Available online at www.sciencedirect.com
SciVerse ScienceDirect
Advances in Space Research 52 (2013) 1047–1062
ADVANCES IN SPACE RESEARCH (a COSPAR publication)
www.elsevier.com/locate/asr

Optimal network topology of relative navigation and communication for navigation sharing in fractionated spacecraft cluster ☆

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Received 6 November 2012; received in revised form 6 May 2013; accepted 12 June 2013
Available online 20 June 2013

Abstract

Navigation sharing as a key technology provides orbit and attitude information broadcasting for the whole fractionated spacecraft cluster (FSC). The navigation sharing concept has already been proposed for some years. However, the problem of achieving the optimal navigation sharing by designing the proper network topology is still unsolved, especially in the case that some members of the cluster have absolute navigation devices, some just have relative navigation devices and some only have communication devices. In this paper, a comprehensive model of describing the navigation sharing problem in FSC is proposed. The model of network topology constructing by relative navigation links and communication links is established. By using the graph theory and genetic algorithm as the tool, the conditions for navigation sharing in FSC with different sharing degrees are obtained. Finally, some examples are presented to test the methods, and it has been found that, the absolute positions and attitudes of all the members can still be determined even some members in FSC have no absolute navigation devices.

Keywords: Navigation sharing; Fractionated spacecraft cluster; Relative navigation; Communication; Graph theory

1. Introduction

Fractionated spacecraft cluster (FSC) has become a hot topic recently, which is researched widely by lots of scientists and engineers (Jamnejad and Silva, 2008; O'Neil and Weigel, 2011; Kichkaylo et al., 2012; Dubos and Saleh, 2011; Wang and Nakasuka, 2012; Yao et al., 2012). The FSC represents a relaxed formation with multiple members which can be incomplete spacecraft assembly. Once receiving the mission command and control signal, the members in FSC can be rapidly gathered and assembled into one or more holonomic spacecrafts. Due to this characteristic, the FSC has more survivability and flexibility than the traditional spacecraft formation. To achieve the aim of distributing the FSC in space, some key technologies have been recognized, for example, the formation flight control technology, the space-based self-organized network technology, and the navigation sharing technology, etc. (Brown, 2004; Brown and Eremenko, 2006a,b). It is clear that the navigation sharing can provide the input information of the control system, which is vitally important to be studied in FSC.

The navigation sharing means that the members in a cluster exchange the navigation data with each other. In a traditional spacecraft formation, to maintain the formation configuration, the member in the formation should know its own inertial (absolute) position and attitude and others' relative position and attitude relative to it. This is done by the absolute navigation device and relative navigation device

☆ Supported by the National Natural Science Foundation of China (No. 11002076), Hunan Provincial Innovation Foundation for Postgraduate, and Graduate Student Innovation Foundation of National University of Defense Technology (No. B120101).
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http://dx.doi.org/10.1016/j.asr.2013.06.009

installed in each spacecraft, respectively. By measuring one spacecraft's absolute position and attitude and relative position and attitude with respect to others, the former can obtain others' absolute navigation data, which is a simple navigation sharing in essence. This concept has also been presented in the DARPA's System F6 program (Future, Fast, Flexible, Fractionated, Free-Flying Spacecraft United by Information Exchange) (Brown and Eremenko, 2006a,b) and the personal navigation application (Isaacson, 2010). It should be noted that System F6 seeks to demonstrate the feasibility and benefits of a satellite architecture wherein the functionality of a traditional "monolithic" spacecraft is delivered by a cluster of wirelessly-interconnected modules capable of sharing their resources and utilizing resources found elsewhere in the cluster. However, in FSC, there exists the member that has no absolute navigation device. In this case, it cannot obtain either its own or others' absolute navigation data. To make up this deficiency, the communication between the deficient member and other normal ones is necessary (Dang and Zhang, 2013). In most cases, the navigation sharing assisted with communication is more efficient than the relative navigation. But it should be noted that there are some cases in which the relative navigation is necessary, for example, when no absolute navigation devices or communication devices can be used.

In our previous work (Dang and Zhang, 2013), we have discussed the concept of navigation sharing in detail. That concept is slightly different to the one stated by some other researchers (Brown and Eremenko, 2006a,b). By a comparison of the concepts given by different researchers, it has been found that our previous work limited the navigation sharing to the case of absolute navigation and communication, but Brown (2004), Brown and Eremenko (2006a,b) addressed themselves to the case in which only absolute navigation and relative navigation are considered. In fact, in real application, absolute navigation, relative navigation, and communication may all exist. In this case, how to achieve the navigation sharing is still an unsolved problem.

This paper tries to investigate the condition of the navigation sharing in FSC. For the communication device, we find two main factors which affect the navigation sharing. One is the communication channels' number which represents the number of the neighbors that one member in the cluster can communicate with. The other is the communication capacity which represents the number of the neighbors whose navigation data can be transmitted in one communication channel. For the relative navigation device, the relative navigation channels' number is the main factor which represents the number of the neighbors for which one member can perform the relative navigation. The relative navigation links and communication links among the members in FSC are modeled as the edges of directed graph, then the Graph Theory is used as the analysis tool to find the conditions for navigation sharing.

The rest of this paper is organized as follows. Section 2 gives the preliminaries and some assumptions. In this section, the concepts of navigation sharing, the definitions of some terminologies about topology graph are introduced. Section 3 presents the models and methods for navigation sharing. Three cases are discussed, then the objective function vectors are presented to optimize the network topology. The genetic algorithm for searching for the optimal network topology constructed by the relative navigation and communication links are also proposed. In Section 4, some examples are studied. The corresponding solutions are obtained and analyzed carefully. Finally, Section 5 gives some conclusions.

2. Preliminaries and assumptions

2.1. Concept and definitions

The preliminaries of navigation sharing, corresponding its definitions and related assumptions used in this paper are presented below. The definitions are used to describe the nomenclatures relative to navigation sharing. The assumptions limit the problems discussed in this paper to a rational range.

Generally, the navigation sharing means that the member in FSC exchanges the navigation data obtained by sensors or received by communication devices with others. Here 'exchange' may be real communication or virtual data transmission using relative navigation devices. Communication between two members veritably send navigation data to each other. The relative navigation can also be used to achieve this aim. This can be comprehended by the fact that the relative navigation device can determine the relative position and attitude data for the owner with respect to the target. If the owner can also determine its own absolute position and attitude, then it will obtain the wanted absolute position and attitude data of the target by adding these two kinds of data. The concept of navigation sharing is shown in Fig. 1.

The Graph Theory (Bass et al., 1974) is selected as the tool to analyze the problem of navigation sharing in this paper. To describe the network topology of relative navigation and communication links for navigation sharing, some definitions are presented as follows (Dang and Zhang, 2013):

a. Node

A node represents a member in FSC. It is denoted by v_i where the subscript i is a natural number. The total number of nodes equal to the total number of members in the cluster and denoted by N. Obviously, for a cluster, $N > 1$.

b. Information

Information or navigation information is the position and attitude data that are transmitted and received between two members. For simplicity, the information is denoted by the symbol e_i. Here e_i represents the position and attitude of the node v_i. But it should be remembered that any node is not confined to only transmit its own information. A node v_i can transmit information $e_j (j \neq i)$ to others as along as it has this information and others need.

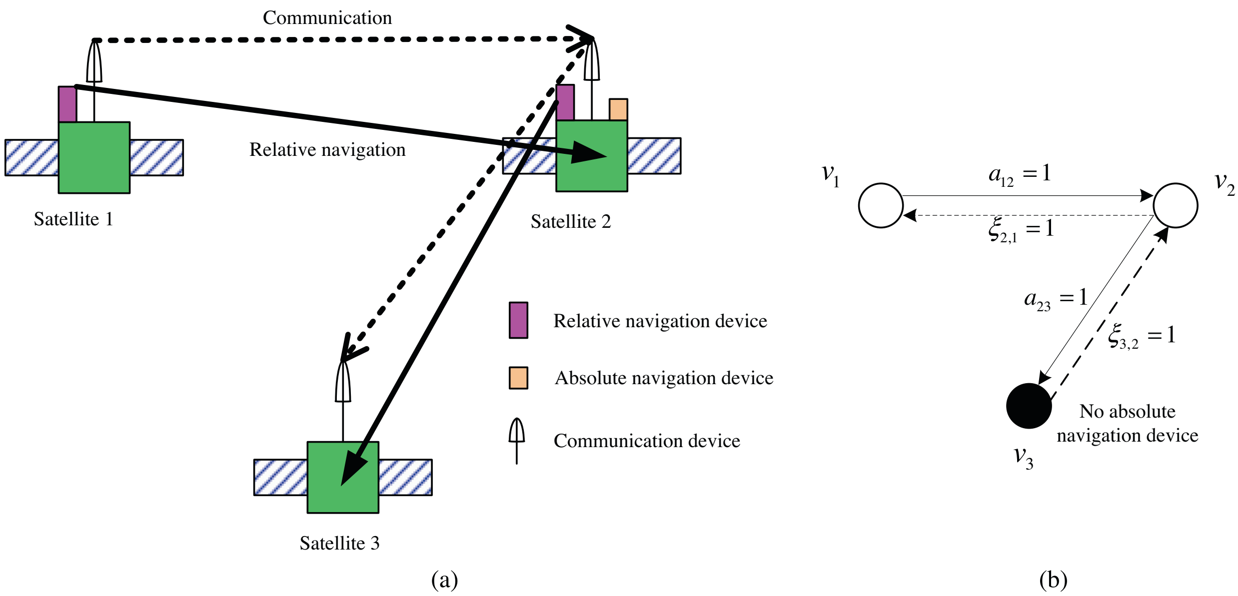

Fig. 1. Navigation sharing concept using absolute navigation, relative navigation, and communication, (a) real scenario, (b) schematic representation of the correspondent network.

