Universität Zürich[UZH]

Bachelor Thesis: Efficiency of Univariate Kernel Density Estimation with TensorFlow

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Contents

1 Abstract

This study aims at comparing the speed and accuracy of different methods for one-dimensional kernel density estimation in Python/TensorFlow, especially concerning applications in high energy physics. Starting from the basic algorithm, several optimizations from recent papers are introduced and combined to ameliorate the effeciency of the algorithm.

2 Introduction

2.1 Kernel Density Estimation

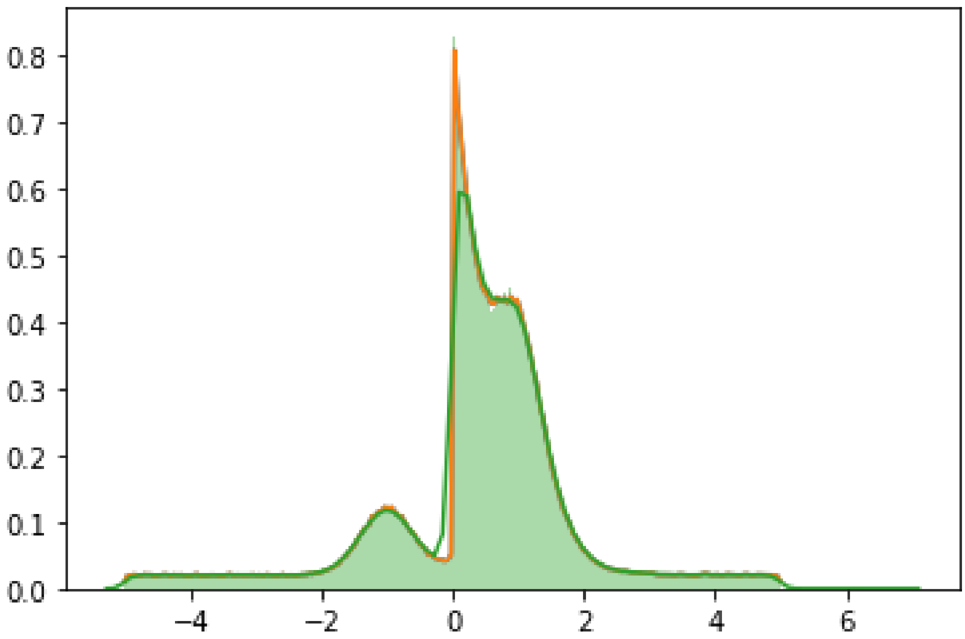
Kernel Density Estimation(Rosenblatt) has improved

```
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import tensorflow as tf
import tensorflow_probability as tfp
from zfit_benchmark.timer import Timer
import zfit as z
```

```
r_seed = 1978239485
n_datapoints = 1000000

tfd = tfp.distributions
mix_3gauss_1exp_1uni = tfd.Mixture(
  cat=tfd.Categorical(probs=[0.1, 0.2, 0.1, 0.4, 0.2]),
  components=[
    tfd.Normal(loc=-1., scale=0.4),
    tfd.Normal(loc=+1., scale=0.5),
    tfd.Normal(loc=+1., scale=0.3),
    tfd.Exponential(rate=2),
    tfd.Uniform(low=-5, high=5)
])

data = mix_3gauss_1exp_1uni.sample(sample_shape=n_datapoints, seed=
    r_seed).numpy()
```

```
%matplotlib inline
ax = plt.gca()

n_testpoints = 200
fac1 = 1.0 / np.sqrt(2.0 * np.pi)
exp_fac1 = -1.0/2.0

y_fac1 = 1.0/(h*n_datapoints)
h1 = 0.01


with Timer ("Benchmarking") as timer:
    with timer.child('tf.simple-kde'):
        @tf.function(autograph=False)
        def tf_kde():

            fac = tf.constant(fac1, tf.float64)
            exp_fac = tf.constant(exp_fac1, tf.float64)
            y_fac = tf.constant(y_fac1, tf.float64)
            h = tf.constant(h1, tf.float64)
            data_tf = tf.convert_to_tensor(data, tf.float64)


            gauss_kernel = lambda x: tf.math.multiply(fac, tf.math.exp(tf.math.multiply(exp_fac, tf.math.square(x))))
            calc_value = lambda x: tf.math.multiply(y_fac, tf.math.reduce_sum(gauss_kernel(tf.math.divide(tf.math.subtract(x, data_tf), h))))

            x = tf.linspace(tf.cast(-5.0, tf.float64), tf.cast(5.0, tf.float64), num=tf.cast(n_testpoints, tf.int64))
            y = tf.zeros(n_testpoints)


            return tf.map_fn(calc_value, x)

        y = tf_kde()
        sns.lineplot(x, y, ax=ax)
        timer.stop()

    with timer.child('simple-kde'):

        fac = fac1
        exp_fac = exp_fac1

        y_fac = y_fac1
        h = h1

        gauss_kernel = lambda x: fac * np.exp(exp_fac * x**2)

        x2 = np.linspace(-5.0, 5.0, num=n_testpoints)
        y2 = np.zeros(n_testpoints)

        for i, x_i in enumerate(x2):
            y2[i] = y_fac * np.sum(gauss_kernel((x_i-data)/h))
        sns.lineplot(x2,y2, ax=ax)
        timer.stop()

    with timer.child('sns.distplot'):
        sns.distplot(data, bins=1000, kde=True, rug=False, ax=ax)
        timer.stop()
```

```
4.848263676998612936586141586
12.504365049993793945759534844
10.825755580997793003916740422
```



png

$$\mathbf{r} \equiv \begin{bmatrix} y \\ \theta \end{bmatrix}$$

References

Rosenblatt, Murray. "Remarks on Some Nonparametric Estimates of a Density Function." *Ann. Math. Statist.*, vol. 27, no. 3, The Institute of Mathematical Statistics, Sept. 1956, pp. 832–37, doi:10.1214/aoms/1177728190.