Linear Regression for house price prediction

In this colab, we will build different linear regression models for california house price prediction:

1. Linear regression (with normal equation and iterative optimization)
2. Polynomial regression
3. Regularized regression models - ridge and lasso.

We will set regularization rate and polynomial degree with hyper-parameter tuning and cross validation.

We will compare different models in terms of their parameter vectors and mean absolute error on train, devel and test sets.

Imports

For regression problem, we need to import classes and utilities from `sklearn.linear_model`.

- This module has implementations for different regression models like `LinearRegression`, `SGDRegressor`, `Ridge`, `Lasso`, `RidgeCV`, and `LassoCV`.

We also need to import a bunch of model selection utilities from `sklearn.model_selection` module and metrics from `sklearn.metrics` module.

The data preprocessing utilities are imported from `sklearn.preprocessing` modules.

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns

from scipy.stats import loguniform
from scipy.stats import uniform

from sklearn.datasets import fetch_california_housing
from sklearn.dummy import DummyRegressor
from sklearn.linear_model import LinearRegression
from sklearn.linear_model import Lasso
from sklearn.linear_model import LassoCV
from sklearn.linear_model import RidgeCV
from sklearn.linear_model import Ridge
from sklearn.linear_model import SGDRegressor
from sklearn.metrics import mean_squared_error
from sklearn.metrics import mean_absolute_error
from sklearn.metrics import mean_absolute_percentage_error
```

```
from sklearn.model_selection import cross_validate
from sklearn.model_selection import cross_val_score
from sklearn.model_selection import train_test_split
from sklearn.model_selection import ShuffleSplit
from sklearn.model_selection import validation_curve
from sklearn.model_selection import GridSearchCV
from sklearn.model_selection import RandomizedSearchCV
from sklearn.preprocessing import PolynomialFeatures
from sklearn.preprocessing import StandardScaler
from sklearn.pipeline import Pipeline
```

Common set up

Set up random seed to a number of your choice.

```
np.random.seed(306)
```

Let's use `ShuffleSplit` as cv with 10 splits and 20% examples set aside as test examples.

```
cv = ShuffleSplit(n_splits=10, test_size=0.2, random_state=42)
```

Data Loading and splitting

We use california housing dataset for this demo. We will load this dataset with `fetch_california_housing` API as a dataframe.

We will load the data and split it into three parts - train, dev and test. Train+Dev will be used for cross validation and test will be used for evaluating the trained models.

```
# fetch dataset
features, labels = fetch_california_housing(as_frame=True, return_X_y=True)

# train-test split
com_train_features, test_features, com_train_labels, test_labels = train_test_split(
    features, labels, random_state=42)

# train --> train + dev split
train_features, dev_features, train_labels, dev_labels = train_test_split(
    com_train_features, com_train_labels, random_state=42)
```

Linear regression with normal equation

Let's use normal equation method to train linear regression model.

We set up pipeline with two stages:

- Feature scaling to scale features and
- Linear regression on the transformed feature matrix.

Throughout this colab, we will have the following pattern for each estimator:

- We will be using `Pipeline` for combining data preprocessing and modeling steps.
- `cross_validate` for training the model with `ShuffleSplit` cross validation and `neg_mean_absolute_error` as a scoring metric.
- Convert the scores to error and report mean absolute errors on the dev set.

```
lin_reg_pipeline = Pipeline([("feature_scaling", StandardScaler()),
                             ("lin_reg", LinearRegression())])
lin_reg_cv_results = cross_validate(lin_reg_pipeline,
                                    com_train_features,
                                    com_train_labels,
                                    cv=cv,
                                    scoring="neg_mean_absolute_error",
                                    return_train_score=True,
                                    return_estimator=True)

lin_reg_train_error = -1 * lin_reg_cv_results['train_score']
lin_reg_test_error = -1 * lin_reg_cv_results['test_score']

print(f"Mean absolute error of linear regression model on the train set:\n"
      f"{lin_reg_train_error.mean():.3f} +/- {lin_reg_train_error.std():.3f}")
print(f"Mean absolute error of linear regression model on the test set:\n"
      f"{lin_reg_test_error.mean():.3f} +/- {lin_reg_test_error.std():.3f}")
```

```
Mean absolute error of linear regression model on the train set:
0.530 +/- 0.002
Mean absolute error of linear regression model on the test set:
0.527 +/- 0.008
```

Both the errors are close, but are not low. This points to underfitting. We can address it by adding more feature through polynomial regression.

Linear regression with SGD

Let's use iterative optimization method to train linear regression model.

We set up pipeline with two stages:

- Feature scaling to scale features and
- SGD regression on the transformed feature matrix.

[ ] ↳ *1 cell hidden*

Polynomial regression

We will train a polynomial model with degree 2 and later we will use `validation_curve` to find out right degree to use for polynomial models.

`PolynomialFeatures` transforms the features to the user specified degrees (here it is 2). We perform feature scaling on the transformed features before using them for training the regression model.

[ ] ↳ *7 cells hidden*

Ridge regression

The polynomial models have a tendency to overfit - if we use higher order polynomial features. We will use `Ridge` regression - which penalizes for excessive model complexity in the polynomial regression by adding a regularization term. Here we specify the regularization rate `alpha` as 0.5 and train the regression model. Later we will launch hyperparameter search for the right value of `alpha` such that it leads to the least cross validation errors.

