Exercise 6

Introduction to Computational Astrophysics, SoSe 2024

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Task 1. *The Kepler problem II*

Solution. a) See source code. The program can be executed by "output" file after running "make" command.
b) The time step size should be smaller than 0.001, as seen from Table 1.

c) Eight planets of solar systems were tested to calculate the orbital period, in units of years. The result is summarized in Table 2 Initial values (x0 and vy0) were first searched from Williams [2024] and then converted to units of au and year. For instance, the input value of Jupiter was x0 = 5.203 au and vy0 = 2.78 au/year.

d) Here, the program is modified to retrieve the orbital period and either semi-major or semi-minor axis and then to return the distances and velocities at perihelion and aphelion respectively. Halley's Comet, as well as Mercury and Jupiter, were tested. The result can be seen in Table 3. Additionally, reference values are given in parentheses for comparison. Here, the distance is expressed in au but the velocity is in units of km/s, as required. The input values(perihelion distance and the orbital period) and the reference values were found in Williams [2024] for planets and Rahe [1982] for Halley's Comet.

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Task 2. *Runge-Kutta method (RK4)*

Solution. The script can be executed by the command "make run". a) As seen from Figure 1, RK4 with double variables (orange) is the most accurate method with smallest change in the energy. This is followed by Euler-Richardson method (skyblue), Euler-Cromer method (green), RK4 with float variables (yellow), and explicit Euler method (purple).

b) Because RK4 works better in maintaining stability, it is more appropriate for such problems.

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Task 5. *The special three-body problem*

Solution. a) See the source code. The script can be executed by the command "make run". b) From Figure 2, it can be clearly seen that Planet 1 (with m1/M = 1E-3 and R0=2.52), represented in blue, proceeds to escape right away. However, Planet 2, shown in violet, maintains a closed

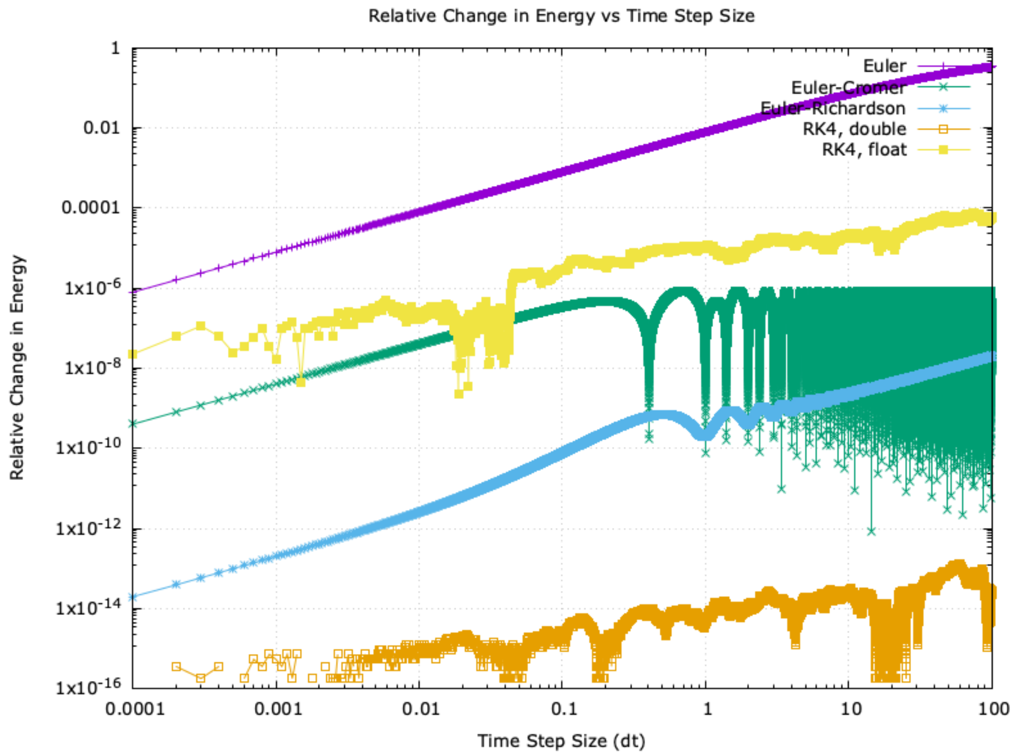


Figure 1: Task 2a

orbit. Therefore, Planet 1 is more influenced. When the program is modified to make use of the Earth and Jupiter data, both planets exhibit closed, circular orbits. c) Since the number of steps relies on pairwise calculations, the complexity of the program grows quadratically as the number of planets increases, i.e. $O\left(N^2\right)$.

$$\binom{N}{2} = \frac{N!}{2!(N-2)!} = \frac{N(N-1)}{2}$$

□

References

J. Rahe. Komet Halley 1985/86. *Sterne und Weltraum*, 21:21–24, January 1982.

David R. Williams. Planetary Fact Sheet, 3 2024. URL https://nssdc.gsfc.nasa.gov/planetary/factsheet/index.html.

