Exercise 9

Introduction to Computational Astrophysics, SoSe 2024

Seha Lee

Task 1. *Interpolation*

Solution. The interpolated curve by a Langrange polynomial is shown in Figure 1. □

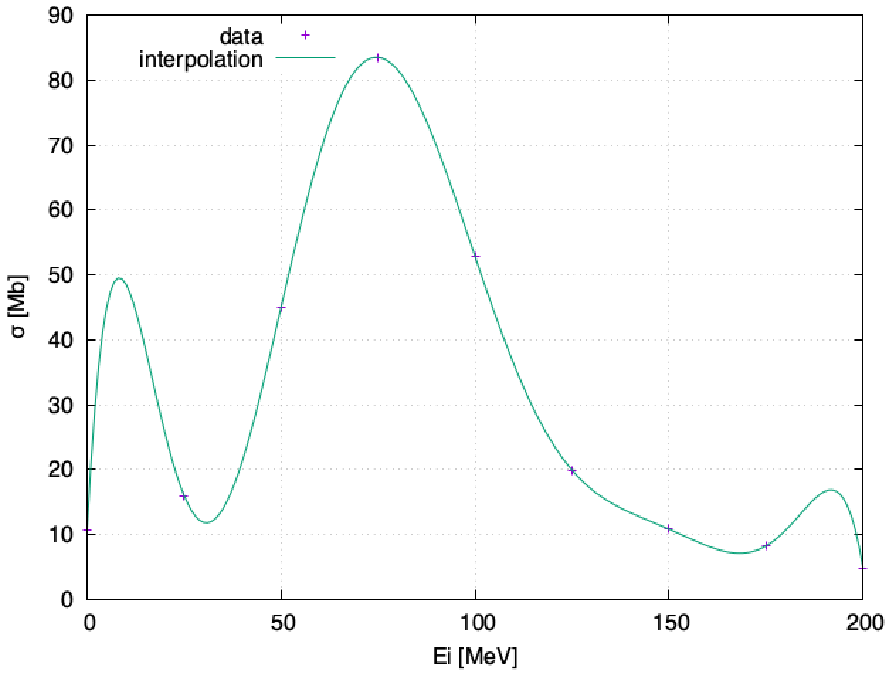

Figure 1: Task 1

Task 2. *2d-Interpolation*

Solution. I will use the same notation as in the lecture notes.

1. linear interpolation in y-direction:

$$f(x_1, y) \approx \frac{y_2 - y}{y_2 - y_1} f(Q_{11}) + \frac{y - y_1}{y_2 - y_1} f(Q_{12}),$$
$$f(x_2, y) \approx \frac{y_2 - y}{y_2 - y_1} f(Q_{21}) + \frac{y - y_1}{y_2 - y_1} f(Q_{22}).$$

2. linear interpolation in x-direction:

$$
\begin{aligned}
f(x,y) &\approx \frac{x_2 - x}{x_2 - x_1} f(x_1, y) + \frac{x - x_1}{x_2 - x_1} f(x_2, y) \\
&= \frac{x_2 - x}{x_2 - x_1} \left(\frac{y_2 - y}{y_2 - y_1} f(Q_{11}) + \frac{y - y_1}{y_2 - y_1} f(Q_{12}) \right) \\
&\quad + \frac{x - x_1}{x_2 - x_1} \left(\frac{y_2 - y}{y_2 - y_1} f(Q_{21}) + \frac{y - y_1}{y_2 - y_1} f(Q_{22}) \right) \\
&= \frac{1}{(x_2 - x_1)(y_2 - y_1)} \big(f(Q_{11})(x_2 - x)(y_2 - y) + f(Q_{12})(x_2 - x)(y - y_1) \\
&\quad + f(Q_{21})(x - x_1)(y_2 - y) + f(Q_{22})(x - x_1)(y - y_1) \big).
\end{aligned}
$$

Thus,

$$
\begin{aligned}
f(x,y) = \frac{1}{(x_2 - x_1)(y_2 - y_1)} &\big(f_1(x_2 - x)(y_2 - y) + f_2(x_2 - x)(y - y_1) \\
&+ f_3(x - x_1)(y_2 - y) + f_4(x - x_1)(y - y_1) \big).
\end{aligned}
$$

□

Task 3. *Kepler's equation*

Solution. The result of the calculation is given in the Table 1. Since the number of iteration is constantly smaller for the Newton's method, we can conclude that it's faster.

Eccentricity	Method	Iterations	Eccentric Anomaly	True Anomaly	Distance
0.047837	Fixed Point	6	3.47142	3.45626	5.44004
	Newton Method	3	3.47142	3.45626	5.44004
0.1	Fixed Point Iteration	8	3.45599	3.42640	5.69943
	Newton Method	3	3.45599	3.42640	5.69943
0.3	Fixed Point Iteration	14	3.40795	3.33758	6.71078
	Newton Method	3	3.40795	3.33758	6.71078

Table 1: Task 3: units are radians and au.

□

Task 4.

Solution. □