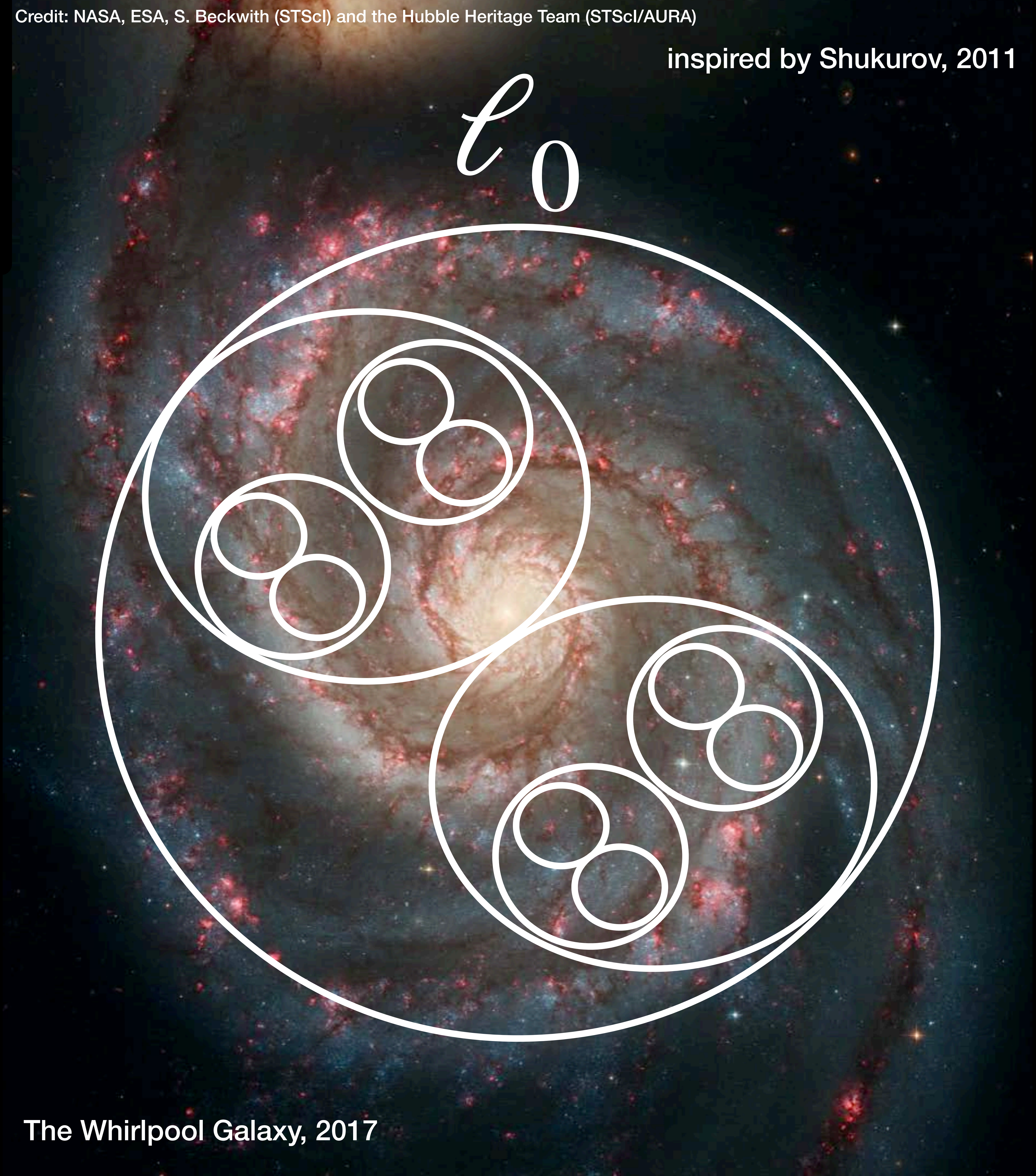
Turbulence

Cascade

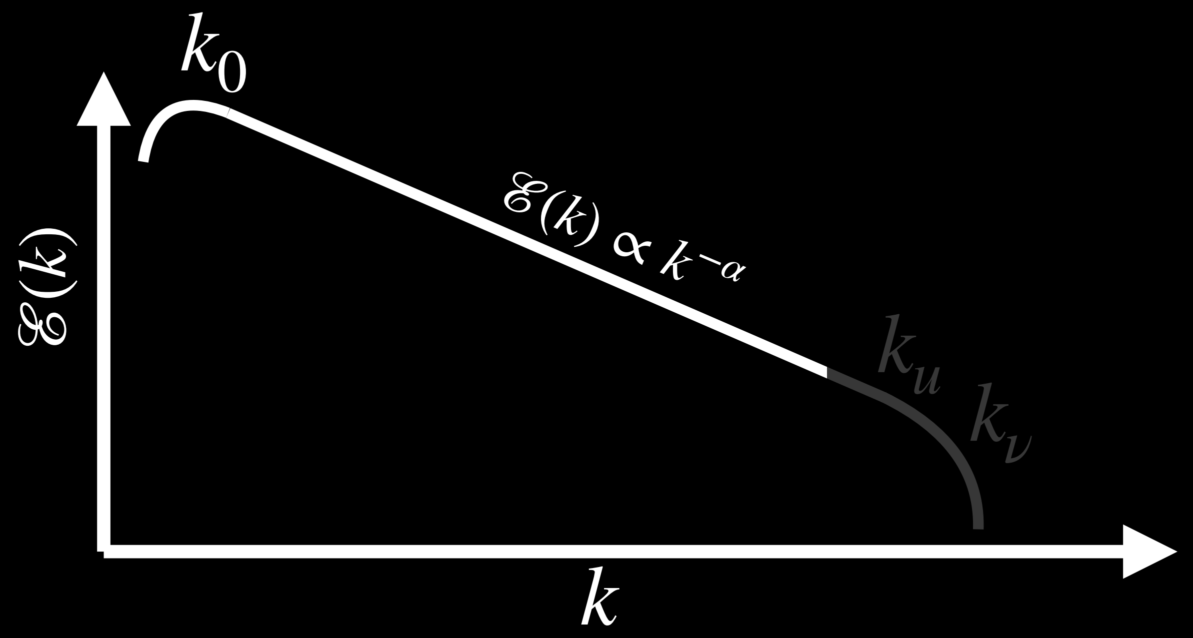


deeper into
the cascade



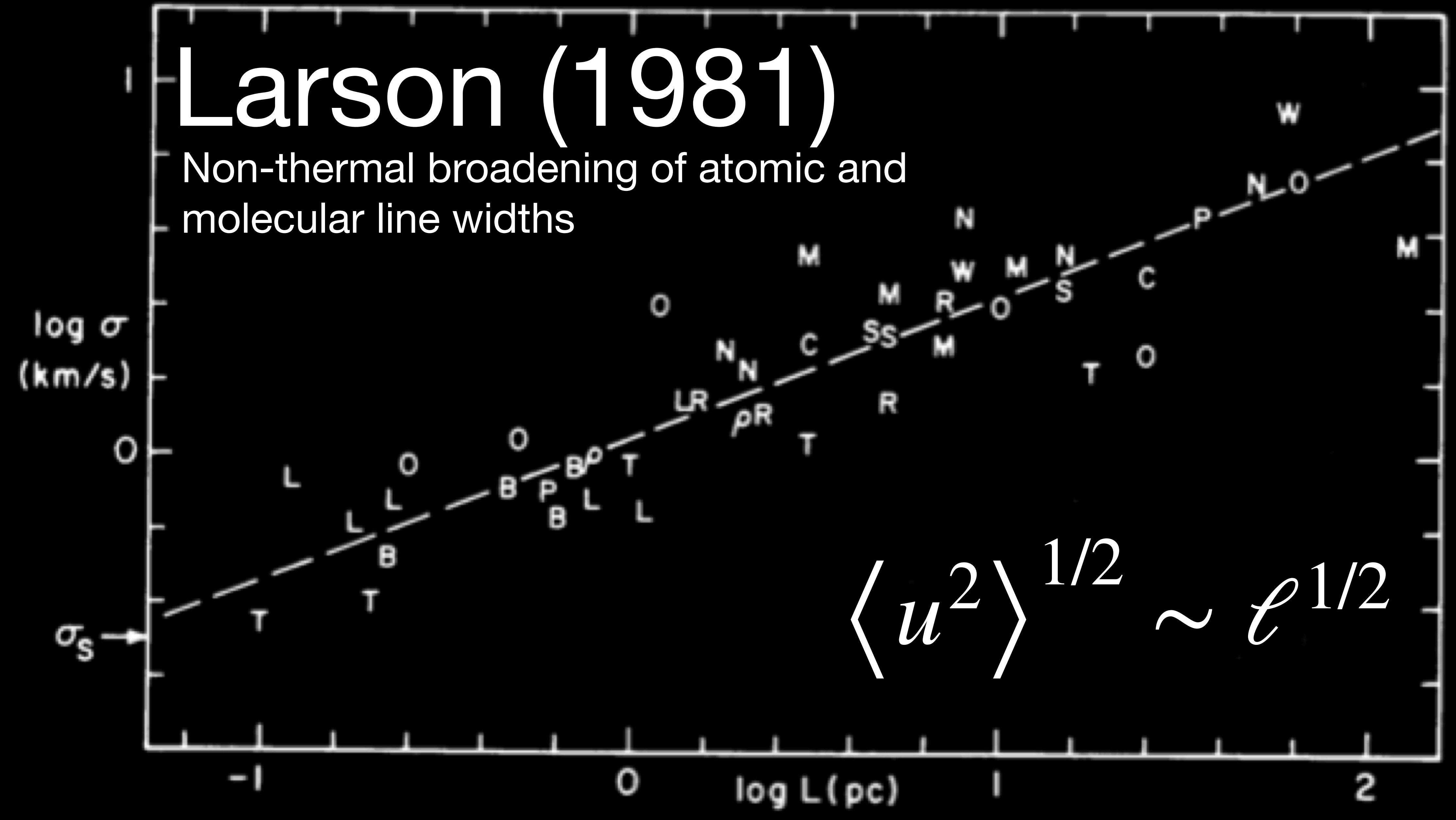
Turbulence

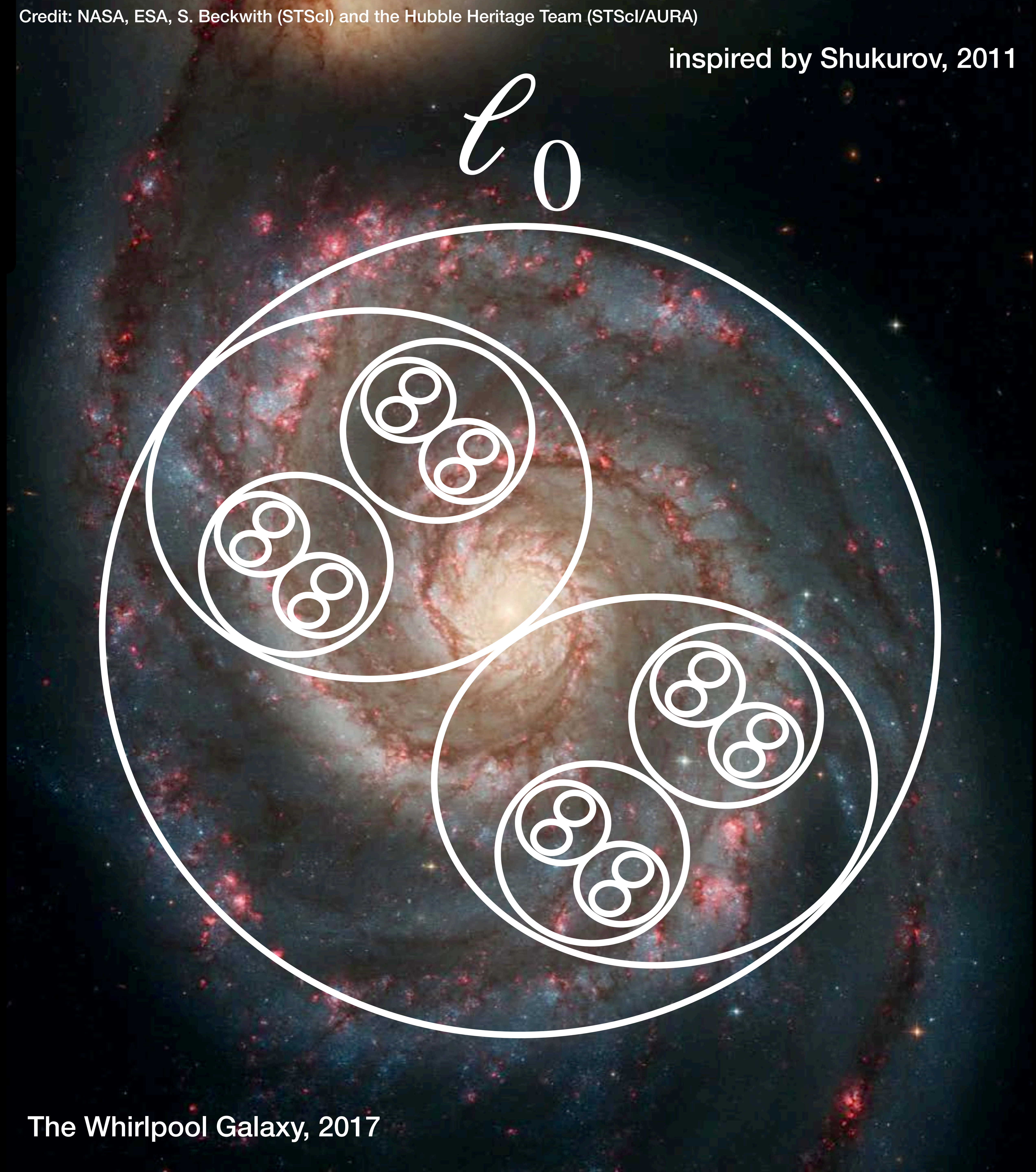
Cascade



Reynolds number is shrinking

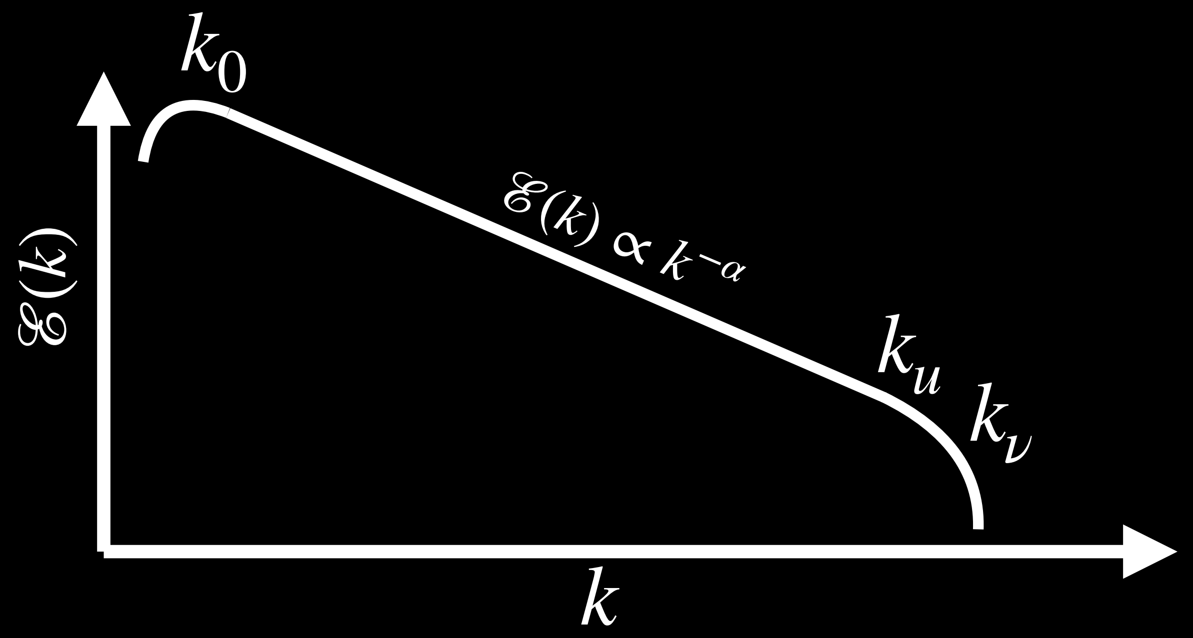
$$\mathrm{Re} \sim \langle u^2 \rangle^{1/2} \ell / \nu \sim \ell^{3/2} / \nu$$





Turbulence

Cascade



until

$$\mathrm{Re} = \frac{|\nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u})|}{|2\nu \nabla \cdot (\rho \mathbb{S})|} = 1$$

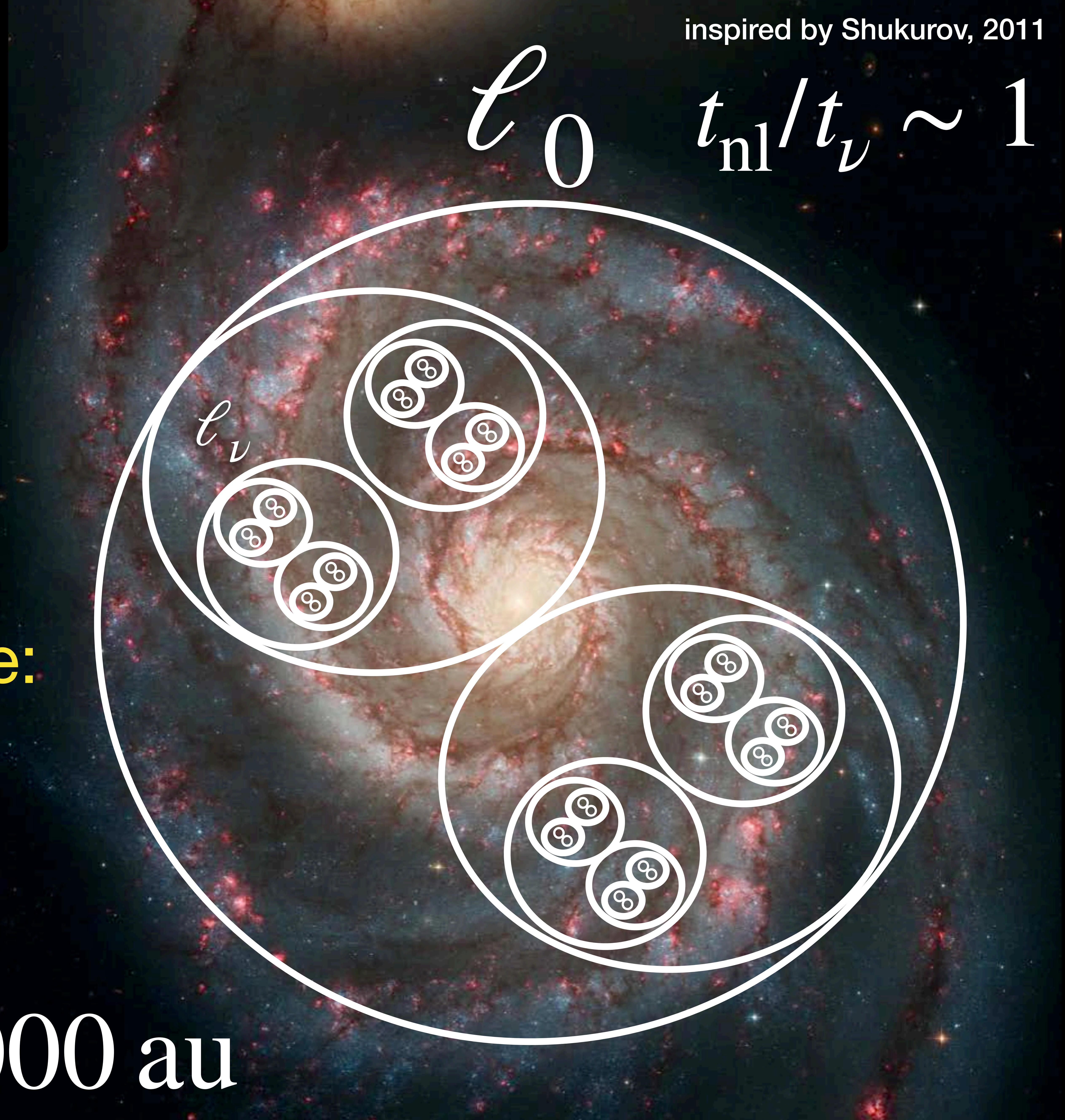
dissipation scales for WIM cascade:

$$\ell_\nu \sim \mathrm{Re}^{-3/4} \ell_0$$

WIM: $\mathrm{Re} \sim 10^7$

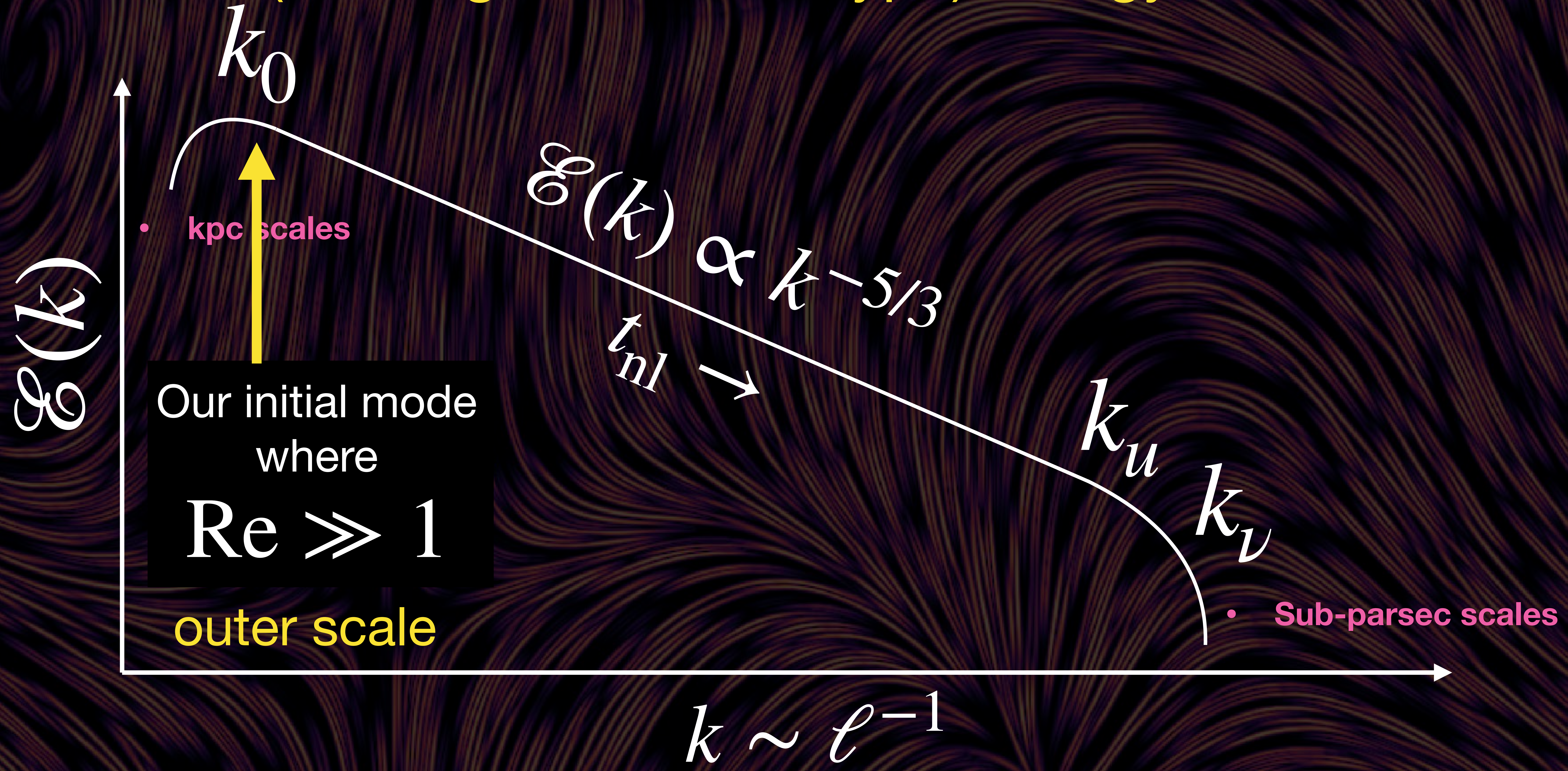
$$\ell_0 \sim 1\,\mathrm{kpc} \implies \ell_\nu \sim 1000\,\mathrm{au}$$

Credit: NASA, ESA, S. Beckwith (STScI) and the Hubble Heritage Team (STScI/AURA)



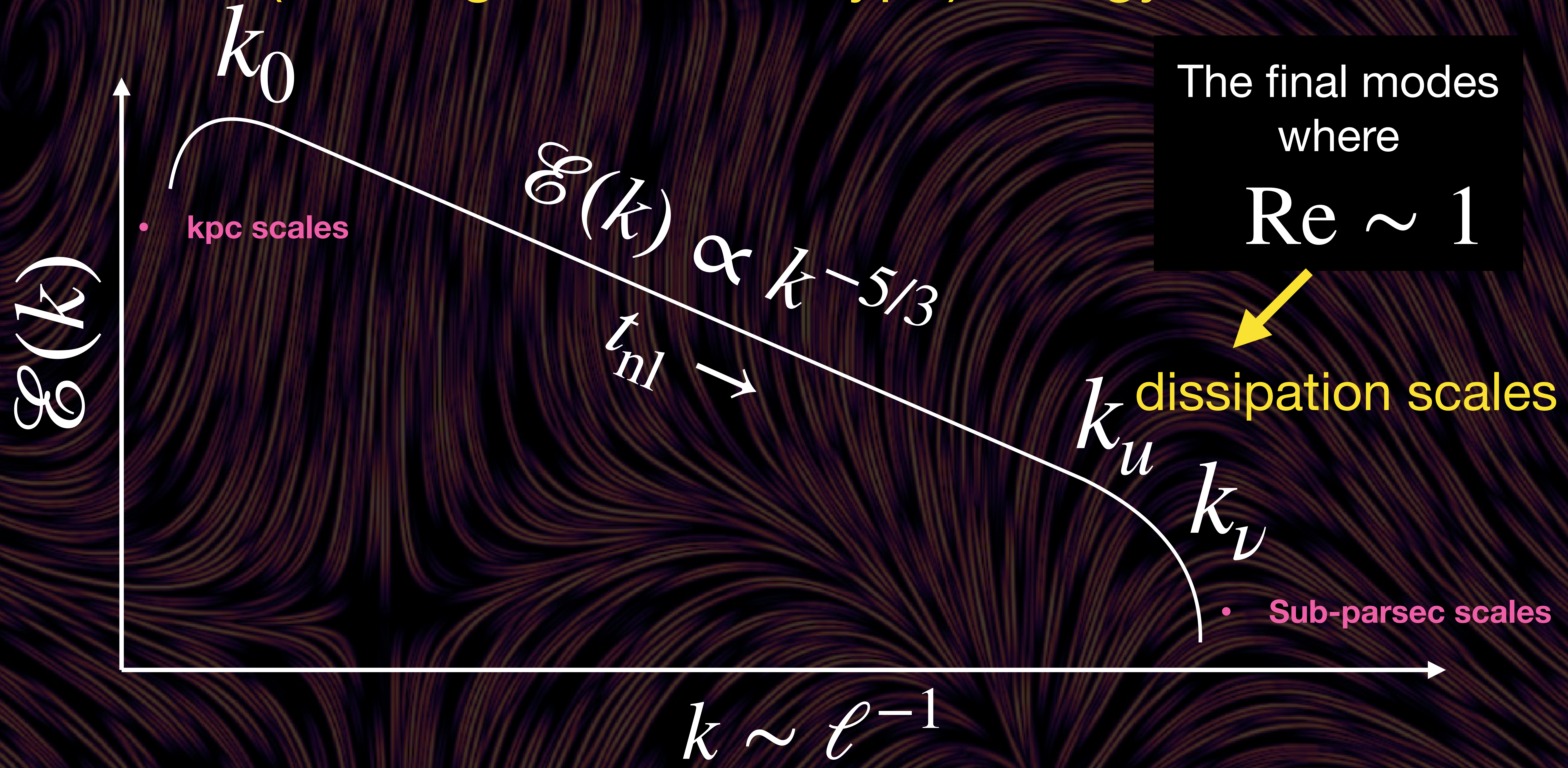
The Whirlpool Galaxy, 2017

The (Kolmogorov, 1941 -type) energy cascade



Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The (Kolmogorov, 1941 -type) energy cascade



