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Interplanetary trajectories

Example: Earth to Mars case

Report

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1 Figure example formats

FIGURE

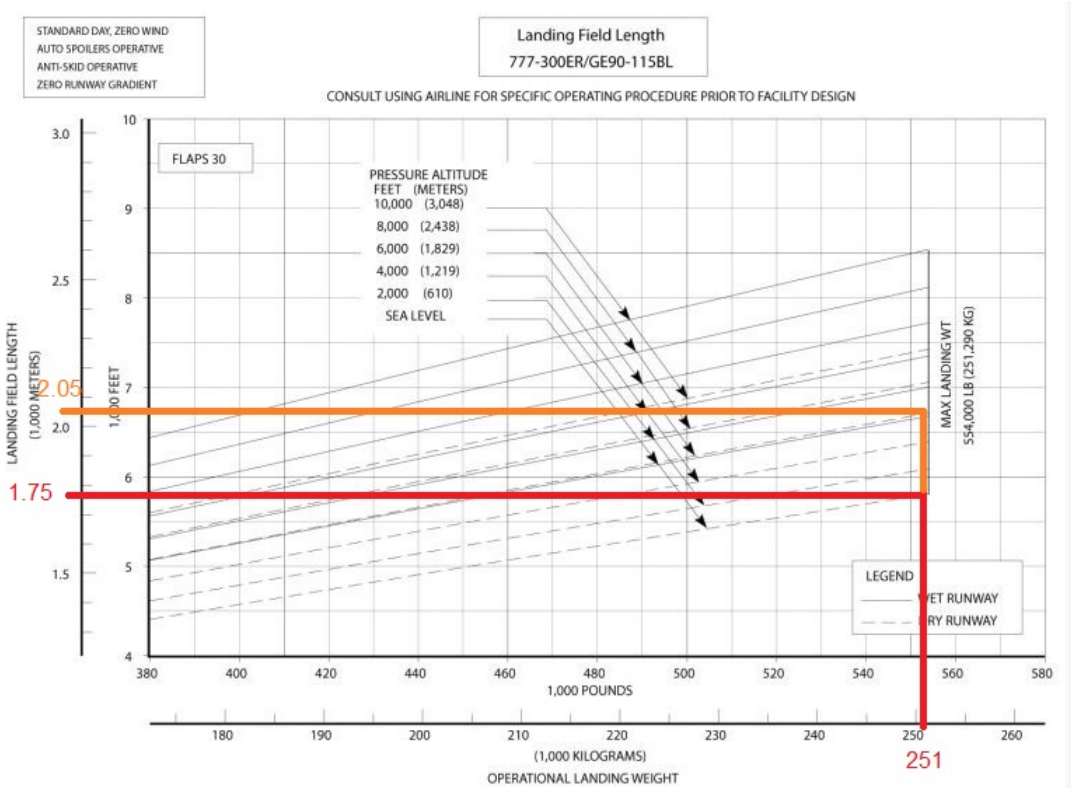

Figure 1.0.1: Landing distance vs MTOW for the Boeing 777.

TABLE

T_1	13 cm
T_2	21 cm
T_3	62 cm
T_t	95 cm

Table 1.0.1: Thickness after the materials correction factor.

2 Aim

This projects aims to compute an interplanetary trajectory which, for a given ecliptic rectangular positions of two planets in two known time instances, is able to carry a spaceship with a unique impulse, from the first planet to the second.

3 | Theoretical background

3.1 Planetary orbits and approximations analysis

In order to calculate the interplanetary trajectory between two planets, several approximations can be used. The simplest approximation, which can be called *aprox. 0* accepts the following hypothesis:

- Circular and coplanar orbits
- No analysis about the exit of the planet of start is done.
- No analysis about entering the planet of arrival is done.

Aprox. 0 is very basic and can be easily improved adding some parameters.
Another approximation widely used is **Patched Conic Approximation (PCA)**. This method improves significantly the results obtained with *aprox. 0* and represents a good starting point for a more precise numerical analysis of the mission. For this reason, in this project the PCA method will be used.

3.1.1 Patched Conic Approximation (PCA)

The Patched Conic Approximation (PCA) consist on the evaluation of an interplanetary trajectory dividing it into three stages. Considering the Earth as the planet of start, this stages are:

- Geocentric phase: Hyperbolic exit of the Earth. This phase takes place while the probe is going through the influence sphere of the Earth.
- Heliocentric phase: Trajectory with the Sun as main influencer.