```
ridge_reg_pipeline = Pipeline([("poly", PolynomialFeatures(degree=2)),
                              ("feature_scaling", StandardScaler()),
                              ("ridge", Ridge(alpha=0.5))])
ridge_reg_cv_results = cross_validate(ridge_reg_pipeline,
                                    com_train_features,
                                    com_train_labels,
                                    cv=cv,
                                    scoring="neg_mean_absolute_error",
                                    return_train_score=True,
                                    return_estimator=True)

ridge_reg_train_error = -1 * ridge_reg_cv_results['train_score']
ridge_reg_test_error = -1 * ridge_reg_cv_results['test_score']

print(f"Mean absolute error of ridge regression model (alpha=0.5) on the train set:\n"
      f"{ridge_reg_train_error.mean():.3f} +/- {ridge_reg_train_error.std():.3f}")
print(f"Mean absolute error of ridge regression model (alpha=0.5) on the test set:\n"
      f"{ridge_reg_test_error.mean():.3f} +/- {ridge_reg_test_error.std():.3f}")
```

```
Mean absolute error of ridge regression model (alpha=0.5) on the train set:
0.481 +/- 0.003
Mean absolute error of ridge regression model (alpha=0.5) on the test set:
0.487 +/- 0.006
```

HPT for ridge regularization rate

```
alpha_list = np.logspace(-4, 0, num=20)
ridge_reg_pipeline = Pipeline([("poly", PolynomialFeatures(degree=2)),
                              ("feature_scaling", StandardScaler()),
                              ("ridge_cv", RidgeCV(alphas=alpha_list,
                                                   cv=cv,
                                                   scoring="neg_mean_absolute_error"))])
ridge_reg_cv_results = ridge_reg_pipeline.fit(com_train_features, com_train_labels)
```

```
print ("The score with the best alpha is:",
       f"{ridge_reg_cv_results[-1].best_score_:.3f}")
print ("The error with the best alpha is:",
       f"{-ridge_reg_cv_results[-1].best_score_:.3f}")
```

```
The score with the best alpha is: -0.473
The error with the best alpha is: 0.473
```

```
print ("The best value for alpha:", ridge_reg_cv_results[-1].alpha_)
```

```
The best value for alpha: 0.007847599703514606
```

RidgeCV with cross validation

Let's search for right value of regularization rate through `RidgeCV`, where we specify the regularization rates to be tried.

```
alpha_list = np.logspace(-4, 0, num=20)
ridge_reg_pipeline = Pipeline([("poly", PolynomialFeatures(degree=2)),
                              ("feature_scaling", StandardScaler()),
                              ("ridge_cv", RidgeCV(alphas=alpha_list,
                                                   store_cv_values=True))])
ridge_reg_cv_results = cross_validate(ridge_reg_pipeline,
                                      com_train_features,
                                      com_train_labels,
                                      cv=cv,
                                      scoring="neg_mean_absolute_error",
                                      return_train_score=True,
                                      return_estimator=True)

ridge_reg_train_error = -1 * ridge_reg_cv_results['train_score']
ridge_reg_test_error = -1 * ridge_reg_cv_results['test_score']

print(f"Mean absolute error of ridge regression model on the train set:\n"
      f"{ridge_reg_train_error.mean():.3f} +/- {ridge_reg_train_error.std():.3f}")
print(f"Mean absolute error of ridge regression model on the test set:\n"
      f"{ridge_reg_test_error.mean():.3f} +/- {ridge_reg_test_error.std():.3f}")
```

```
Mean absolute error of ridge regression model on the train set:
0.470 +/- 0.011
```

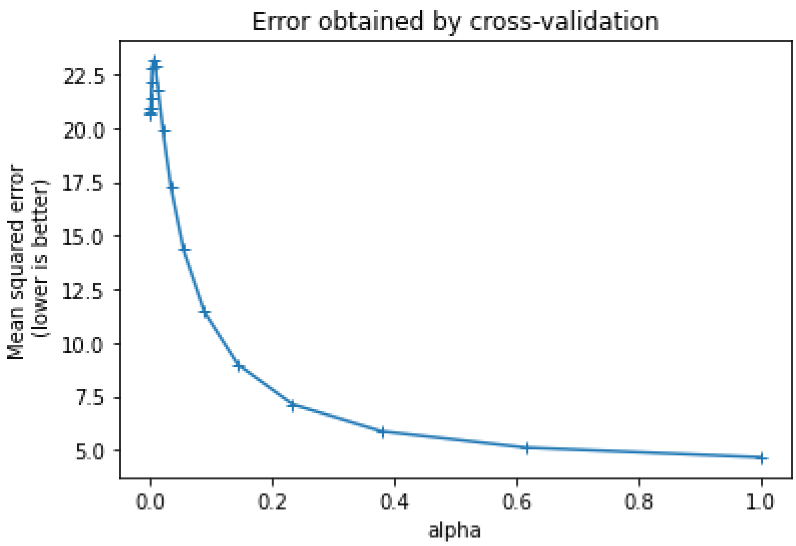
```
Mean absolute error of ridge regression model on the test set:
0.474 +/- 0.011
```

Let's look at the mean of mean absolute errors at different values of regularization rate across different cross validation folds.