c. Communication

When we say communication, it means the transmitting and receiving process of the navigation data between two members. The communication is assumed to have a direction. So v_i communicating with v_j is different with v_j communicating with v_i.

d. Relative navigation

When we say relative navigation, it means the process that one member determines its own relative position and relative attitude relative to another different member by relative navigation device. The relative navigation is also directed.

e. Communication network topology

A communication network topology describes the whole communication connection relations among nodes. We adopt the related concept of Graph Theory to describe the topology. Therefore, a topology is represented by an adjacency matrix whose entries denote whether the connection between two nodes exists. All the entries can only be valued with 1 or 0, where the former denotes the connection exists, and the latter does not. For example, $a_{ij} = 1$ means that the node v_i sends information to the node v_j.

f. Relative navigation network topology

A relative navigation network topology describes the whole relative navigation relations among nodes. This topology is similar to the communication network topology. It is also represented by an adjacency matrix.

g. Communication edge

A communication edge represents the communication connection relation between two nodes. The edge is directed. If one edge $\overrightarrow{v_i v_j}$ exists, it means $a_{ij} = 1$, or else $a_{ij} = 0$.

h. Relative navigation edge

A relative navigation edge represents the relative navigation relation between two nodes. This edge is also directed, and its definition is similar with the communication edge.

i. Communication path

A communication path is a series of alternate nodes and communication edges. On a communication path, every two adjacent communication edges have a common node which is the end of the former edge and the head of the latter edge. For example, $v_i \to v_{i+1} \to v_{i+2} \to \cdots \to v_j$ is a path connecting the nodes v_i and v_j, where the edges like $\overrightarrow{v_{i+t} v_{i+t+1}}$ and $\overrightarrow{v_{i+t+1} v_{i+t+2}}$ $(t = 0, 1, \ldots, j - i - 2)$ all exist.

j. Communication capacity

The communication capacity of a node (v_i) represents the maximum number of the nodes (v_j) whose navigation data can be sent by the former node (v_i) in a single communication edge. Here the edge must be the neighborhood edge of the former node (i.e. v_i). We denote this maximum number as p_i. For any edge starting from the node v_i, like $\overrightarrow{v_i v_j}$ $(j \neq i)$, the communication capacity is same, viz. p_i.

k. Number of communication channels

Any communication edge starting from the same node is named as a communication channel with respect to this node. Therefore, the number of communication channels for a specific node v_i is the number of the edges originating from v_i. The maximum number of communication channels relative to a node v_i is denoted as q_i. It should be noted that the maximum number of communication channels cannot be too large due to the physical constraints of the communication device.

l. Number of relative navigation channels

Any relative navigation device can only determine limited numbers' relative state between two nodes. The maximum number that one node can perform relative navigation is denoted as the number of relative navigation channels. This number is denoted as g_i.

m. Head and end

If an edge starts from the node v_i, then the node v_i is the head corresponding to this edge. Similarly, the end is the node to which an edge points.

n. Number of absolute navigation node

The number of nodes who can determine its own absolute position and attitude by absolute navigation device is denoted as the number of absolute navigation nodes. It can be represented by the sign n_a. In the application field of FSC, only a definited number of members can determine its own absolute position and attitude.

o. Number of relative navigation nodes

The number of nodes that equipped with relative navigation device is the number of relative navigation nodes. Similarly, not all the members have the relative navigation device.

2.1.1. Definitions on navigation sharing

For convenience, the exact definitions about navigation sharing are listed as follows:

a. Complete navigation sharing

The complete navigation sharing (CNS) is achieved if each member can obtain all other ones' absolute navigation data.

b. Incomplete navigation sharing

The incomplete navigation sharing (ICNS) means that there exists at least one member that can only obtain limited neighbors' absolute navigation data.

c. Sharing degree

The sharing degree (SD) is proposed to describe the degree of the achieved navigation sharing. It can easily distinguish different kinds of navigation sharing. The SD is defined as follows:

$$SD = \frac{\min_i(n_i)}{N} \tag{1}$$

where n_i represents the number of the members who can obtain the absolute position and attitude of the node v_i, and N is the total members' number of the cluster.

Remark 1. It can be seen that $0 \leqslant SD \leqslant 1$. $SD = 0$ indicates that there is at least one member in FSC whose navigation data cannot be obtained by any other member (including itself). This is the worst case in which the cluster has some difficulties in running in a collaboration way. $SD = 1$ indicates that any member in the cluster can be known by all the others. In this case, the precise formation can be easily formed. This case is the so-called complete navigation sharing.

Remark 2. Using the above definitions, we can describe the problem of navigation sharing in various kinds of constraints. The mathematical models for different constraints will be given in the next section. It should be emphasized that the aim of this paper is to find the optimal network topology of the communication and relative navigation links for navigation sharing.

2.2. Assumptions

In specific applications, the navigation sharing of FSC would be affected by many factors. In this paper, however, only three main factors and their main properties are analyzed. This can make the problem of navigation sharing be more easily understood. At the same time, these factors in this paper can represent a large quantity of real scenarios. The main assumptions used in this paper are summarized as follows:

a. Static scenario

It assumes that the communication links and relative navigation relations keep unchanged with time. Once the network topology of these devices are established, it will be fixed. Even when the formation configuration of the members is changed due to orbital motions, the communication links and relative navigation relation will still keep the same.

b. Short distance

In specific applications, if the inter-distance between two members of FSC is too large, the communication and relative navigation devices may be invalid. To make the problem relatively simple, we assume that the members of FSC are gathered in a small space area. In this case, the relative distance between any two members are short enough to make the communication and relative navigation devices in an operating range.

c. Direct measurement principle

If one member of the cluster has an absolute navigation device, it must determine its absolute position and attitude by its own. This principle is called the direct measurement principle. In fact, however, one member can also receive its absolute position and attitude by others. It is clear that the latter case makes a waste of either communication or relative navigation channels compared with the former case. So this assumption is significant for optimal navigation sharing.

Remark 3. It must be noted that the above three assumptions only correspond to a limited real scenarios. In the real application, the conditions may be different with the one stated here. Especially, the second assumption is not always right in real world. This means, we only discuss a special case in this paper.

2.3. An important lemma

There presents an important lemma to support the main theorems of this paper. This lemma is summarized as follows:

Lemma 1 Bass et al. (1974). *If the matrix A is the adjacency matrix of the graph G, and the index k is a positive integer, then the entry in position (i,j) of the matrix A^k, viz. $a_{i,j}^{(k)}$, is the number of paths with length k in G, joining the node v_i to the node v_j.*

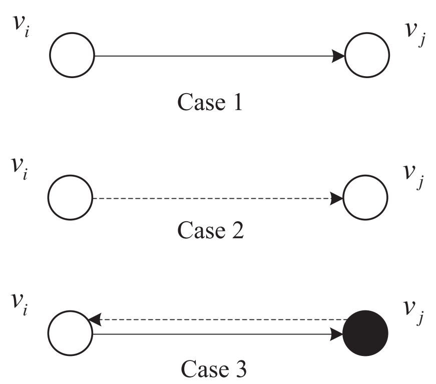

Fig. 2. Three cases for navigation sharing.

Proof. The result is true for $k = 0$ (since $A^0 = I$) and for $k = 1$ (since $A^1 = A$ is the adjacency matrix). Suppose that the result is true for $k = K$. From the identity

$$(A^{K+1})_{i,j} = \sum_{h=1}^{N} (A^K)_{i,h} a_{h,j} \tag{2}$$

we deduce that $(A^{K+1})_{i,j}$ is the number of paths of length $K + 1$ joining v_i to v_j, whence the result for all k follows by induction. □

3. Models and methods for navigation sharing

3.1. Navigation sharing conditions and constraints

There are three cases to be considered when studying the navigation sharing problem. In each case, the navigation sharing can be achieved if some conditions are satisfied. These three cases are distinguished according to the devices that are used, i.e. absolute navigation device, relative navigation device, and communication device. If only absolute navigation and relative navigation are adopted, it is the Case 1. When the relative navigation is replaced by the communication, it becomes the Case 2. These two cases correspond to some special scenarios. For a general scenario in which all three kinds of devices are incorporated, it is the Case 3.

The explanation of Fig. 2 is shown below. Case 1: any node has not communication device, and its relative navigation data is obtained by others who have relative navigation devices; Case2: any node has not relative navigation device, and it sends its absolute navigation data to others by communication devices; Case 3: the hybrid scenario of the former two cases, that means some nodes have relative devices, and some others have communication devices. It also noted that in all three cases, some nodes may lose the absolute navigation devices, which is shown by solid black circles.