- Planet-centred phase: Hyperbolic arrival to the planet of destination. Similarly to the geocentric phase, this phase starts when the probe enters the sphere of influence of the planet.

The influence spheres mentioned are the space close to the planets where it can be considered that the influence of the Sun is negligible in comparison with that of the planet in question. The Laplace criteria will be considered to calculate this sphere. In Table 3.1.1 the radius of the sphere of influence of the solar system's planets are shown.

Planet	$R_I \times 10^6$km	$R_I \times 10^{-3}$ UA	R_I Radius of the planet
Mercury	0.111	0.740	45
Venus	0.616	4.11	100
Earth	0.924	6.16	145
Mars	0.577	3.85	170
Jupiter	48.157	321.0	677
Saturn	54.796	365.3	901
Uranus	91.954	346.4	2025
Neptune	80.196	534.6	3866

Table 3.1.1: Radius of influence of the planets

In order to set out the problem and begin with the resolution of it using the PCA method, the times and positions of the planets at the beginning and end of the trajectory are needed and some hypothesis are taken under consideration. The hypothesis are:

- The spheres of influences of the planets are not considered during the heliocentric phase. This hypothesis is admissible because the radius of the sphere are very small in comparison with the distance between planets.
- The spheres of influence are considered infinite from the point of view of the planet. This is assumed due to the fact that the radius of influence of the planets are much larger than the radious of the planet itself, as can be seen in Tabl 3.1.1.
- The duration of the trajectory can be taken as the duration of the heliocentric phase.

With this data the trajectory can be found through the orbital elements of the trajectories and the thrust required.

3.1.1.1 1st. Geocentric stage

3.1.1.2 2nd. Heliocentric stage

In this section the equations and assumptions done in order to obtain the orbital elements of the trajectory will be explained. The objective of the calculations done regarding this stage is to find:

- Ω : Right ascension of the ascending node.
- *e*: Eccentricity.
- *i*: Inclination to the ecliptic plane.
- *a*: Semimajor axis.
- ω : Argument of the perihelion.

As said previously, the times of departure and arrival are provided, together with the position of the planets. The steps to be followed to achieve the aim of this section are now explained.

Longitude, latitude and distance The position vector is defined as:

$$\overrightarrow{r} = (x_k, y_k, z_k) \tag{3.1.1}$$

Where:

$$x_k = rcos\beta cos\lambda \tag{3.1.2}$$

$$y_k = rcos\beta sin\lambda \tag{3.1.3}$$

$$z_k = rsin\beta \tag{3.1.4}$$

Then longitude, latitude and distance are computed with:

$$r = |\overrightarrow{r}| \tag{3.1.5}$$

$$\beta = \arcsin\left(\frac{z_k}{r}\right) \tag{3.1.6}$$

$$\lambda = \arctan\left(\frac{y_k}{x_k}\right) \tag{3.1.7}$$

The difference between λ at the beginning and at the end of the trajectory is an important magnitude that will be used. Taking into account that subscript 1 refers to the start position and subscript 2 to the end:

$$\Delta\lambda = \lambda_2 - \lambda_1 \tag{3.1.8}$$

Inclination, right ascension of the ascending node and true anomaly variation Trigonometry has to be used to compute this elements. A general case will be considered, that is to say, that no assumption will be done on whether the two planets are on the ecliptic or not. As shown in reference [1], the equations to be used are:

$$\cos\Delta\theta = \sin\beta_1 \sin\beta_2 + \cos\beta_1 \cos\beta_2 \cos\Delta\lambda \tag{3.1.9}$$

$$\sin A = \cos\beta_2 \frac{\sin\Delta\lambda}{\sin\Delta\theta} \tag{3.1.10}$$

$$\cos i = \sin A \cos\beta_1 \tag{3.1.11}$$

$$\sin l = \frac{\tan\beta_1}{\tan i} \tag{3.1.12}$$

$$\tan\sigma = \frac{\tan\beta_1}{\cos A} \tag{3.1.13}$$

$$\Omega = \lambda_1 - l \tag{3.1.14}$$

Eccentricity, semimajor axis and true anomaly of the starting point With the aim of obtaining this data three equations can be stated. Due to the complexity of the equations, the resolution will be done iteratively. Two cases will be considered: elliptic and hyperbolic. Its equations and iteration process are now shown:

