University of Queensland, PHYS3080/7080, Astrophysics III

COSMOLOGY PROJECT

Overview

Part I: Expansion history

In this project you will begin by calculating the expansion history of the universe (how scalefactor changes with time), and experiment with different compositions (different amounts of matter, radiation and dark energy) to get a feel for the behaviour of different cosmological models. You then have the option of calculating the behaviour of many other features of the universe, such as distance as a function of redshift and how densities change with time. You need to choose at least two items from each of the two lists shown in Table 1. You are not expected to calculate everything on the lists.

Part II: Measuring cosmological parameters

After you have calculated the basics you will apply that knowledge to some (fake) data. In the second part of this project you will perform a modern cosmological analysis, combining data from supernovae (SNe), cosmic microwave background (CMB), and baryon acoustic oscillations (BAO) to measure the composition of the universe. You will learn about fitting models to data and also about how to include 'priors' (extra knowledge) to your fits.

Available resources

Our primary aim with this project is to develop your understanding of cosmology, but a secondary aim is to develop your coding and numerical skills that can be used in a wide range of applications. For example, the functions that you need to solve have no (simple) analytical solutions, so in order to do the calculations you will need to do numerical integration. This project will also give you experience in how to perform the basic statistics necessary to test models against data, and how to present your results in a professional manner with polished plots and paper-style write-up. To aid in these aims we have provided you with some basic code to get you started. This can be found on the Cosmology Project website here `https://astrouq.github.io/PHYS3080/` You are not obliged to use this code, feel free to do the numerical integrations and plotting in any way you prefer. The code snippets are just there as a helping hand if you need it, and to make clear some simple algorithms.

Assessment

For the final project report you will present a few of your best plots from Part I and your results for Part II in the form of a **scientific paper**. Scientific papers typically consist of an abstract (outlining the motivation and results), an introduction (summarising the relevant theory and necessary background information, which is where some plots from Part I would go), an analysis section (outlining the techniques used), a results section (detailing all the major results), and discussion / conclusions (where you summarise the major insights). Equations and plots should be included throughout to demonstrate your points. This final project report is worth **20% of your final grade**.

Mid-way through the project you may hand in a draft introduction with the plots for Part I to get some feedback from Tamara. This is optional, but I recommend it – and it is a good gauge to see if you are on track to finish on time. We encourage you to work in groups but your reports should be submitted, and will be graded, individually. It is good to include plots your teammates made (share the load), but be clear in the captions who created which figures. It is important that your writeup demonstrates understanding of the physics behind all figures you include. Final report grades will be awarded based on the "Cosmology Project Marking Rubric" below.

Items to consider when handing in your report:

- Have you handed in something that passes the minimum criteria (at least two plots each from each list in Table 1, and the results from the fitting to supernovae, with descriptions of how they were made and what they show)?
- Does the behaviour of the models you have plotted make sense?
- Are the plots clearly explained?
- Do the choice of plots and descriptions of the plots demonstrate:
  - understanding of the underlying processes?
  - insight into the results for different possible cosmologies (e.g. different matter and dark energy densities)?
  - insight into the features of different analysis choices (e.g. different types of distances)?
- Are the plots well labeled and easy to understand?
- Is the presentation easy to follow?
- Are there error bars on your cosmology results? (There should be!)
- Have you acknowledged your teammates?

PHYS3080/7080 Cosmology Project Marking Rubric

Each of the criteria below will receive a grade out of 7, which is then scaled to the amount that criterion is worth (i.e. out of 6 or 8 marks). The project is worth 20% in total.