To make the description of the navigation sharing problem be clear, it should define some related symbols. Let ε_i be the variable indicating whether the node v_i contains absolute navigation device. When v_i contains absolute navigation device, then $\varepsilon_i = 1$, or else $\varepsilon_i = 0$. Let $\eta_{i,j,k}$ represent whether the communication edge $\overrightarrow{v_j v_k}$ contains the navigation data of the node v_i. If the answer is yes, then $\eta_{i,j,k} = 1$, or else $\eta_{i,j,k} = 0$. Let $\xi_{i,j}$ be the variable representing whether the node v_j has the relative navigation device and determines the relative position and attitude between its own and the node v_i. Only these two conditions both hold, $\xi_{i,j} = 1$; or else $\xi_{i,j} = 0$ (see Fig. 3).

For a cluster of fractionated spacecrafts, once they are launched into space, the absolute and relative navigation devices are determined. Therefore, for the navigation sharing problem, ε_i is all known. And for the node v_j without relative navigation device, $\xi_{i,j} = 0(\forall i)$. In contrast, when the node v_j has the relative navigation device, $\xi_{i,j}$ can be assigned to 1. It can be seen that if all the variables $\eta_{i,j,k}$ and $\xi_{i,j}$ are fixed, the corresponding network topology constructed by the relative navigation and communication links will be determined. Therefore the aim of the mathematical model is to describe the relations among these variables.

3.1.1. Case 1: absolute navigation plus relative navigation

When the members in the fractionated spacecraft cluster have no communication devices, to achieve the aim of complete navigation sharing, the unique solution is that every member has both the absolute navigation device and the relative navigation device. Since the number of the members is N, it requires that the number of relative navigation channels of each node must be greater than $N - 2$, i.e. $g_i \geqslant N - 1$. As a result, the sharing degree is $SD = 1$.

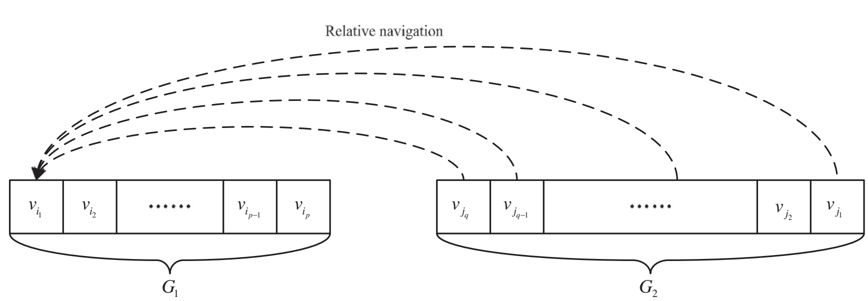

Fig. 3. Scheme of relative navigation for Case 1.

When $g_i < N-1$, however, the complete navigation sharing cannot be achieved. Based on the definition of sharing degree, we can get

$$SD = \max_{\xi_{i,j}} \frac{\min_i \sum_{j=1}^{N} \varepsilon_j \xi_{i,j}}{N}$$

$$s.t. \quad \sum_{i=1}^{N} \xi_{i,j} \leqslant g_j, \quad \forall j$$

where $\sum_{i=1}^{N} \xi_{i,j} \leqslant g_j$ is the relative navigation channels' constraint.

The following theorem indicates that the sharing degree SD, when all ε_j are positive, can be easily found.

Theorem 1. *The maximum sharing degree of Case* 1 *when all members have absolute devices is* $SD = \frac{1}{N}\min_j(g_j)$.

Proof. The proof can be divided into two steps. In the first step, we will prove that $SD \leqslant \frac{1}{N}\min_j(g_j)$. In the second step, we will prove that $SD \geqslant \frac{1}{N}\min_j(g_j)$. Therefore, these two steps will easily lead to the conclusion that $SD = \frac{1}{N}\min_j(g_j)$. □

The first step

We denote v_{i_1} as the member whose number of relative navigation channels is equal to $\min_j(g_j)$. Since v_{i_1} can only receive a maximum $\min_j(g_j)$ of other members' relative navigation data, this means there are at most $\min_j(g_j)$ members whose absolute navigation data can be shared by the other members. Therefore, $SD \leqslant \frac{1}{N}\min_j(g_j)$ can be obtained.

The second step

In fact, we can construct a scenario in which there are $\min_j(g_j)$ members whose absolute navigation data are shared by the other members. To that end, we firstly rearrange the order of the members like $v_{i_1}, v_{i_2}, \ldots, v_{i_p}, v_{j_q}, v_{j_{q-1}}, \ldots, v_{j_1}$ where $g_{i_1} = \min_j g_j$, $g_{j_1} = \max_j g_j$, and $g_{i_1} \leqslant g_{i_2} \leqslant \cdots \leqslant g_{i_p} \leqslant g_{j_q} \leqslant g_{j_{q-1}} \leqslant \cdots \leqslant g_{j_1}$. Let $q = g_{i_1}$. It is clear that $p = N - q$. We collect the first p members in the set G_1, which leads to $G_1 = \{v_{i_1}, v_{i_2}, \ldots, v_{i_p}\}$. Similarly, collect the rest of the members in the set G_2, which results in $G_2 = \{v_{j_q}, v_{j_{q-1}}, \ldots, v_{j_1}\}$. Since v_{i_1} has g_{i_1} channels of relative navigation, it can determine the absolute navigation data of all the members in G_2. Furthermore, any member in G_1 can determine the absolute navigation data of the members in G_2. According to the definition of G_2, any member in it has the ability of determining the absolute navigation data of each other. By this way, we can easily find that the navigation data of the members in G_2 have been known by all the members. This means that at least there are $q = \min_j(g_j)$ members whose navigation data can be shared, i.e. $SD \geqslant \frac{1}{N}\min_j(g_j)$.

It is also clear that, in a cluster with known g_j($j = 1, 2, \ldots, N$), to obtain the biggest sharing degree, the network topology of relative navigation links should be established as the way of constructing sets G_1 and G_2 in Theorem 1.

3.1.2. Case 2: absolute navigation plus communication

When all relative navigation devices are replaced by communication devices, the navigation sharing can also be obtained. Since the communication can transmit the navigation data between two members, it is easier to achieve the navigation sharing in this case.

For the complete navigation sharing, our previous work (Dang and Zhang, 2013) has discovered the corresponding conditions. The following theorem gives the conclusion.

Theorem 2 Dang and Zhang (2013). *If the Eqs*. (4) and (5) *hold, then the complete navigation sharing can be achieved for Case* 2, *without considering any other constraints*

$$\sum_{t=1}^{N-1} \eta_{i,i,j}^{(t)} \geqslant 1, \quad \forall i, j \neq i \tag{4}$$

where

$$\begin{cases} \eta_{i,j,k}^{(1)} = \eta_{i,j,k} \\ \eta_{i,j,k}^{(t+1)} = \sum_{s=1}^{N} \eta_{i,j,s}^{(t)} \cdot \eta_{i,s,k} \end{cases}, \quad \forall i, j, k$$

Proof. See Dang and Zhang (2013). □

Corollary 1. *If Eq.* (4) *does not hold, then the sharing degree for Case* 2 *can be determined by the following equation*

$$SD = \frac{\min_i \left\{ \sum_{j=1}^{N} \min\left(\sum_{t=1}^{N-1} \eta_{i,i,j}^{(t)}, 1 \right) \right\}}{N} \tag{6}$$

Proof. It is clear that if $\sum_{t=1}^{N-1} \eta_{i,i,j}^{(t)} \geqslant 1$, then it means node v_i sends its absolute navigation data to node v_j. Therefore, $\sum_{j=1}^{N} \min(\sum_{t=1}^{N-1} \eta_{i,i,j}^{(t)}, 1)$ represents the number of the memebers who can obtain the absolute navigation data of node v_i. Taking the minimum value of these numbers and divided by the total number of the members results in the navigation sharing degree. That just complete the conclusion of Corollary 1. □

Although the Eqs. (4) and (5) can lead to the navigation sharing, it may contradict with the real conditions. The following constraints should be considered (Dang and Zhang, 2013).

Constraint 1:

The number of the navigation data in any edge originating from a same node should be less than the node's communication capacity, viz.

$$\sum_{i=1}^{N} \eta_{i,j,k} \leqslant p_j, \quad \forall j, k \neq j \tag{7}$$

Constraint 2:

For any node v_j, if it can only establish a maximum q_j edges, then the following constraint must be satisfied,

$$\sum_{k=1}^{N}\left\{1-\prod_{i=1}^{N}(1-\eta_{i,j,k})\right\} \leqslant q_j, \quad \forall j \tag{8}$$

Constraint 3:

Any $\eta_{i,j,k}$ can only be 0 or 1, viz.

$$\eta_{i,j,k}(1-\eta_{i,j,k})=0, \quad \forall i,j,k \tag{9}$$

Constraint 4:

Any node cannot point to itself, viz.

$$\eta_{i,j,j}=0, \quad \forall i,j \tag{10}$$

Constraint 5:

Any node v_j cannot send the navigation data of v_i to v_i itself, viz.

$$\eta_{i,j,i}=0, \quad \forall i,j \tag{11}$$

3.1.3. Case 3: general scenario with absolute navigation, relative navigation, and communication

The former two cases correspond to some real possible applications, but a more general scenario will contain all these three kinds of devices, i.e. absolute navigation device, relative navigation device, and communication device. Similarly, we give the following theorem to find the navigation sharing condition.