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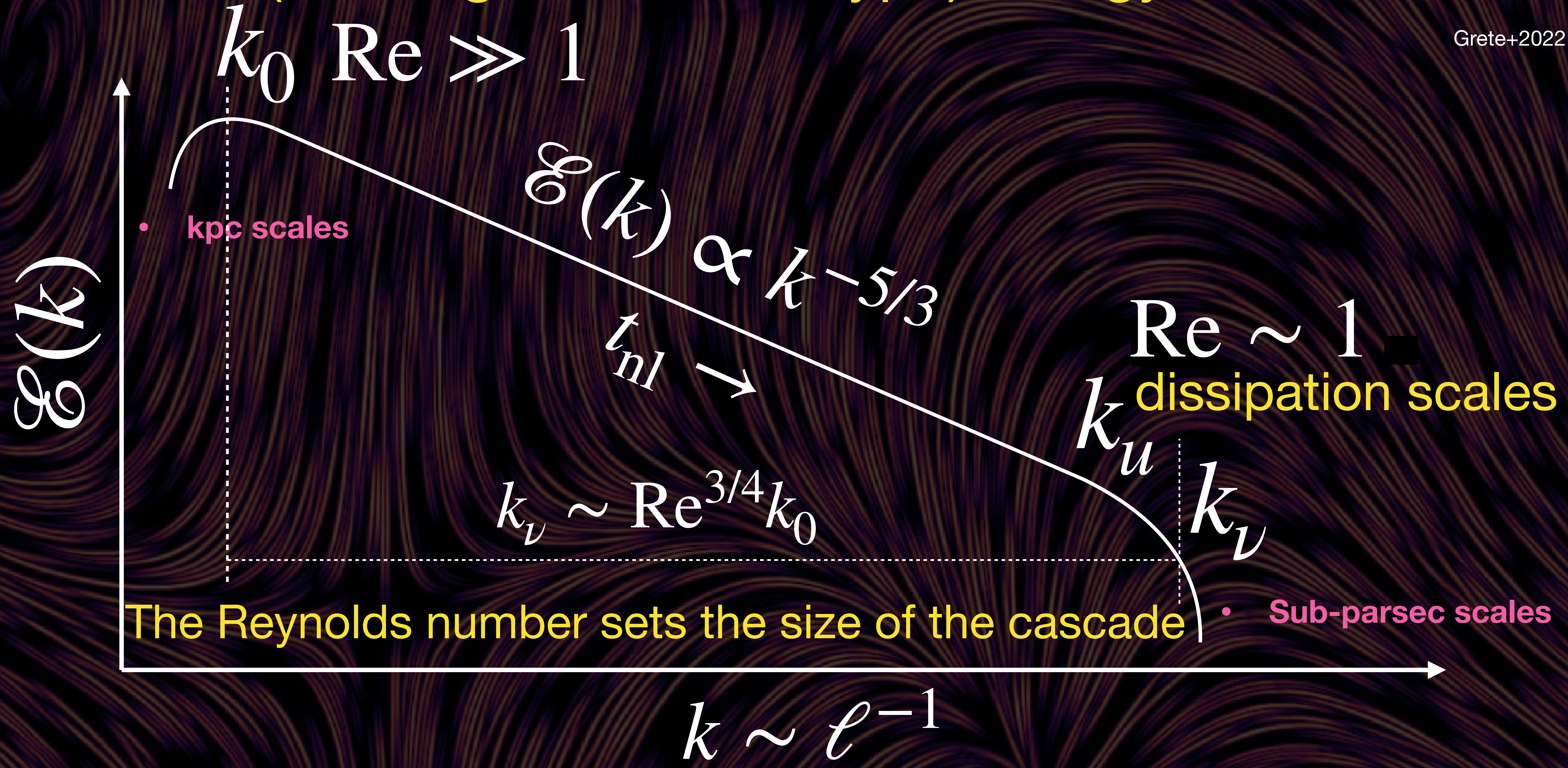
Grete+2022



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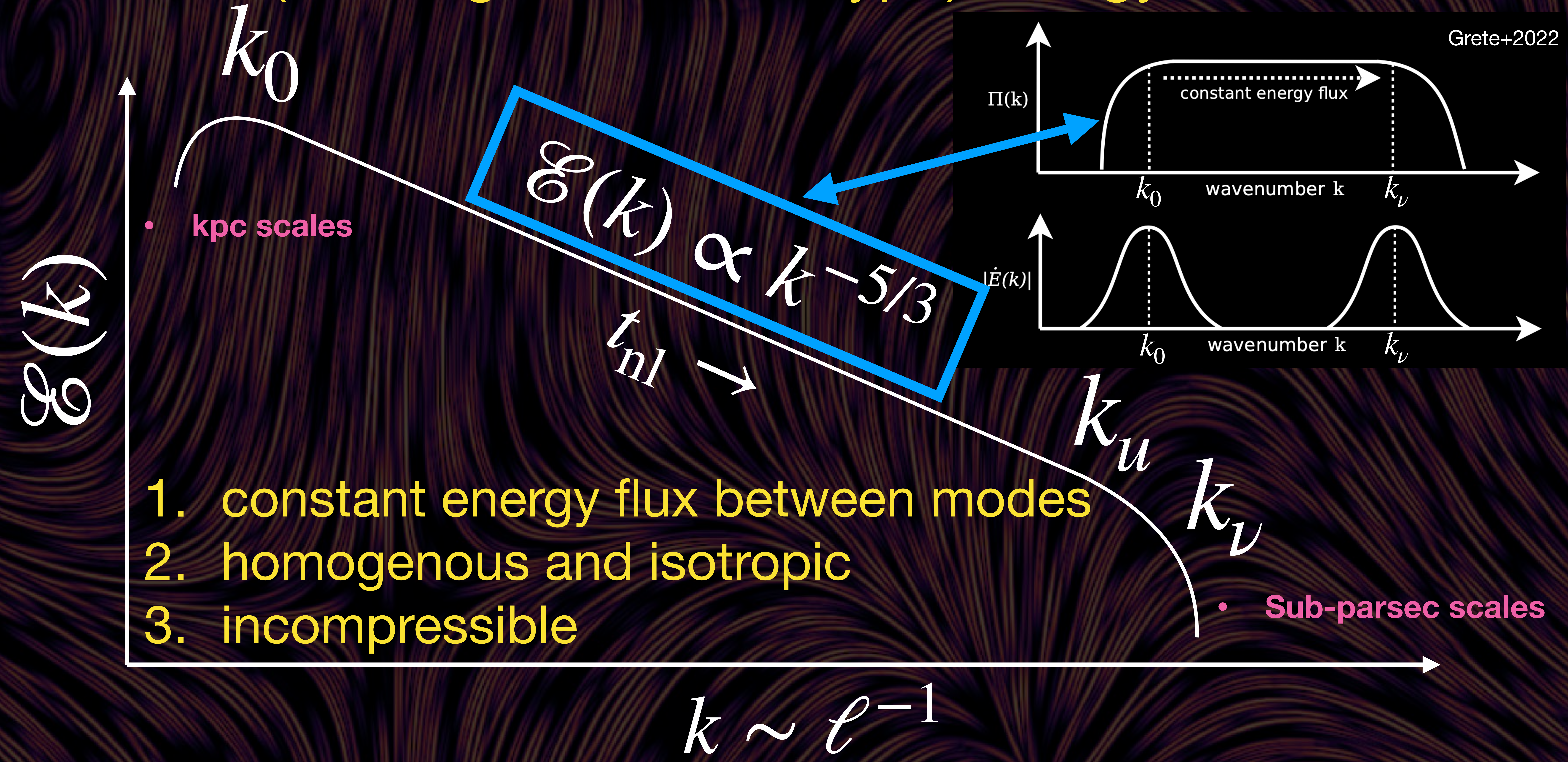
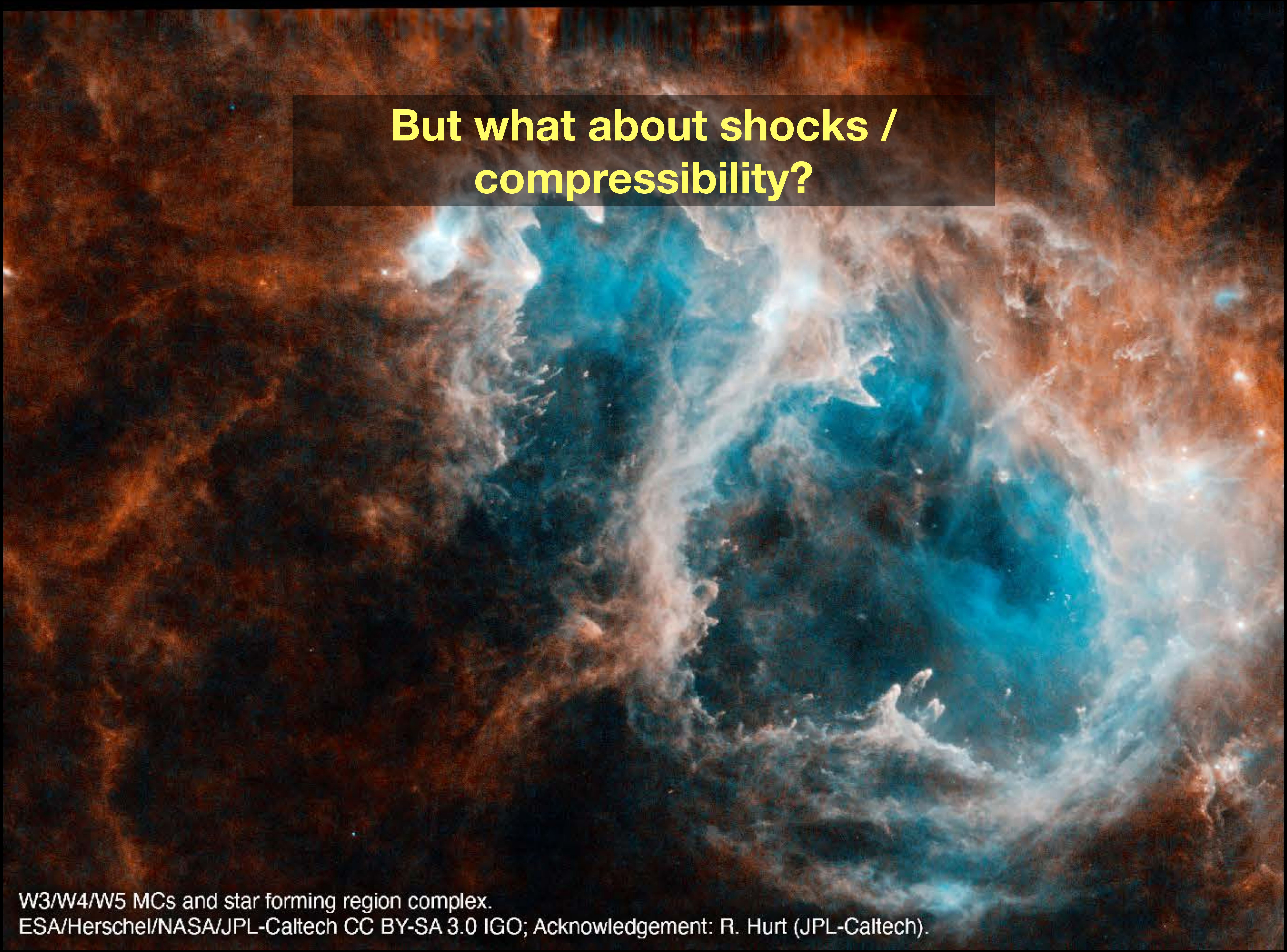


1. constant energy flux between modes
2. homogenous and isotropic
3. incompressible

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

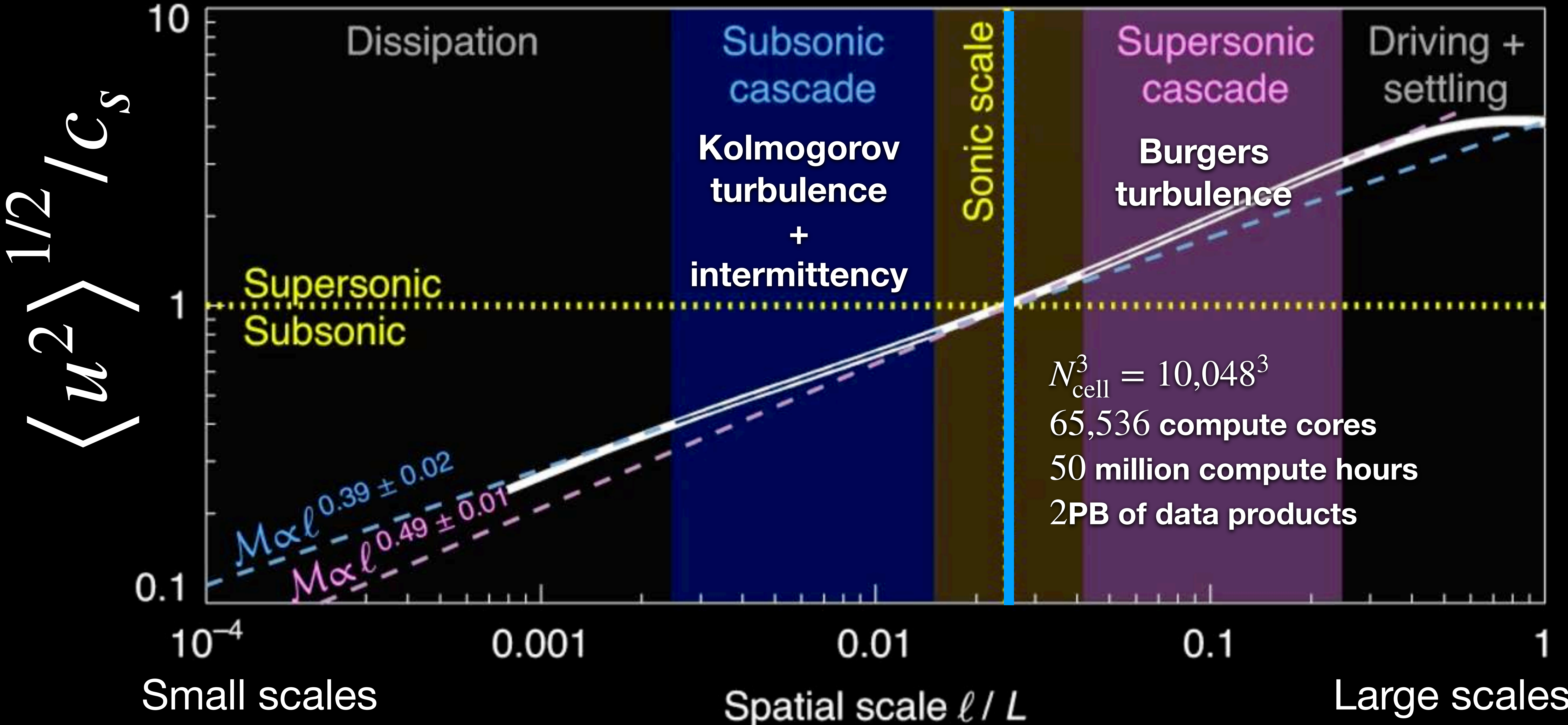
But what about shocks / compressibility?



W3/W4/W5 MCs and star forming region complex.
ESA/Herschel/NASA/JPL-Caltech CC BY-SA 3.0 IGO; Acknowledgement: R. Hurt (JPL-Caltech).

The energy cascade in supersonic hydrodynamical turbulence

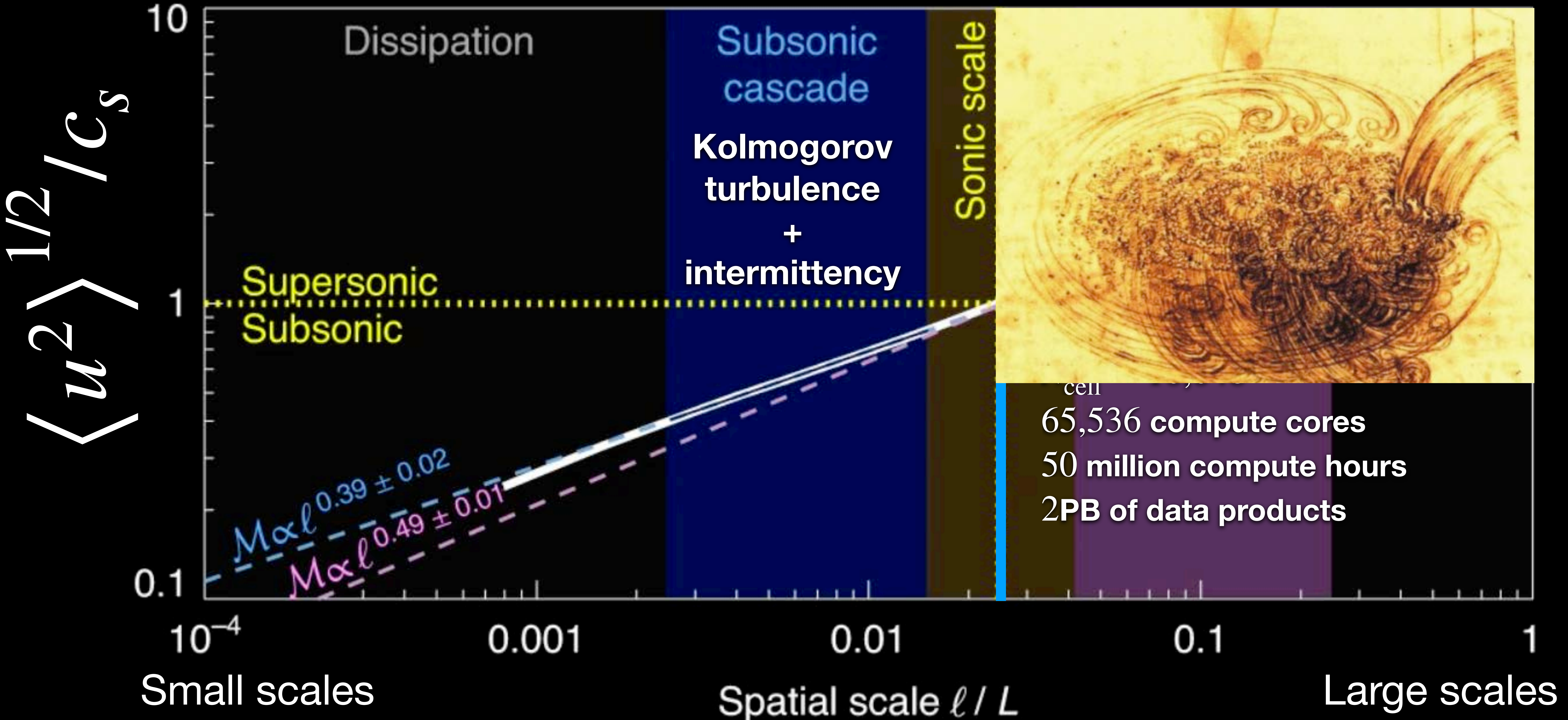
Federrath, Klessen, Ipachio & Beattie (2021)
Nature Astronomy



The energy cascade in supersonic hydrodynamical turbulence

Federrath, Klessen, Ipachio & Beattie (2021)
Nature Astronomy

$$\mathcal{E}(k) \sim k^{-5/3}$$



The energy cascade in supersonic hydrodynamical turbulence

Federrath, Klessen, Ipachio & Beattie (2021)
Nature Astronomy

$$\mathcal{E}(k) \sim k^{-2}$$

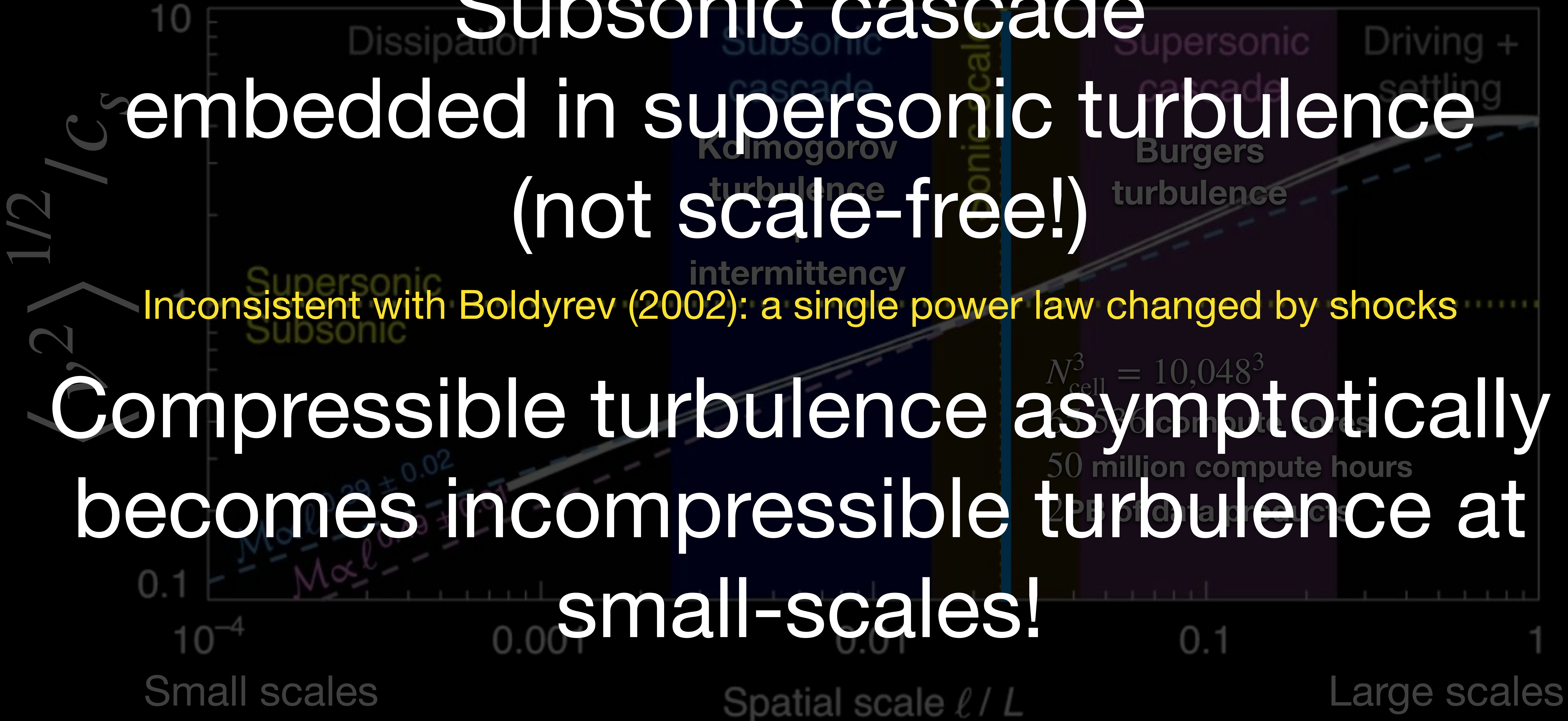
The energy cascade in supersonic hydrodynamical turbulence

Federrath, Klessen, Ipachio & Beattie (2021)
Nature Astronomy

Subsonic cascade
embedded in supersonic turbulence
(not scale-free!)

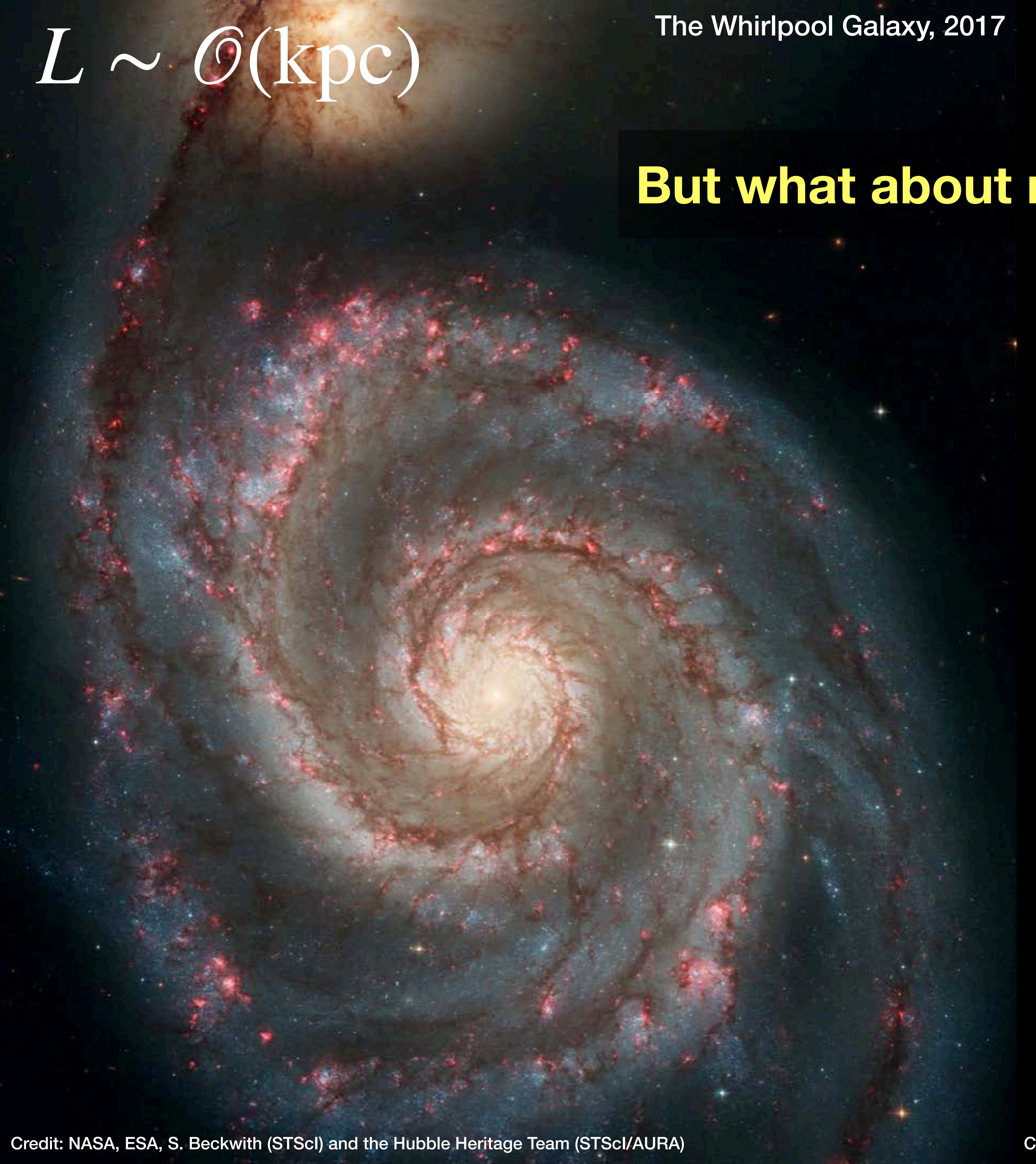
Inconsistent with Boldyrev (2002): a single power law changed by shocks

Compressible turbulence asymptotically
becomes incompressible turbulence at
small-scales!



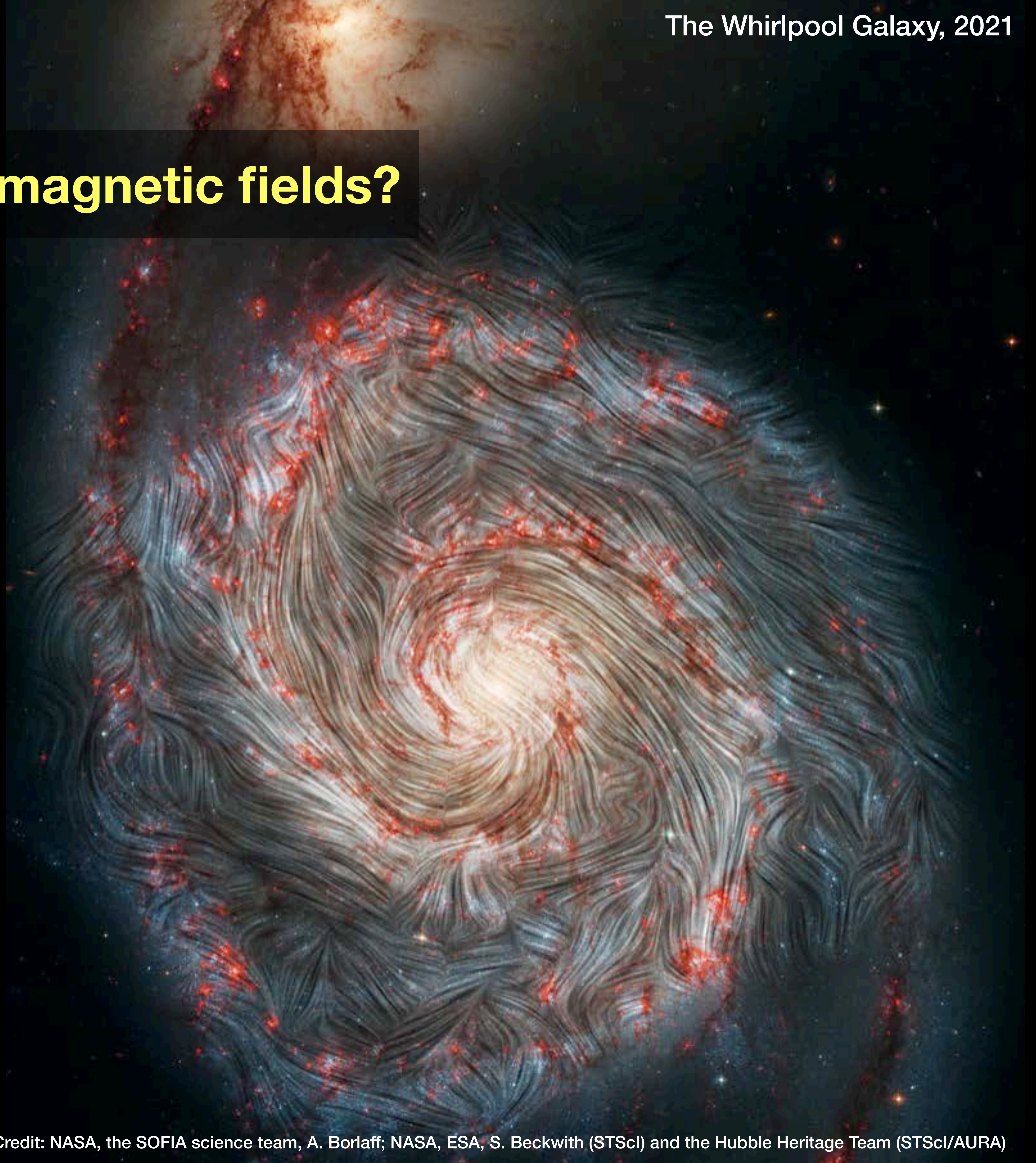
$L \sim \mathcal{O}(\mathrm{kpc})$

The Whirlpool Galaxy, 2017



Credit: NASA, ESA, S. Beckwith (STScI) and the Hubble Heritage Team (STScI/AURA)

The Whirlpool Galaxy, 2021



Credit: NASA, the SOFIA science team, A. Borlaff; NASA, ESA, S. Beckwith (STScI) and the Hubble Heritage Team (STScI/AURA)

But what about magnetic fields?

$L \sim \mathcal{O}(\mathrm{pc})$

Soler, 2019, A&A

Irreducible — add magnetic flux to the cascade on all scales

straight, strong fields ($\kappa \propto B^{-1/2}$; Schekochihin+2004) penetrating through Orion A

Orion A in infrared; ESA/Herschel/Planck; J. D. Soler, MPIA

Magnetic Reynolds Number

What is it?