Grade	Basic Analysis (Completing and adding adaptions to the parts of the project for which I supplied programs.) /6	Extended Analysis (Everything beyond the basic analysis.) /8	Presentation and Language /6
7	Completed all the basic analysis steps, with several inventive adaptions. Demonstrated thorough understanding of all concepts.	Completed many extensions, including some of the more complex ones and showed insight or inventiveness in the choice of extensions. Demonstrated thorough understanding of all concepts. Explained those concepts clearly.	Virtually indistinguishable from a publishable paper. Accurate, clear, and concise descriptions. Well presented figures with clear informative and insightful captions. Well laid out and easy to understand discussion. Close to perfect grammar.
6	Completed all the basic analysis steps, with some adaptions. Demonstrated thorough understanding of all concepts.	Completed several extensions, including some of the more complex ones. Demonstrated thorough understanding of all concepts. Explained those concepts clearly.	Accurate, clear, and concise descriptions throughout most of the paper. Well presented figures with informative captons. Easy to follow discussion. Very good grammar.
5	Completed all the basic analysis steps, with some adaptions. Demonstrated understanding of most concepts.	Completed several extensions. Demonstrated understanding of most concepts. Explained those concepts clearly.	Mostly accurate, clear, and concise, with some weaknesses. Understandable figures, with descriptive captions. Discussion adequately presented. Good grammar.
4	Completed all the basic analysis steps, with some adaptions. Demonstrated understanding of some concepts, but may have some misconceptions.	Completed some extensions. Demonstrated understanding of some concepts, but may have some misconceptions.	Inaccurate, unclear or not cohesive in places. Figures may be confusing or missing adequate description in captions. Discussion somewhat difficult to follow. Grammar mostly sound.
3	Completed some of the basic analysis steps, with some adaptions. Demonstrates inadequate understanding of some concepts.	Completed some extensions. Demonstrates inadequate understanding of some concepts.	Often inaccurate, unclear, and incohesive in places. Figures poorly presented and/or difficult to understand. Discussion difficult to follow. Grammar uneven.
2	Completed some of the basic analysis steps, with no or few adaptions. Demonstrates misuderstanding of some concepts.	Completed at least one extension. Demonstrates active misuderstanding of some concepts.	Largely inaccurate, unclear, and incohesive. Figures poorly presented and difficult to understand. Grammar poor.
1	Completed some of the basic analysis steps with no adaptions. Demonstrates little understanding of concepts and/or significant misconceptions.	Completed at least one extension. Demonstrates little understanding of concepts and/or significant misconceptions.	Largely inaccurate, unclear, incohesive, and incomplete. Figures uninteligible. Grammar poor.
0	Did not complete.	Did not complete.	Did not complete.

1 PART I: Expansion history

It is the 1920's and your name is Alex Friedmann.[1] You recently used Albert Einstein's inspiring new theory of general relativity to derive dynamics of an homogeneous, isotropic universe. In doing so you uncovered a major problem with the theory: a static homogeneous, isotropic universe is unstable. It has to either expand or contract. But that is in direct violation of the stationary nature of the observed stars. You published your results and have been eagerly awaiting the response from Albert.

In the mail today you find a letter from Albert himself. It reads.

> *"Dear Professor Friedmann,*
>
> *I read your article on the instability of an homogeneous, isotropic universe with much interest and initial consternation. I am pleased to say I have found a resolution. By including the mathematically permissable (yet inelegant) constant in my equations I have been able to create a stable universe that must resemble the one we live in. I have called this addition to general relativity the "cosmological constant" and describe it using the symbol* Λ*. I hope you find the enclosed paper on the subject of interest.*
>
> *Yours in Science,*
>
> *Albert Einstein"*

Quickly you rush to see how this new part of the theory impacts your equations, and find that,

$$H^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} - \frac{k}{R^2}, \tag{1}$$

so the cosmological constant basically acts as an extra density parameter with

$$\rho_\Lambda = \frac{\Lambda}{8\pi G}. \tag{2}$$

You realise your definition of critical density as $k = 0$ no longer corresponds to the transition between whether a universe will expand forever or recontract, but it does still correspond to flatness, so you decide to keep it. The critical density remains,

$$\rho_{\text{crit}} = \frac{3H^2}{8\pi G}. \tag{3}$$

Translating your equations into normalised densities with $\Omega_{\text{x}} = \rho_{\text{x}}/\rho_{\text{crit}}$ you find,[2]

$$\frac{da}{dt} = H_0\left[1 + \Omega_M\left(\frac{1}{a} - 1\right) + \Omega_\Lambda(a^2 - 1)\right]^{1/2}. \tag{4}$$

Realising that this is starting to look a little ungainly (especially when you add radiation as well), you figure out that by generalising a little you can make things look much simpler. You define an equation of state $w = p/\rho c^2$ that describes how each of the density components evolves as the universe expands. (An equation of state, after all, just describes how the pressure, p, of a substance reacts to density changes.)