Theorem 3. *If the following Eqs. (12) and (13) hold, the complete navigation sharing for Case 3 is achieved, without considering any other constraints*

$$\begin{aligned}&\varepsilon_i\cdot\left(\sum_{t=1}^{N-1}\eta_{i,i,j}^{(t)}+\xi_{i,j}\cdot\varepsilon_j^*\right)+(1-\varepsilon_i)\\&\cdot\left[\sum_{s=1,s\neq j}^{N}\left(\xi_{i,s}\cdot\varepsilon_s^*\cdot\sum_{t=1}^{N-1}\eta_{i,s,j}^{(t)}\right)+\xi_{i,j}\cdot\varepsilon_j^*\right]\\&\geqslant 1, \quad \forall i,j\cdot\varepsilon_i\neq i\end{aligned} \tag{12}$$

where

$$\varepsilon_j^*=\frac{\max\left\{\left[\varepsilon_j+\sum_{s=1,s\neq j}^{N}\left(\xi_{j,s}\cdot\varepsilon_s^*\cdot\sum_{t=1}^{N-1}\eta_{j,s,j}^{(t)}\right)\right],0\right\}}{\max\left\{\left[\varepsilon_j+\sum_{s=1,s\neq j}^{N}\left(\xi_{j,s}\cdot\varepsilon_s^*\cdot\sum_{t=1}^{N-1}\eta_{j,s,j}^{(t)}\right)\right],1\right\}}, \quad \forall j \tag{13}$$

Proof. To achieve the complete navigation sharing, the absolute navigation data of any node v_i should be obtained by others. There are two scenarios to be considered.

When $\varepsilon_i=1$, the node v_i can get its own absolute position and attitude by its absolute navigation device. In this scenario, for any node $v_j(j\neq i)$, it can get the navigation data of v_i by two ways. The first way is that v_i sends its navigation data to v_j by a series of communication links. The second way is that the relative navigation data of v_i relative to v_j is obtained by v_j by the relative navigation device. But it should be noted that the second way depends on that v_j has got its own absolute navigation data, i.e. $\varepsilon_j^*=1$. This leads to $\sum_{t=1}^{N-1}\eta_{i,i,j}^{(t)}+\xi_{i,j}\cdot\varepsilon_j^*\geqslant 1$.

When $\varepsilon_i=0$, it must firstly determine the absolute position and attitude of the node v_i. This can be done by the nodes who can obtain the relative navigation data between v_i and them. Then these kinds of nodes further send the relative navigation data and their absolute navigation data to v_i. At the same time, these kinds of nodes also send the above two types of data to the node v_j. Or else, v_j must resort to its own to perform the relative navigation between itself and the node v_i. This leads to $\sum_{s=1,s\neq j}^{N}\left(\xi_{i,s}\cdot\varepsilon_s^*\cdot\sum_{t=1}^{N-1}\eta_{i,s,j}^{(t)}\right)+\xi_{i,j}\cdot\varepsilon_j^*\geqslant 1$.

Therefore, to achieve the complete navigation sharing, the total condition should be the summarization of these two scenarios. This just leads to the results of Eq. (12). But it should be noted that, in Eq. (12), some $\varepsilon_t(t=1,2,\ldots,N)$ are replaced by $\varepsilon_t^*(t=1,2,\ldots,N)$ with the expression of Eq. (13). This is because that maybe this kind of nodes should be firstly navigating under other ones' assist. □

Corollary 2. *When the complete navigation sharing of the Case 3 is achieved, it has $\varepsilon_j^*=1(j=1,2,\ldots,N)$.*

Proof. It is clear that, when the complete navigation sharing of the Case 3 is achieved, the absolute navigation data of every node can be known by all the nodes in the cluster. Of course its absolute navigation data are also known by itself. This means $\varepsilon_j^*=1$. □

Corollary 3. *If Eq. (12) does not hold, the sharing degree for Case 3 can be determined by the following equation*

$$SD=\frac{\min_i\left\{\sum_{j=1}^{N}\min(f_{i,j},1)\right\}}{N} \tag{14}$$

where $f_{i,j}=\varepsilon_i\cdot\left(\sum_{t=1}^{N-1}\eta_{i,i,j}^{(t)}+\xi_{i,j}\cdot\varepsilon_j^*\right)+(1-\varepsilon_i)\cdot\left[\sum_{s=1,s\neq j}^{N}\left(\xi_{i,s}\cdot\varepsilon_s^*\cdot\sum_{t=1}^{N-1}\eta_{i,s,j}^{(t)}\right)+\xi_{i,j}\cdot\varepsilon_j^*\right]$.

Proof. The proof of this corollary is very similar with the one of Corollary 1. So the details are ignored here. □

Similar to Case 2, there are some constraints which should be included when considering the navigation sharing of the Case 3. The Constraints 1–4 of Case 2 are still valid for Case 3. The Constraint 5 in Case 2 is no longer valid for Case 3. Then some other constraints should be further considered.

Constraint 5:

Any ε_i can only be 0 or 1, viz.

$$\varepsilon_i(1-\varepsilon_i)=0, \quad \forall i \tag{15}$$

Constraint 6:

Any $\xi_{i,j}$ can only be 0 or 1, viz.

$$\xi_{i,j}(1-\xi_{i,j})=0, \quad \forall i,j\neq i \tag{16}$$

Constraint 7:

Any node cannot point to itself for the relative navigation, viz.

$$\xi_{i,i} = 0, \quad \forall i \tag{17}$$

Constraint 8:

The number of relative navigation links for any node should be less than the node's relative navigation ability, viz.

$$\sum_{i=1, i \neq j}^{N} \xi_{i,j} \leqslant g_j, \quad \forall j \tag{18}$$

3.2. Optimization models for navigation sharing

3.2.1. Objective function

There are three kinds of optimization objects, i.e. minimum communication connections' number (MCCN) (Dang and Zhang, 2013), minimum relative navigation connections' number (MRNCN), and the hybrid of MCCN and MRNCN.

The objective functions (OF) are summarized as follows:

(1) Communication connections' number:

$$J_1 = \sum_{j=1}^{N} \sum_{k=1}^{N} \left\{ 1 - \prod_{i=1}^{N} (1 - \eta_{i,j,k}) \right\} \tag{19}$$

where $1 - \prod_{i=1}^{N}(1 - \eta_{i,j,k})$ represents whether the edge $\overrightarrow{v_j v_k}$ exists. When $1 - \prod_{i=1}^{N}(1 - \eta_{i,j,k}) = 1$, it means that the edge $\overrightarrow{v_j v_k}$ exists. Or else if $1 - \prod_{i=1}^{N}(1 - \eta_{i,j,k}) = 0$, the edge $\overrightarrow{v_j v_k}$ does not exist.

It is clear that J_1 represents the total communication connections' number in FSC. By minimizing this objective function, it can find the MCCN optimal network.

(2) Relative navigation connections' number:

$$J_2 = \sum_{j=1}^{N} \sum_{i=1, i \neq j}^{N} \xi_{i,j} \tag{20}$$

It can be easily found that J_2 denotes the total relative navigation connections' number. By minimizing this objective function, it can obtain the MRNCN optimal network.

(3) Weighted sum of communication connections' number and relative navigation connections' number

$$J_3 = w \cdot \sum_{j=1}^{N} \sum_{k=1}^{N} \left\{ 1 - \prod_{i=1}^{N} \left(1 - \eta_{i,j,k}\right) \right\} + (1 - w) \cdot \sum_{j=1}^{N} \sum_{i=1, i \neq j}^{N} \xi_{i,j} \tag{21}$$

where w is the weight for the two types' objective function vectors.

In some cases, the cost of the communication and relative navigation may be different and both need be evaluated, the weighted sum of the two kinds of connections' number should be considered. This leads to the objective function J_3.

3.2.2. Optimization models

(1) Absolute navigation plus relative navigation
In this case, there are clear solutions for navigation sharing. Three simple types are shown in Table 1.