```
mse_alphas = [est[-1].cv_values_.mean(axis=0)
              for est in ridge_reg_cv_results["estimator"]]
cv_alphas = pd.DataFrame(mse_alphas, columns=alpha_list)
cv_alphas
```

	0.000100	0.000162	0.000264	0.000428	0.000695	0.001129	0.001833	0.
0	79.825105	83.556979	89.052274	96.713331	106.577419	117.958587	129.309233	138.
1	14.065799	13.742714	13.245802	12.504952	11.450787	10.048351	8.345884	6.
2	5.267710	5.272331	5.279626	5.290956	5.308105	5.333104	5.367714	5.
3	5.853085	5.867043	5.888945	5.922643	5.972982	6.045051	6.142429	6.
4	8.134646	7.993974	7.781172	7.471108	7.042443	6.489075	5.830605	5.
5	66.100008	60.047159	52.002193	42.279138	31.879141	22.232981	14.513912	9.
6	2.309882	2.253722	2.168544	2.044094	1.871808	1.650275	1.391207	1.
7	19.019937	20.494095	22.742542	26.033396	30.548208	36.182667	42.368884	48.
8	4.351827	4.345297	4.335085	4.319448	4.296236	4.263277	4.219299	4.
9	3.963850	3.952362	3.934504	3.907436	3.867958	3.813571	3.744526	3.

```
cv_alphas.mean(axis=0).plot(marker="+")
plt.ylabel("Mean squared error\n (lower is better)")
plt.xlabel("alpha")
_ = plt.title("Error obtained by cross-validation")
```



```
best_alphas = [est[-1].alpha_ for est in ridge_reg_cv_results["estimator"]]
best_alphas
```

```
[1.0,
 0.3792690190732246,
 0.0001,
 0.0001,
 0.08858667904100823,
 0.08858667904100823,
 0.0206913808111479,
 1.0,
 0.0206913808111479,
 0.012742749857031334]
```

The optimal regularization strength is not necessarily the same on all cross-validation iterations. But since we expect each cross-validation resampling to stem from the same data distribution, it is common practice to use the average value of the best alpha found on different cross-validation folds as our final estimate for the tuned alpha.

```
print(f"The mean optimal alpha leading to the best generalization performance is:\n"
      f"{np.mean(best_alphas):.2f} +/- {np.std(best_alphas):.2f}")
```

```
The mean optimal alpha leading to the best generalization performance is:
0.26 +/- 0.38
```

Ridge HPT through `GridSearchCV`

```
ridge_grid_pipeline = Pipeline([("poly", PolynomialFeatures(degree=2)),
                              ("feature_scaling", StandardScaler()),
                              ("ridge", Ridge())])

param_grid = {'poly__degree': (1, 2, 3),
              'ridge__alpha': np.logspace(-4, 0, num=20)}
ridge_grid_search = GridSearchCV(ridge_grid_pipeline,
                                 param_grid=param_grid,
                                 n_jobs=2,
                                 cv=cv,
                                 scoring="neg_mean_absolute_error",
                                 return_train_score=True)
ridge_grid_search.fit(com_train_features, com_train_labels)
```

```
GridSearchCV(cv=ShuffleSplit(n_splits=10, random_state=42, test_size=0.2,
train_size=None),
             estimator=Pipeline(steps=[('poly', PolynomialFeatures()),
                                       ('feature_scaling', StandardScaler()),
                                       ('ridge', Ridge())]),
             n_jobs=2,
             param_grid={'poly__degree': (1, 2, 3),
                         'ridge__alpha': array([1.00000000e-04, 1.62377674e-04,
2.63665090e-04, 4.28133240e-04,
```

```
            6.95192796e-04, 1.12883789e-03, 1.83298071e-03, 2.97635144e-03,
            4.83293024e-03, 7.84759970e-03, 1.27427499e-02, 2.06913808e-02,
            3.35981829e-02, 5.45559478e-02, 8.85866790e-02, 1.43844989e-01,
            2.33572147e-01, 3.79269019e-01, 6.15848211e-01, 1.00000000e+00])},
                  return_train_score=True, scoring='neg_mean_absolute_error')
```

`ridge_grid_search.best_index_` gives us the index of the best parameter in the list.

```
mean_train_error = -1 * ridge_grid_search.cv_results_['mean_train_score'][ridge_grid_searc
mean_test_error = -1 * ridge_grid_search.cv_results_['mean_test_score'][ridge_grid_search.
std_train_error = ridge_grid_search.cv_results_['std_train_score'][ridge_grid_search.best_
std_test_error = ridge_grid_search.cv_results_['std_train_score'][ridge_grid_search.best_i

print(f"Best Mean absolute error of polynomial ridge regression model on the train set:\n'
      f"{mean_train_error:.3f} +/- {std_train_error:.3f}")
print(f"Mean absolute error of polynomial ridge regression model on the test set:\n"
      f"{mean_test_error:.3f} +/- {std_test_error:.3f}")
```

```
Best Mean absolute error of polynomial ridge regression model on the train set:
0.463 +/- 0.004
Mean absolute error of polynomial ridge regression model on the test set:
0.474 +/- 0.004
```

```
print ("Mean cross validated score of the best estimator is: ", ridge_grid_search.best_scc
print ("Mean cross validated error of the best estimator is: ", -ridge_grid_search.best_sc
```

```
Mean cross validated score of the best estimator is:  -0.47386511769919126
Mean cross validated error of the best estimator is:  0.47386511769919126
```

Note that this is same as `RidgeCV` that we carried out earlier.