With relief you realise that lets you write your equations as,

$$H^2(a) = H_0^2 \sum \Omega_i a^{-3(1+w_i)}, \tag{5}$$

or in terms of redshift, z, which obeys $1 + z = 1/a$,

$$H^2(z) = H_0^2 \sum \Omega_i (1+z)^{3(1+w_i)}. \tag{6}$$

[1] WARNING: Imaginary history! Similarity to actual historical figures is not coincidental, but should not be taken as fact!

[2] The scalefactor $R(t)$ has dimensions of distance, and its current day value is R_0. This equation uses the scalefactor $a(t) = R(t)/R_0$, which has been normalised to equal 1 at the present day.

Denoting the present day values with subscripts zero,

$$w = -1 \;\rightarrow\; \Omega_\Lambda = \Omega_{\Lambda_0} \quad \text{(cosmological constant)}, \tag{7}$$
$$w = -\frac{1}{3} \;\rightarrow\; \Omega_\mathrm{k} = \Omega_{\mathrm{k}_0} a^{-2} \quad \text{(curvature)}, \tag{8}$$
$$w = 0 \;\rightarrow\; \Omega_\mathrm{m} = \Omega_{\mathrm{m}_0} a^{-3} \quad \text{(matter)}, \tag{9}$$
$$w = +\frac{1}{3} \;\rightarrow\; \Omega_\mathrm{r} = \Omega_{\mathrm{r}_0} a^{-4} \quad \text{(radiation)}. \tag{10}$$
$$\tag{11}$$

And you notice that using something like $E(z) = H(z)/H_0$ might come in handy later if you want to calculate things without having to put in a value for H_0.

The curvature term, Ω_k, is given by $\Omega_\mathrm{k} = 1 - \Omega_\mathrm{m} - \Omega_\Lambda - \Omega_\mathrm{r}$, which is handy because it means that when you sum all the density terms you find Ω_k is greater than, less than, or equal to zero for negatively curved, positively curved, or flat universes, respectively (notice Ω_k is negative for a positively curved universe, which may seem unintuitive). It is also related to the current radius of curvature of the universe according to,

$$R_0 = \frac{c}{H_0} \left| \frac{1}{1 - \Omega_\mathrm{m} - \Omega_\Lambda - \Omega_\mathrm{r}} \right|^{1/2}. \tag{12}$$

More generally you can replace $-\Omega_\mathrm{m} - \Omega_\Lambda - \Omega_\mathrm{r}$ with $-\sum \Omega_i$. When the universe is flat the current radius of curvature is infinite (a sphere with infinite radius is flat), but in a flat universe R_0 cancels out perfectly in all equations, so you don't need to worry about the infinity!

Curiously it looks like the cosmological constant has a negative pressure. That must be how Albert intends to stabilise the universe. Immediately inspired you embark on the task of plotting graphs of how the scalefactor evolves with time. To get the scalefactor with respect to time you have to integrate your equation for $\dot{a} = da/dt$,

$$t(a) = \int_0^a \frac{da}{\dot{a}}. \tag{13}$$

(Luckily you have someone from the future to help you with the integration... they've got fancy things called computers that can do even complicated integration quickly. They've even posted an example program up on the github site for this project.)

Clearly most of the possibilities still indicate an expanding or contracting universe. It would take a pretty fine balance between the matter, radiation, and cosmological constant to hold the universe static. Even though the observers tell you that stars don't appreciably move away or toward us, you wonder "what if they're wrong"?