(2) Absolute navigation plus communication
Since Case 2 has no relative navigation device, so only the MCCN type's OF can be used. We can write the whole description of the MCCN communication network topology for the navigation sharing as follows

$$\begin{cases} \text{find} \quad \eta_{i,j,k} \\ \min \quad J_1 \\ \text{s.t} \quad \sum_{t=1}^{N-1} \eta_{i,i,j}^{(t)} \geqslant 1, \quad \forall i \in G_n, j \neq i \\ \qquad \sum_{i=1}^{N} \eta_{i,j,k} \leqslant p_j, \quad \forall j, k \neq j \\ \qquad \sum_{k=1}^{N} \{1 - \prod_{i=1}^{N} (1 - \eta_{i,j,k})\} \leqslant q_j, \quad \forall j \\ \qquad \eta_{i,j,k}(1 - \eta_{i,j,k}) = 0, \quad \forall i, j, k \neq j \\ \qquad \eta_{i,j,j} = 0, \quad \forall i, j \\ \qquad \eta_{i,j,i} = 0, \quad \forall i, j \end{cases} \tag{22}$$

(3) General scenario with absolute navigation, relative navigation, and communicationSince there are three kinds of OFs, we denote any one of them as the symbol J_i. Then the final optimization model can be written as follows

$$\begin{cases} \text{find} \ \ \eta_{i,j,k}, \xi_{i,j} \\ \min \ \ J_i (i = 1, 2, 3) \\ \text{s.t} \ \ \varepsilon_i \cdot (\sum_{t=1}^{N-1} \eta_{i,i,j}^{(t)} + \xi_{i,j} \cdot \varepsilon_j^*) \\ \quad + (1 - \varepsilon_i) \cdot \left[\sum_{s=1, s \neq j}^{N} (\xi_{i,s} \cdot \varepsilon_s^* \cdot \sum_{t=1}^{N-1} \eta_{i,s,j}^{(t)}) + \xi_{i,j} \cdot \varepsilon_j^* \right] \geqslant 1, \ \ \forall i \in G_n, j \cdot \varepsilon_i \neq i \\ \quad \sum_{i=1}^{N} \eta_{i,j,k} \leqslant p_j, \ \ \forall j, k \neq j \\ \quad \sum_{k=1}^{N} \left\{ 1 - \prod_{i=1}^{N} (1 - \eta_{i,j,k}) \right\} \leqslant q_j, \ \ \forall j \\ \quad \eta_{i,j,k}(1 - \eta_{i,j,k}) = 0, \ \ \forall i, j, k \neq j \\ \quad \eta_{i,j,j} = 0, \ \ \forall i, j \\ \quad \varepsilon_i (1 - \varepsilon_i) = 0, \ \ \forall i \\ \quad \xi_{i,j}(1 - \xi_{i,j}) = 0, \ \ \forall i, j \neq i \\ \quad \xi_{i,i} = 0, \ \ \forall i \\ \quad \sum_{i=1, i \neq j}^{N} \xi_{i,j} \leqslant g_j, \ \ \forall j \end{cases} \tag{23}$$

Table 1
The three simple types of the Case 1.

No.	Absolute navigation devices' number	Minimum relative navigation channels' number	Navigation sharing degree
1	N	$\geqslant N - 1$	1
2	N	g_m	g_m/N
3	$<N$	g_m	Solved by Eq. (3)

3.3. Solving methods

3.3.1. The generating algorithm for the coupled ε_j^*

Eq. (13) exposures an obvious problem that the values of ε_j^* are coupled with each other. To solve this problem, we propose a generating algorithm. It can be easily found that there is at least one node v_t who has an absolute navigation device to make the navigation sharing problem have a solution. So we can determine the values of each ε_j^* one by one from the positive valued ε_i. The algorithm is presented as follows

Algorithm 2.

Step 1. Classify the nodes into two groups. For any node v_j $(j = 1,2,\ldots,N)$, if $\varepsilon_j = 1$, then the node v_j is collected in the first set G_1. Or else, v_j is allocated into the second set G_2. These two sets can be expressed as $G_1 = \{v_{i_1}, v_{i_2}, \ldots, v_{i_p}\}$ and $G_2 = \{v_{j_1}, v_{j_2}, \ldots, v_{j_q}\}$. It is clear that $p + q = N$. Let $\varepsilon_{i_t}^* = 1 (\forall t = 1,2,\ldots,p)$ and $\varepsilon_{j_t}^* = 0 (\forall t = 1,2,\ldots,q)$. Let $l = 1$.

Step 2. For each $j_t(t = 1,2,\ldots,q)$, calculate the values of $\varepsilon_{j_t}^*$ using the following equations,

$$\varepsilon_{j_t}^* = \frac{\max\left\{\sum_{s\in G_1}\left(\xi_{j_t,s}\cdot\varepsilon_s^*\cdot\sum_{t=1}^{N-1}\eta_{j_t,s,j_t}^{(t)}\right),0\right\}}{\max\left\{\sum_{s\in G_1}\left(\xi_{j_t,s}\cdot\varepsilon_s^*\cdot\sum_{t=1}^{N-1}\eta_{j_t,s,j_t}^{(t)}\right),1\right\}},\quad \forall j_t\in G_2 \tag{24}$$

Denote n_l as the numbers of all the nodes like $v_{j_t}(t = 1,2,\ldots,q)$ with $\varepsilon_{j_t}^* = 1$. Transfer these n_l nodes of G_2 to G_1. Let $p \leftarrow p + n_l$, and $q \leftarrow q - n_l$.

Step 3. If $n_l > 0$ and $q > 0$, let $l = l + 1$ and go to Step 2. Or else, stop and output the current values of each $\varepsilon_j^*(j = 1,2,\ldots,N)$.

3.3.2. Genetic algorithm for optimization problem

In Section 3.2, the navigation sharing problem has been transformed into a nonlinear 0–1 integer programming problem (0–1 NIP). To solve this kind of problem, the Genetic Algorithm (GA) can be used. For adopting the GA to solve the navigation sharing problem, it must first design a proper representation and fitness measure. Next, the termination criterion should be carefully devised. The flow chart of GA is shown in Fig. 4 (Goldberg, 1989; Renner and Ekart, 2003).

Algorithm 3.

(1) Representation
The representation of the navigation sharing problem is the first step of using a genetic algorithm. Considering the property of navigation sharing problem, the fixed length bit string is adopted. We collect all the variables $\eta_{i,j,k}$ and $\varepsilon_{i,j}$ in a vector $\boldsymbol{x}$. Since $\eta_{i,j,j} = 0$, $\eta_{i,j,i} = 0$ (except for Case 3) and $\xi_{i,i} = 0$ for any i and j, we eliminate all these variables from the vector $\boldsymbol{x}$. Because that the entries in $\boldsymbol{x}$ can only be 0 or 1, they need not to be encoded any longer.

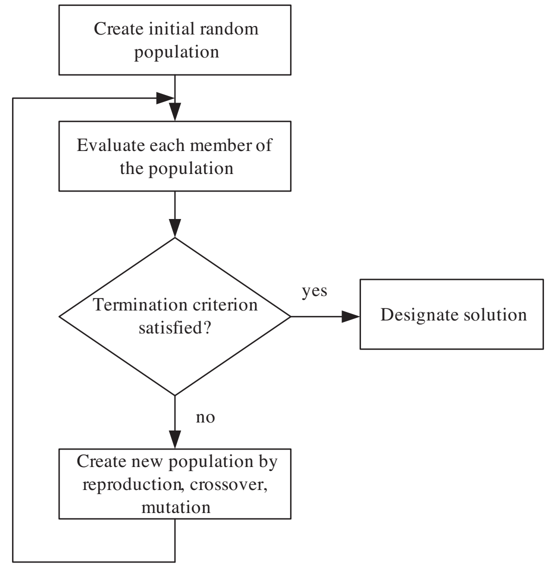

Fig. 4. Flow chart of Genetic Algorithm.

(2) Genetic operators
In each generation, three genetic operators are applied to select proper individuals from the current population in order to create a better population. The arithmetical crossover, non-uniform mutation and proportion selections are applied.Crossover represents the process that the new individuals are created as the offspring of two parents. In this process, one or more so-called crossover points are selected randomly within the chromosome of each parent, at the same place in each.Mutation means a new individual is created by making modifications to one selected individual. This can be done by changing one or more values in the representation or in adding/deleting parts of the representation.A proportion selection is the process that a part of the new population is created by simply copying without change selected individuals from the present population. This can bring the possibility of survival for already developed fit solutions. The more details of these three genetic operations can be found in reference Goldberg (1989).

(3) Fitness evaluation functionThe original GA algorithm is defined for unconstrained problems. However the resulted optimization problem in this paper is constrained. It is important to add the capability of dealing with the constraints. According to our previous work

(Dang and Zhang, 2013) and the methods of Luo et al. (2006), it can deal with these constraints by introducing of a penalty function. For the constrained optimization problem, i.e.

$$\begin{cases} \text{find} & \boldsymbol{x} \\ \min & f(\boldsymbol{x}) \\ \text{s.t} & g_i(\boldsymbol{x}) \leqslant 0, \quad i = 1, 2, \ldots, s \\ & h_j(\boldsymbol{x}) = 0, \quad j = 1, 2, \ldots, t \end{cases} \tag{25}$$

the hybrid restated, penalized and non-constrained optimization problem yields

$$\min f'(x) = f(\boldsymbol{x}) + M_1 \sum_{i=1}^{s} \max(0, g_i(\boldsymbol{x})) + M_2 \sum_{j=1}^{t} |h_j(\boldsymbol{x})| \tag{26}$$

where the two coefficients satisfy $M_1 > 0$ and $M_2 > 0$.