```
print ("The best parameter value is:", ridge_grid_search.best_params_)
```

```
The best parameter value is: {'poly__degree': 2, 'ridge__alpha': 0.00784759970351460
```

Lasso regression

Baseline model with fixed learning rate

```
lasso_reg_pipeline = Pipeline([("poly", PolynomialFeatures(degree=2)),
                               ("feature_scaling", StandardScaler()),
                               ("lasso", Lasso(alpha=0.01))])
lasso_reg_cv_results = cross_validate(lasso_reg_pipeline,
                                      com_train_features,
                                      com_train_labels,
                                      cv=cv,
```

```
                                    scoring="neg_mean_absolute_error",
                                    return_train_score=True,
                                    return_estimator=True)

lasso_reg_train_error = -1 * lasso_reg_cv_results['train_score']
lasso_reg_test_error = -1 * lasso_reg_cv_results['test_score']

print(f"Mean absolute error of linear regression model on the train set:\n"
      f"{lasso_reg_train_error.mean():.3f} +/- {ridge_reg_train_error.std():.3f}")
print(f"Mean absolute error of linear regression model on the test set:\n"
      f"{lasso_reg_test_error.mean():.3f} +/- {ridge_reg_test_error.std():.3f}")
```

```
/usr/local/lib/python3.7/dist-packages/sklearn/linear_model/_coordinate_descent.py:6
  coef_, l1_reg, l2_reg, X, y, max_iter, tol, rng, random, positive
/usr/local/lib/python3.7/dist-packages/sklearn/linear_model/_coordinate_descent.py:6
  coef_, l1_reg, l2_reg, X, y, max_iter, tol, rng, random, positive
/usr/local/lib/python3.7/dist-packages/sklearn/linear_model/_coordinate_descent.py:6
  coef_, l1_reg, l2_reg, X, y, max_iter, tol, rng, random, positive
/usr/local/lib/python3.7/dist-packages/sklearn/linear_model/_coordinate_descent.py:6
  coef_, l1_reg, l2_reg, X, y, max_iter, tol, rng, random, positive
/usr/local/lib/python3.7/dist-packages/sklearn/linear_model/_coordinate_descent.py:6
  coef_, l1_reg, l2_reg, X, y, max_iter, tol, rng, random, positive
/usr/local/lib/python3.7/dist-packages/sklearn/linear_model/_coordinate_descent.py:6
  coef_, l1_reg, l2_reg, X, y, max_iter, tol, rng, random, positive
/usr/local/lib/python3.7/dist-packages/sklearn/linear_model/_coordinate_descent.py:6
  coef_, l1_reg, l2_reg, X, y, max_iter, tol, rng, random, positive
/usr/local/lib/python3.7/dist-packages/sklearn/linear_model/_coordinate_descent.py:6
  coef_, l1_reg, l2_reg, X, y, max_iter, tol, rng, random, positive
/usr/local/lib/python3.7/dist-packages/sklearn/linear_model/_coordinate_descent.py:6
  coef_, l1_reg, l2_reg, X, y, max_iter, tol, rng, random, positive
Mean absolute error of linear regression model on the train set:
0.529 +/- 0.011
Mean absolute error of linear regression model on the test set:
0.528 +/- 0.011
/usr/local/lib/python3.7/dist-packages/sklearn/linear_model/_coordinate_descent.py:6
  coef_, l1_reg, l2_reg, X, y, max_iter, tol, rng, random, positive
```

HPT for lasso regularization rate

With cross validation

```
alpha_list = np.logspace(-6, 0, num=20)
lasso_reg_pipeline = Pipeline([("poly", PolynomialFeatures(degree=2)),
                              ("feature_scaling", StandardScaler()),
                              ("lasso_cv", LassoCV(alphas=alpha_list, cv=cv))])
lasso_reg_cv_results = cross_validate(lasso_reg_pipeline,
                                    com_train_features,
                                    com_train_labels,
                                    cv=cv,
                                    scoring="neg_mean_absolute_error",
```

```
                                    return_train_score=True,
                                    return_estimator=True)

lasso_reg_train_error = -1 * lasso_reg_cv_results['train_score']
lasso_reg_test_error = -1 * lasso_reg_cv_results['test_score']

print(f"Mean absolute error of linear regression model on the train set:\n"
      f"{lasso_reg_train_error.mean():.3f} +/- {lasso_reg_train_error.std():.3f}")
print(f"Mean absolute error of linear regression model on the test set:\n"
      f"{lasso_reg_test_error.mean():.3f} +/- {lasso_reg_test_error.std():.3f}")
```

```
best_alphas = [est[-1].alpha_ for est in lasso_reg_cv_results["estimator"]]
best_alphas
```

```
[0.012742749857031322,
 0.012742749857031322,
 0.00615848211066026,
 0.00615848211066026,
 0.00615848211066026,
 1e-06,
 0.012742749857031322,
 0.0003359818286283781,
 0.00615848211066026,
 0.026366508987303555]
```