Maxwell's equations + fluid = induction equation

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{b} - \eta \mathbf{j}), \quad \mathbf{j} = \frac{1}{4\pi} \nabla \times \mathbf{b}$$

Creating nonlinear magnetic things in the fluid

$$\mathrm{Rm} = \frac{|\nabla \times (\mathbf{u} \times \mathbf{b})|}{|\nabla \times \eta \mathbf{j}|} \sim \frac{UL}{\eta}$$

Dissipating nonlinear magnetic things in the fluid

Re & Rm Landscape

Rincon (2019)

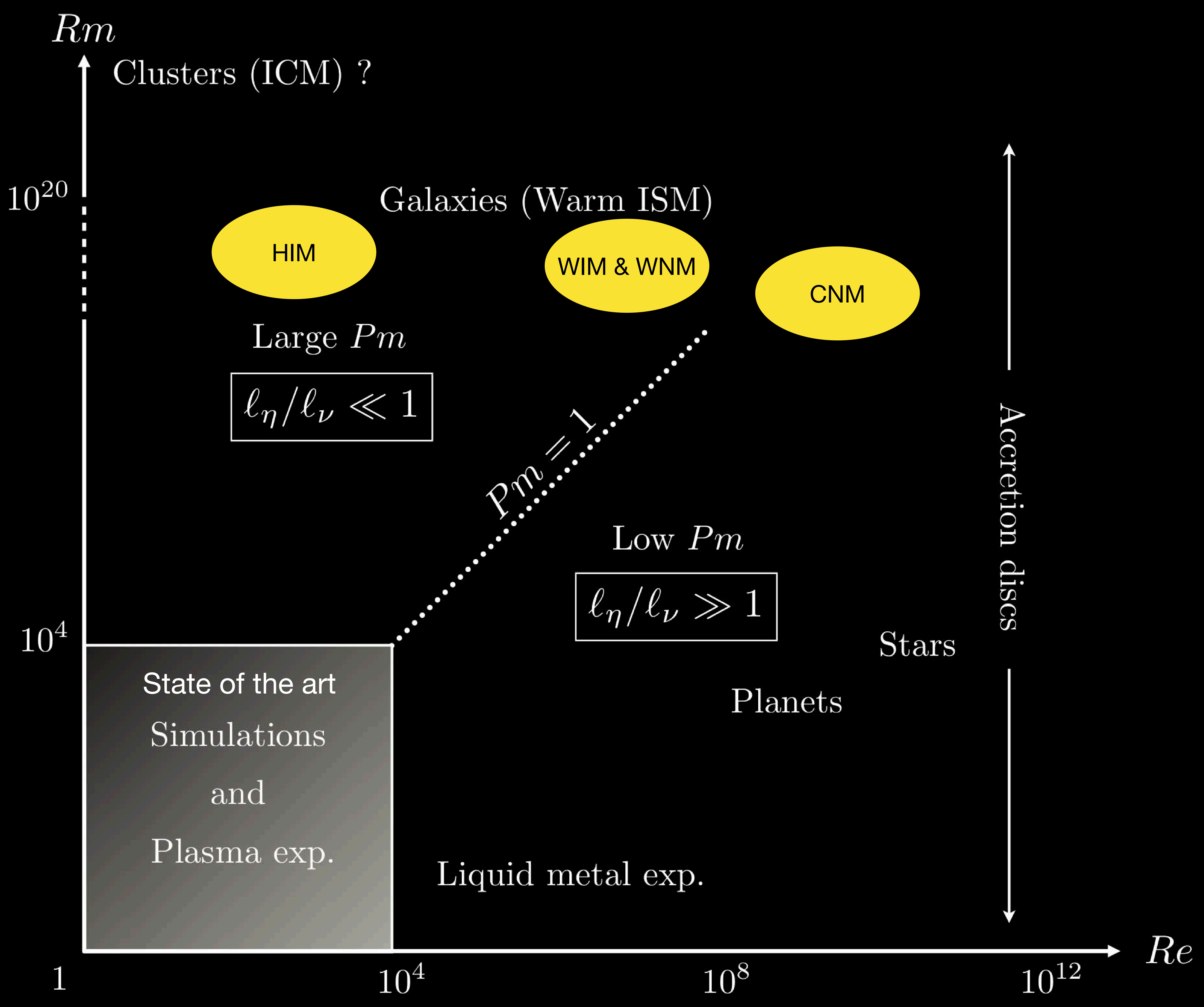
WIM: $\mathrm{Rm} \sim 10^{18}$

WNM: $\mathrm{Rm} \sim 10^{18}$

CNM: $\mathrm{Rm} \sim 10^{15}$

Ferrière, 2020; Plasma Physics and Controlled Fusion

$$\mathrm{Pm} = \frac{\mathrm{Rm}}{\mathrm{Re}}$$



Creating the simplest compressible MHD turbulence simulation at $\mathrm{Re} \gtrsim 10^6$

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

$10{,}080^3$ magnetised interstellar medium turbulence simulation

Beattie, Federrath, Mocz, Klessen, Cielo & Bhattacharjee

1. **What is the scaling of the energy cascade in compressible MHD turbulence with no net b flux?**
2. **How are the characteristic scales organised in the ISM turbulence?**
3. **What are the saturation physics of the turbulent dynamo?**

PI of a 100million core-hour project on superMUC-NG

ILES of compressible MHD turbulence

Turbulence: $\sigma_V / c_s \approx 4,\ \ell_0 = L/2$

Magnetic fields: $B = b_{\rm turb},\ \mathcal{M}_{\rm A} \approx 2$

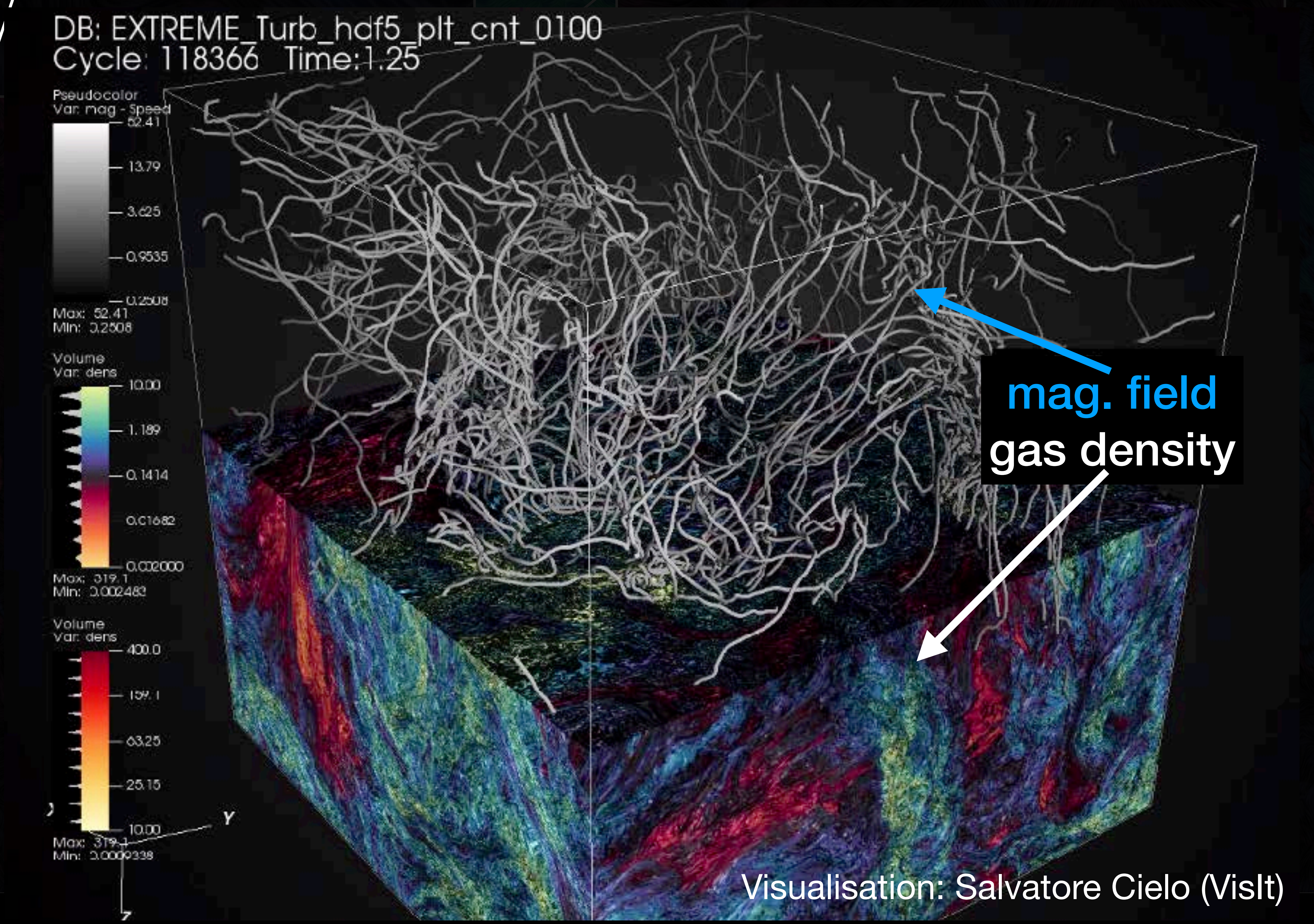
Three experiments for convergence tests:

- **LOW-RES:** $2{,}520^3$ **(0.3Mcore-h, 8,640cores)**
- **MID-RES:** $5{,}040^3$ **(4.0Mcore-h, 34,560cores)**
- **HIGH-RES:** $10{,}080^3$ **(80.0Mcore-h, 148,240cores)**
    - **Resolving 10pc down to ~200au everywhere on the grid**
    - **Factor of 4 higher linear grid resolution than IllustrisTNG**

3.45**PB in data products**

Broad Code details:

- Highly-modified version of finite volume code *FLASH*, second-order in space approximate Riemann (PPM) solver with framework outlined in Bouchut+2010, tested in *FLASH* in Waagen+2011.
- Ideal (ILES) compressible non-helical, isothermal MHD turbulence with finite correlation time (OU process; Federrath2022).



Visualisation: Salvatore Cielo (VisIt)

Stochastically forced Compressible MHD equations (ILES)

$$\partial_t \rho + \nabla \cdot \left(\rho \mathbf{u}\right) = 0$$

$$\partial_t(\rho \mathbf{u}) + \nabla \cdot \left(\rho \mathbf{u} \otimes \mathbf{u} + p\mathbb{I} - \frac{1}{4\pi}\mathbf{b} \otimes \mathbf{b}\right) = \rho \mathbf{f} + \nabla \cdot \mathbb{D}_\nu(\rho \mathbf{u})$$

$$\partial_t \mathbf{b} + \nabla \cdot (\mathbf{u} \otimes \mathbf{b} - \mathbf{b} \otimes \mathbf{u}) = \nabla \cdot \mathbb{D}_\eta(\mathbf{b})$$

$$\nabla \cdot \mathbf{b} = 0$$

$$p = c_s^2 \rho + \frac{1}{8\pi}\mathbf{b} \cdot \mathbf{b}$$

$$10{,}080^3 \implies \mathrm{Re} \sim \mathrm{Rm} \sim (1-3) \times 10^6$$

Computed for our solver using results from Lakshmi+2023

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Rincon (2019)

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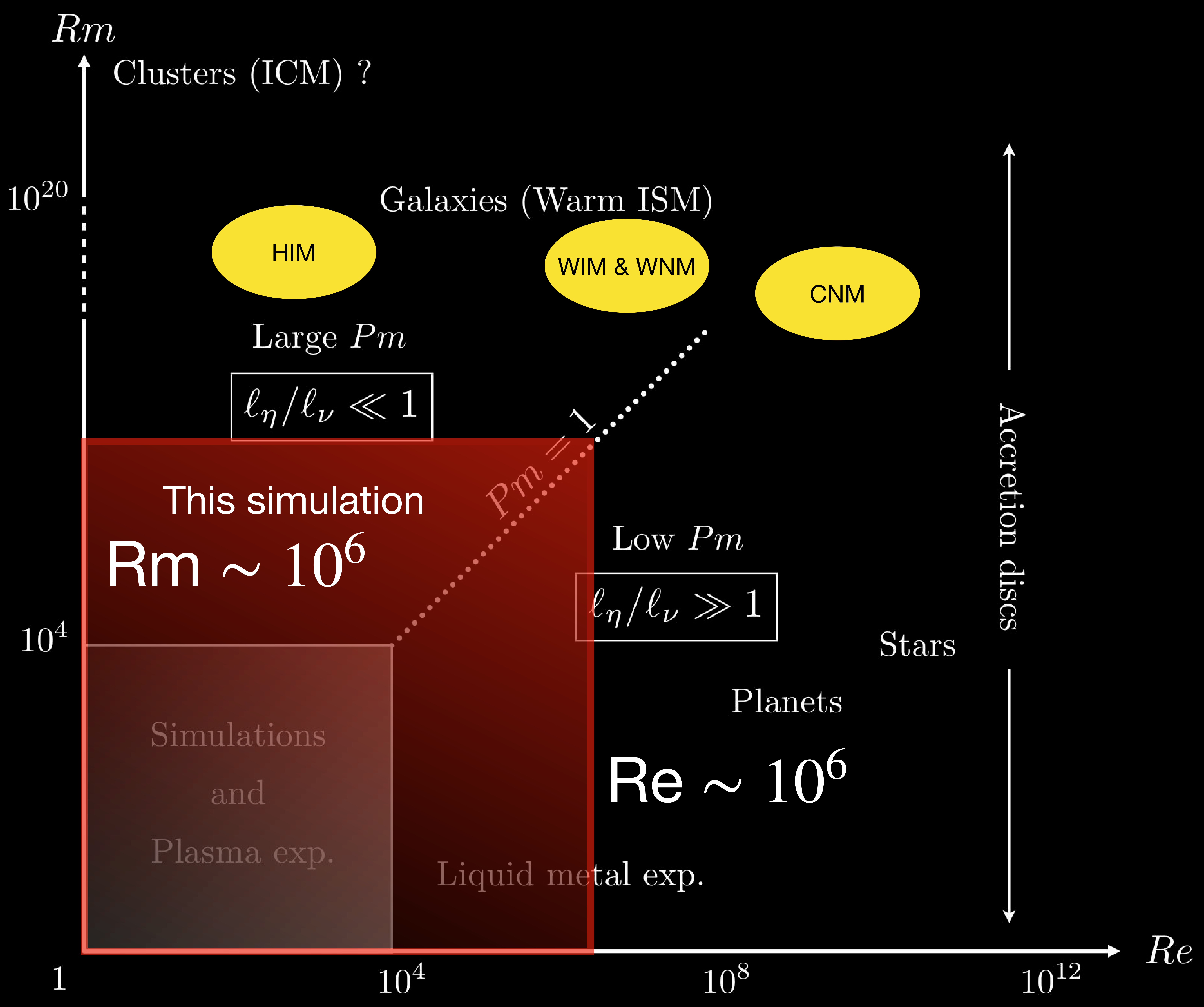
Ferrière, 2020; Plasma Physics and Controlled Fusion



Re & Rm Landscape

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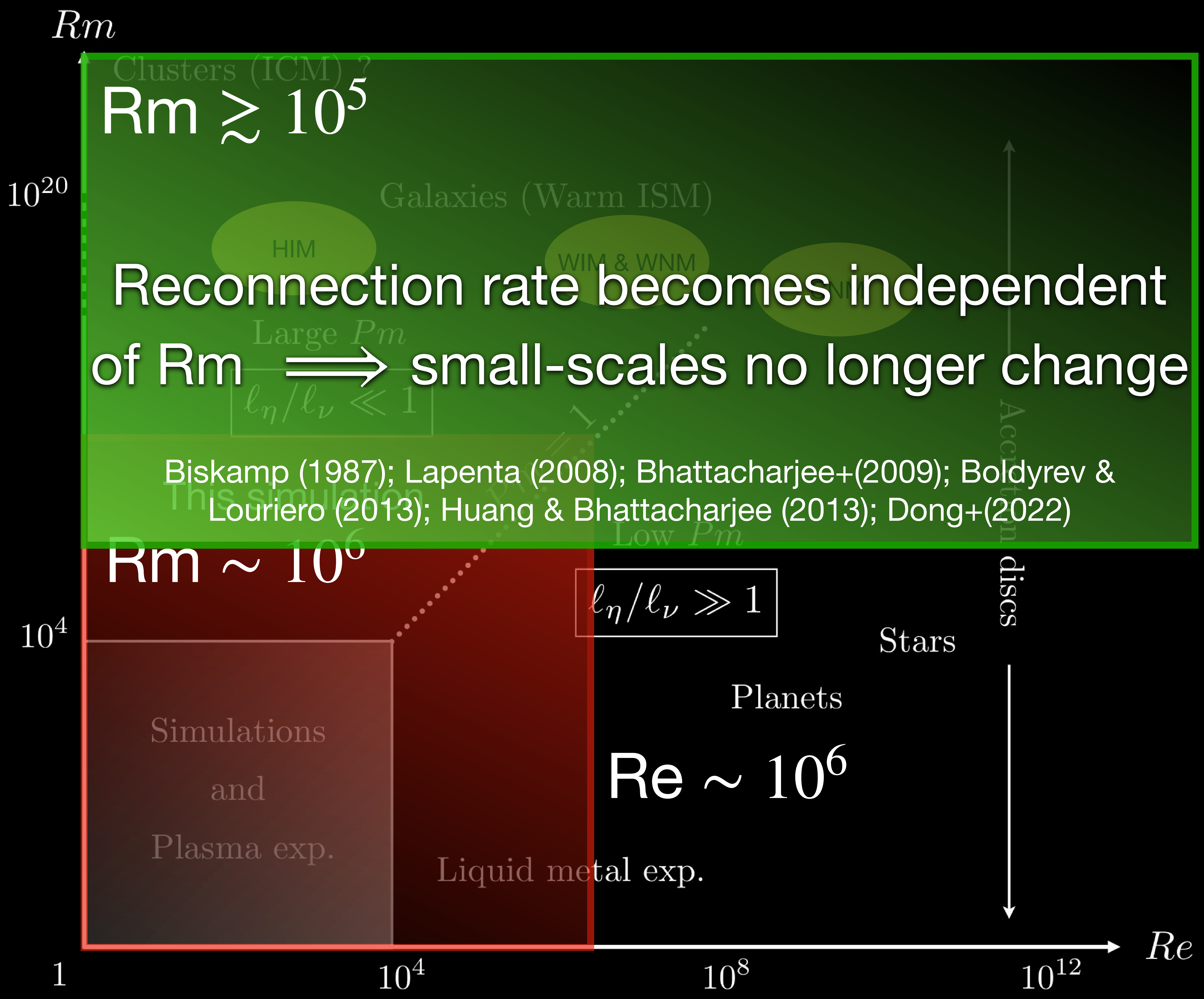
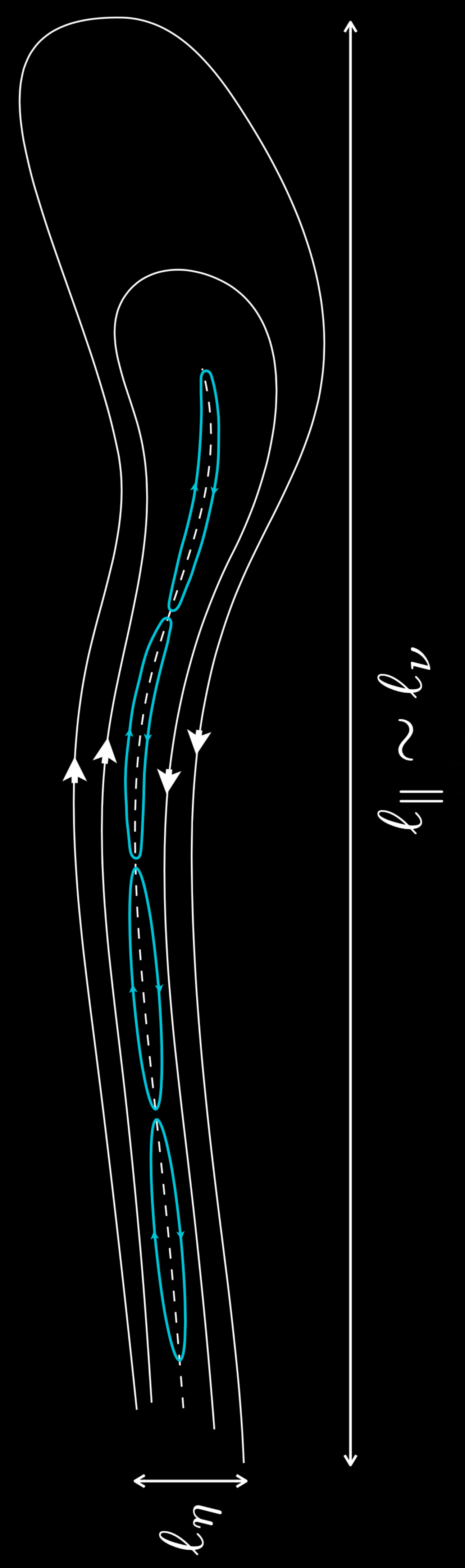
Ferrière, 2020; Plasma Physics and Controlled Fusion

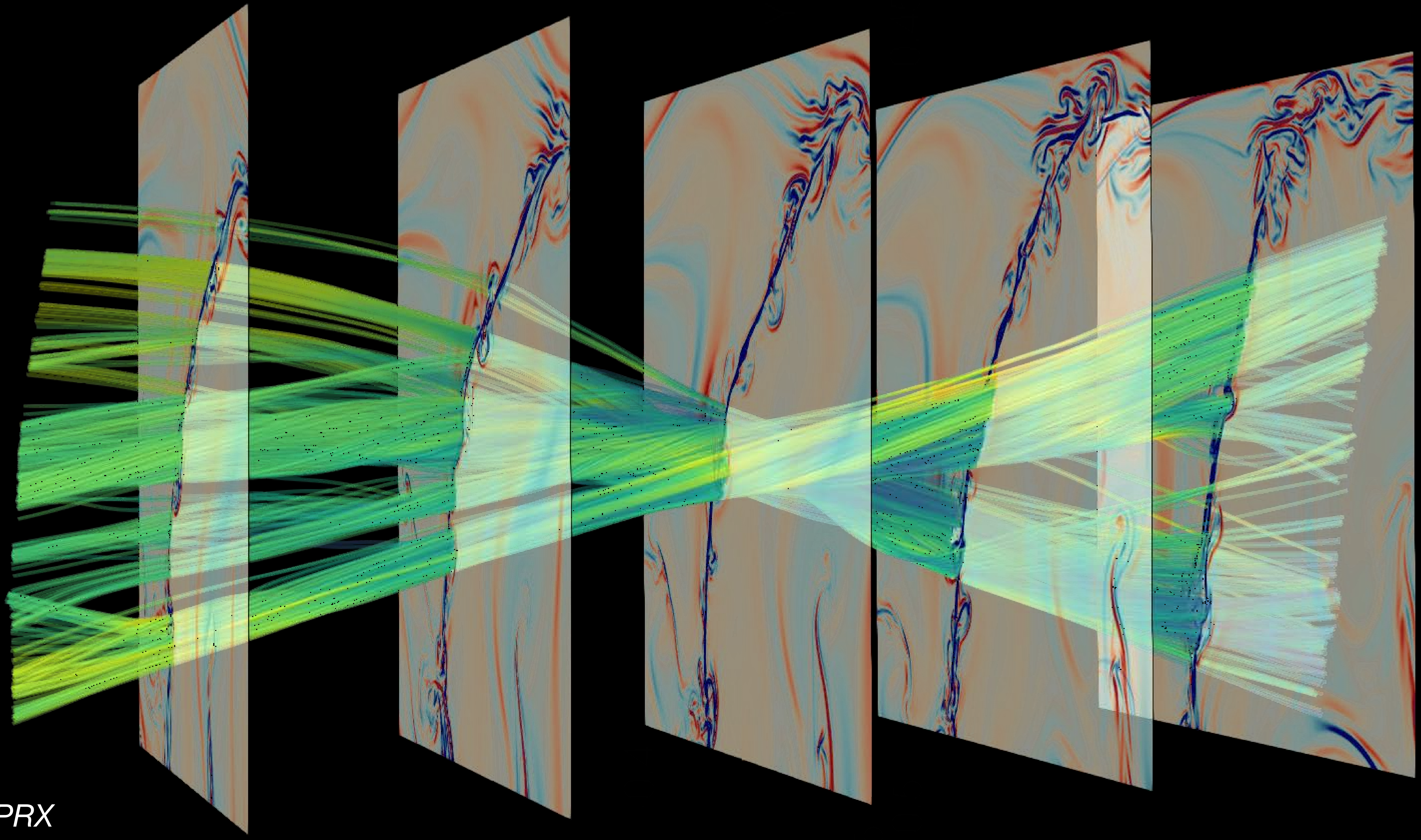


The magnetic energy cascade

Dong+(2022) *Science Advances*



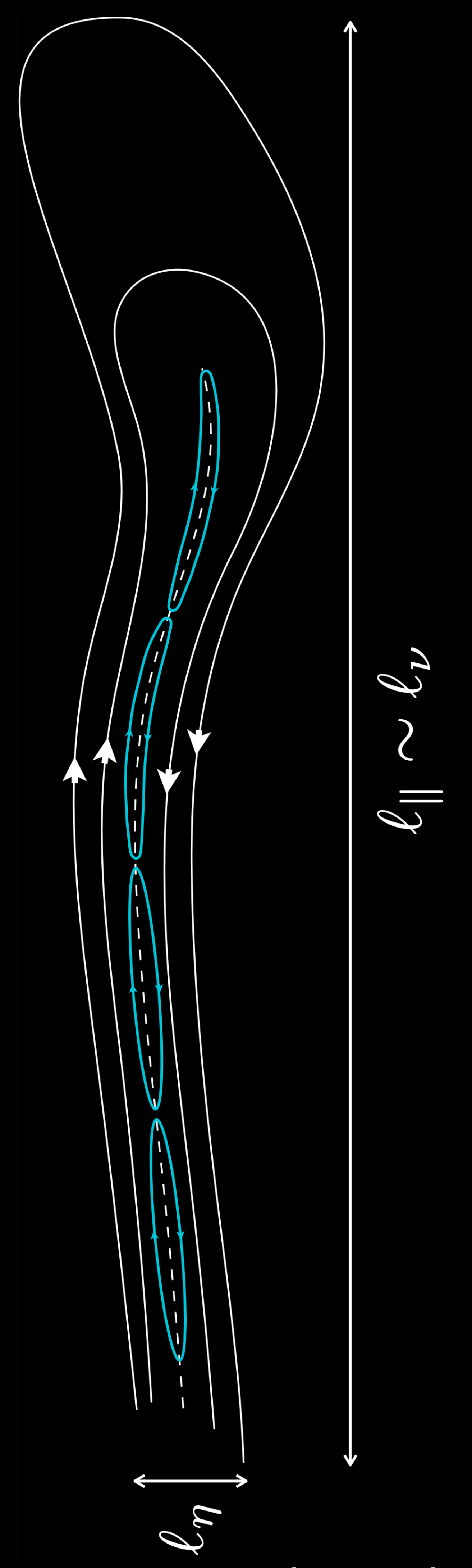
Galishnikova+ (2022) *PRX*

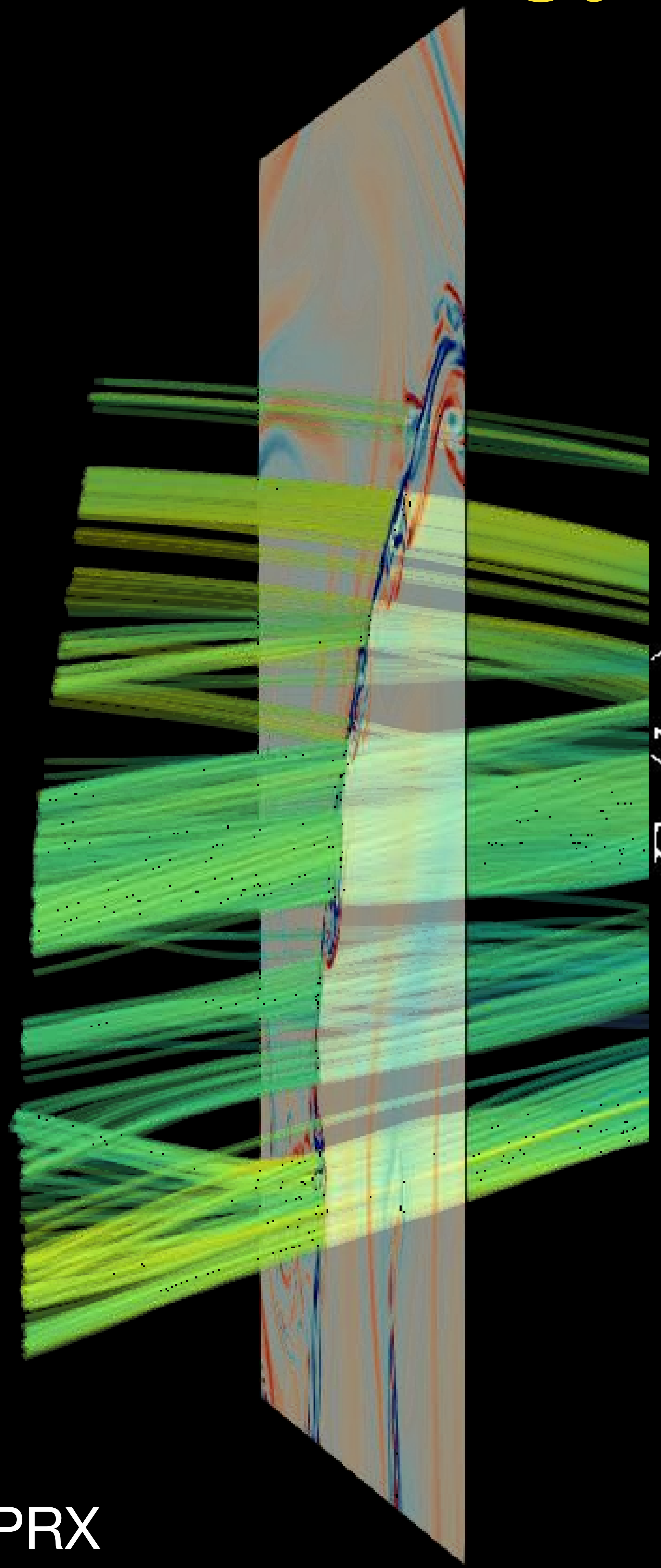


tearing instabilities growing inside a 3D current sheet

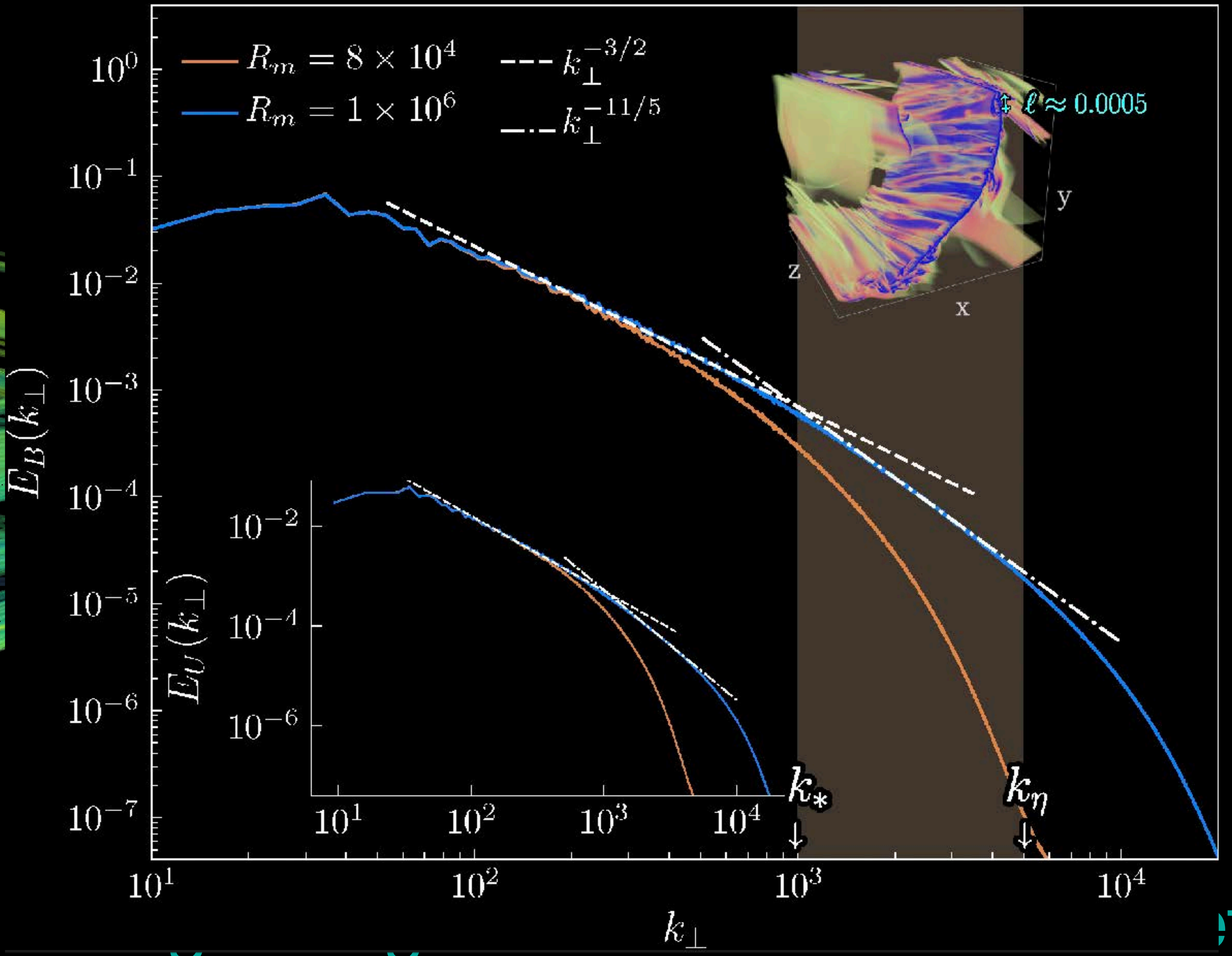
The magnetic energy cascade

Dong+(2022) *Science Advances*





Galishnikova+ (2022) PRX



plasmoid instabi

et

Stochastically forced Compressible MHD equations (ILES)