The fitness functions for the two kinds of navigation sharing problems are chosen as

(a) Absolute navigation plus communication

$$Fit(\mathbf{x}) = J_1 + M_1 \left\{ \begin{array}{l} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \max\left(0, 1 - \sum_{t=1}^{N-1} \eta_{i,i,j}^{(t)}\right) \\ + \sum_{j=1}^{N} \sum_{k=1, k \neq j}^{N} \max\left(0, \sum_{i=1}^{N} \eta_{i,j,k} - p_j\right) \\ + \sum_{j=1}^{N} \max\left(0, \sum_{k=1}^{N} \left\{1 - \prod_{i=1}^{N} (1 - \eta_{i,j,k})\right\} - q_j\right) \end{array} \right\} \tag{27}$$

(b) General scenario with absolute navigation, relative navigation, and communication

where M_1 is a positive number. Note that the equality constraint $\eta_{i,j,k}(1 - \eta_{i,j,k}) = 0$ and $\xi_{i,j}(1 - \xi_{i,j}) = 0$ are not regulated into the fitness function. This is because that we directly take the variables $\eta_{i,j,k}$ and $\xi_{i,j}$ as the binary code in encoding operation.

On the base of the above three operations' illustration, we can encode the GA in computer to solve the navigation sharing problem using the standard procedure.

4. Examples and discussions

In this section, some examples will be presented to test the models and corresponding solving methods. Without loss of generality, a fractionated spacecraft cluster with five members is selected as a basic scenario. In different cases, i.e. Cases 1–3, the members in the cluster have different devices. For example, in Case 1, all members only have absolute navigation devices and/or relative navigation devices. In each case, when the objective functions are different, the results will be different too, which will be analyzed in a dependent sub-section.

It should be explained that, in the following figures, the sub-figures (a)–(e) represent the information flow of each independent node. For example, the sub-figure (a) in Fig. 4 shows the flow of the absolute navigation data of node v_1. The last sub-figure (f) in each figure represents the final total network topology of the information flow. To be clear, the dotted line represents the relative navigation link, and the solid line represents the communication link. The full circle represents the node that has not absolute navigation devices. Meanwhile, the empty circle represents the node that has absolute navigation devices.

4.1. Comparison of the results in different cases

In this subsection, the objective function is selected as the same, i.e. J_3. Three examples will be solved to test the algorithms developed in this paper. These examples corre-

$$Fit(\mathbf{x}) = J_i + M_1 \cdot \left\{ \begin{array}{l} \sum_{i=1}^{N} \sum_{j=1, j \cdot \varepsilon_i \neq i}^{N} \max\left(0, 1 - \varepsilon_i \cdot \left(\sum_{t=1}^{N-1} \eta_{i,i,j}^{(t)} + \xi_{i,j} \cdot \varepsilon_j^*\right) - (1 - \varepsilon_i) \cdot \left[\sum_{s=1, s \neq j}^{N} \left(\xi_{i,s} \cdot \varepsilon_s^* \cdot \sum_{t=1}^{N-1} \eta_{i,s,j}^{(t)}\right) + \xi_{i,j} \cdot \varepsilon_j^*\right]\right) \\ + \sum_{j=1}^{N} \sum_{k=1, k \neq j}^{N} \max\left(0, \sum_{i=1}^{N} \eta_{i,j,k} - p_j\right) \\ + \sum_{j=1}^{N} \max\left(0, \sum_{k=1}^{N} \left\{1 - \prod_{i=1}^{N} (1 - \eta_{i,j,k})\right\} - q_j\right) \\ + \sum_{j=1}^{N} \max\left(0, \sum_{i=1, i \neq j}^{N} \xi_{i,j} - g_j\right) \end{array} \right\} \tag{28}$$

Table 2
Basic conditions for navigation sharing in Cases 1/2/3.

Node	v_1	v_2	v_3	v_4	v_5
ε_i	1/1/1	1/1/1	1/1/0	1/1/1	1/1/0
g_i	5/0/5	5/0/5	5/0/5	5/0/5	5/0/5
p_i	0/5/5	0/5/5	0/5/5	0/5/5	0/5/5
q_i	0/5/5	0/5/5	0/5/5	0/5/5	0/5/5

spond to the three cases that are distinguished in this paper. The basic conditions are set as in the Table 2.

The results of Case 1 are presented in Fig. 5. In Case 1, there are no communication devices. Therefore, the unique solution of the optimal network topology is a complete graph. Sub-figures (a)–(e) illustrate the relative navigation data's flow of each independent node. Sub-figure (f) illustrates the total network topology of these relative navigation links. It is clear that the total links' number is 20.

The results of Case 2 are presented in Fig. 6. In this case, there are only communication devices to be used. Therefore the optimal solution is a ring. The sub-figures (a)–(e) show the network topologies of the communication links for each independent node, which are used to transmit the navigation data of each node to others. Sub-figure (f) is the final topology of these communication links. The total links' number can be calculated to be 5.

The results of Case 3 are shown in Fig. 7. In this case, the nodes v_3 and v_5 have no absolute navigation devices. However, all the nodes have full relative navigation and communication ability. The nodes v_3 and v_5 determine their own absolute navigation data by the assist of the nodes v_2 and v_4, respectively, using the relative navigation devices. Then their navigation data are transmitted along the ring communication topology. The sub-figures (a)–(e) show the information flow of each independent node. Sub-figure (f) illustrates that the total topology is a ring adding with two relative navigation links. The total links' number is 7.

In these three cases, the complete navigation sharing are all achieved. This is because that the conditions are enough to achieve this aim.

4.2. Comparison of the results in different objective functions

In this subsection, the results in the same conditions with different objective functions are compared. Two different scenarios will be distinguished. The first one is the so-called 'without constraints'. In this scenario, all members have absolute navigation devices, relative navigation devices, and communication devices. Furthermore, the relative navigation channels, the communication capacity, and the communication channels in each member are enough. The second scenario is the so-called 'with constraints'. In this scenario, some members have no absolute navigation devices. The relative navigation channels, the communication capacity, and the communication channels in each member are also limited.

(1) Without constraints
The basic conditions for this scenario are set in Table 3. From the table, it can be seen that all the nodes have absolute navigation devices, relative navigation devices, and communication devices.
When adopting the MCCN type's OF, the solution is same as the one shown in Fig. 5. This is because that for the MCCN type's OF, the communication links should be replaced by the relative navigation links as much as possible. So the condition in Table 3 has the same effects as the one for Case 1 in Table 2.

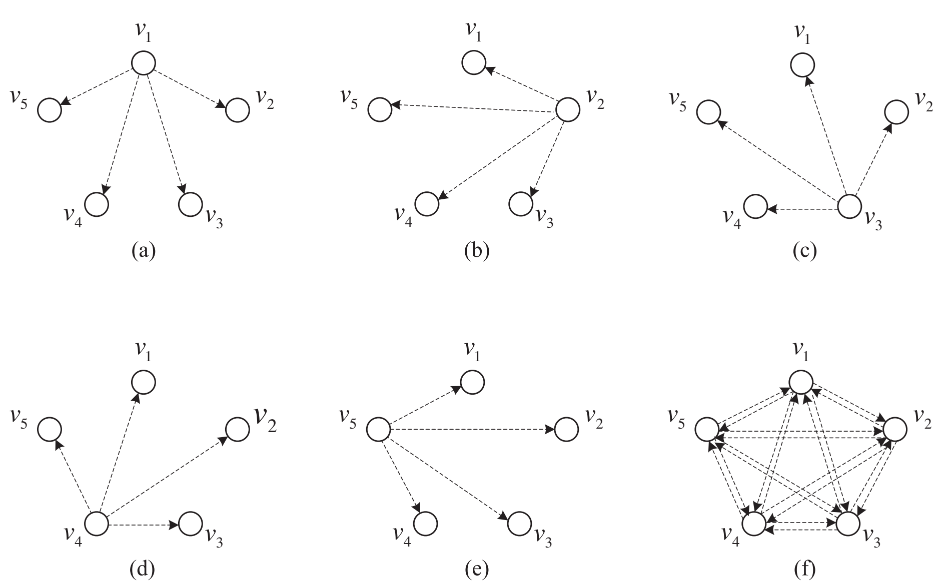

Fig. 5. Optimal network topology of relative navigation links for Case 1.

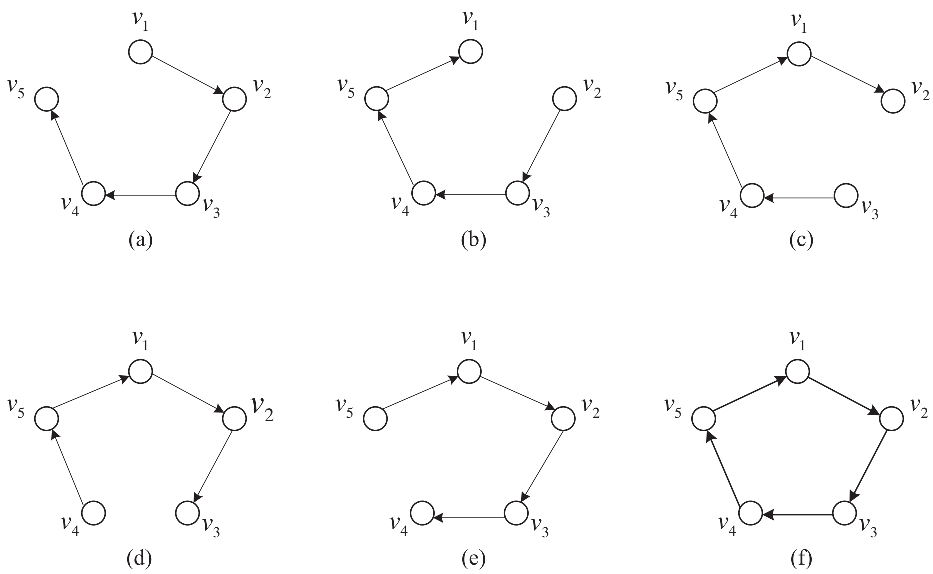

Fig. 6. Optimal network topology of communication links for Case 2.