So you decide to calculate the redshift of objects as a function of their distance, assuming the universe were expanding or contracting as your equations seem to indicate.

Starting from the metric,[3]

$$ds^2 = -c^2 dt^2 + R(t)^2 [d\chi^2 + S_k^2(\chi) d\psi^2], \tag{14}$$

you see the radial ($d\psi = 0$) distance along a constant time-slice ($dt = 0$) is $ds = R d\chi$, which upon integrating gives the proper distance,

$$D = R\chi. \tag{15}$$

Instead, looking along the path of a photon ($ds = 0$),

$$c dt = R(t) d\chi, \tag{16}$$

[3] Where c is the speed of light, dt is the time separation, $d\chi$ is the comoving coordinate separation and $d\psi^2 = d\theta^2 + \sin^2\theta d\phi^2$, where θ and ϕ are the polar and azimuthal angles in spherical coordinates. The scalefactor, R, has dimensions of distance. The function $S_k(\chi) = \sin\chi$, χ or $\sinh\chi$ for closed ($k = +1$), flat ($k = 0$), or open ($k = -1$) universes respectively.

which makes it clear that to calculate the comoving distance χ between two points on the photon's path you need to integrate

$$\int_{t_\mathrm{e}}^{t_\mathrm{o}} d\chi = c\int_{t_\mathrm{e}}^{t_\mathrm{o}} \frac{dt}{R(t)}, \tag{17}$$

where t_e is the time the photon was emitted, and t_o the time it was observed (we don't have to be the observers, t_o could be anytime). By changing the limits of the integral you can calculate the particle horizon (the distance light can travel between the beginning of the universe and now), or the event horizon (the distance light can travel between now and the end of the universe, which may be $t_\mathrm{o} \to \infty$).

You realise that it may be useful to express that integral with respect to the scale factor, instead of time, since we don't necessarily know an easy function for $R(t)$, but we do know da/dt as a function of a (Eq. 4 or 5). (Recall $a(t) = R(t)/R_0$, and $\dot{a} \equiv da/dt$.) Equation 17 therefore becomes,

$$R_0\Delta\chi = c\int_{a_\mathrm{e}}^{a_\mathrm{o}} \frac{da}{a\dot{a}}. \tag{18}$$

Alternatively you could integrate from now backwards to the time a photon was emitted to get the distance to the emitter. Realising that's not particularly practical for an observer, you replace all your times with redshifts and use $1 + z = R_0/R = 1/a$ to get,

$$R_0\chi(z) = c\int_0^{z_\mathrm{obs}} \frac{dz}{H(z)}. \tag{19}$$

Finally having all the information you need, you go ahead and calculate every aspect of your expanding and contracting universes that you can think of. Most importantly you experiment with a wide variety of matter densities, cosmological constant values, and even different dark energy equations of state. It occurs to you that perhaps even a time-varying dark energy equation of state may be possible.

Brainstorming quickly you scribble down a list of things you could potentially plot....

Table 1: Examples of features you could plot.

List One: Scalefactor & time	**List Two: Distances**
Scalefactor as a function of time, $R(t)$	Comoving distance vs redshift, $R_0\chi(z)$
Redshift as a function of scalefactor, $z(a)$	Proper distance vs redshift and time, $D(t, z) = R(t)\chi(z)$
The Hubble parameter vs time, $H(t)$	Luminosity dist. vs redshift, $D_\mathrm{L}(z) = R_0S_k(\chi)(1 + z)$
The normalised densities, Ω_r, Ω_m, Ω_Λ vs time	Ang. diam. dist. vs redshift, $D_\mathrm{A}(z) = R_0S_k(\chi)/(1 + z)$
Age of the universe vs Ω_m or Ω_Λ	Distance to the particle horizon
Lookback time vs redshift	Distance to the event horizon
Bounce time. Crunch time	Distance to the Hubble sphere (where $v_\mathrm{recession} = c$)

In the end you find your results so exciting that you **write them up in a scientific paper**. In the paper it is important to demonstrate depth of understanding, so each plot should come with an explanation of what is being plotted, a description of the most interesting features, and the equations needed to be able to generate the plot. Where analytic solutions are possible it may be good to show some if they help demonstrate your point.