$$\partial_t \rho + \nabla \cdot \left(\rho \mathbf{u}\right) = 0$$

$$\partial_t \left(\rho \mathbf{u}\right) + \nabla \cdot \left(\rho \mathbf{u} \otimes \mathbf{u} + p \mathbb{I} - \frac{1}{4\pi} \mathbf{b} \otimes \mathbf{b}\right) = \boxed{\rho \mathbf{f}} + \nabla \cdot \mathbb{D}_\nu (\rho \mathbf{u})$$

$$\partial_t \mathbf{b} + \nabla \cdot (\mathbf{u} \otimes \mathbf{b} - \mathbf{b} \otimes \mathbf{u}) = \nabla \cdot \mathbb{D}_\eta (\mathbf{b})$$

$$\nabla \cdot \mathbf{b} = 0$$

$$p = c_s^2 \rho + \frac{1}{8\pi} \mathbf{b} \cdot \mathbf{b}$$

the turbulence source function

Stochastically forced Compressible MHD equations (ILES)

$$d\hat{\mathbf{f}}(\mathbf{k},t) = f_0(\mathbf{k})\mathbb{P}(\mathbf{k}) \cdot d\mathbf{W}(t) - \hat{\mathbf{f}}(\mathbf{k},t)\frac{dt}{t_0}$$

$d\mathbf{W}(t)$ Weiner process that draws delta correlated from $\sim \mathcal{N}(0,1)$

K space projection tensor

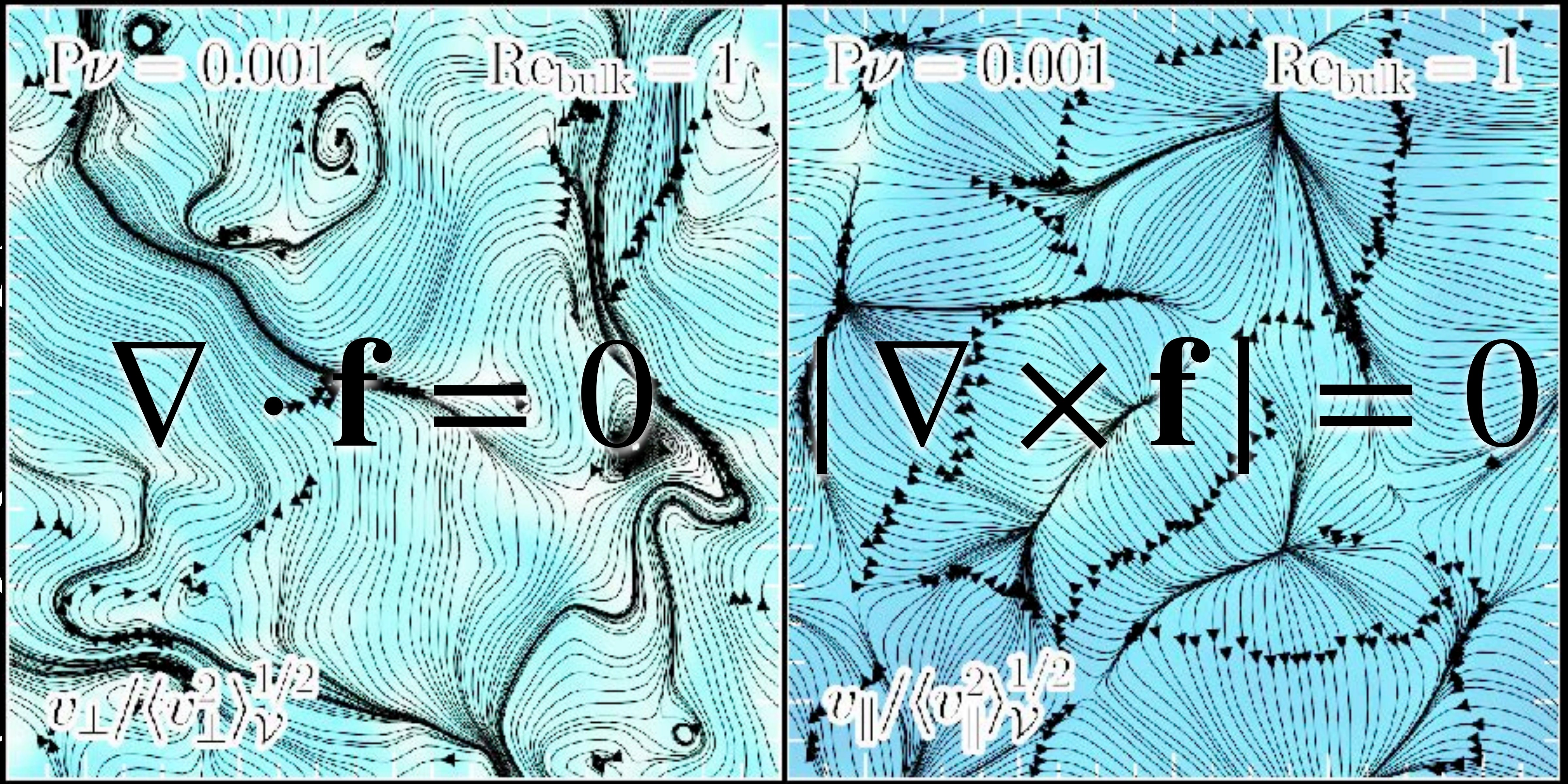
$$\mathbb{P} = \zeta\mathbb{P}^{\perp} + (1-2\zeta)\mathbb{P}^{\parallel} = \zeta\mathbb{I} + (1-2\zeta)\frac{\mathbf{k}\otimes\mathbf{k}}{|\mathbf{k}|^2}$$

$$\zeta = 1 \implies \nabla\cdot\mathbf{f} = 0$$

$$\zeta = 0 \implies \nabla\times\mathbf{f} = 0$$

t_0 e-folding time of the forcing / correlation time / outer-scale turbulent turnover time

Stochastically forced Compressible MHD equations (ILES)



$$d\hat{\mathbf{f}}(\mathbf{k}, \ldots \mathbf{k}, t)\frac{dt}{t_0}$$

$d\mathbf{W}($ ... ed from $\sim \mathcal{N}(0,1)$

K space projectior

$$\mathbb{P} = \zeta \mathbb{P}^{\perp} + (1 - 2\zeta)\mathbb{P}^{\parallel} = \zeta \mathbb{I} + (1 - 2\zeta)\frac{\mathbf{k} \otimes \mathbf{k}}{|\mathbf{k}|^2}$$

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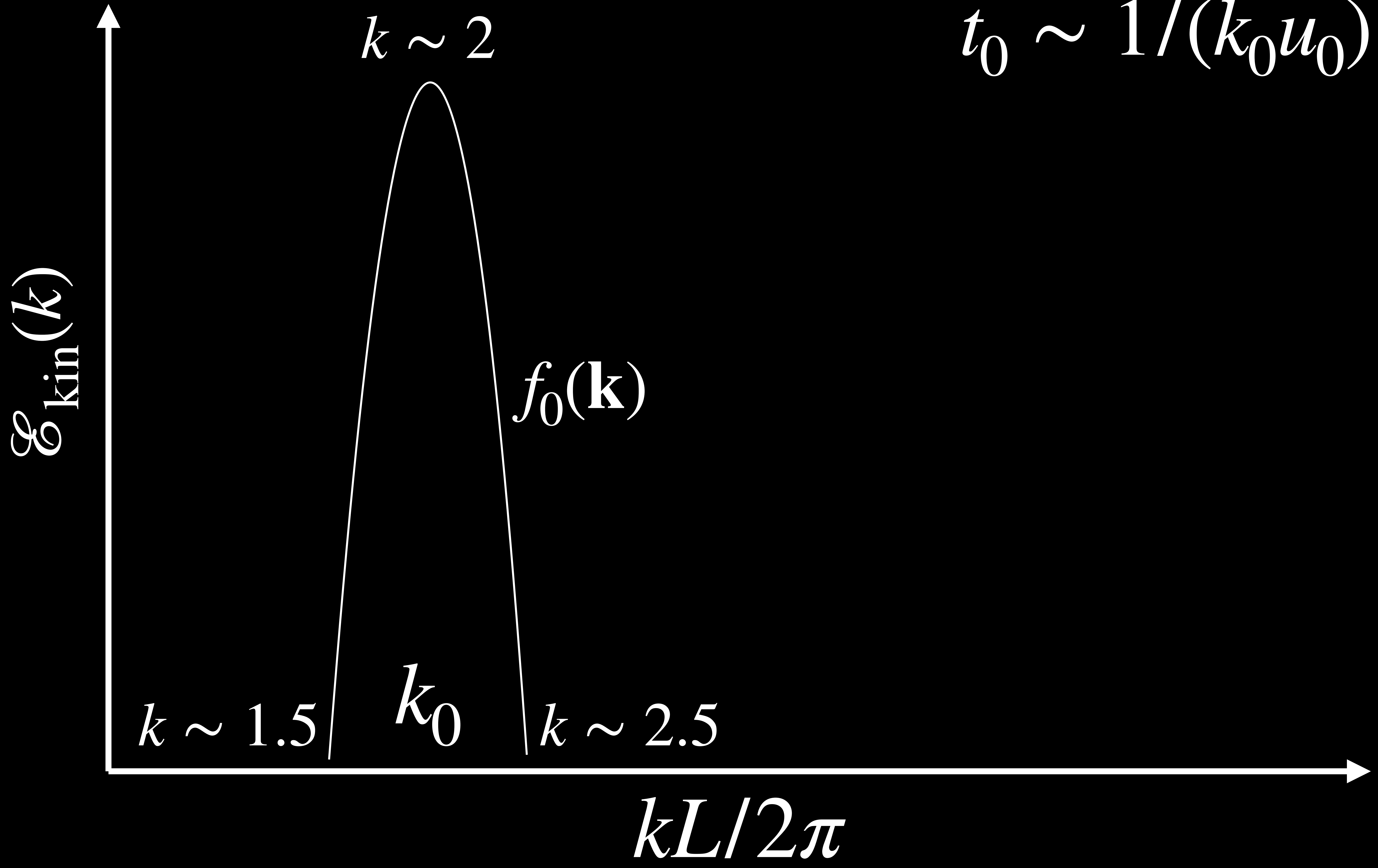
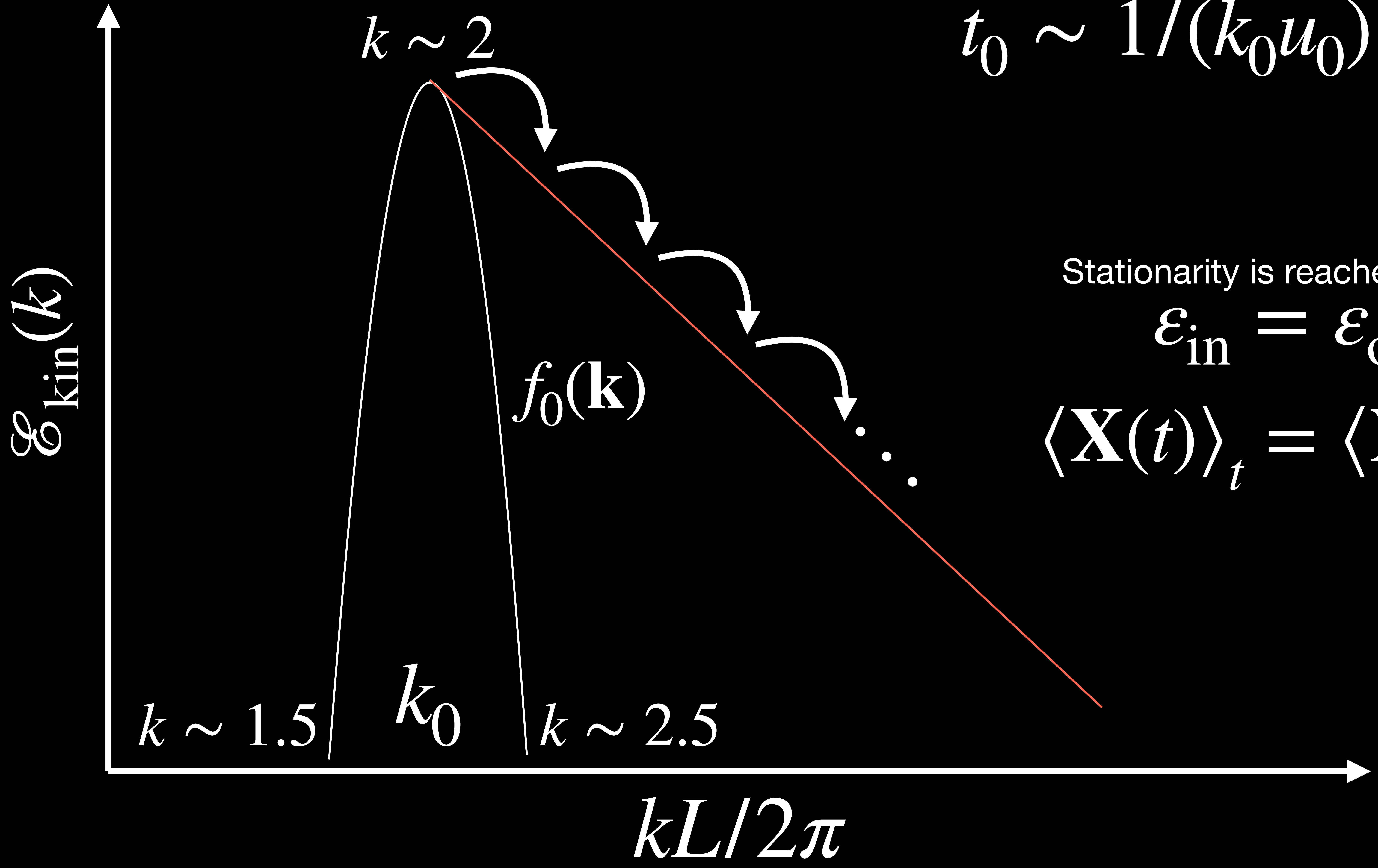


$t_0 \sim 1/(k_0 u_0)$

Stochastically forced Compressible MHD equations (ILES)



$$t_0 \sim 1/(k_0 u_0)$$

Stationarity is reached when

$$\varepsilon_{\mathrm{in}} = \varepsilon_{\mathrm{out}}$$

$$\langle \mathbf{X}(t) \rangle_t = \langle \mathbf{X}(t+\tau) \rangle_t$$

$10{,}080^3$ magnetised interstellar medium turbulence simulation

Beattie, Federrath, Mocz, Klessen &

1. What is the scaling of the energy cascad compressible MHD turbulence with no n
2. How are the characteristic scales organi ISM turbulence?
3. What are the saturation physics of the tu dynamo?

PI of a 100million core-hour project on super

ILES of compressible MHD turbulence

Turbulence: $\sigma_V/c_s \approx 4,\ \ell_0 = L/2$

Magnetic fields: $B = b_{\rm turb},\ \mathcal{M}_{\rm A} \approx 2$

Three experiments for convergence tests:

- LOW-RES: $2{,}520^3$ (0.3Mcore-h, 8,640
- MID-RES: $5{,}040^3$ (4.0Mcore-h, 34,56
- HIGH-RES: $10{,}080^3$ (80.0Mcore-h, 148,
  - Resolving 10pc down to ~200au everywhere on
  - Factor of 4 higher linear grid resolution than Illu

3.45PB in data products

Broad Code details:

- Highly-modified version of finite volume code *FLASH*, second-order in space approximate Riemann (PPM)

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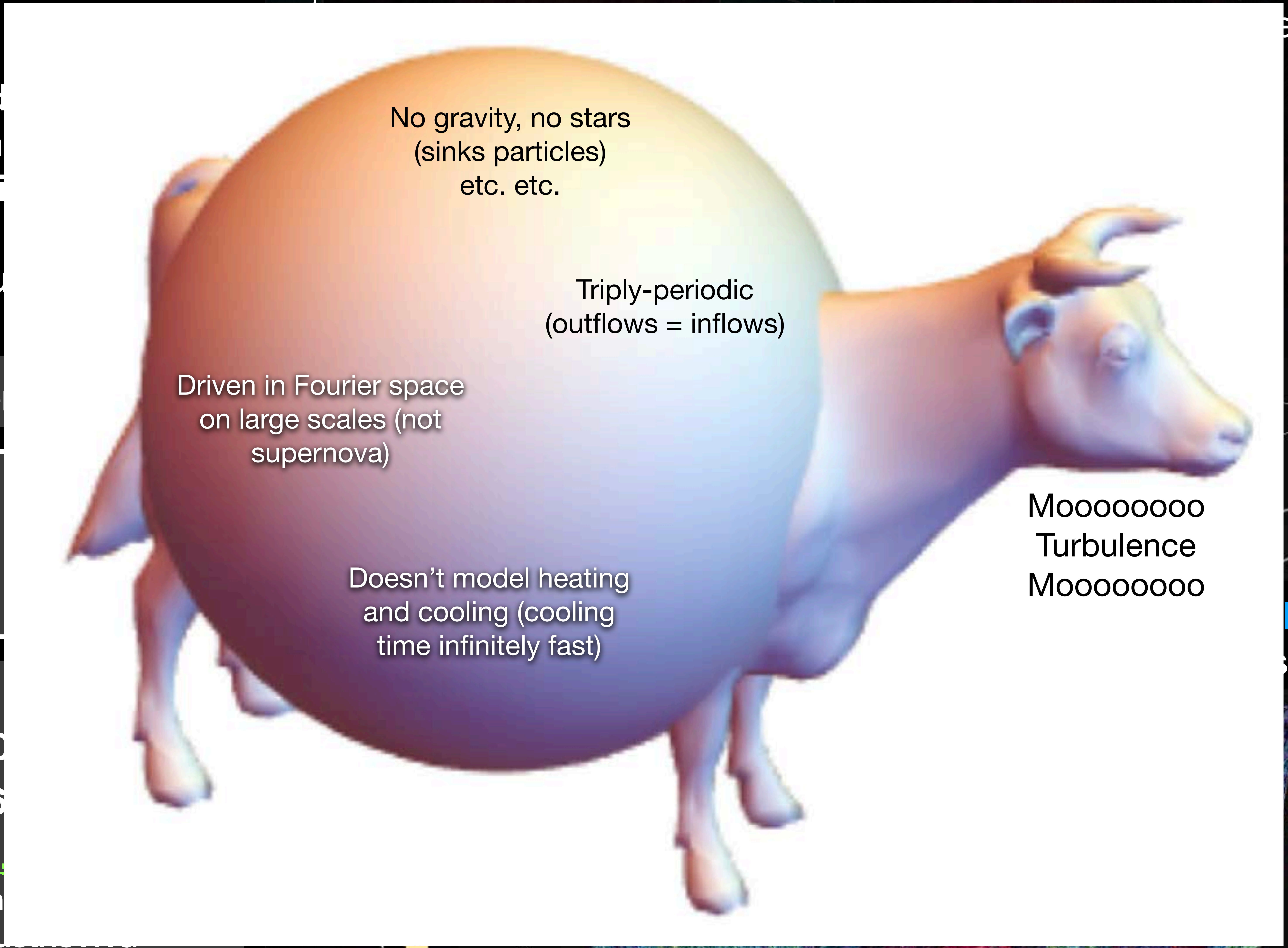


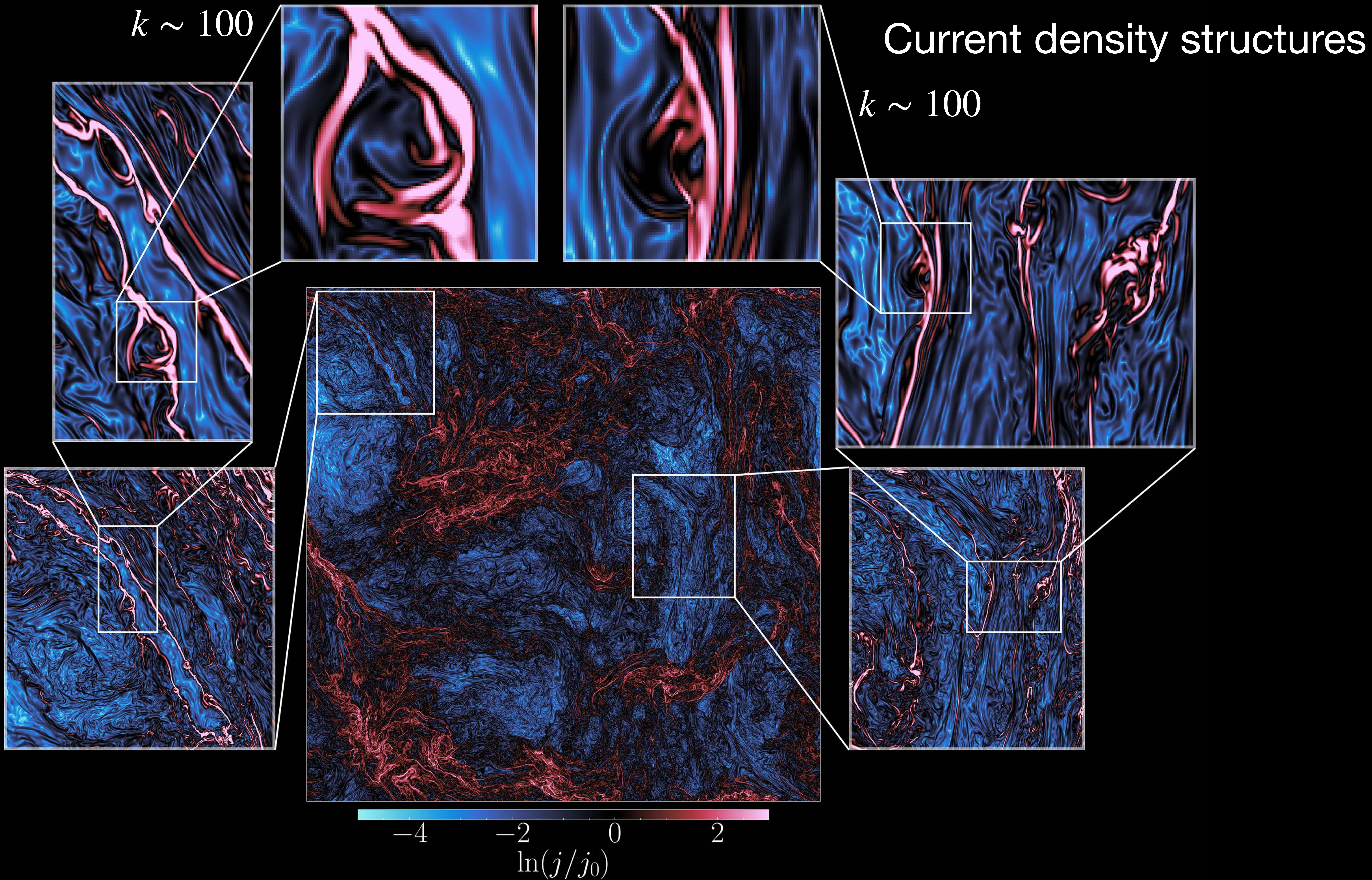
Visualisation: Salvatore Cielo (VisIt)

Volume integral statistics

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).