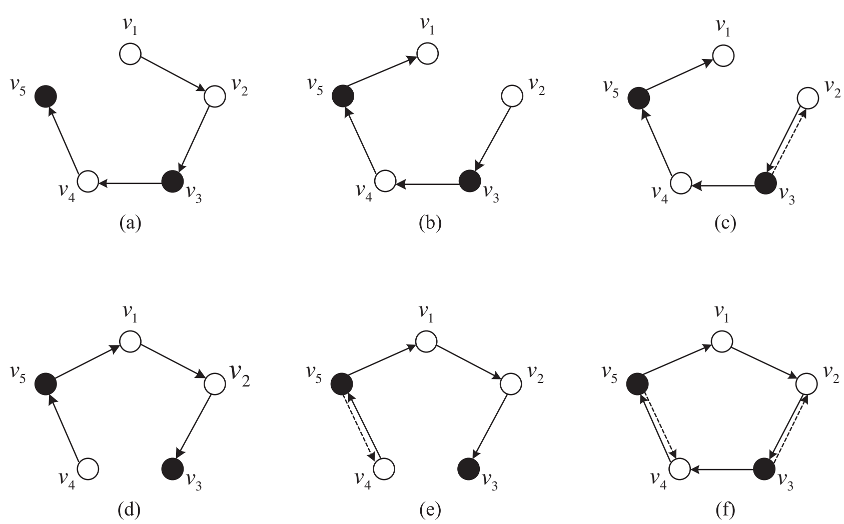

Fig. 7. Optimal network topology of the relative navigation and communication links for Case 3.

When adopting the MRNCN type's OF, the solution is same as the one shown in Fig. 6. The reason is that for the MRNCN type's OF, the relative navigation links should be replaced by the communication links as much as possible. Therefore, the condition in Table 3 will lead to the same results under the condition for the Case 2 in Table 2.

When adopting the Hybrid OF with a weight $w = 0.5$, the solution is same as the one with MCCN type's OF. This is due to the fact that the communication can transmit more data than the relative navigation link. When $w = 0.5$, however, a communication link is equal with a relative navigation link. So the optimal solution will require more communication links but not the relative navigation links. This conclusion is correct for all kinds of weights in the scenario of 'without constraint'.

(2) With constraints

The basic conditions for this scenario are shown in Table 4. From the table, it can be seen that node v_3 has no absolute navigation device. This means that it cannot obtain its absolute navigation data by itself. Node v_5 has no relative navigation device since the relative navigation channels are 0. Node v_1 cannot send out any navigation data to other nodes for that $p_1 = 0$ and $q_1 = 0$.

Using the algorithms developed in this paper, the solutions for three kinds of objective functions are shown as follows. Fig. 8 shows the optimal network topology of relative navigation links and communication links when the objective function is selected as the MCCN type's OF. It can be seen that the nodes v_1 and v_3 are all assisted by the node v_2 with the relative navigation device. In this way, the absolute navigation data of v_1 and v_3 are known by the node v_2. Since the node v_3 does not know its own absolute navigation data, the node v_2 sends this data to it by communication. Due to the fact that the node v_1 has not the ability of sending out its navigation data, the node v_2 undertakes the responsibility to transmit the navigation data of v_1 to others, see the sub-figure (a) in Fig. 8. The final network topology is shown in the sub-figure (f) in Fig. 8. It can be seen that the communication links' number and the relative navigation links' number are 4 and 7, respectively.

Fig. 9 illustrates the network topology of relative navigation links and communication links for the scenario with MRNCN type's OF. It is clear that the network topology is different to the one of MCCN type's OF. The treatment of the node v_1 and v_3, however, is same as the one in the scenario of MCCN type's OF. The final network topology is shown in the sub-figure (f) in Fig. 9. It can be seen that the communication links' number increases to 8 which is more than the one (4) in the scenario of MCCN type's OF. However, the relative navigation links' number is only 2.

The solution for the Hybrid OF is same as the one for MRNCN type's OF. This is because that the communication link has more advantages than the relative navigation on transmitting the navigation data. To reduce the total links, it is best to reduce the relative navigation links. In this sense, the MRNCN type's OF is equal to the Hybrid OF. However, the relative navigation device cannot be cut down, especially in the case that some nodes have no absolute navigation devices or cannot send out navigation data by communication devices. In a fractionated spacecraft cluster, there are some members, who have no navigation devices. In this situation, the relative navigation and communication can make up this deficiency. The above results are summarized in the Table 5.

Table 3
Basic conditions for navigation sharing without constraints.

Node	v_1	v_2	v_3	v_4	v_5
ε_i	1	1	1	1	1
g_i	5	5	5	5	5
p_i	5	5	5	5	5
q_i	5	5	5	5	5

Table 4
Basic conditions for navigation sharing with constraints.

Node	v_1	v_2	v_3	v_4	v_5
ε_i	1	1	0	1	1
g_i	3	3	3	3	0
p_i	0	4	4	4	4
q_i	0	2	2	2	2

Table 5
Comparison of the solution in different objective function vectors.

Objective function vector	Relative navigation links' number	Communication links' number	Total links' number
MCCN	7	4	11
MRNCN	2	6	8
Hybrid	2	6	8

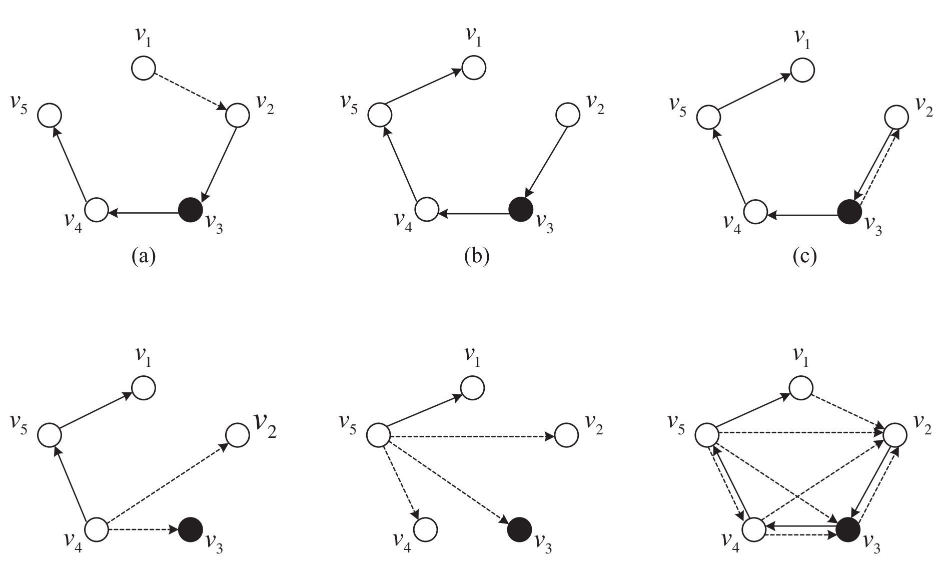

Fig. 8. Optimal network topology of relative navigation and communication links for MCCN type's OF.

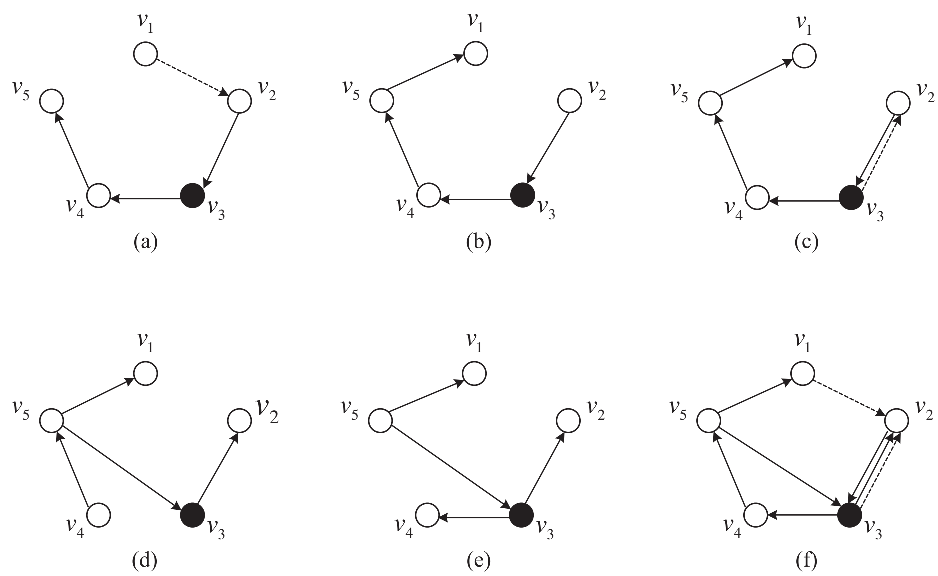

Fig. 9. Optimal network topology of relative navigation and communication links for MRNCN type's OF.