HINT: Get started by looking at the example code on `https://astrouq.github.io/PHYS3080/`

Notes:

2 PART II: Measuring cosmological parameters

Fast forward to today. You're Friedmann's great-granddaughter/son and following in your ancestor's footsteps. Now the data on the expansion are so good that it is hard to believe physicists such as Einstein once believed that the universe was static.

These days we're on the hunt for dark energy. It looks like the expansion is accelerating, but we don't know what's causing it. You're part of one of the largest supernova searches around, the University of Queensland Spectacular Supernova Search (UQS^3) using the Very Imaginary Telescope (VIT) to discover hundreds of supernovae. You plan to make the best ever measurement of the "Hubble diagram" (magnitude vs redshift). Your job is to take that data and figure out what it implies about the nature of dark energy.

You will take data of observed magnitudes of type Ia supernovae and compare them to a variety of cosmological models to figure out which is the best fit. To start with you will find the most likely values of two cosmological parameters: matter density (Ω_m) and cosmological constant (Ω_Λ), and plot probability contours on the Ω_m - Ω_Λ plane.

In addition you can add prior knowledge from measurements of the Cosmic Microwave Background and/or Baryon Acoustic Oscillations.

As an optional extra you can extend your analysis to a wider variety of dark-energy types and determine whether your data is good enough to distinguish between a cosmological constant and something like quintessence (time-varying dark energy).

Theory

2.1 From observations to distance modulus

This part just gives the details of how to go from something you observe to something you can predict theoretically. Let $\mathcal{P}$ denote a set of cosmological parameters,

$$\mathcal{P} = (H_0, \Omega_\mathrm{m}, \Omega_\Lambda), \tag{20}$$

where Ω_m and Ω_Λ are the present-day energy density parameters of matter and the cosmological constant respectively, and H_0 is Hubble's constant. The apparent magnitude we observe the supernova to have is related to its absolute magnitude by,[4]

$$m(\mathcal{P}, z) = 5\log_{10}[D_\mathrm{L}(\mathcal{P}, z)] + 25 + M. \tag{22}$$

The distance modulus is something we can derive theoretically. It's defined as,

$$\mu(\mathcal{P}, z) \equiv m(\mathcal{P}, z) - M = 5\log_{10}[D_\mathrm{L}(\mathcal{P}, z)] + 25. \tag{23}$$

You can plot either μ or m as your data, as we will see below, because we don't actually care what the number is on the μ axis of the μ vs z plot, we just care about how the slope changes with redshift.

[4]In detail we have to consider which wavelength filter (band) the supernova was observed in. The apparent magnitude in the ith band, m_i, of a supernova at redshift, z, relates to the absolute magnitude in band j by,

$$m_i(\mathcal{P}, z) = 5\log_{10}[D_\mathrm{L}(\mathcal{P}, z)] + 25 + M_j + K_{ij}(z), \tag{21}$$

where $K_{ij}(z)$ is the K-correction required to convert the magnitude observed to the magnitude that would have been observed in the equivalent filter at zero redshift. Eq. 22 is true after the K-correction has been applied.

2.2 From cosmological model to distance modulus

In order to compare your data with cosmological models we need to calculate the distance modulus as a function of redshift for those models. To do this we first calculate the comoving distance as a function of observed redshift, $z_{\rm obs}$,

$$\begin{aligned} R_0\chi(z_{\rm obs}) &= c\int_0^{z_{\rm obs}} \frac{dz}{H(z)} && (24) \\ &= \frac{c}{H_0}\int_0^{z_{\rm obs}} \frac{dz}{E(z)}, && (25) \end{aligned}$$

(recall $E(z) = H(z)/H_0$). The model parameters enter through the equation for Hubble's constant as a function of redshift (Eq. 6). Now define the curvature parameter, $\Omega_{\rm k} = 1-\Omega_{\rm m}-\Omega_\Lambda$. The radius of curvature of the universe is given by,

$$R_0 = \frac{c}{H_0}\frac{1}{\sqrt{|\Omega_{\rm k}|}}. \tag{26}$$

Note that for a flat universe this radius is undefined, but as we will see it cancels out.