Gas density structures
$\ln(\rho/\rho_0)$


magnetic field structures

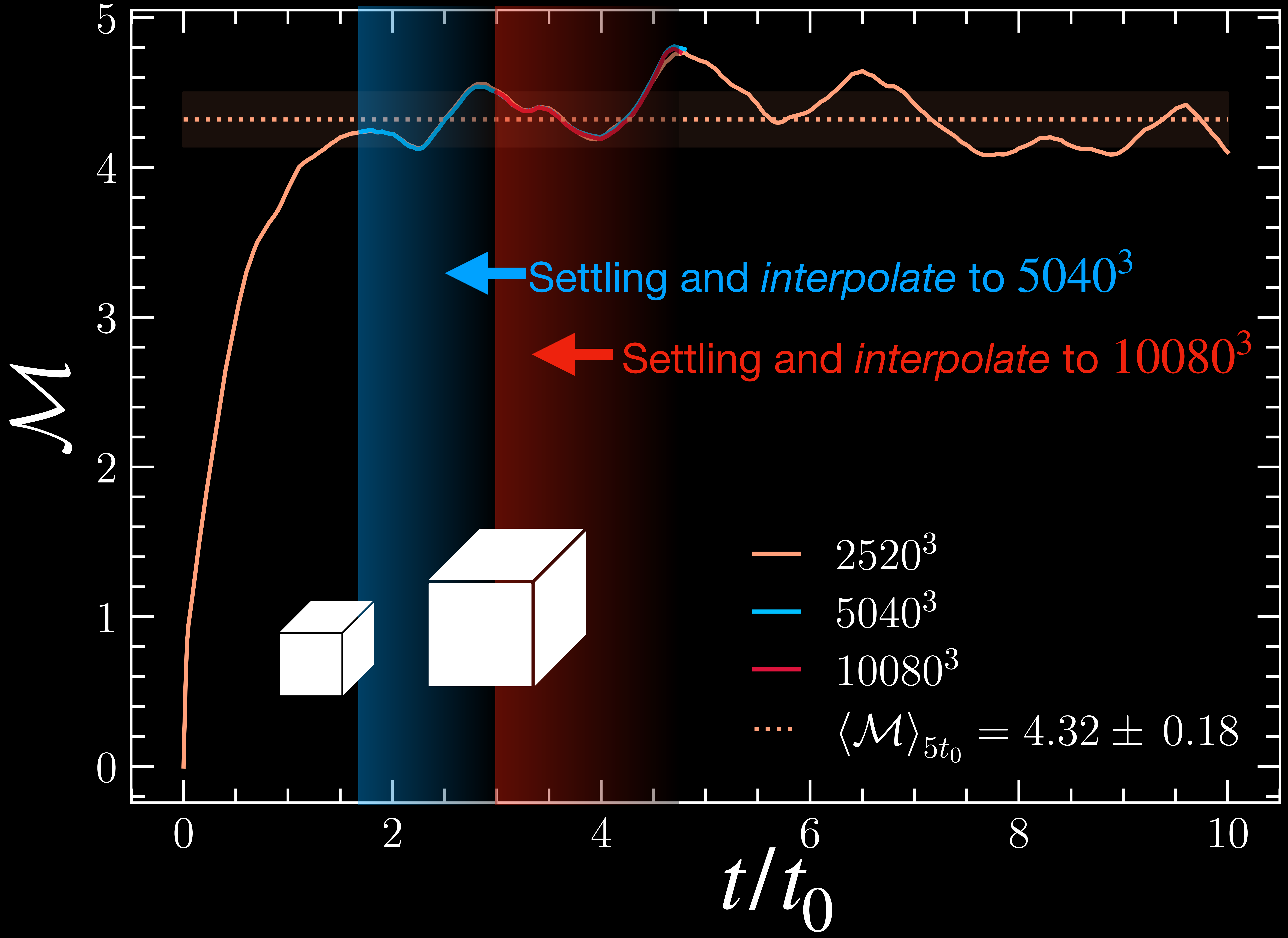


Current density structures
$k \sim 100$
$k \sim 100$
-4
-2
0
2
$\ln(j/j_0)$

Volume integral Quantities

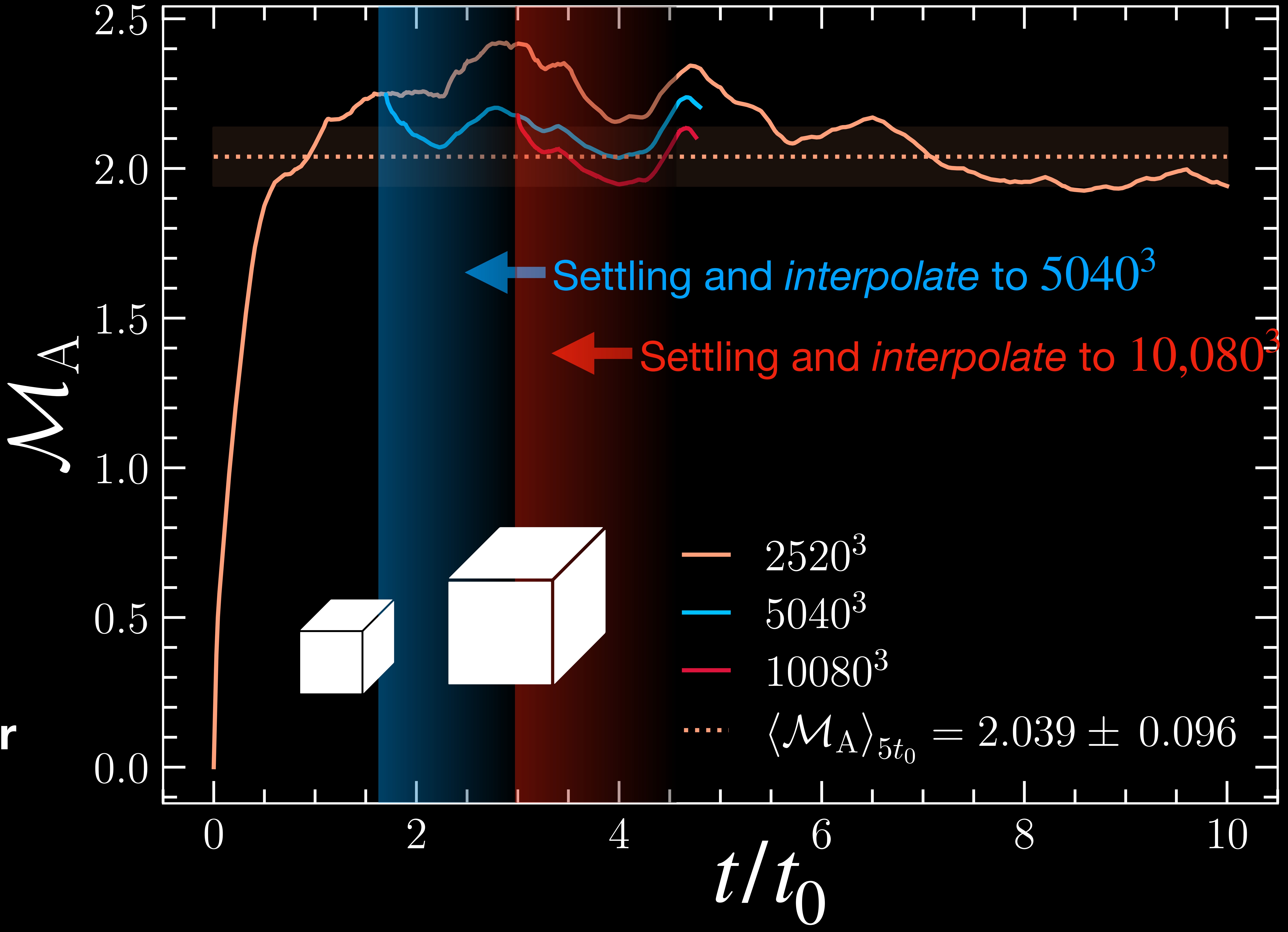
$$\mathscr{M} = \frac{\left\langle u^2 \right\rangle_{\mathscr{V}}^{1/2}}{c_s}$$

Interpolation to generate ICs for successively higher resolution experiments



Volume integral Quantities

$$\mathscr{M}_A = \sqrt{\frac{E_{\mathrm{kin}}}{E_{\mathrm{mag}}}}$$



Interpolation to generate ICs for successively higher resolution experiments

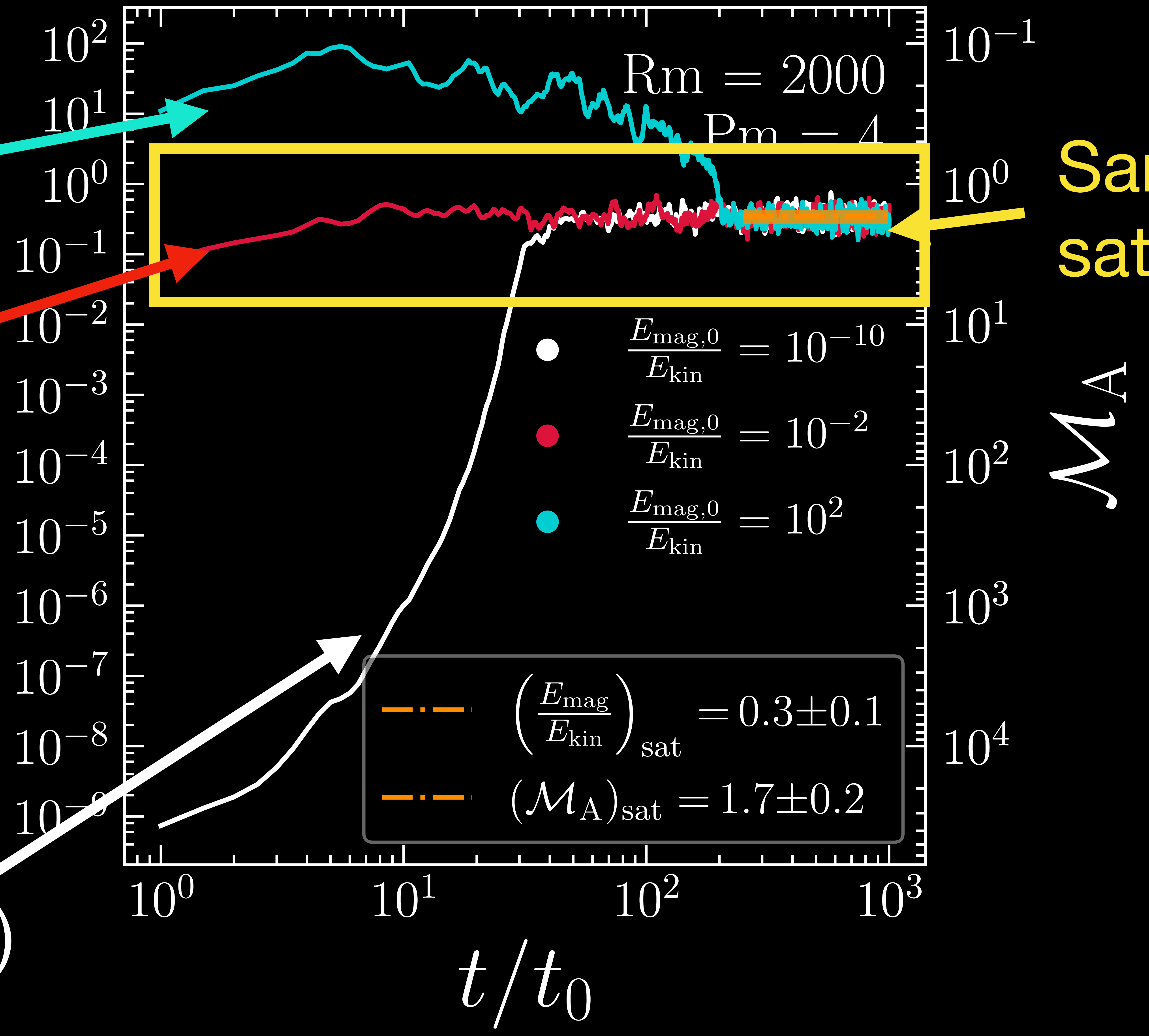
Magnetic ICs.

Decay into the saturation

Initialise in the saturation

Grow into the saturation (classical turbulent dynamo)

$$\mathscr{M}_A = \sqrt{\frac{E_{\mathrm{kin}}}{E_{\mathrm{mag}}}}$$



Same saturation

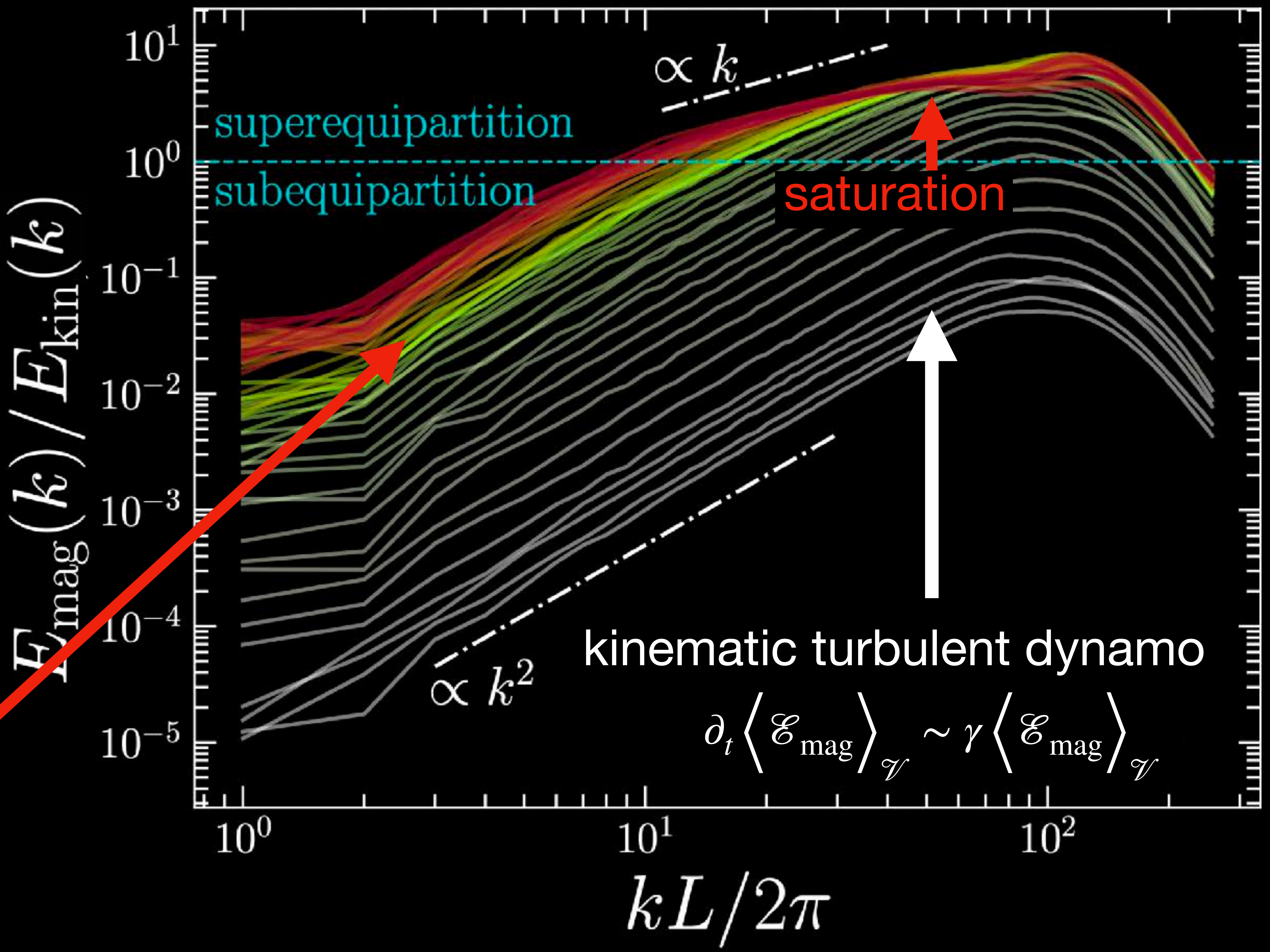
Beattie et al. (2023, MNRAS). *Growth or Decay — I: Universality of the turbulent dynamo saturation*

Magnetic ICs.

$$\langle b \rangle = 0$$

Completely turbulent magnetic field maintained by a turbulent dynamo

ICs taken from a saturated dynamo experiment

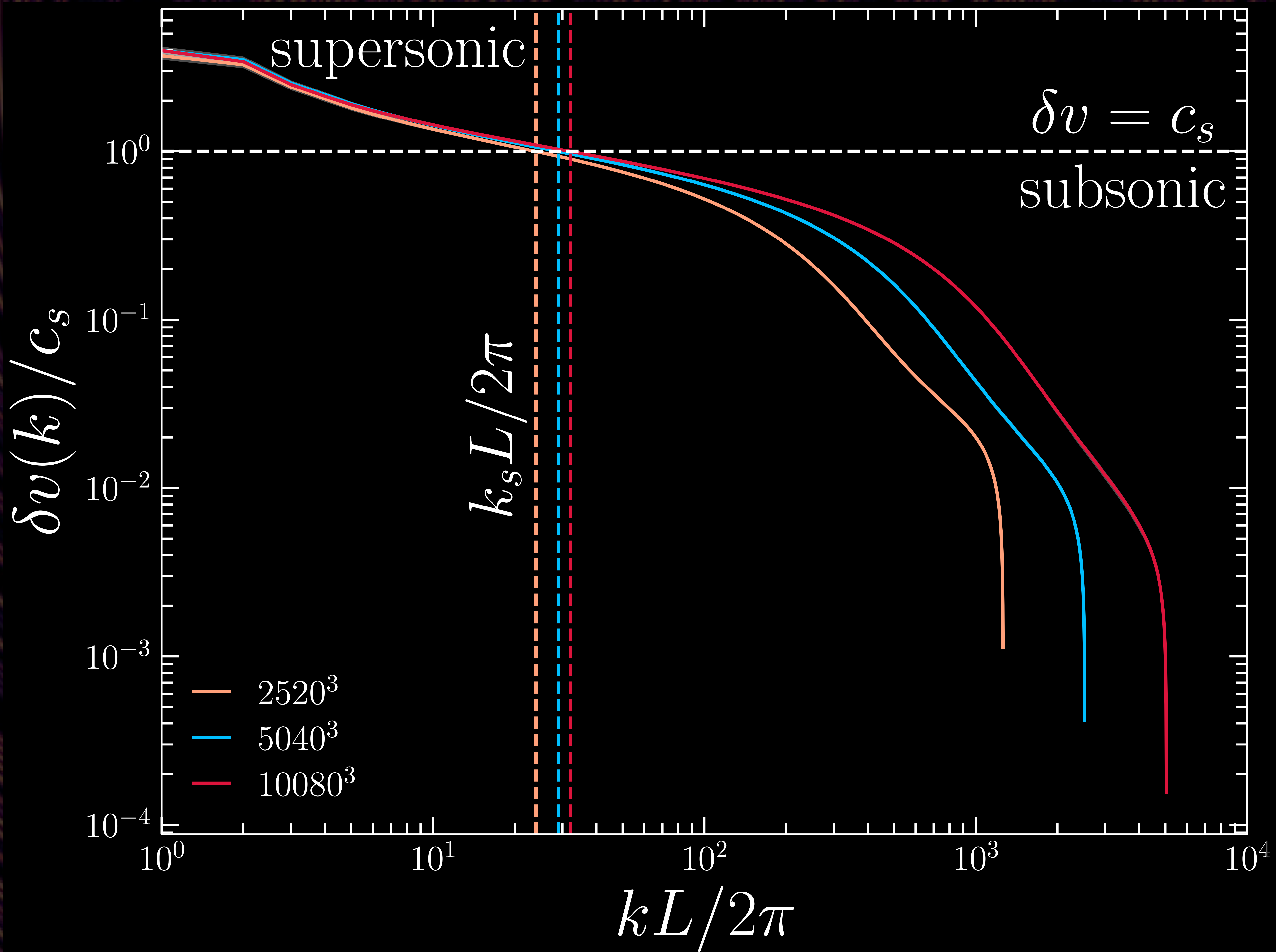


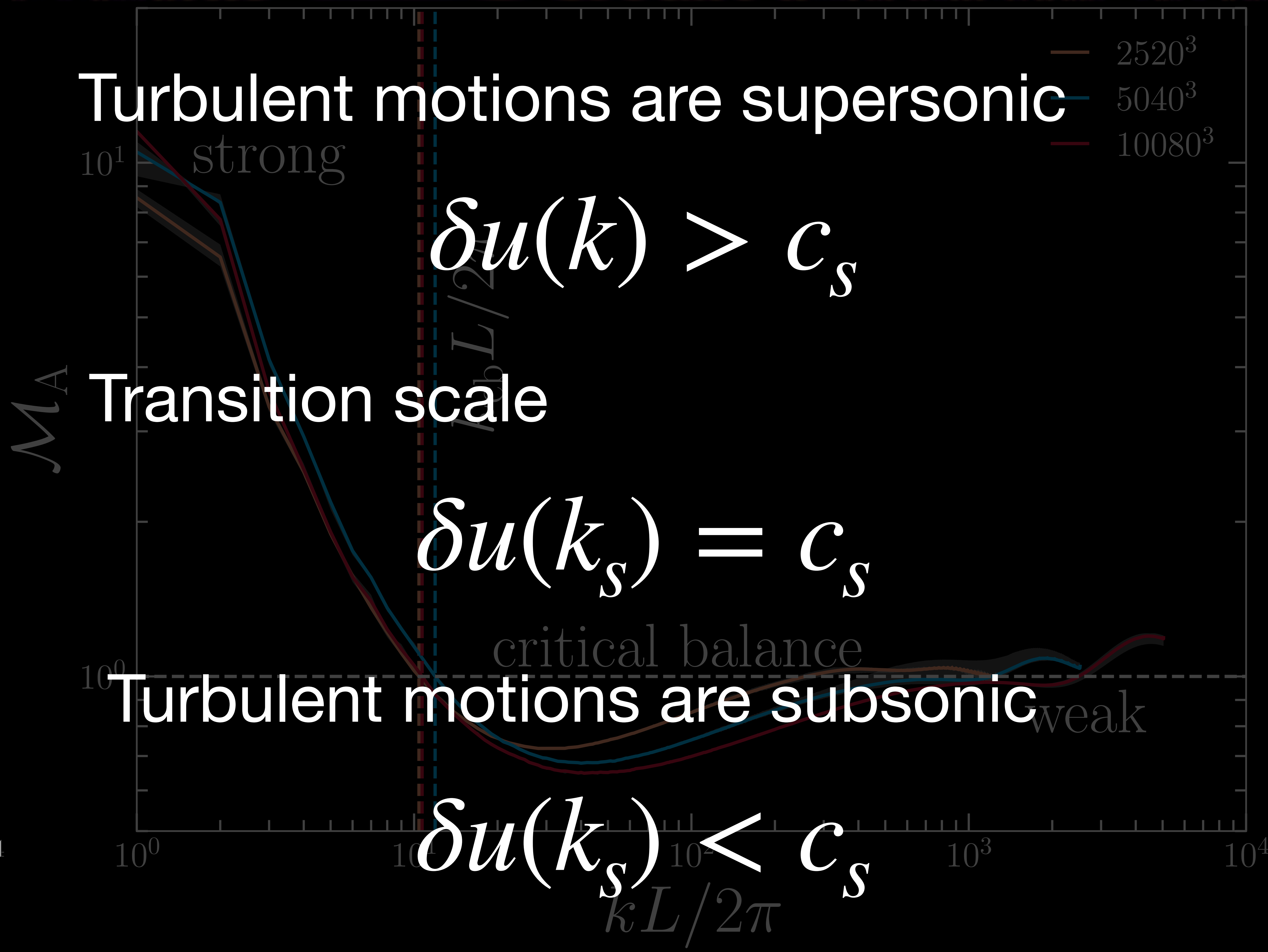
Beattie+. (2024, subm.). *Taking control of compressible modes: bulk viscosity and the turbulent dynamo*

Turbulent spectra and scales

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Two important scales





Turbulent motions are supersonic

$$\delta u(k) > c_s$$

Transition scale

$$\delta u(k_s) = c_s$$

Turbulent motions are subsonic

$$\delta u(k_s) < c_s$$

First measurement of magnetosonic scale

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Two important scales

Kinetic energy dominates, $\mathscr{E}_{\rm mag} < \mathscr{E}_{\rm kin}$

$$\mathscr{M}_A > 1$$

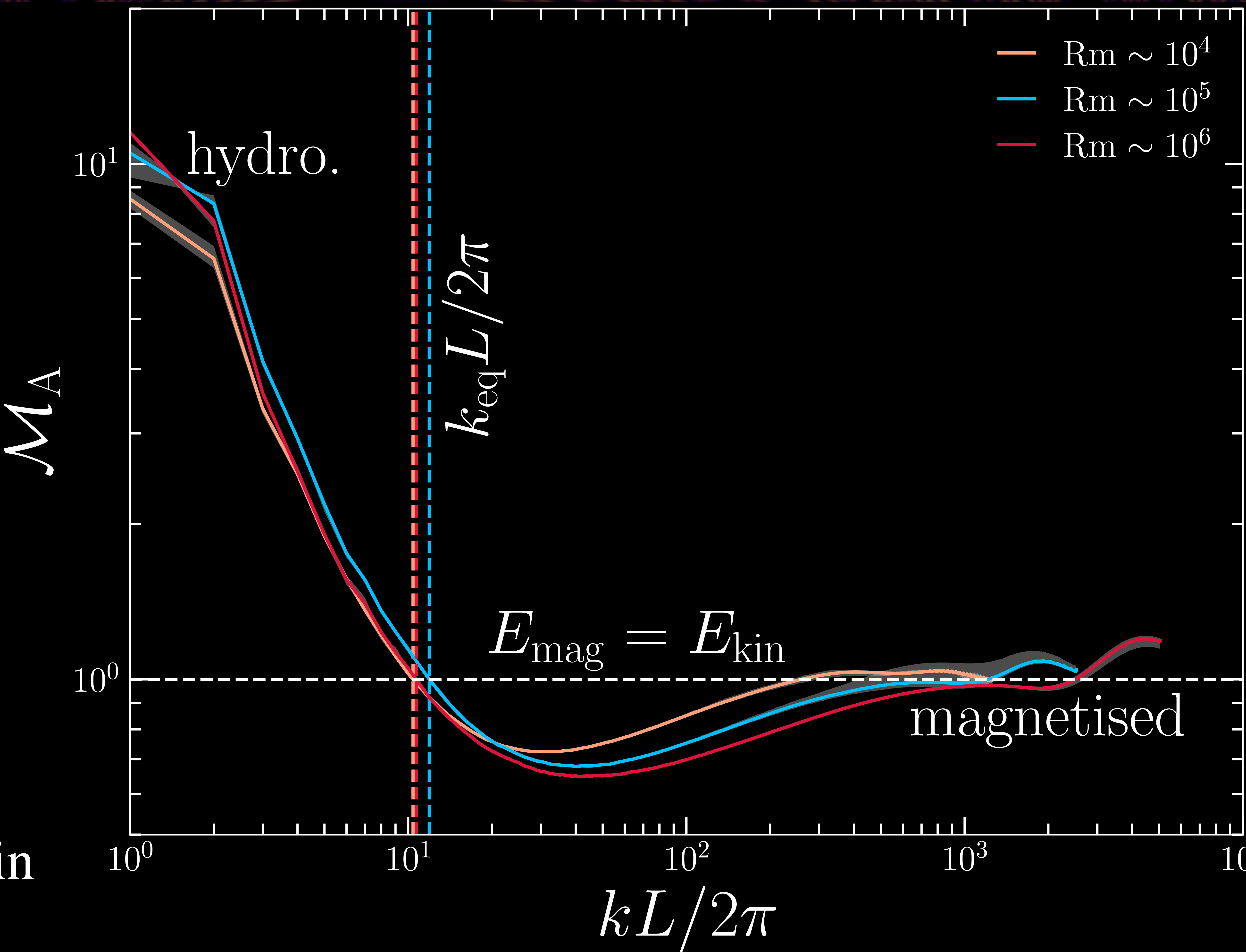
Transition into magnetically dominated turbulence $\mathscr{E}_{\rm mag} = \mathscr{E}_{\rm kin}$

$$\mathscr{M}_A = 1$$

Magnetic energy dominates, $\mathscr{E}_{\rm mag} > \mathscr{E}_{\rm kin}$

$$\mathscr{M}_A < 1$$

Energy equipartition scale



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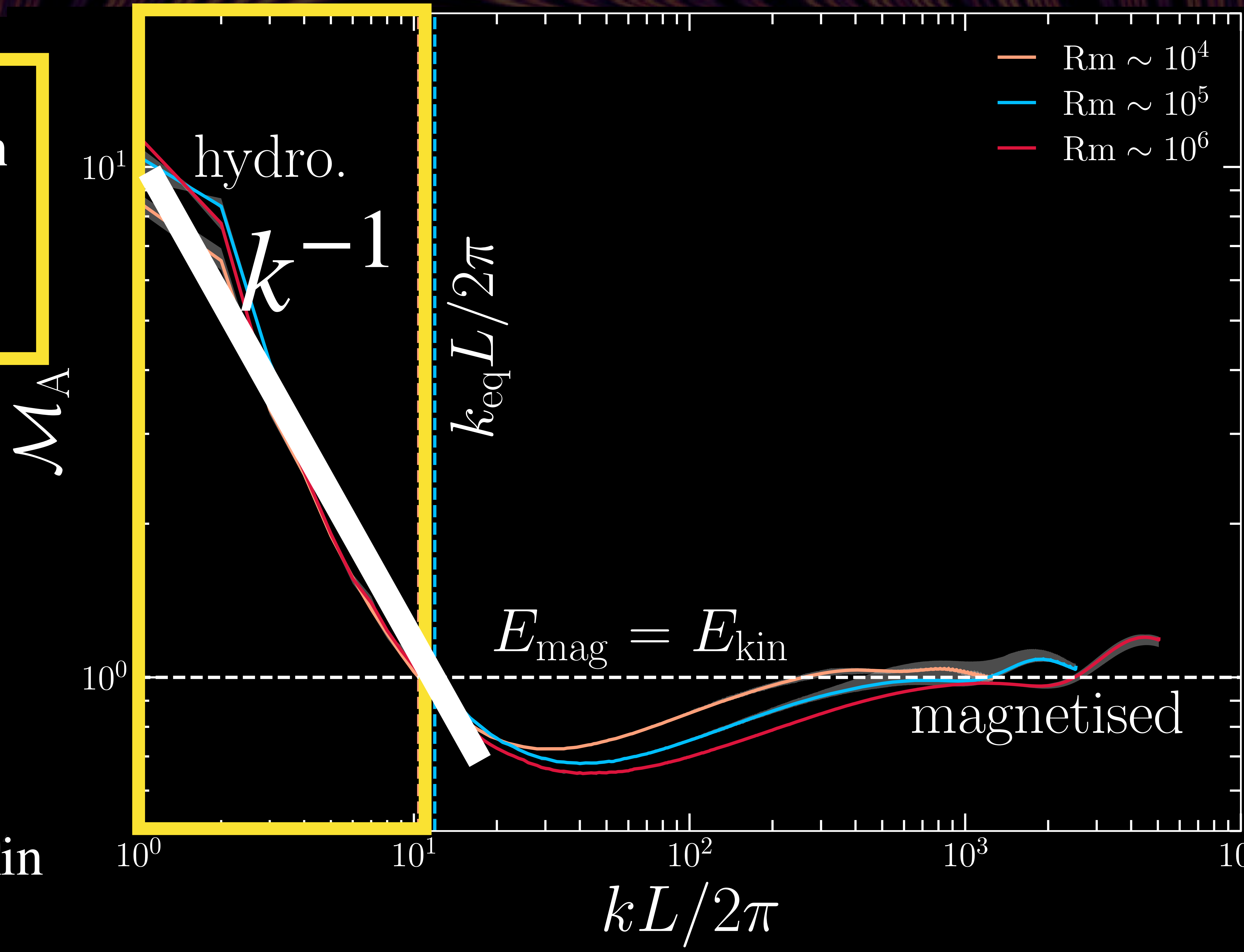