Table 6
Basic conditions for navigation sharing for three kinds of scenarios.

Node	v_1	v_2	v_3	v_4	v_5
ε_i	1/1/1	1/1/1	1/1/1	1/1/1	1/1/1
g_i	0/0/0	0/0/0	0/0/0	0/0/0	0/0/0
p_i	2/2/2	2/2/2	2/2/2	2/2/2	2/2/2
q_i	2/2/2	2/2/2	2/1/1	2/2/2	2/2/1

4.3. Comparison of the results in different sharing degrees

In this subsection, we will show that in different conditions, the sharing degree may be different too. Three scenarios are selected. The conditions are shown in Table 6. In these three scenarios, all the nodes have absolute navigation devices but no relative navigation devices. They all have communication devices, too. In all scenarios, the communication capacities are all set to be 2. In the first scenario, all the nodes have 2 communication channels. In the second scenario, the node v_3 has only one communication channel. In the third scenario, the nodes v_3 and v_5 both have one communication channel.

The solutions for the optimal network topology of communication links are shown below. In the first scenario, the 5 nodes all can send their navigation data to others. Therefore, the total communication topology is a bidirectional ring, seeing Fig. 10. The corresponding sharing degree is

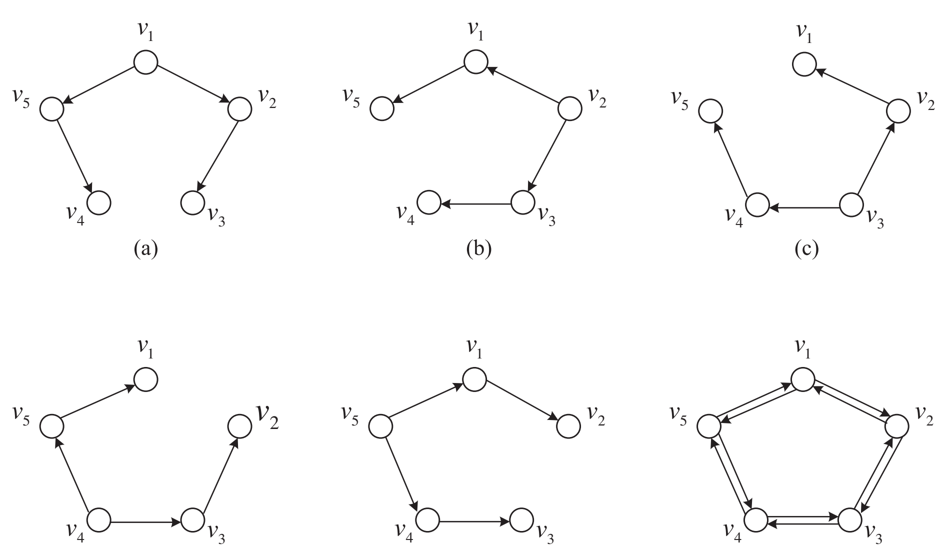

Fig. 10. Optimal network topology of communication links with a sharing degree of 5.

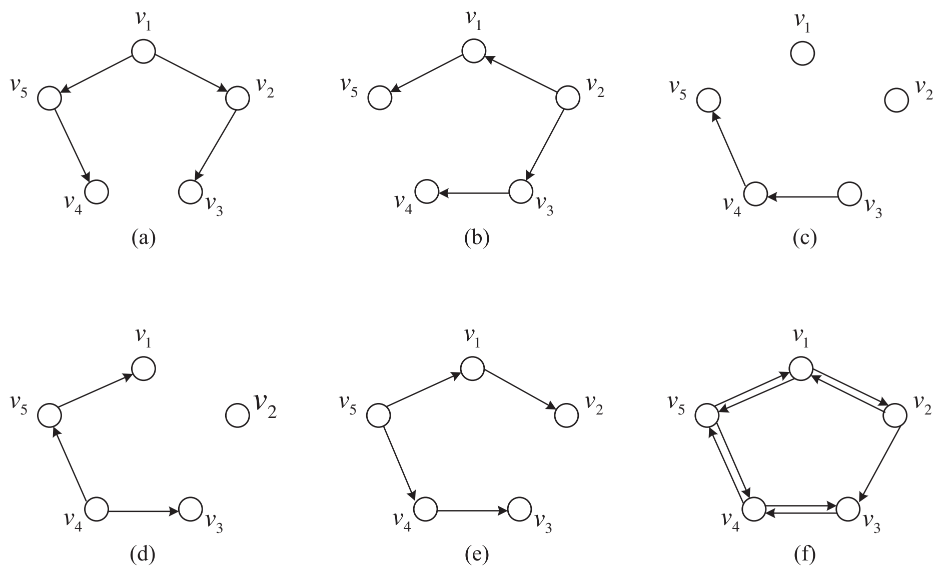

Fig. 11. Optimal network topology of communication links with a sharing degree of 3.

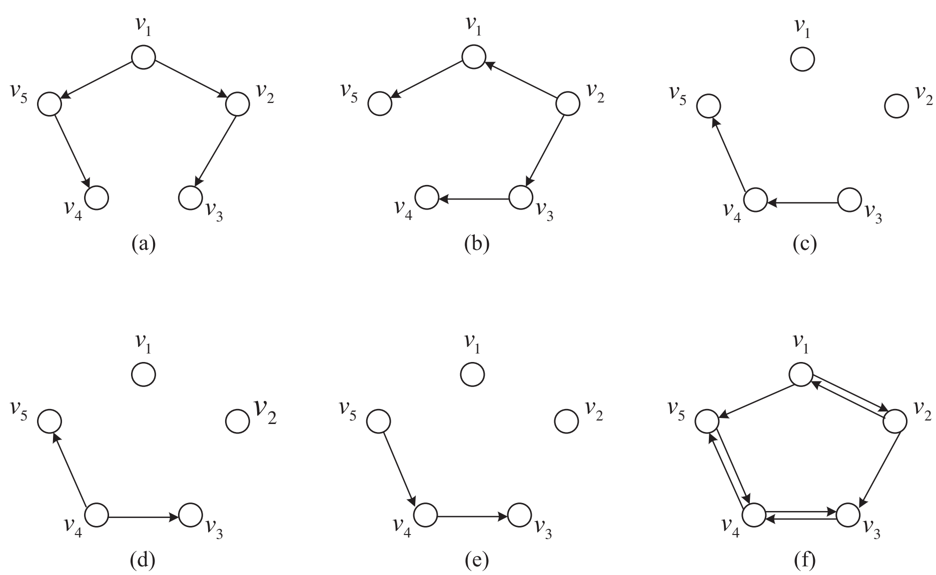

Fig. 12. Optimal network topology of communication links with a sharing degree of 3.

5. In the second scenario, due to the reason that node v_3 has only one communication channel, the navigation data of the nodes v_3 and v_4 cannot send their navigation data to all the other nodes. Therefore, the total sharing degree for this scenario is only 3, seeing Fig. 11. In the third scenario, the nodes v_3 and v_5 both have only one communication channel, which leads to the result that only the nodes v_1 and v_2 can send their navigation data to the others. However, according to the definition of sharing degree, the total sharing degree is 3, seeing Fig. 12.

5. Conclusions

The navigation sharing model for the fractionated spacecraft cluster with absolute navigation device, relative navigation device and communication device is established. The sharing degree is defined to describe the degree on how the navigation sharing is achieved. For given absolute navigation devices, the relative navigation channels' number, the communication capacity and the communication channels' number are three important variables affecting the navigation sharing. By distinguishing the different factors, three cases are recognized, i.e. absolute navigation plus relative navigation, absolute navigation plus communication, and general case with all three kinds of devices. The problem of navigation sharing in Case 1 can be solved by the method of Theorem 1. To solve the problem of navigation sharing in the Cases 2 and 3, however, the genetic algorithm is needed. Some examples are presented to test the methods. The results show the effectiveness of the methods.

From the results, it is also found that, even in the case that some members have no absolute navigation devices, they can also obtain their own absolute navigation data by navigation sharing. Furthermore, in the case that some members cannot send out their navigation data to others, their navigation data can also be obtained by others using relative navigation devices. These results show the advantages and potential application values of the navigation sharing technology.

In fact, the navigation sharing problem in this paper is not same as the one in real scenario where more constraints should be considered. For example, the communication between satellites has some delay in real scenario. This may affect the navigation sharing. In the next paper, we may discuss the validiness of the optimization method for achieving navigation sharing by building a hardware-in-loop simulation testbed.

Acknowledgement

The authors would like to thank the reviewers and the editor for many suggestions that helped to improve this paper.

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