```
print(f"The mean optimal alpha leading to the best generalization performance is:\n"
      f"{np.mean(best_alphas):.2f} +/- {np.std(best_alphas):.2f}")
```

```
The mean optimal alpha leading to the best generalization performance is:
0.01 +/- 0.01
```

```
lasso_reg_pipeline = Pipeline([("poly", PolynomialFeatures(degree=2)),
                              ("feature_scaling", StandardScaler()),
                              ("lasso", Lasso(alpha=0.01))])
lasso_reg_pipeline.fit(com_train_features, com_train_labels)
train_error = mean_absolute_error(com_train_labels,
                                 lasso_reg_pipeline.predict(com_train_features))

print(f"Mean absolute error of Lasso CV model on the train set:", train_error)
```

```
Mean absolute error of Lasso CV model on the train set: 0.5291330037868303
/usr/local/lib/python3.7/dist-packages/sklearn/linear_model/_coordinate_descent.py:6
  coef_, l1_reg, l2_reg, X, y, max_iter, tol, rng, random, positive
```

with GridSearchCV

```
lasso_grid_pipeline = Pipeline([("poly", PolynomialFeatures()),
                              ("feature_scaling", StandardScaler()),
                              ("lasso", Lasso())])
```

```
param_grid = {'poly__degree': (1, 2, 3),
              'lasso__alpha': np.logspace(-4, 0, num=20)}
lasso_grid_search = GridSearchCV(lasso_grid_pipeline,
                                 param_grid=param_grid,
                                 n_jobs=2,
                                 cv=cv,
                                 scoring="neg_mean_absolute_error",
                                 return_train_score=True)
lasso_grid_search.fit(com_train_features, com_train_labels)
```

```
/usr/local/lib/python3.7/dist-packages/sklearn/linear_model/_coordinate_descent.py:6
  coef_, l1_reg, l2_reg, X, y, max_iter, tol, rng, random, positive
GridSearchCV(cv=ShuffleSplit(n_splits=10, random_state=42, test_size=0.2,
train_size=None),
             estimator=Pipeline(steps=[('poly', PolynomialFeatures()),
                                       ('feature_scaling', StandardScaler()),
                                       ('lasso', Lasso())]),
             n_jobs=2,
             param_grid={'lasso__alpha': array([1.00000000e-04, 1.62377674e-04,
2.63665090e-04, 4.28133240e-04,
       6.95192796e-04, 1.12883789e-03, 1.83298071e-03, 2.97635144e-03,
       4.83293024e-03, 7.84759970e-03, 1.27427499e-02, 2.06913808e-02,
       3.35981829e-02, 5.45559478e-02, 8.85866790e-02, 1.43844989e-01,
       2.33572147e-01, 3.79269019e-01, 6.15848211e-01, 1.00000000e+00]),
                         'poly__degree': (1, 2, 3)},
             return_train_score=True, scoring='neg_mean_absolute_error')
```

```
mean_train_error = -1 * lasso_grid_search.cv_results_['mean_train_score'][lasso_grid_searc
mean_test_error = -1 * lasso_grid_search.cv_results_['mean_test_score'][lasso_grid_search.
std_train_error = lasso_grid_search.cv_results_['std_train_score'][lasso_grid_search.best_
std_test_error = lasso_grid_search.cv_results_['std_train_score'][lasso_grid_search.best_i

print(f"Best Mean absolute error of polynomial ridge regression model on the train set:\n'
      f"{mean_train_error:.3f} +/- {std_train_error:.3f}")
print(f"Mean absolute error of polynomial ridge regression model on the test set:\n"
      f"{mean_test_error:.3f} +/- {std_test_error:.3f}")
```

```
Best Mean absolute error of polynomial ridge regression model on the train set:
0.462 +/- 0.003
Mean absolute error of polynomial ridge regression model on the test set:
0.488 +/- 0.003
```

```
print ("Mean cross validated score of the best estimator is: ", lasso_grid_search.best_scc
```

```
Mean cross validated score of the best estimator is:  -0.48798304453391356
```

```
print ("The best parameter value is:", lasso_grid_search.best_params_)
```

```
The best parameter value is: {'lasso__alpha': 0.0001, 'poly__degree': 3}
```

SGD: Regularization and HPT

We can also perform regularization with SGD. `SGDRegressor` has many hyperparameters that require careful tuning to achieve the same performance as with `LinearRegression`.

```
poly_sgd_pipeline = Pipeline([("poly", PolynomialFeatures()),
                              ("feature_scaling", StandardScaler()),
                              ("sgd_reg", SGDRegressor(
                                 penalty='elasticnet',
                                 random_state=42))])
poly_sgd_cv_results = cross_validate(poly_sgd_pipeline,
                                    com_train_features,
                                    com_train_labels,
                                    cv=cv,
                                    scoring="neg_mean_absolute_error",
                                    return_train_score=True,
                                    return_estimator=True)

poly_sgd_train_error = -1 * poly_sgd_cv_results['train_score']
poly_sgd_test_error = -1 * poly_sgd_cv_results['test_score']

print(f"Mean absolute error of linear regression model on the train set:\n"
      f"{poly_sgd_train_error.mean():.3f} +/- {poly_sgd_train_error.std():.3f}")
print(f"Mean absolute error of linear regression model on the test set:\n"
      f"{poly_sgd_test_error.mean():.3f} +/- {poly_sgd_test_error.std():.3f}")
```

```
Mean absolute error of linear regression model on the train set:
10824283052.546 +/- 4423288211.832
Mean absolute error of linear regression model on the test set:
10946788540.250 +/- 5396536227.703
```

Let's search for the best set of parameters for polynomial + SGD pipeline with `RandomizedSearchCV`.

> Remember in `RandomizedSearchCV`, we need to specify distributions for hyperparameters.