The luminosity distance is given by,

$$D_{\rm L} = R_0 S_k(\chi)(1+z), \tag{27}$$

where,

$$S_k(\chi) = \begin{cases} \sinh(\chi) & \Omega_{\rm k} > 0, \\ \chi & \Omega_{\rm k} = 0, \\ \sin(\chi) & \Omega_{\rm k} < 0. \end{cases} \tag{28}$$

Using $D_{\rm L} = \frac{c}{H_0}\, D'_{\rm L}$ and $\mathcal{P} = (\Omega_{\rm m}, \Omega_\Lambda)$ the distance modulus can be rewritten as,

$$\mu(\mathcal{P}, z) = 5\log_{10}[D'_{\rm L}(\mathcal{P}, z)] + 5\log_{10}[c/H_0] + 25, \tag{29}$$

which leaves all the parameters we want to test in the first term, while Hubble's constant is only an additive constant.

This is useful because it separates out the additive constants from the part we're interested in – the cosmological parameters. Unfortunately, even though we're interested in H_0 it is a nuisance parameter in the context of measuring dark energy, because the absolute magnitude of type Ia supernovae is not known well enough, and so there's an uncertainty in M (call it Δ_M) that also ends up in the equation for distance modulus (Eq. 23). Combining all the constants together into a new constant called $\mathcal{M}$ allows us to *marginalise* over our lack of knowledge of those constants, known as *nuisance parameters.* (The statistics notes explain marginalisation.)

$$\begin{aligned} \mu(\mathcal{P}, z) &= 5\log_{10}[D'_{\rm L}(\mathcal{P}, z)] + 5\log_{10}[c/H_0] + 25 + \Delta_M, && (30) \\ &= 5\log_{10}[D'_{\rm L}(\mathcal{P}, z)] + \mathcal{M}. && (31) \end{aligned}$$

2.3 Comparing data and model

To calculate how well a model matches the data, we use a χ^2 test. The value of χ^2 for a particular data/model combination is given by,

$$\chi_0^2 = \sum_i \left(\frac{\mu_{\rm model} - \mu_{\rm i}}{\sigma_{\rm i}} \right)^2 , \tag{32}$$

where $\mu_{\rm model}$ is the value predicted by your cosmological model and $\mu_{\rm i} \pm \sigma_{\rm i}$ is the ith data point and its uncertainty.

If we have prior information about a particular parameter that goes into this model, then we want to weight the χ^2 value so it prefers parameter values close to our prior. We do this by adding an extra term in the χ^2 equation. For example, say we have a prior on a parameter, $\mathcal{P}$, such that we know $\mathcal{P} = \mathcal{P}_{\rm prior} \pm \sigma_{\rm prior}$. (For example $\mathcal{P}_{\rm prior}$ could be $\Omega_{\rm m} = 0.30 \pm 0.03$ or maybe $\Omega_{\rm m} + \Omega_\Lambda = 1.00 \pm 0.05$.) The revised equation reads,

$$\chi^2 = \chi_0^2 + \left(\frac{\mathcal{P}_{\rm model} - \mathcal{P}_{\rm prior}}{\sigma_{\rm prior}} \right)^2 . \tag{33}$$

To get an approximate measure of goodness of fit use the reduced χ^2. To get the reduced χ^2 divide χ^2 by the number of degrees of freedom: $n_{\rm dof} = n_{\rm data} - 1 - n_{\rm params}$, where $n_{\rm data}$ is the number of data points and $n_{\rm params}$ is the number of parameters you are fitting for, 2 in our case. For a good fit the result should be close to 1. This is not rigorously defined rule, but rather a rule of thumb. A reduced χ^2 much less than 1 indicates that the error bars are probably overestimated (too large), while a reduced χ^2 of much larger than 1 indicates that the data aren't really a very good fit to your model (or you've underestimated your uncertainties).