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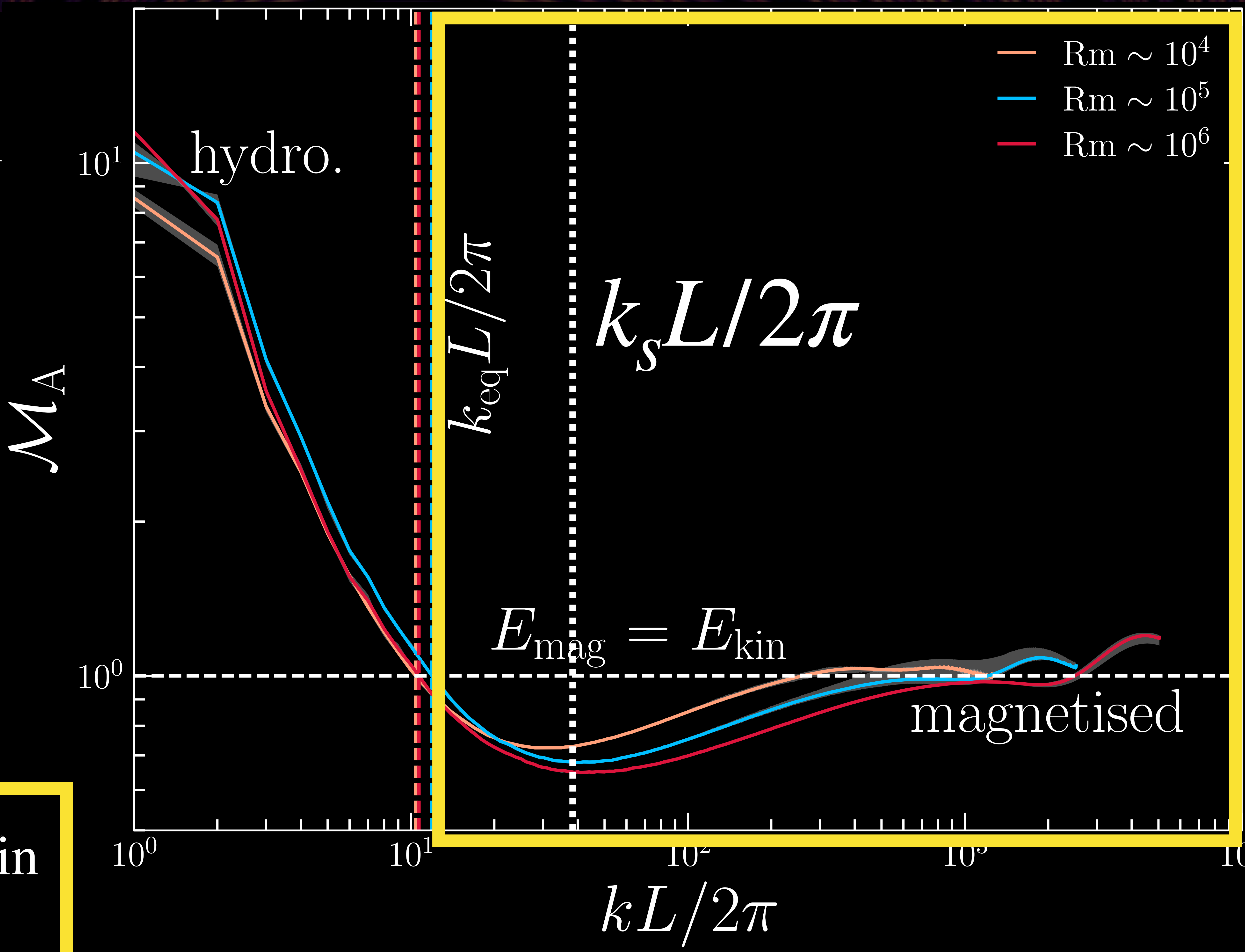

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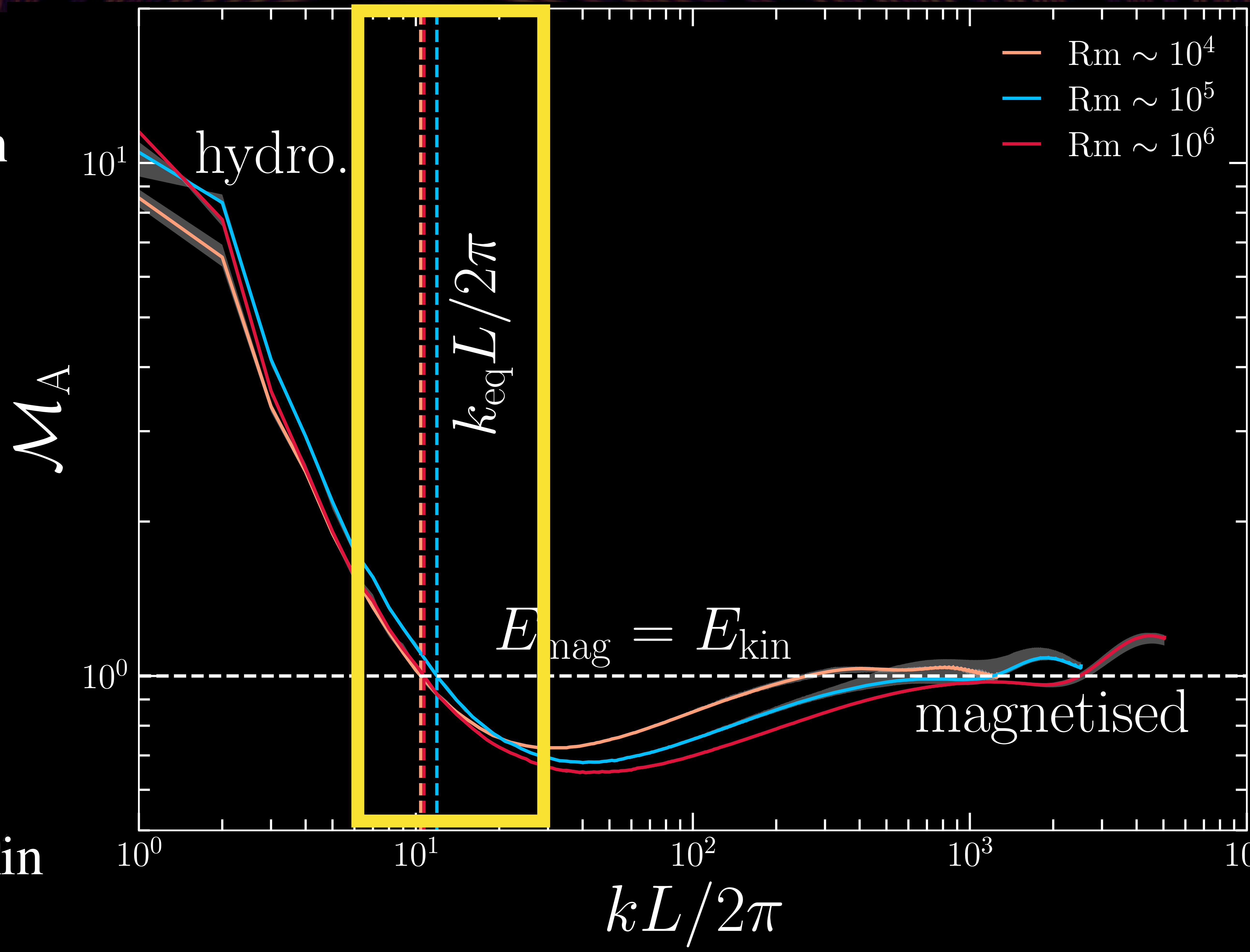
Energy equipartition scale



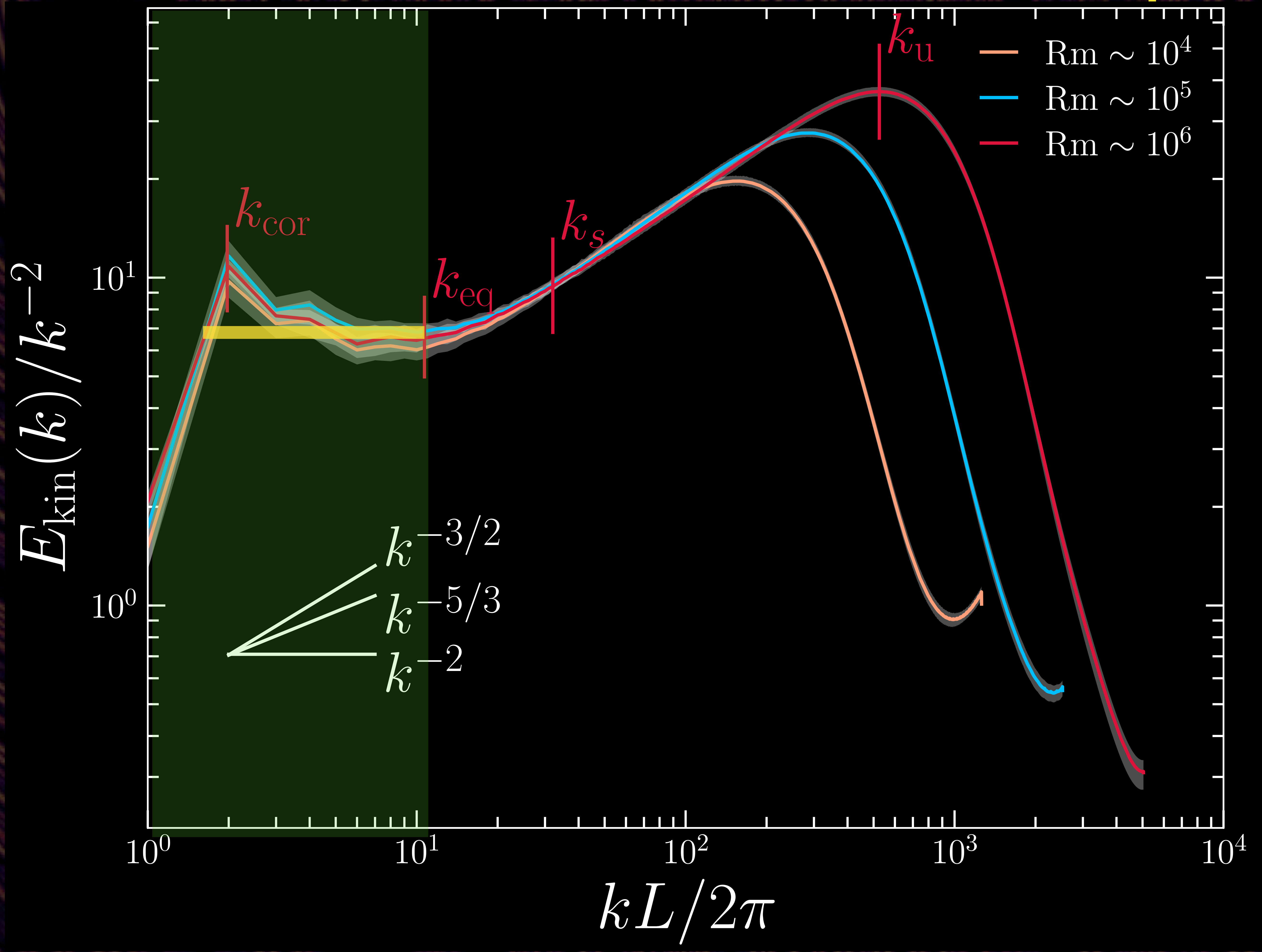
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

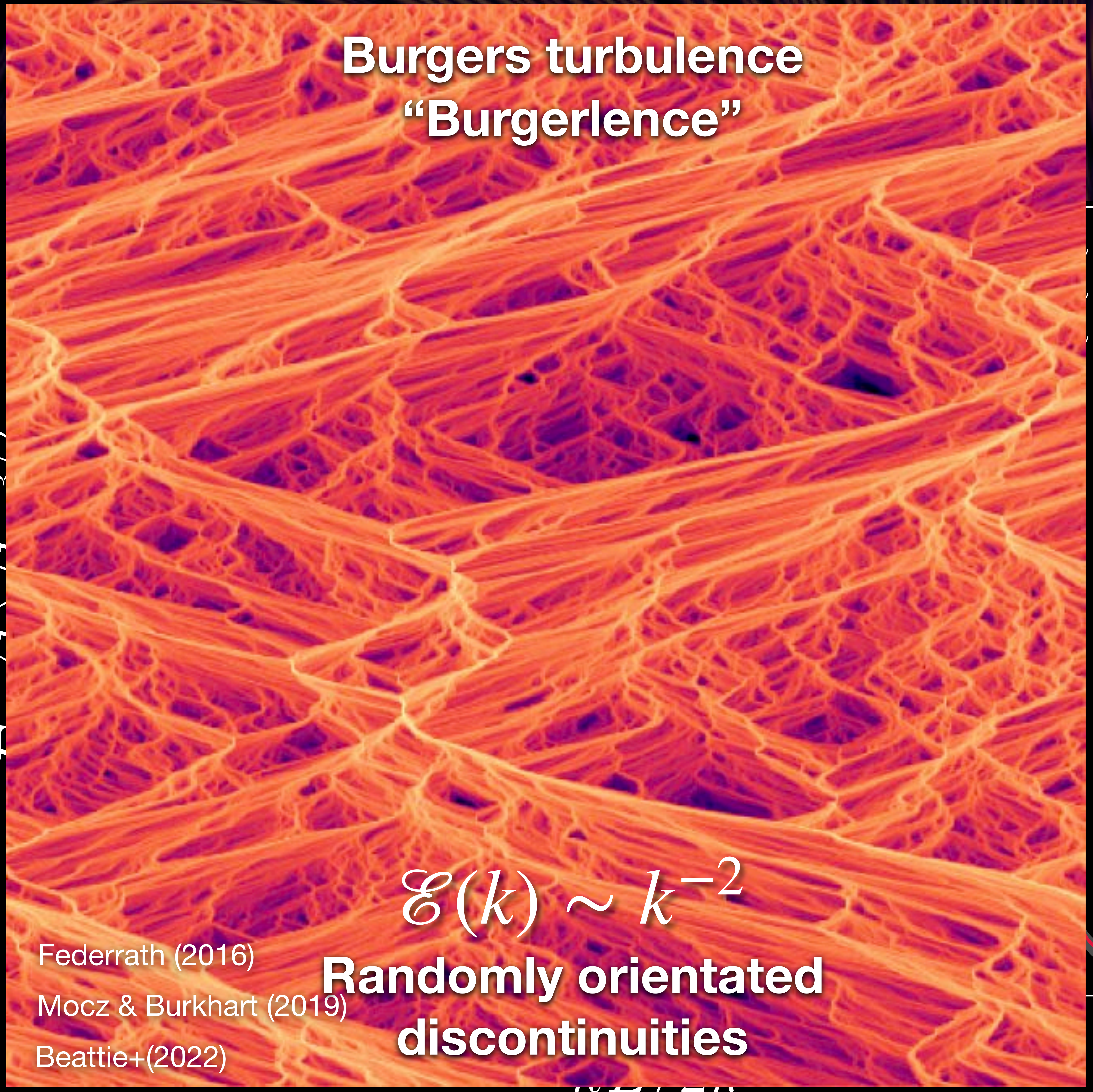
The kinetic energy cascade

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The supersonic energy cascade

$$\mathscr{E}(k) \sim k^{-2},\ k \leq k_{\mathrm{eq}}$$





Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

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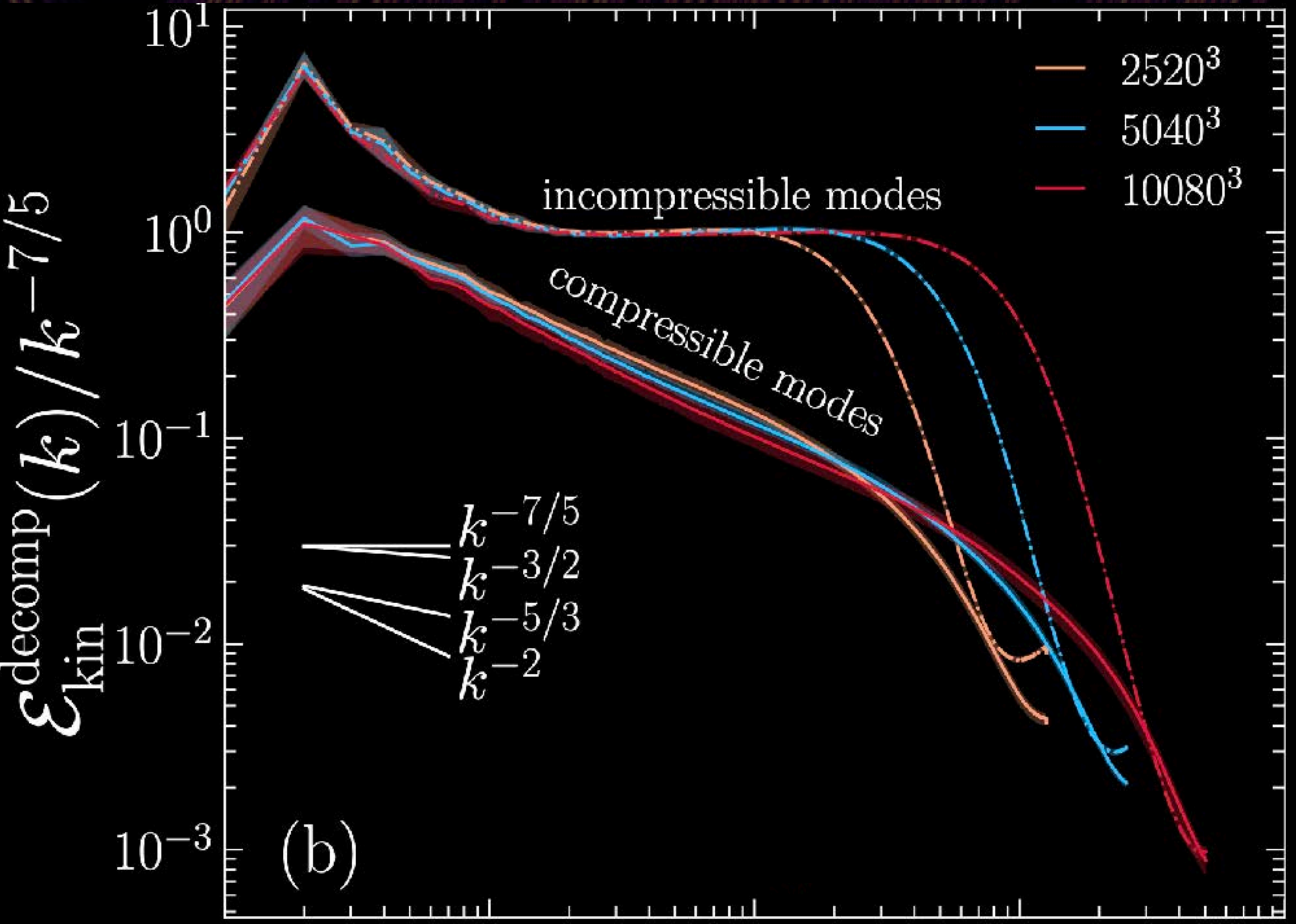
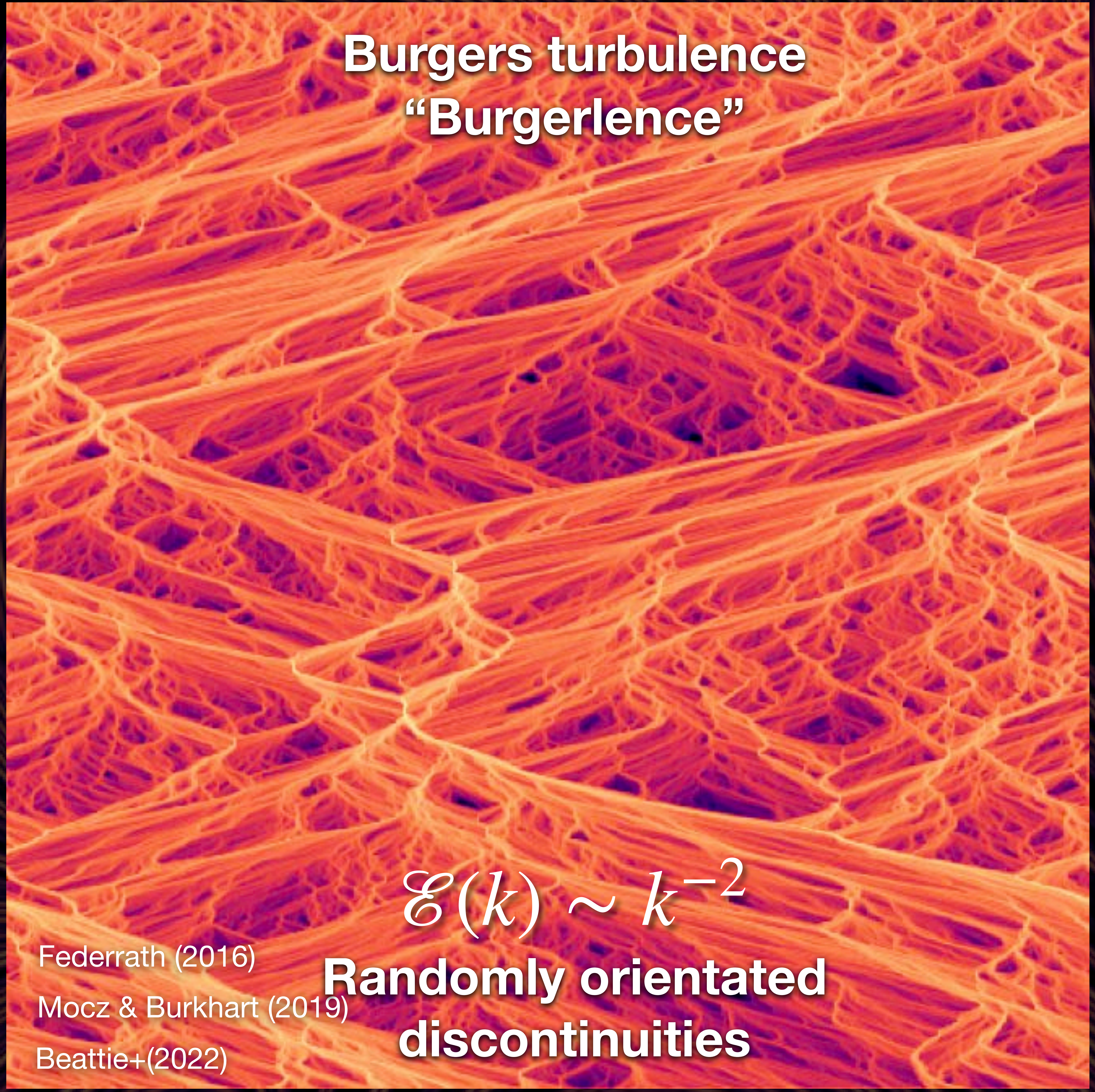




Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The subsonic energy cascade

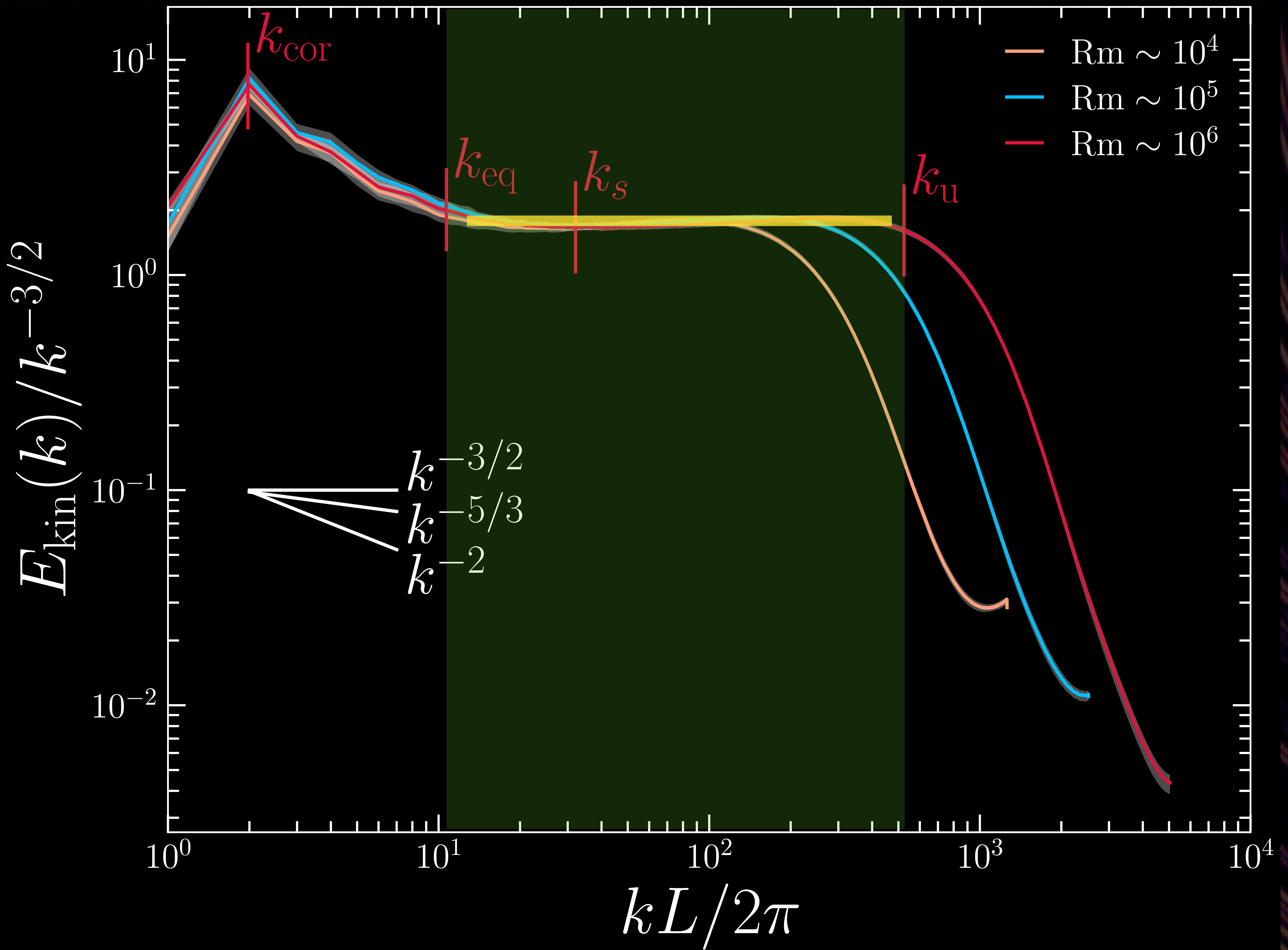
$$\mathscr{E}(k) \sim k^{-3/2},\ k > k_{\mathrm{eq}}$$

IK-type turbulence (Kolmogorov + b flux)?

Dynamical alignment?

?

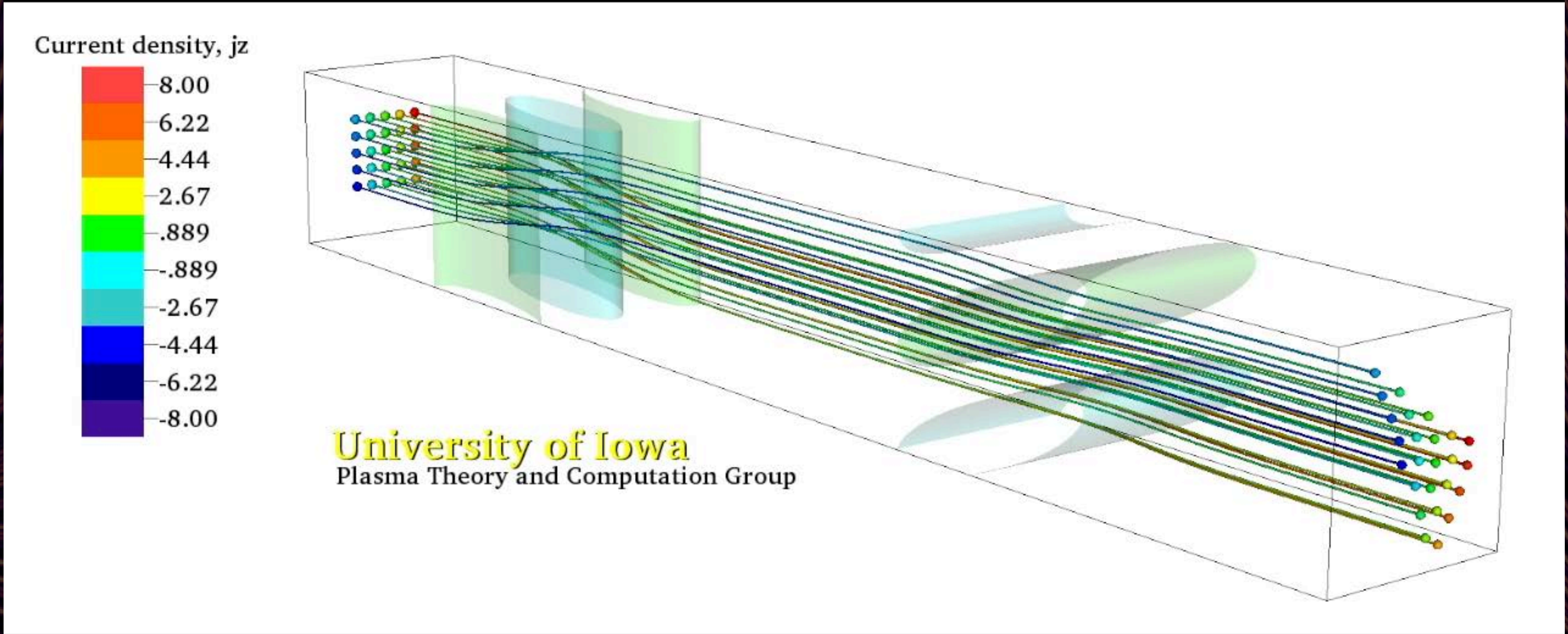
A mechanism for depleting nonlinearities in the turbulence