```
class uniform_int:
    """Integer valued version of the uniform distribution"""
    def __init__(self, a, b):
        self._distribution = uniform(a, b)

    def rvs(self, *args, **kwargs):
        """Random variable sample"""
        return self._distribution.rvs(*args, **kwargs).astype(int)
```

Let's specify `RandomizedSearchCV` set up.

```
param_distributions = {
    'poly__degree': [1, 2, 3],
    'sgd_reg__learning_rate': ['constant', 'adaptive', 'invscaling'],
```

```
    'sgd_reg__l1_ratio': uniform(0, 1),
    'sgd_reg__eta0': loguniform(1e-5, 1),
    'sgd_reg__power_t': uniform(0, 1)
}

poly_sgd_random_search = RandomizedSearchCV(
    poly_sgd_pipeline, param_distributions=param_distributions,
    n_iter=10, cv=cv, verbose=1, scoring='neg_mean_absolute_error'
)
poly_sgd_random_search.fit(com_train_features, com_train_labels)
```

```
Fitting 10 folds for each of 10 candidates, totalling 100 fits
/usr/local/lib/python3.7/dist-packages/sklearn/linear_model/_stochastic_gradient.py:
  ConvergenceWarning,
/usr/local/lib/python3.7/dist-packages/sklearn/linear_model/_stochastic_gradient.py:
  ConvergenceWarning,
RandomizedSearchCV(cv=ShuffleSplit(n_splits=10, random_state=42, test_size=0.2,
train_size=None),
                   estimator=Pipeline(steps=[('poly', PolynomialFeatures()),
                                             ('feature_scaling',
                                              StandardScaler()),
                                             ('sgd_reg',
                                              SGDRegressor(penalty='elasticnet',
                                                           random_state=42))]),
                   param_distributions={'poly__degree': [1, 2, 3],
                                        'sgd_reg__eta0':
<scipy.stats._distn_infrastructure.rv_frozen object at 0x7fc4739d4a10>,
                                        'sgd_reg__l1_ratio':
<scipy.stats._distn_infrastructure.rv_frozen object at 0x7fc473897e10>,
                                        'sgd_reg__learning_rate': ['constant',
                                                                   'adaptive',
                                                                   'invscaling'],
                                        'sgd_reg__power_t':
<scipy.stats._distn_infrastructure.rv_frozen object at 0x7fc47384aa90>},
                   scoring='neg_mean_absolute_error', verbose=1)
```

The best score can be obtained as follows:

```
poly_sgd_random_search.best_score_
```

```
-0.5282899475024619
```

The best set of parameters are obtained as follows:

```
poly_sgd_random_search.best_params_
```

```
{'poly__degree': 1,
 'sgd_reg__eta0': 8.074204494282093e-05,
 'sgd_reg__l1_ratio': 0.5830694513861019,
 'sgd_reg__learning_rate': 'constant',
 'sgd_reg__power_t': 0.2575849132301107}
```

And the best estimator can be accessed with `best_estimator_` member variable.

Comparison of weight vectors

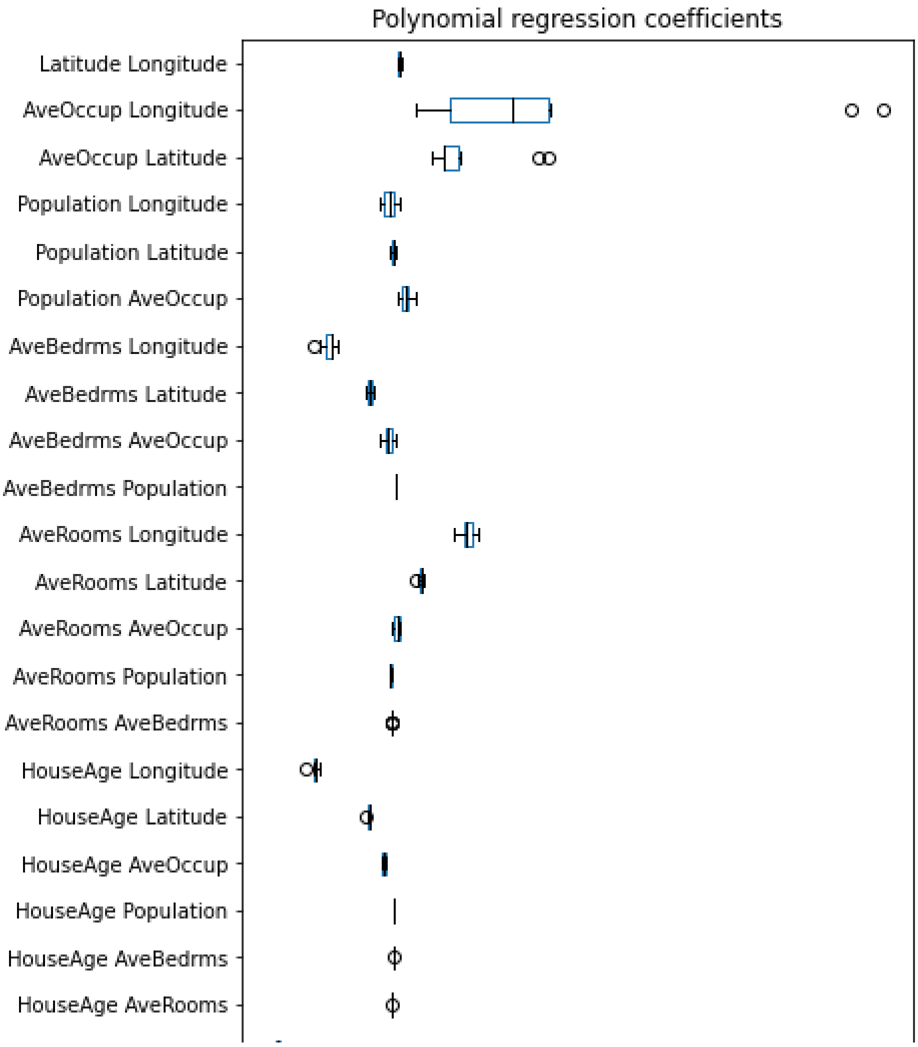
Let's look at the weight vectors produced by different models.