When comparing between models you look at the difference in χ^2 between them, $\Delta\chi^2$. When you're fitting one parameter $\Delta\chi^2 = [1, 4, 9]$ correspond to the first, second, and third standard deviations, respectively. When you're fitting two parameters (as we are here) the first three standard deviations are given by $\Delta\chi^2 = [2.30, 6.18, 11.83]$.

Remember when we calculate χ^2 for each of our models we need to marginalize over the arbitrary additive scaling factor, $\mathcal{M}$, that is in our data. So for each combination of $(\Omega_{\rm m}, \Omega_\Lambda)$ that we test, we need to allow $\mathcal{M}$ to vary, and then use whichever value of $\mathcal{M}$ gives the lowest χ^2 for that model (that works as long as the uncertainties are Gaussian). More detail in the statistics notes.

Note that there are some theory notes, and some statistics notes, that are separate hand-outs that will also help you with some of the details of this project.

Method for cosmological parameter estimation

1. Start with a table of data that you can download from (https://astrouq.github.io/PHYS3080/), which contains for each supernova: their redshift (z), distance modulus (μ) and uncertainty in distance modulus (σ).

2. Plot μ vs z.

3. Overplot a variety of cosmological models with a range of different Ω_m and Ω_Λ values. (Optional: Plot the models and the data relative to the $(\Omega_\mathrm{m}, \Omega_\Lambda) = (0, 0)$ model in order to emphasize the differences between the models.)

4. Calculate the χ^2 value of each model (marginalize over $\mathcal{M}$). Which model is the best fit? Is this model a good fit?

5. Make a contour plot of the likelihood of Ω_m and Ω_Λ values, i.e. calculate likelihood from χ^2 and plot over a grid of Ω_m and Ω_Λ values.

6. What are the best-fit values of Ω_m and Ω_Λ based on this supernova data alone?

7. What are the best-fit values of Ω_m and Ω_Λ when we assume a prior of flatness from the cosmic microwave background experiments (i.e. we assume, $\Omega_\mathrm{m} + \Omega_\Lambda = 1.00 \pm 0.05$)?

8. What is the best-fit value of Ω_Λ when we assume a prior of, $\Omega_\mathrm{m} = 0.30 \pm 0.05$ from galaxy clustering and baryon density oscillation data?

9. Optional extension: Extend this analysis to other types of dark energy (e.g the equation of state $w \neq -1$, or w time varying).

10. Prepare a paper detailing your results. Include all the plots outlined above, with captions explaining the meaning of what has been plotted. This should continue on as part of the paper you began in Part I of this project.

HINT: This is not intended to be primarily a programming exercise, but rather an exercise to understand the science. We have therefore provided you with a programming framework in that you can build on and adapt. See the project website `https://astrouq.github.io/PHYS3080/` for the code. This will show you how to:

- Do basic numerical integration (e.g. to plot scalefactor vs time, or distance vs redshift).
- Plot a basic Hubble diagram.
- Calculate χ^2 for a range of models.
- Make a contour plot to show the best fit parameters.

References

1. Kessler et al. (2009), The Astrophysical Journal Supplement, Volume 185, Issue 1, pp. 32-84, http://jp.arxiv.org/abs/0908.4274
2. Goliath et al. (2001) Astron. Astrophys., 380, 6–18, *Supernovae and the nature of dark energy.* (Note: they provide equations for a time varying dark energy.)

Another useful reference may be Carroll, Press and Turner (1992), Ann. Rev. Astron. Astrophys. 30, 499–542, *The Cosmological Constant.* In which they discuss the effect of a cosmological constant on the expansion of the universe. They provide equations that tell you whether a universe will bounce or recollapse etc... (see their Eq. 11 and 12). **Also see the Theory Notes and Statistics Notes.**