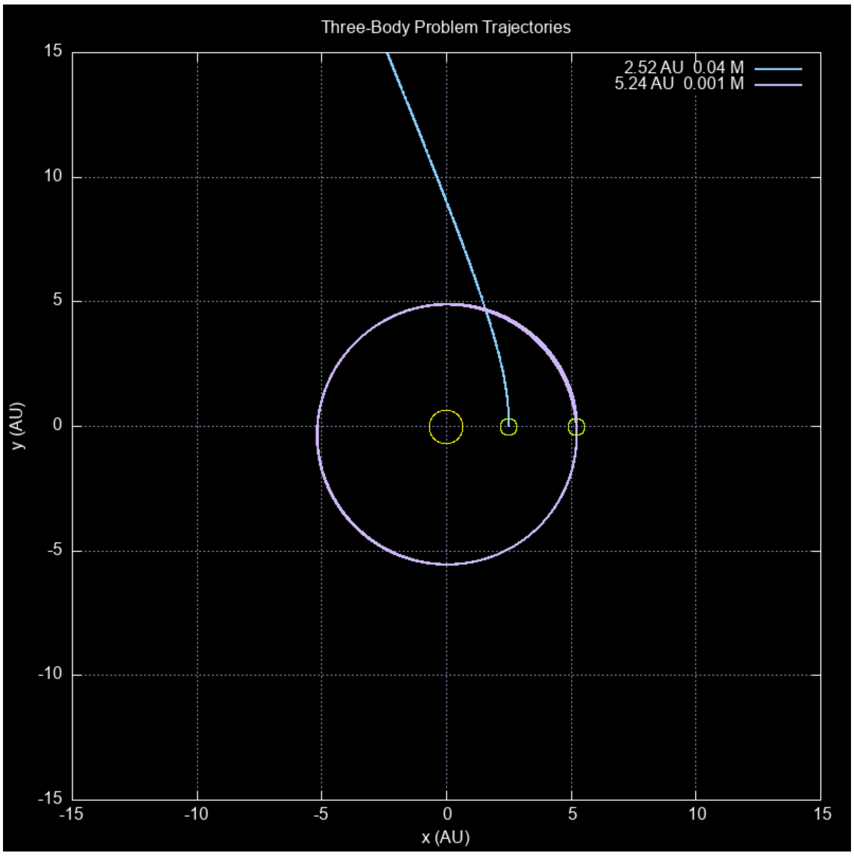


Figure 2: Task 5b Original Planets

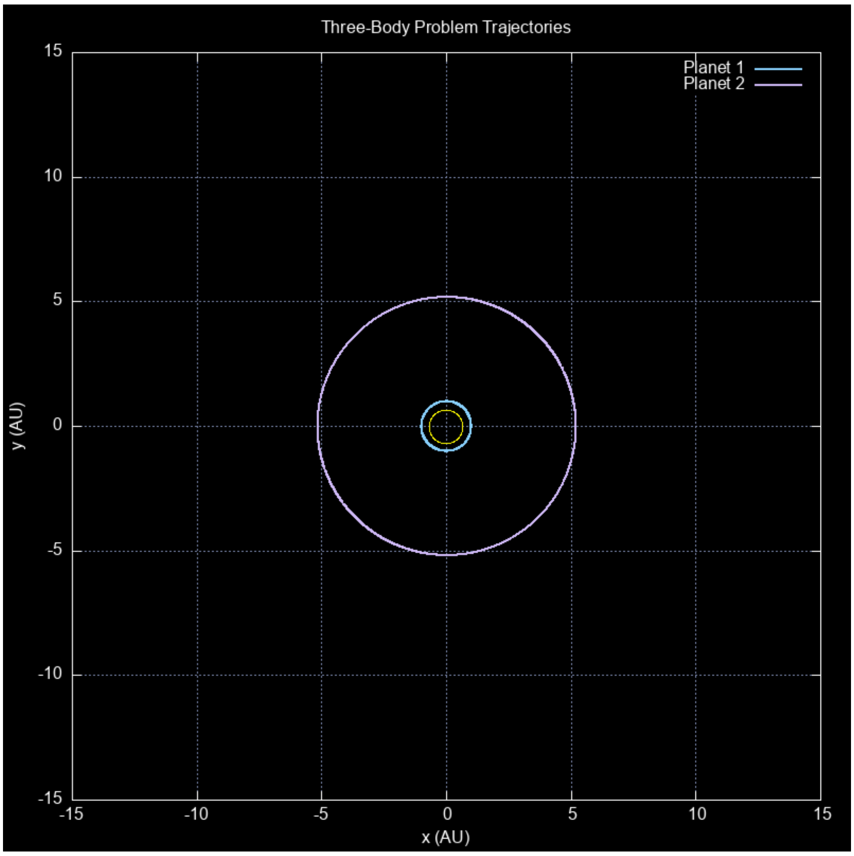


Figure 3: Task 5b Earth and Jupiter

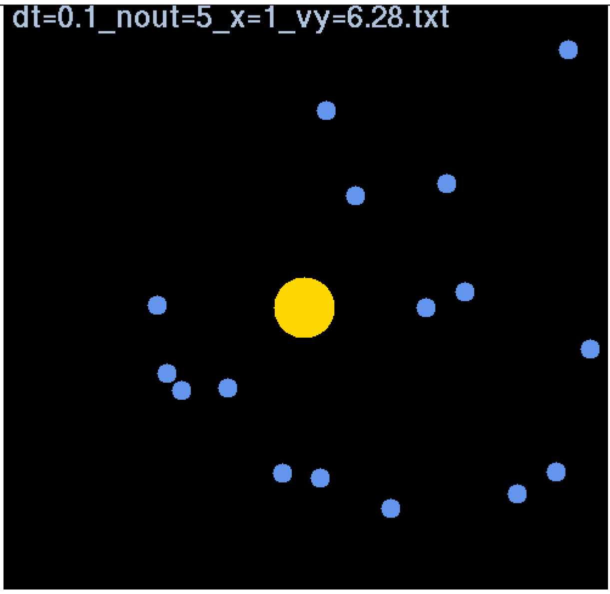
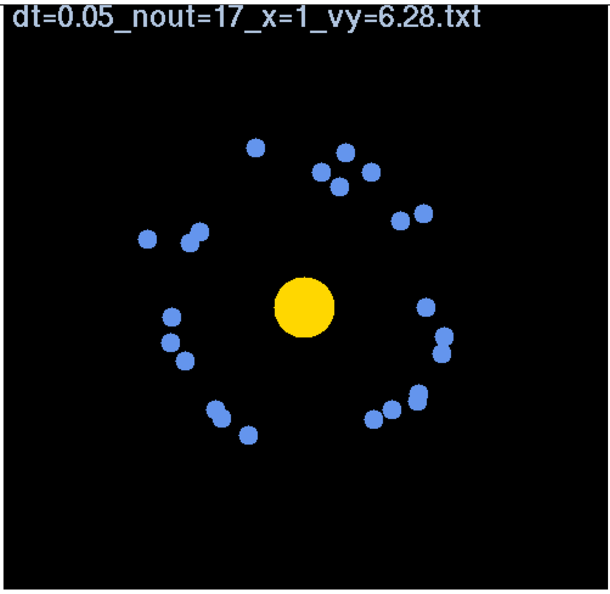
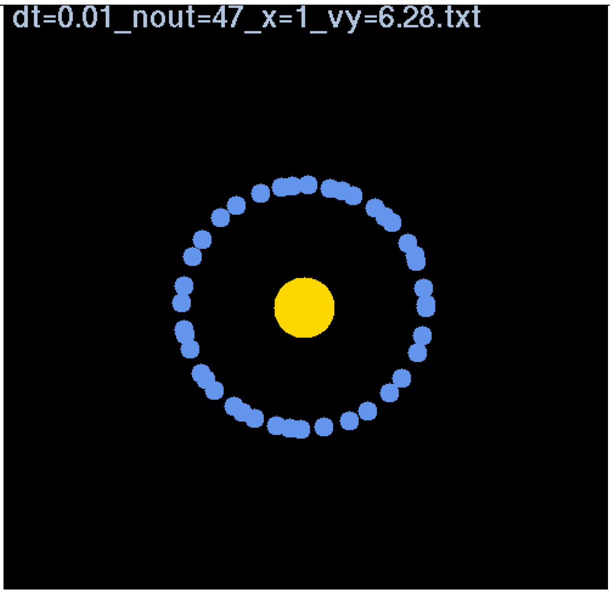
dt	**Orbit**
0.1	
0.01	
0.001	

Table 1: Task 1b

Planet	Numerically Calculated	Theoretically Derived	Reference Value
Mercury	0.2317	0.24075	0.241
Venus	0.6023	0.614763	0.615
Earth	0.99922	1	1
Mars	1.829	1.90452	1.881
Jupiter	12.198	12.5475	11.862
Saturn	31.05	32.457	29.457
Uranus	85.1	86.0995	84.011
Neptune	162.4	164.892	164.79

Table 2: Task 1c

Celestial Object	Distance at Perihelion	Distance at Aphelion	Velocity at Perihelion	Velocity at Aphelion
Halley's Comet	0.59 (0.59)	35.2944 (35.2944)	54.4237 (55)	0.909775 (1)
Mercury	0.313 (0.313)	0.461536 (0.459)	58.1599 (58.97)	39.4424 (38.86)
Saturn	9.229 (9.229)	9.84707 (9.905)	9.96886 (10.14)	9.34315 (9.14)

Table 3: Task 1d