Exercise 1

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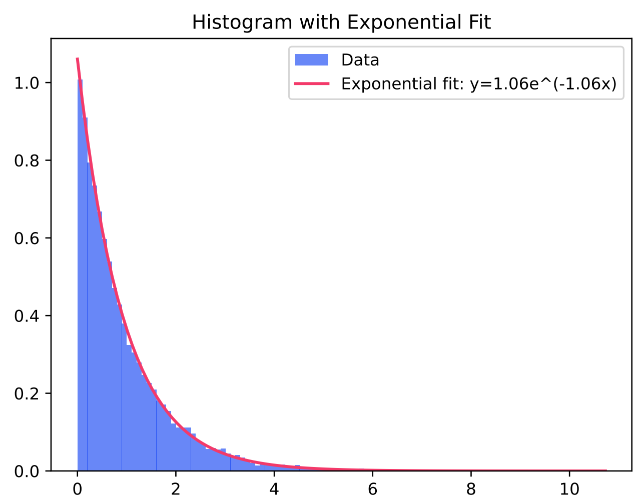
Task 1.



Task 2.

Solution. a)
The differential equation for the radioactive decay

$$-\frac{dN}{dt} = \lambda N(t)$$

$$\int \frac{dN}{N} = -\lambda \int dt$$

$$\ln(N) = -\lambda t + C$$

C is the integration constant.

$$N(t) = N_0 e^{-\lambda t}$$

with $N_0 = e^C$.

The relationship between λ and $t_{1/2}$
When $t = t_{1/2}$:

$$N(t_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

Dividing both sides by N_0:

$$\frac{1}{2} = e^{-\lambda t_{1/2}}$$

Taking the natural logarithm:

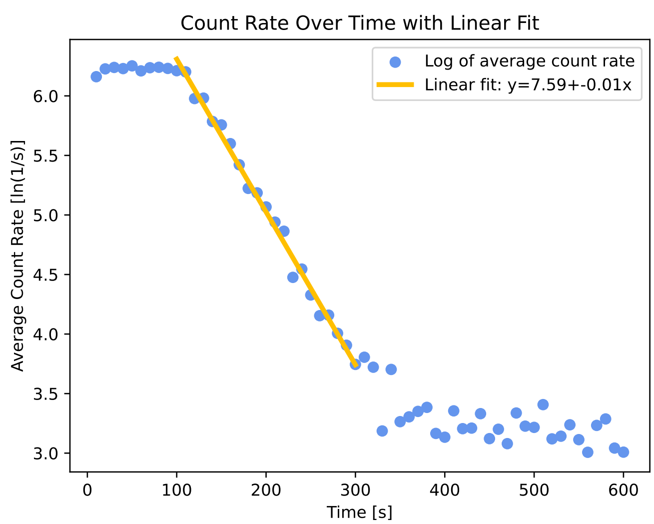
$$t_{1/2} = \frac{\ln(2)}{\lambda}$$

Activity A of a radioactive substance:

$$A = -\frac{dN}{dt} = \lambda N(t) = \lambda N_0 e^{-\lambda t}$$

The activity also decreases exponentially over time. Since the activity is proportional to the remaining number of nuclei, by measuring how the activity decreases to half its initial value, we can determine $t_{1/2}$. □

Solution. b)



Consider the linear decay function given by:

$$y = 7.59 - 0.01x$$

Suppose we have two points on this function, (x_1, y_1) and (x_2, y_2), where $y_2 - y_1 = \ln 2$. The half-life $t_{1/2}$ can be calculated by:

$$t_{1/2} = |x_1 - x_2|$$

Given the slope of the line -0.01:

$$\frac{y_2 - y_1}{x_2 - x_1} = -0.01$$

Thus,

$$t_{1/2} = 100 \times \ln(2) \approx 69.3 \text{ s}$$

This result is comparable to the empirical value of 55.6 s as in Tokonami (2020).

$\square$

References

Tokonami, S. (2020). Characteristics of thoron (220rn) and its progeny in the indoor environment. *International Journal of Environmental Research and Public Health*, 17(23).