- Elliptic trajectory: The equations of the elliptic trajectory are:

$$e = \frac{r_2 - r_1}{r_1 \cos\theta_1 - r_2 \cos(\theta_1 + \Delta\theta)} \tag{3.1.15}$$

$$a = \frac{r_1 \left(1 + e\cos\theta_1\right)}{1 - e^2} \tag{3.1.16}$$

$$t_2 - t_1 = \frac{365.25}{2\pi} a^{\frac{3}{2}} \left(2\arctan\left(\sqrt{\frac{1-e}{1+e}} \tan\frac{\theta_1 + \Delta\theta}{2}\right) - \frac{e\sqrt{1-e^2}\sin(\theta_1 + \Delta\theta)}{1 + e\cos(\theta_1 + \Delta\theta} - 2\arctan\left(\sqrt{\frac{1-e}{1+e}} \tan\frac{\theta_1}{2}\right.\right. \tag{3.1.17}$$

The iteration process done to solve the equations will deal with the difference between the time of the mission calculated and the real time of the mission, that is a known value. An error criteria ϵ is defined as the convergence value. The flow chart of this iteration is shown in Figure 3.1.1.

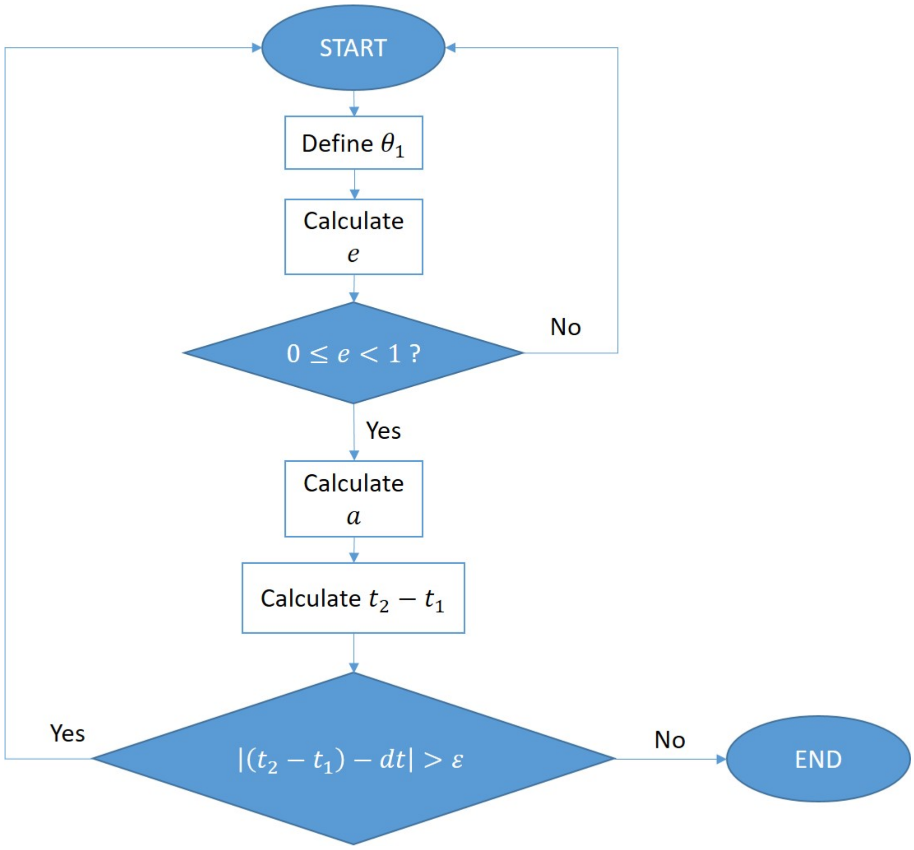

Figure 3.1.1: Flow chart for the elliptic trajectory resolution.