```
feature_names = poly_reg_cv_results["estimator"][0][0].get_feature_names_out(
    input_features=train_features.columns)
feature_names
```

```
array(['1', 'MedInc', 'HouseAge', 'AveRooms', 'AveBedrms', 'Population',
       'AveOccup', 'Latitude', 'Longitude', 'MedInc HouseAge',
       'MedInc AveRooms', 'MedInc AveBedrms', 'MedInc Population',
       'MedInc AveOccup', 'MedInc Latitude', 'MedInc Longitude',
       'HouseAge AveRooms', 'HouseAge AveBedrms', 'HouseAge Population',
       'HouseAge AveOccup', 'HouseAge Latitude', 'HouseAge Longitude',
       'AveRooms AveBedrms', 'AveRooms Population', 'AveRooms AveOccup',
       'AveRooms Latitude', 'AveRooms Longitude', 'AveBedrms Population',
       'AveBedrms AveOccup', 'AveBedrms Latitude', 'AveBedrms Longitude',
       'Population AveOccup', 'Population Latitude',
       'Population Longitude', 'AveOccup Latitude', 'AveOccup Longitude',
       'Latitude Longitude'], dtype=object)
```

```
coefs = [est[-1].coef_ for est in poly_reg_cv_results["estimator"]]
weights_polynomial_regression = pd.DataFrame(coefs, columns=feature_names)
```

```
color = {"whiskers": "black", "medians": "black", "caps": "black"}
weights_polynomial_regression.plot.box(color=color, vert=False, figsize=(6, 16))
_ = plt.title("Polynomial regression coefficients")
```

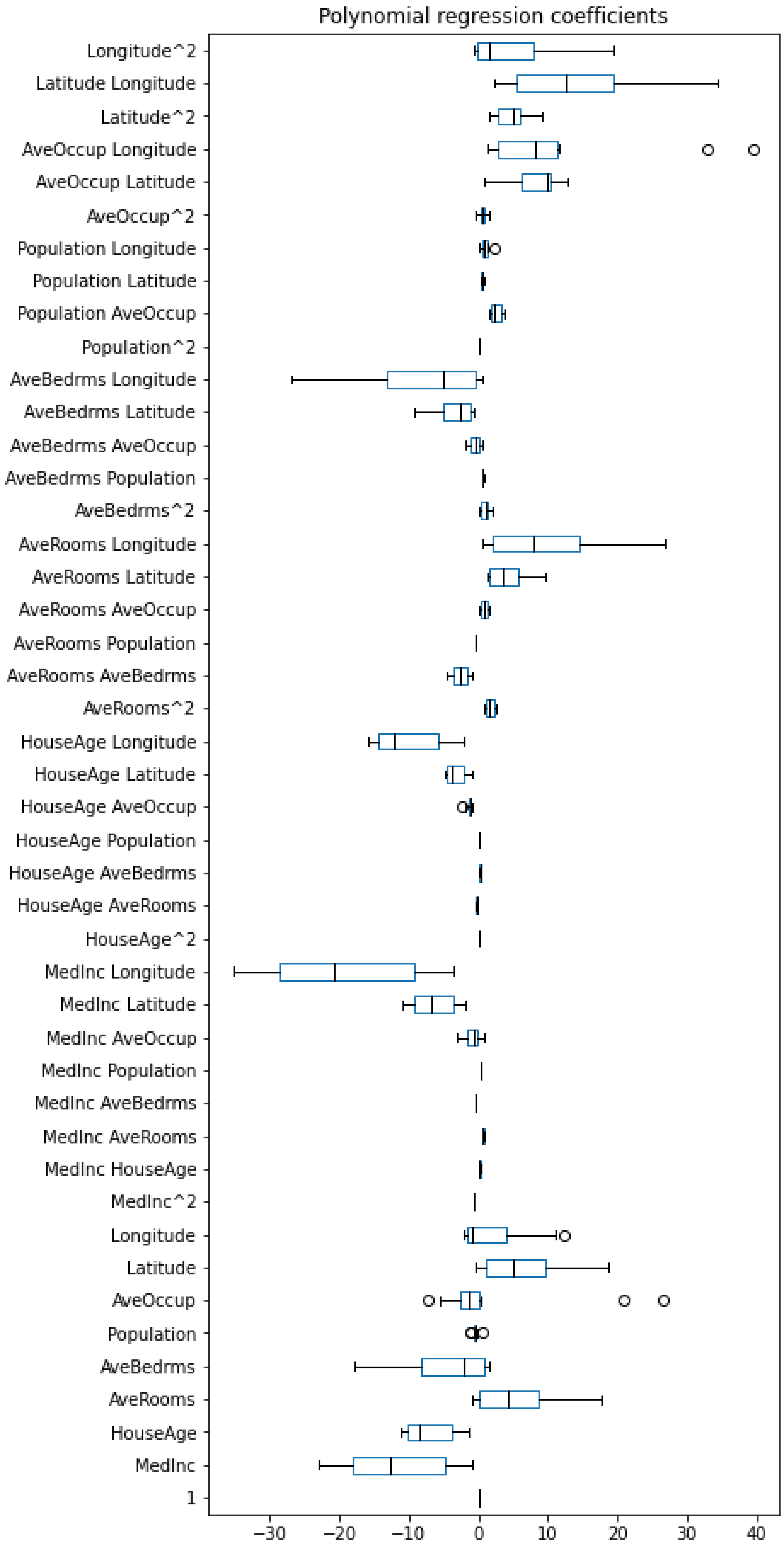