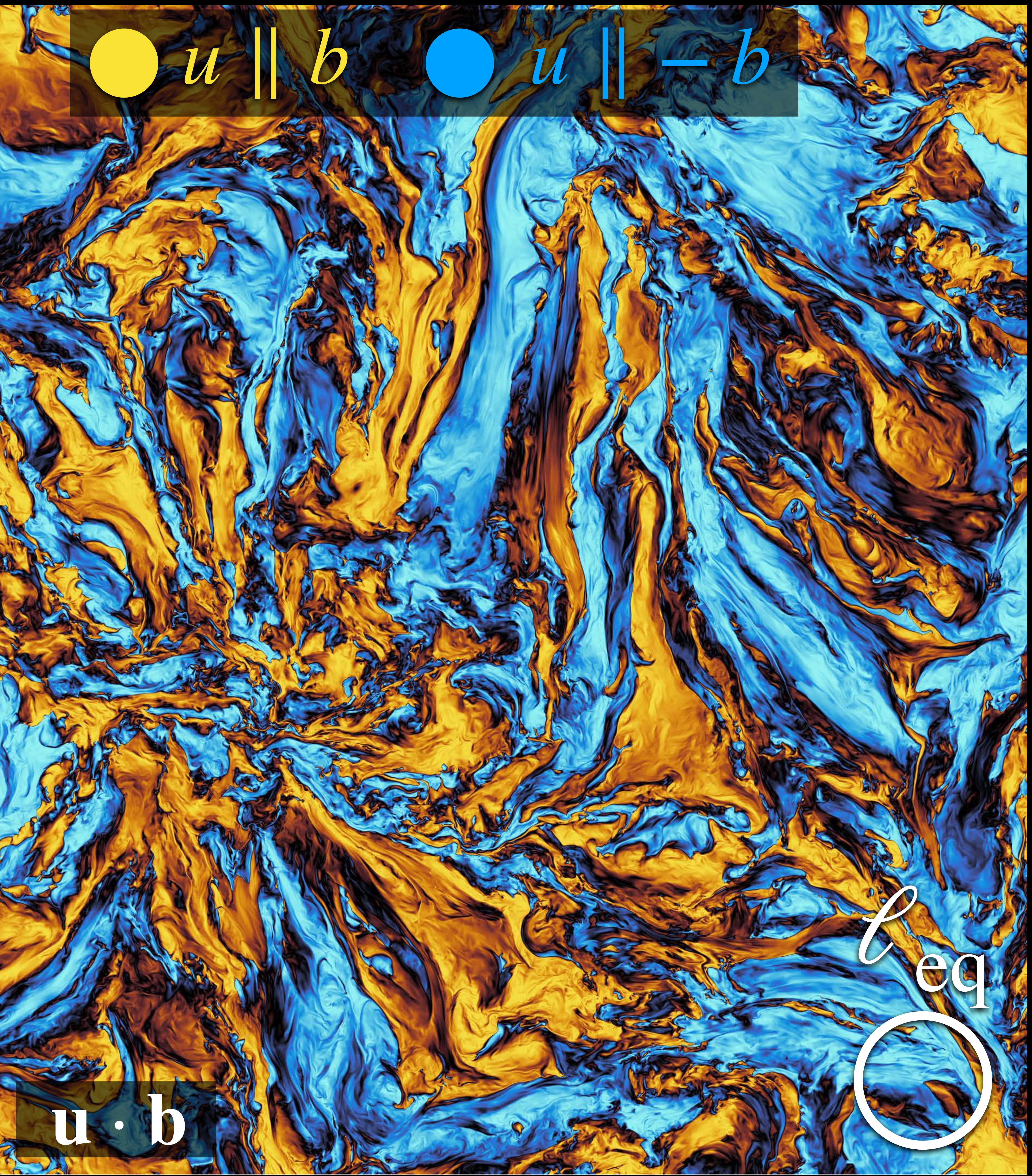
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The incompressible MHD energy cascade — new timescales

Interacting Alfvén waves (visualization from Greg G. Howes)

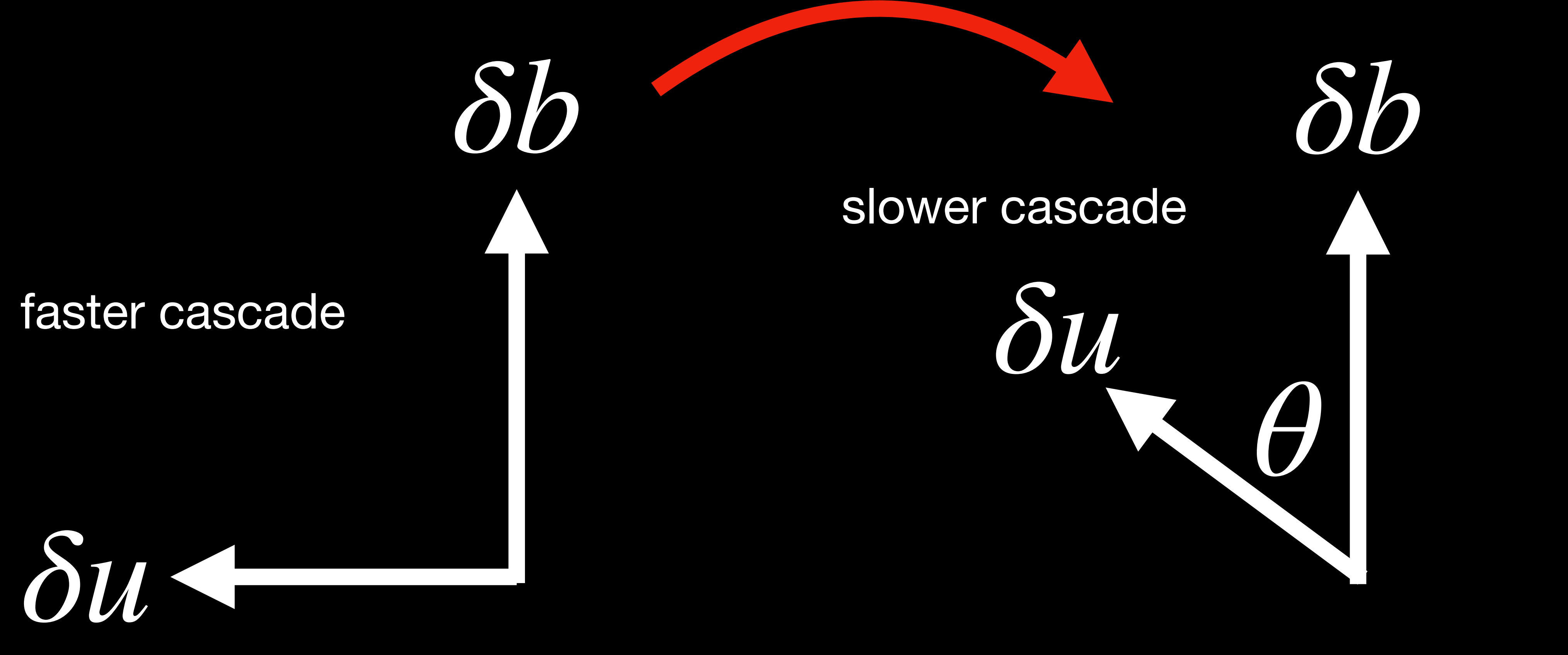


Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).



Dynamic alignment?

Shearing events between counter-propagating Alfvén waves



$E_{\mathrm{kin}}(k_\perp) \propto k_\perp^{-5/3}$

Goldreich & Shridar (1995)

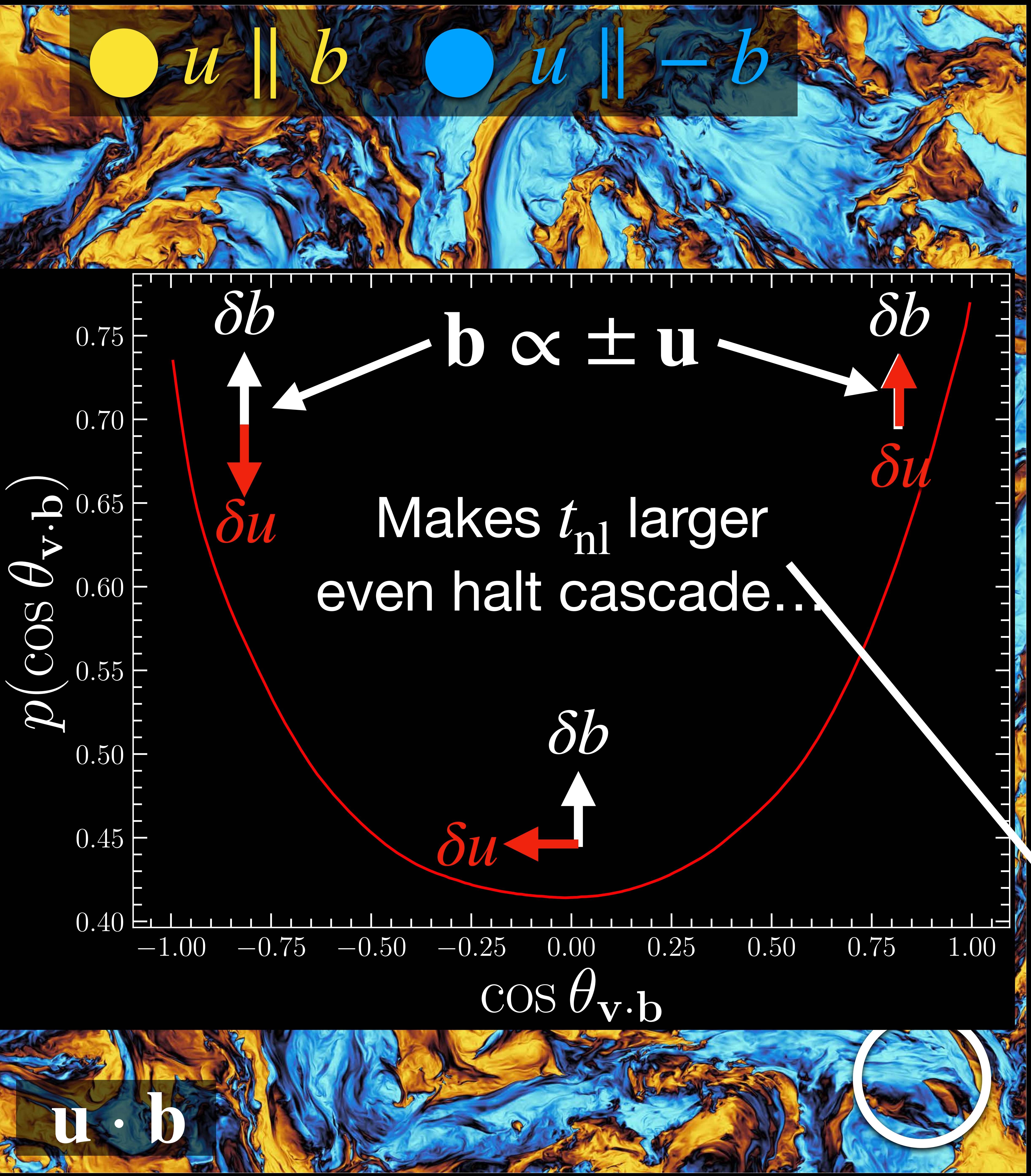
$E_{\mathrm{kin}}(k_\perp) \propto k_\perp^{-3/2}$

Boldyrev (2006)

Modifies the cascade timescale

$$t_{\mathrm{nl}} \sim \frac{\ell_\perp}{z^{\mp} \sin \theta_{z^{\mp}}}, \quad z^{\mp} = (u \mp b)$$

Boldyrev (2006)



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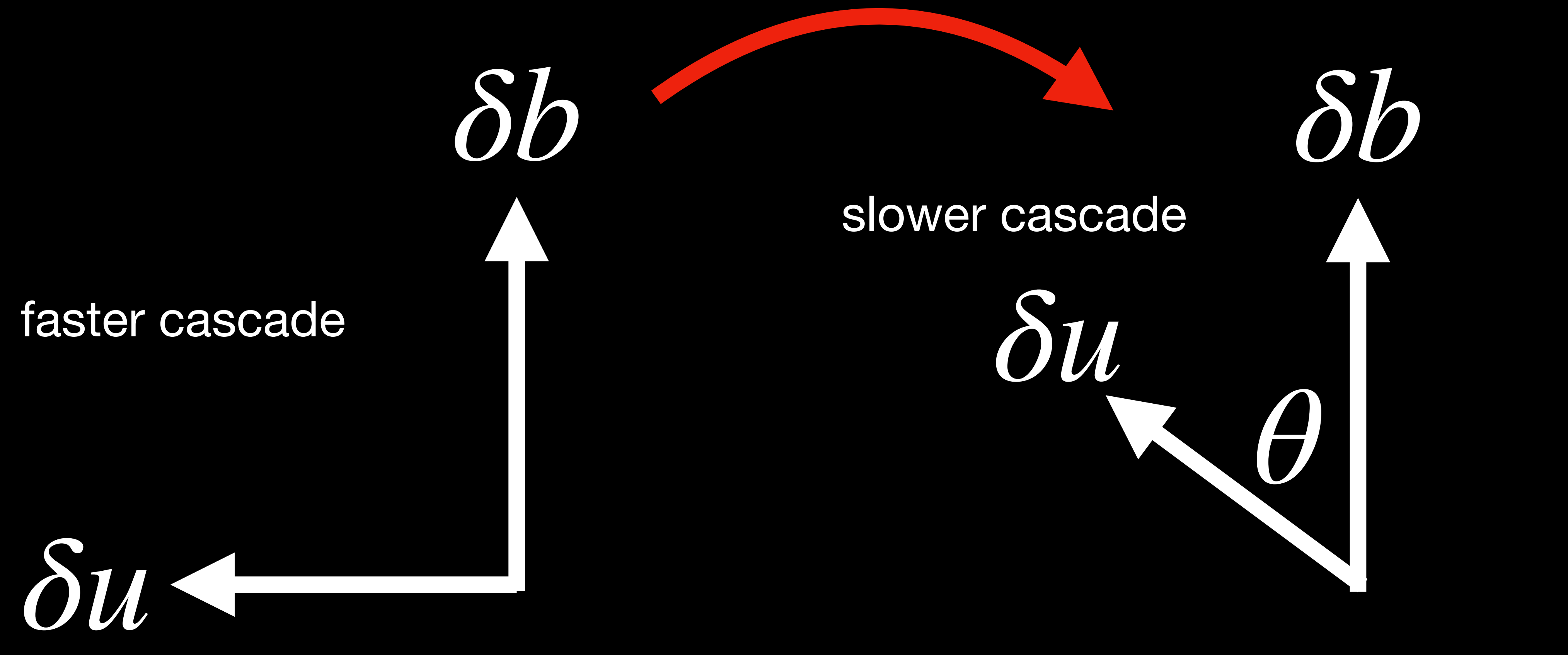

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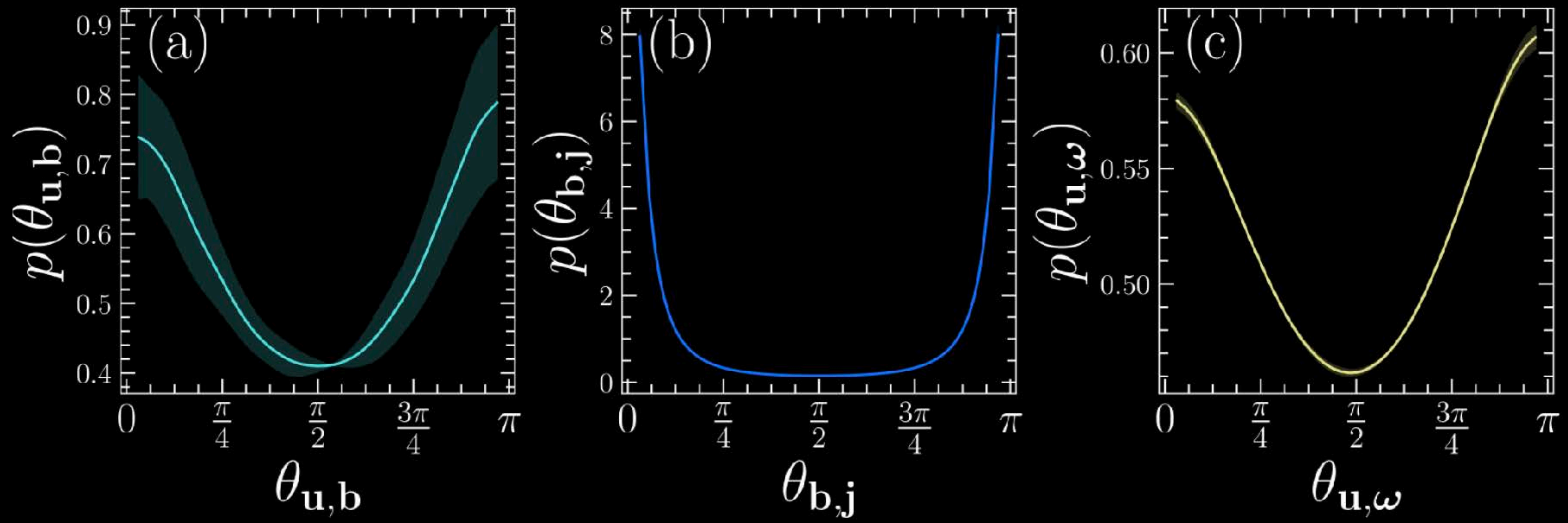
$$t_{\mathrm{nl}} \sim \frac{\ell_\perp}{z^\mp \sin\theta_{z^\mp}}, \quad z^\mp = (u \mp b)$$

Boldyrev (2006)

But even more alignment than just u and b
Searching to weaken the nonlinearities

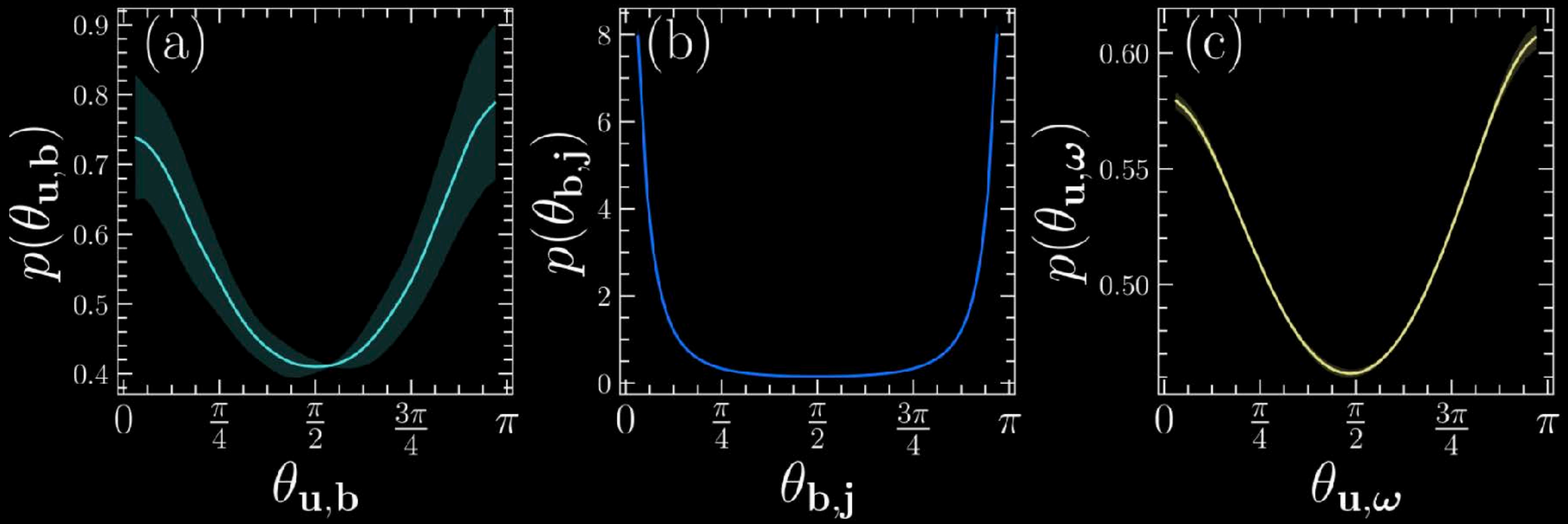
$$\boldsymbol{\omega} = \nabla \times \mathbf{u}$$



$$\mathbf{u} \propto \mathbf{b} \propto \mathbf{j} \propto \boldsymbol{\omega}$$

But even more alignment than just u and b

Searching to weaken the nonlinearities



$\nabla \times (\mathbf{u} \times \mathbf{b})$ — Induction

$\mathbf{j} \times \mathbf{b}$ — Lorentz force

$\nabla \cdot (\mathbf{u} \otimes \mathbf{u}) \sim \omega \times \mathbf{u}$ — Reynolds nonlinearity

Magnetic Relaxation — main idea?

Searching to weaken the nonlinearities

$$H_m = \langle \mathbf{a} \cdot \mathbf{b} \rangle$$

$$H_c = \langle \mathbf{u} \cdot \mathbf{b} \rangle$$

Define constraint equation based on quadratic (ideal) MHD invariants

total energy

cross helicity

$$\mathscr{E} - \lambda_1 H_m - \lambda_2 H_c = \text{const}\,.$$

magnetic helicity

Use variational principle on magnetic energy eq., for perturber δ

$$\delta\left(\mathscr{E} - \lambda_1 H_m - \lambda_2 H_c\right) = 0,$$

Minimize to find minimum energy state.

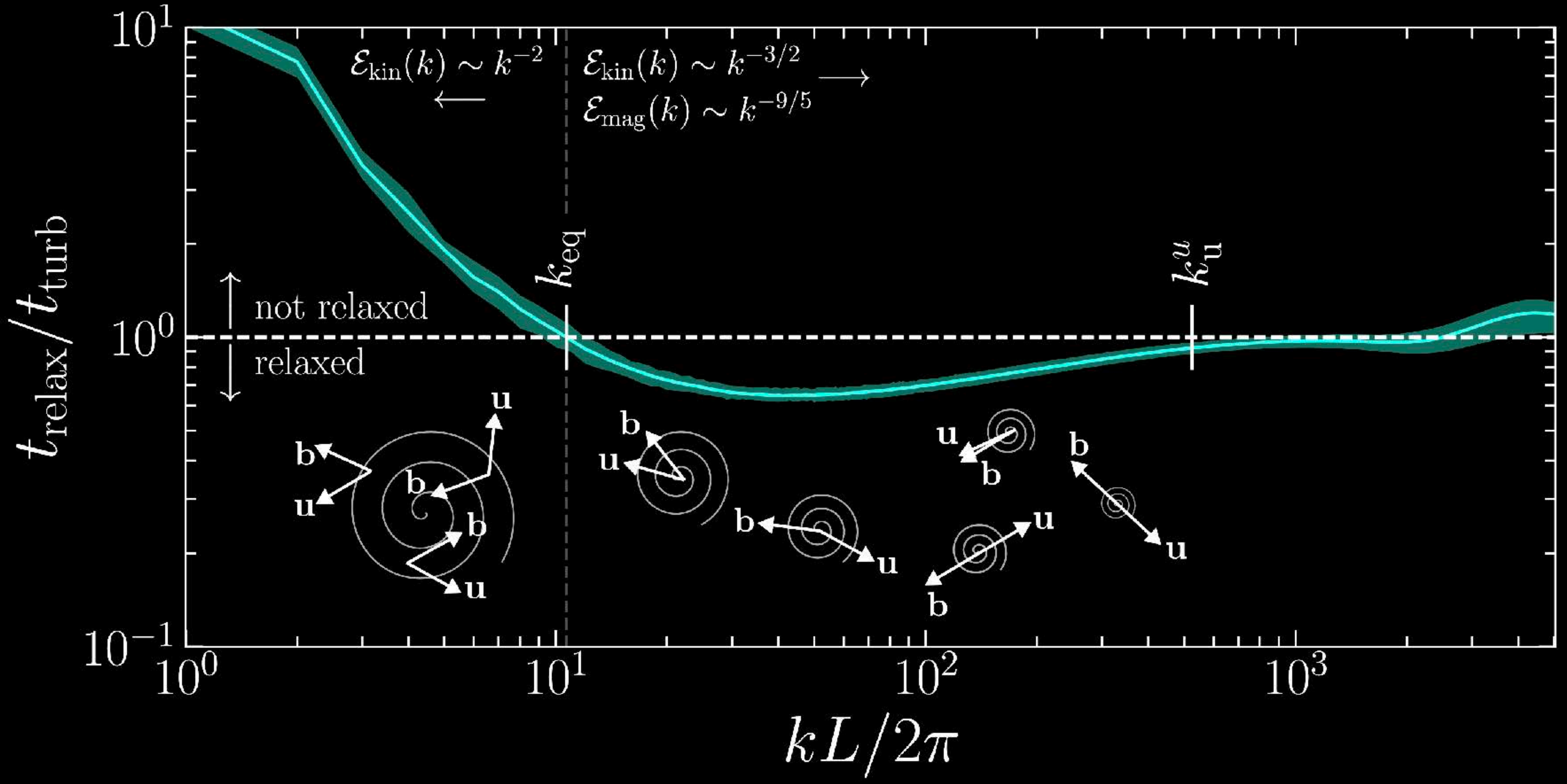
$$\omega = \nabla \times \mathbf{u}$$

$$\mathbf{u} = \lambda_1 \mathbf{b}, \quad \mathbf{j} = 2\lambda_2 \mathbf{b} + \lambda_1 \omega$$

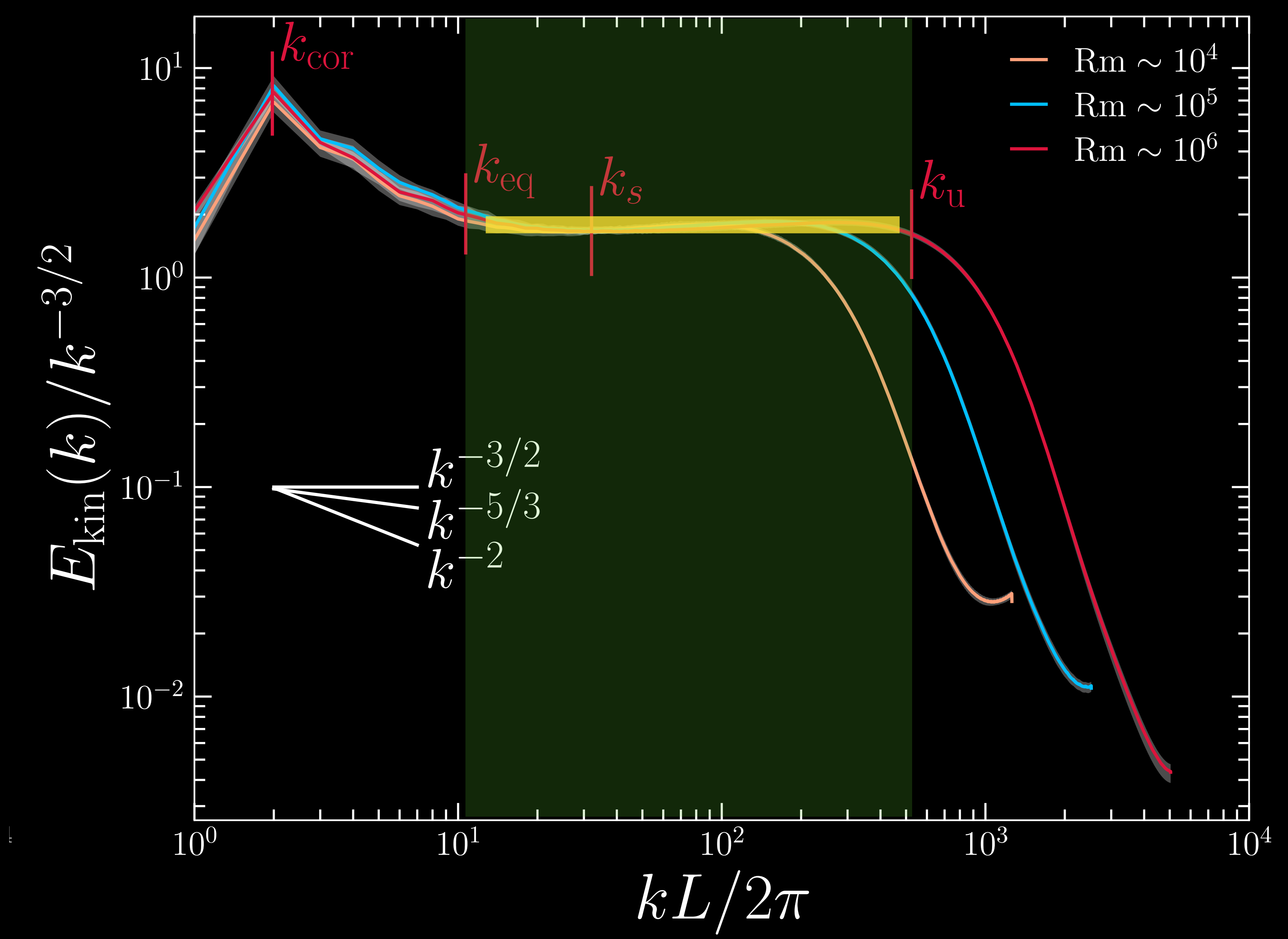
Banerjee+(2023)

Competitive relaxation: turbulence versus relaxation

Can we relax faster than the turbulence can perturb us away from minimum energy?



On these scales…



$$\partial_t \rho + \nabla \cdot \left(\rho \mathbf{u}\right) = 0$$

$$\partial_t(\rho \mathbf{u}) + \nabla \cdot \left(\cancel{\rho \mathbf{u} \otimes \mathbf{u}} + p\mathbb{I} - \cancel{\frac{1}{4\pi}\mathbf{b} \otimes \mathbf{b}} \right) = \cancel{\rho \mathbf{f}} + \nabla \cdot \mathbb{D}_\nu(\rho \mathbf{u})$$

$$\partial_t \mathbf{b} + \cancel{\nabla \cdot (\mathbf{u} \otimes \mathbf{b} - \mathbf{b} \otimes \mathbf{u})} = \nabla \cdot \mathbb{D}_\eta(\mathbf{b})$$

$$\nabla \cdot \mathbf{b} = 0$$

$$p = c_s^2 \rho + \cancel{\frac{1}{8\pi}\mathbf{b} \cdot \mathbf{b}}$$

The subsonic energy cascade

$$\mathcal{E}(k) \sim k^{-3/2},\ k > k_{\mathrm{eq}}$$

IK-type turbulence (Kolmogorov + b flux)?

Dynamical alignment?