- Hyperbolic trajectory: The equations of the hyperbolic trajectory are:

$$e = \frac{r_2 - r_1}{r_1 \cos\theta_1 - r_2 \cos(\theta_1 + \Delta\theta)} \tag{3.1.18}$$

$$a = \frac{r_1 \left(1 + e\cos\theta_1\right)}{e^2 - 1} \tag{3.1.19}$$

$$t_2 - t_1 = \frac{365.25}{2\pi} a^{\frac{3}{2}} \left(\frac{e\sqrt{e^2-1}\sin(\theta_1 + \Delta\theta}{1 + e\cos(\theta_1 + \Delta\theta} - ln\left| \frac{\tan\frac{\theta_1+\Delta\theta}{2} + \sqrt{\frac{e+1}{e-1}}}{\tan\frac{\theta_1+\Delta\theta}{2} - \sqrt{\frac{e+1}{e-1}}} \right| - \frac{e\sqrt{e^2-1}\sin\theta_1}{1 + e\cos\theta_1} + ln\left| \frac{\tan\frac{\theta_1}{2} + \sqrt{\frac{e+1}{e-1}}}{\tan\frac{\theta_1}{2} - \sqrt{\frac{e+1}{e-1}}} \right| \right. \tag{3.1.20}$$

The resolution is similar to that of the elliptic case, but the acceptable values of the eccentricity change. The flow chart is shown in Figure 3.1.2.

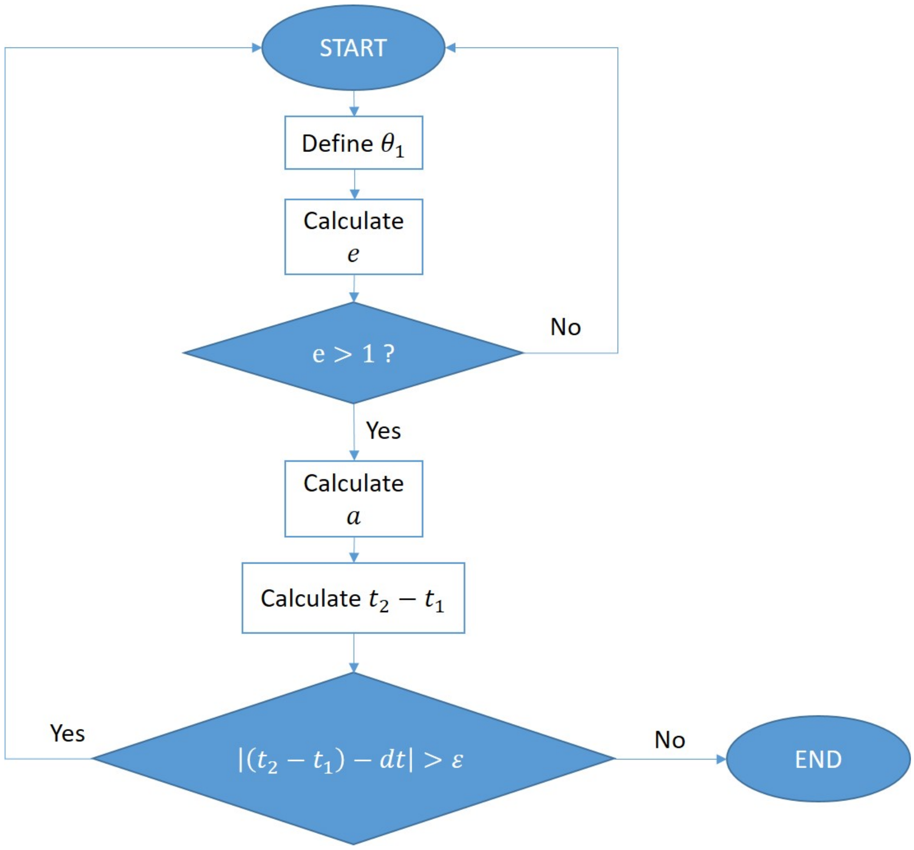

Figure 3.1.2: Flow chart for the hyperbolic trajectory resolution.

Argument of the perihelion The only remaining orbit element that needs to be computed is ω. It can be calculated using results from the previous steps:

$$\omega = 2\pi - (\theta_1 - \sigma) \tag{3.1.21}$$

3.1.1.3 3rd. Planetocentric stage

4 | Calculations and results

4.1 Verification calculations

4.2 Main interplanetary orbit calculations

5 Conclusions

6 Bibliography

[1] J. Calaf, “Trajectòries interplanetàries: Patched Conic Approximation,” 2017.

[2] ——, “Trajectòries interplanetàries,” 2017.

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