```
feature_names = ridge_reg_cv_results["estimator"][0][0].get_feature_names_out(
    input_features=train_features.columns)
feature_names
```

```
array(['1', 'MedInc', 'HouseAge', 'AveRooms', 'AveBedrms', 'Population',
       'AveOccup', 'Latitude', 'Longitude', 'MedInc^2', 'MedInc HouseAge',
       'MedInc AveRooms', 'MedInc AveBedrms', 'MedInc Population',
       'MedInc AveOccup', 'MedInc Latitude', 'MedInc Longitude',
       'HouseAge^2', 'HouseAge AveRooms', 'HouseAge AveBedrms',
       'HouseAge Population', 'HouseAge AveOccup', 'HouseAge Latitude',
       'HouseAge Longitude', 'AveRooms^2', 'AveRooms AveBedrms',
       'AveRooms Population', 'AveRooms AveOccup', 'AveRooms Latitude',
       'AveRooms Longitude', 'AveBedrms^2', 'AveBedrms Population',
       'AveBedrms AveOccup', 'AveBedrms Latitude', 'AveBedrms Longitude',
       'Population^2', 'Population AveOccup', 'Population Latitude',
       'Population Longitude', 'AveOccup^2', 'AveOccup Latitude',
       'AveOccup Longitude', 'Latitude^2', 'Latitude Longitude',
       'Longitude^2'], dtype=object)
```

```
coefs = [est[-1].coef_ for est in ridge_reg_cv_results["estimator"]]
weights_ridge_regression = pd.DataFrame(coefs, columns=feature_names)
```

```
color = {"whiskers": "black", "medians": "black", "caps": "black"}
weights_ridge_regression.plot.box(color=color, vert=False, figsize=(6, 16))
_ = plt.title("Polynomial regression coefficients")
```



Performance on the test set

Baseline

```
baseline_model_median = DummyRegressor(strategy='median')
baseline_model_median.fit(train_features, train_labels)
mean_absolute_percentage_error(test_labels,
                               baseline_model_median.predict(test_features))
```

```
0.5348927548151625
```

Linear regression with normal equation

```
mean_absolute_percentage_error(test_labels,
                               lin_reg_cv_results['estimator'][0].predict(
                                   test_features))
```

```
0.32120472175482906
```

```
mean_absolute_percentage_error(test_labels,
                               poly_sgd_random_search.best_estimator_.predict(
                                   test_features))
```

```
0.32020169843649454
```

Polynomial regression

```
poly_reg_pipeline.fit(com_train_features, com_train_labels)
mean_absolute_percentage_error(test_labels,
                               poly_reg_pipeline.predict(test_features))
```

```
0.28199759082657244
```

Ridge regression

```
mean_absolute_percentage_error(test_labels,
                               ridge_grid_search.best_estimator_.predict(test_features))
```

```
0.27110336451421363
```

Lasso regression

Let's retrain the lasso model with `alpha` identified through hyper-parameter and evaluate it on the test data.

```
mean_absolute_percentage_error(test_labels,
                               lasso_grid_search.best_estimator_.predict(test_features))
```

```
0.28074969263810107
```

Summary

We trained multiple linear regression models on housing dataset. Set their hyperparamters through hyper-parameter optimization. Retrained models with the best values of hyper-parameters and then evaluated their performance on the test data (that was hold back until final evaluation).

This is how most of the real world problems are solved starting from simple models to more sophisticated models.