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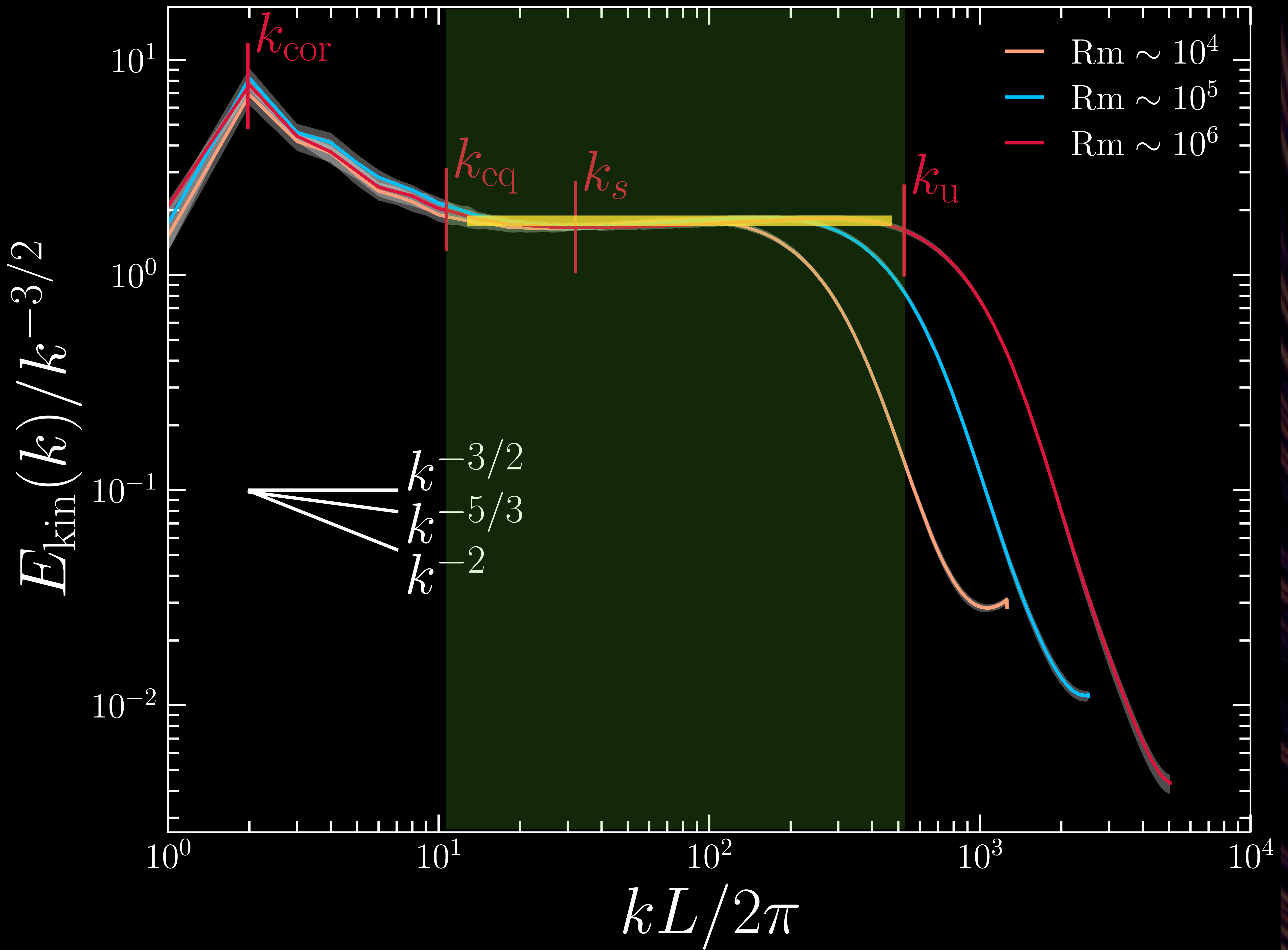
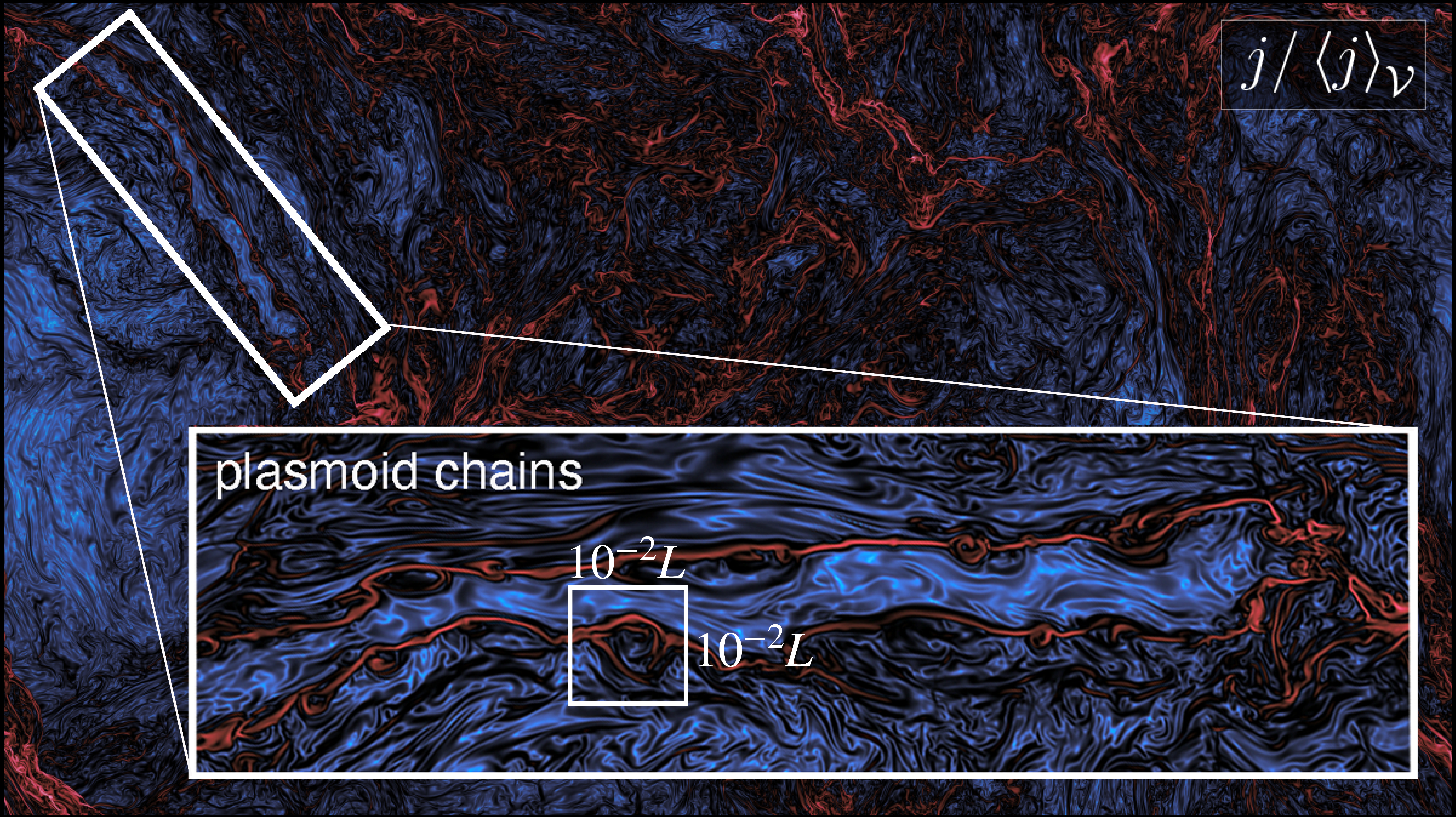
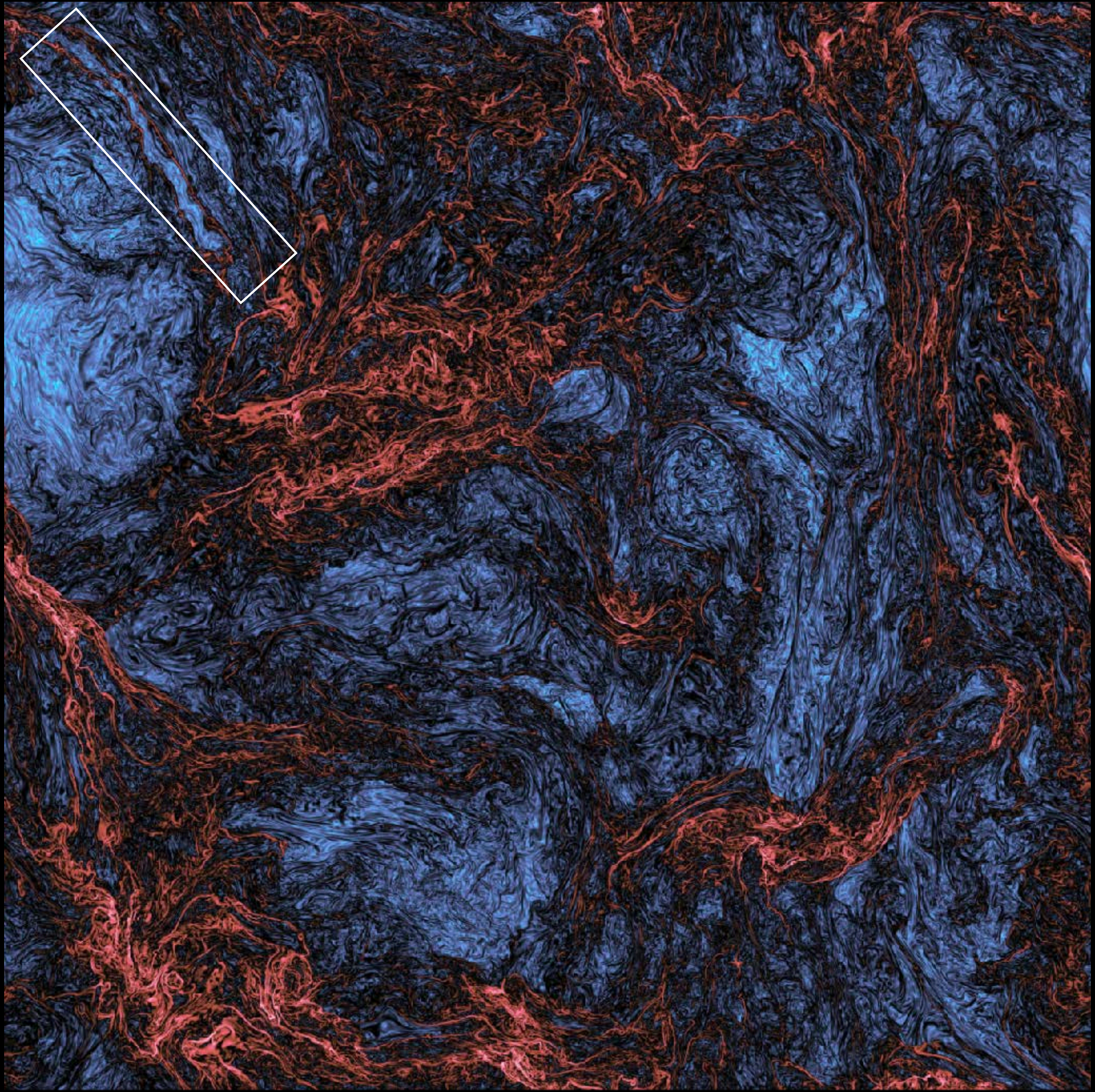


Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The magnetic energy cascade





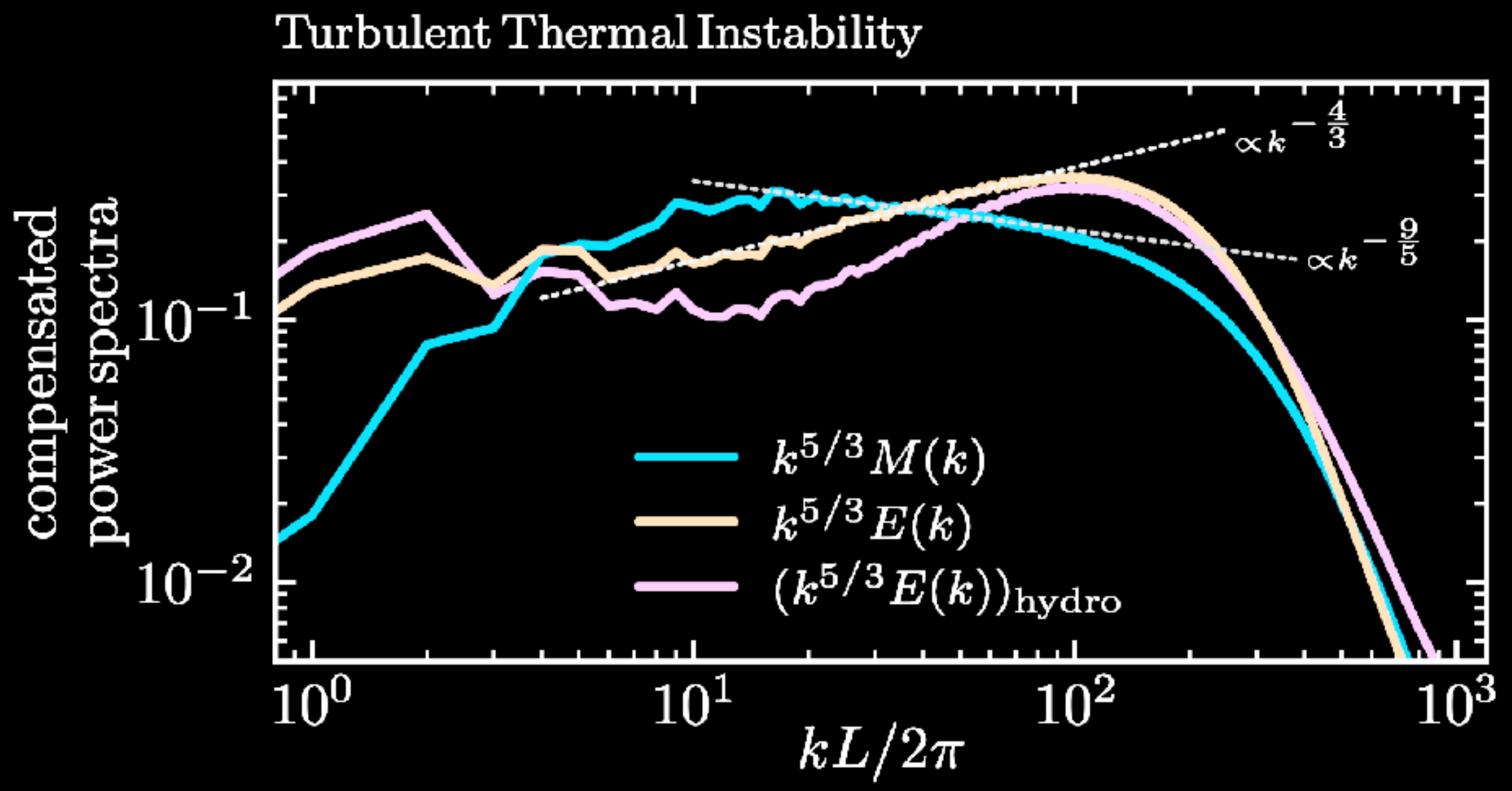
Occupy a very small volume filling-factor in this regime

The magnetic energy cascade

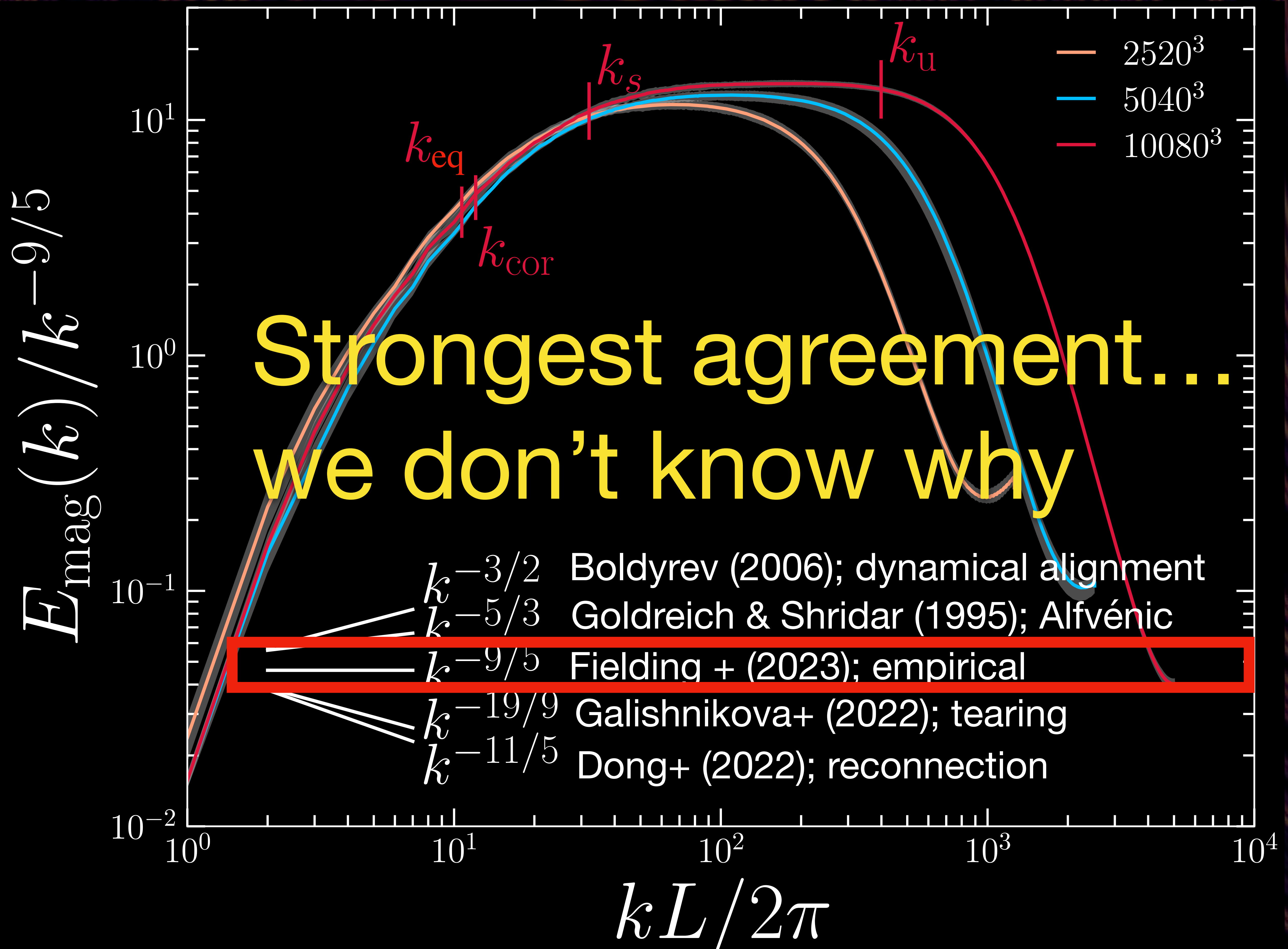
Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The magnetic energy cascade

Fielding+2024 (bistable medium)



Magnetic fluctuations don't seem to care about phases



Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The magnetic energy cascade: small scale structure

Magnetic field is intrinsically a small-scale field! Need a lot of resolution to resolve it properly (energetically)!

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Next steps with observers

james.beattie@princeton. @astro_magnetism

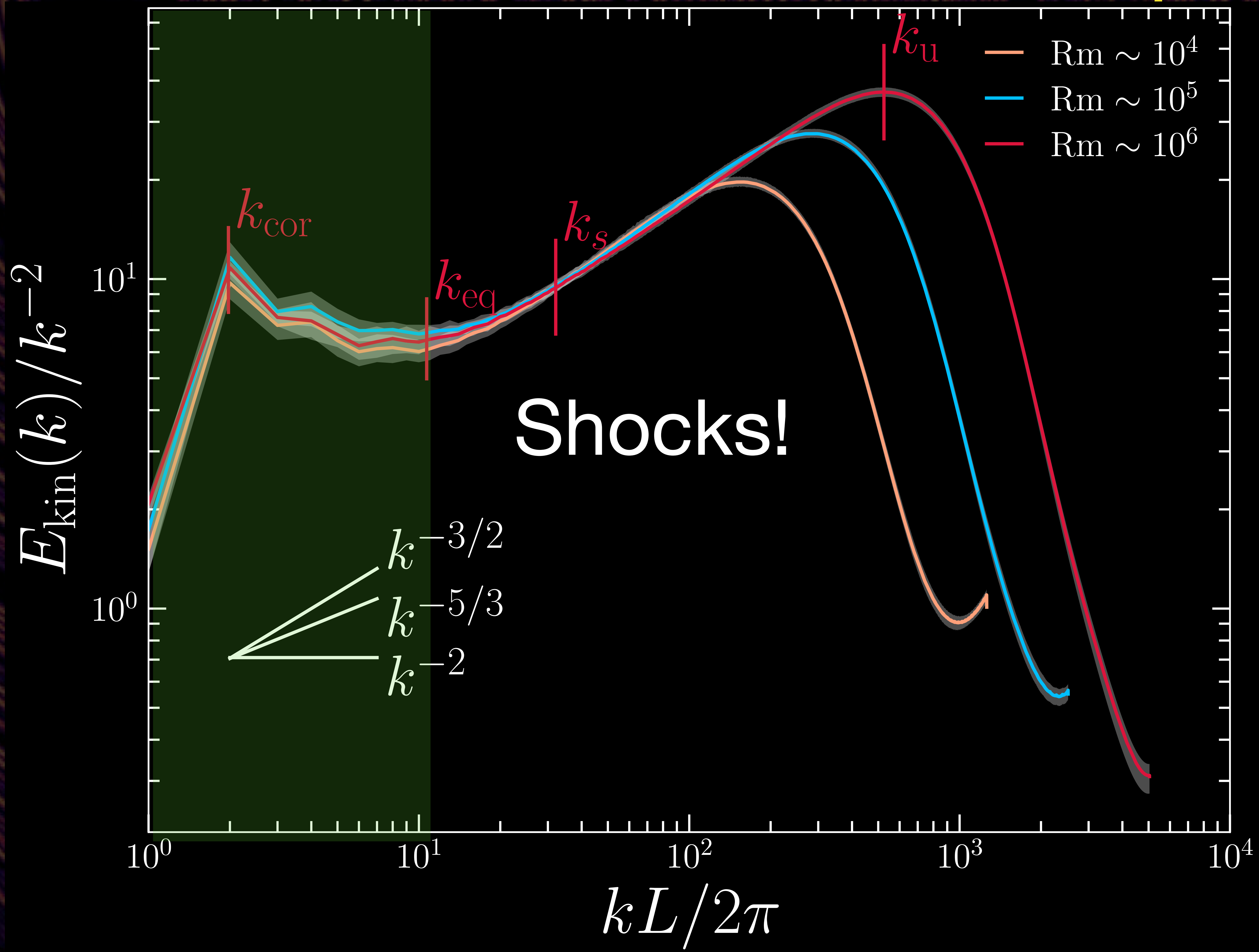
- Post-process to better map to observables (tell me what's interesting, I can make this work at $\sim 10{,}000^3$ but will take some collaboration + time)

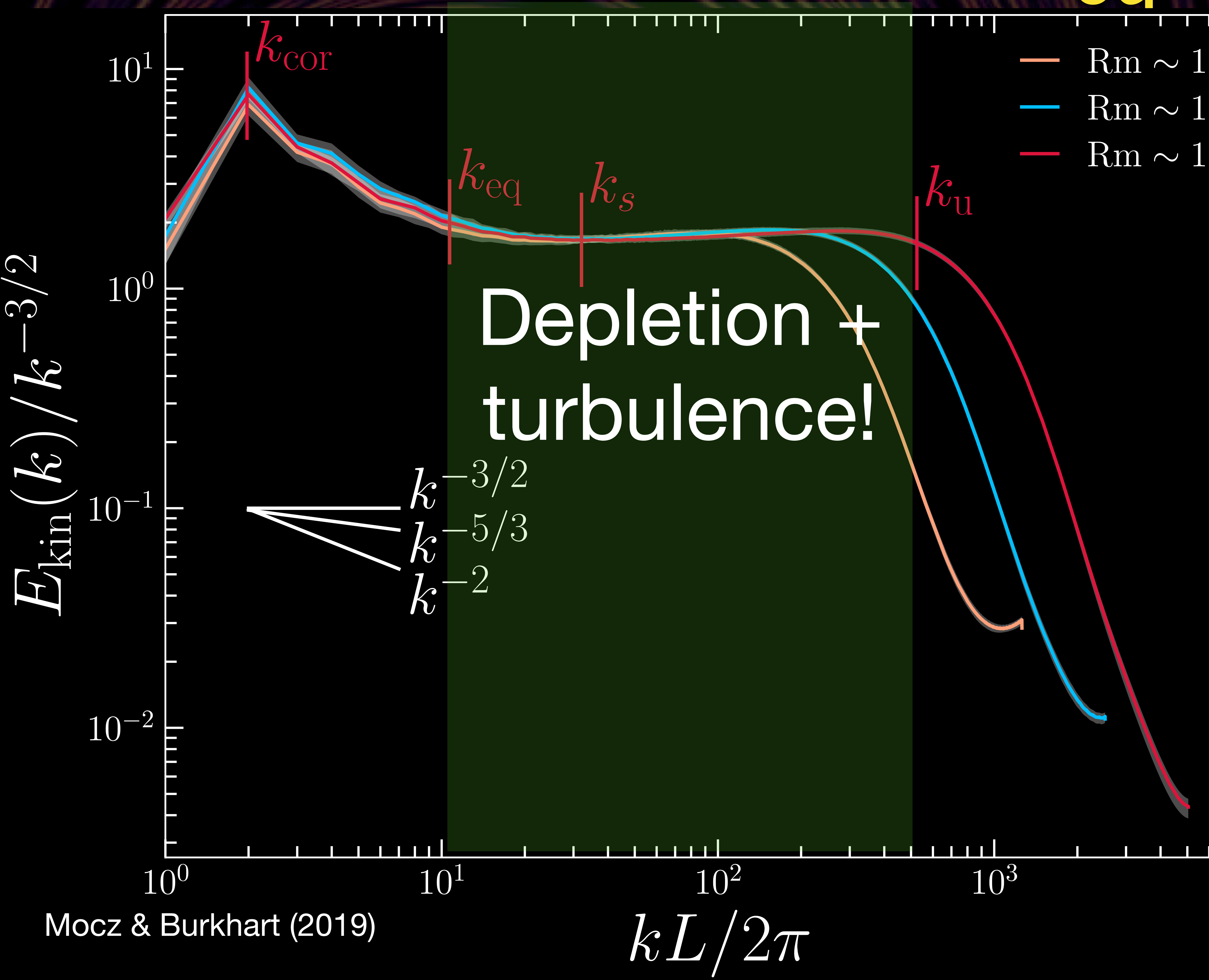
- I want observers to tell me what they can access about alignment — seems ubiquitous in a turbulent dynamo generated magnetic field (related to the saturation of the ISM dynamo), and is in tension with models that have a strong large scale field embedded in ICs (e.g., Lazarian's velocity gradient methods)

Carina Nebula, JWST

Final thoughts…

$$\mathcal{E}(k) \sim k^{-2},\ k \leq k_{\rm eq} \qquad \mathcal{E}(k) \sim k^{-3/2},\ k > k_{\rm eq}$$

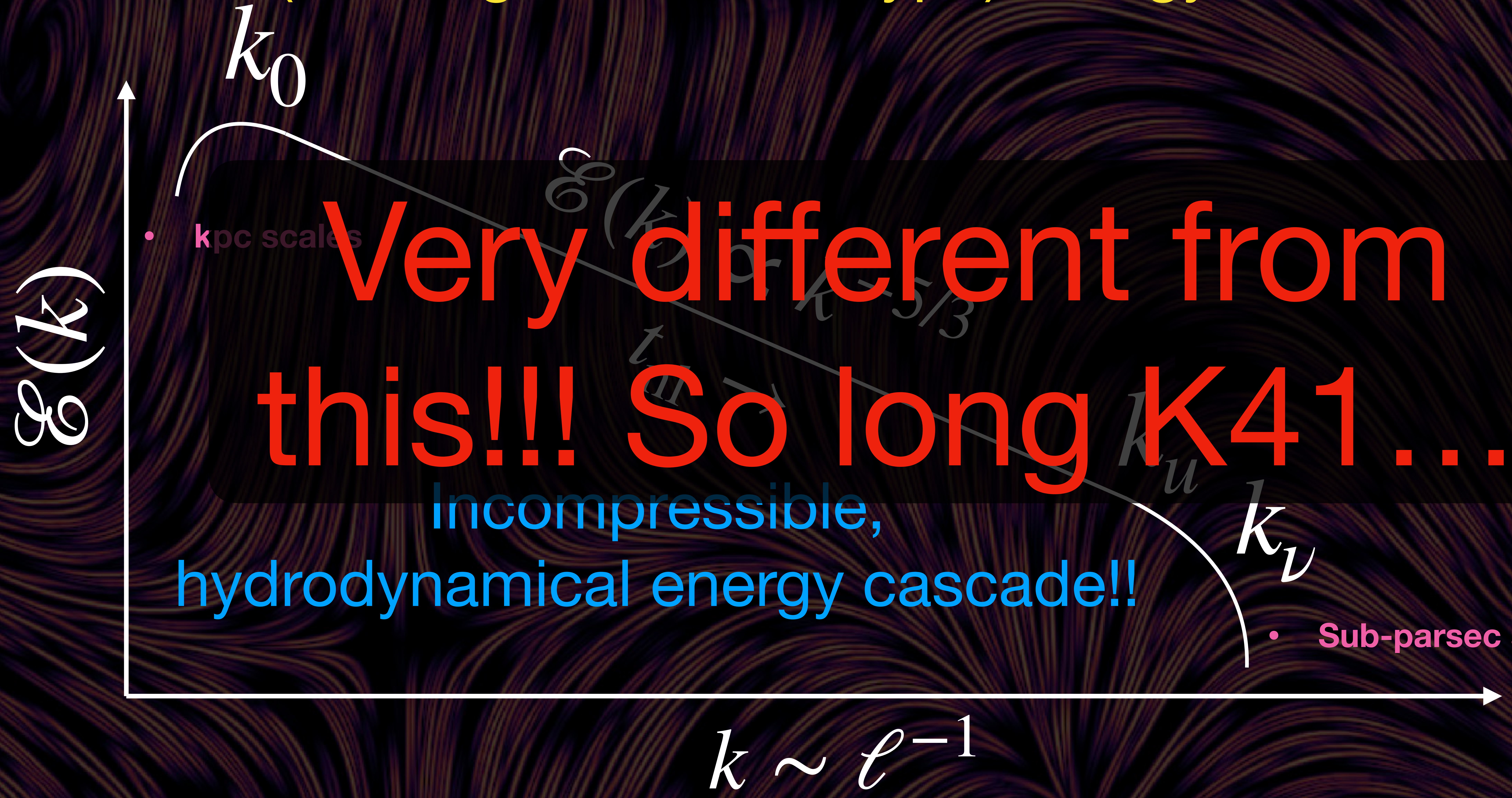




Mocz & Burkhart (2019)

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

The (Kolmogorov, 1941 -type) energy cascade



Very different from this!!! So long K41…

Incompressible, hydrodynamical energy cascade!!

Magnetic reconnection in a turbulent plasma (visualisation by J. Beattie).

Thanks, questions?

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