INTRODUCTION TO PLASMA PHYSICS FOR ELECTRIC PROPULSION

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Outline

- Why Electric Propulsion?
- Definition of Plasma
- Plasma parameters
- Single particle motion
- Plasma as a fluid
- Diffusion models
- Plasma production (DC, CAPACITIVE, INDUCTIVE, RF WAVE-HEATING)
- Plasma acceleration: magnetic nozzles

Chemical vs Electric Propulsion

- In chemical rockets, thrust is obtained by nozzled expansion of a propellant previously heated by its own chemical reaction.
- In EP, thrust is obtained by acceleration of gases by electrical heating and/or by electric and magnetic body forces.

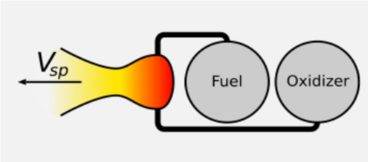

Figure: Chemical rockets

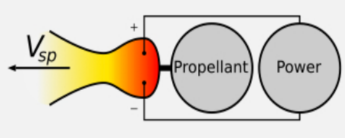

Figure: Electric thruster

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- Impulsive maneuvers

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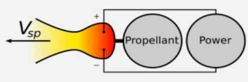

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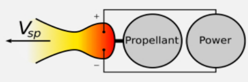

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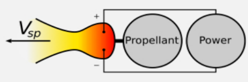

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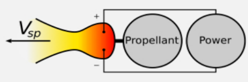
- **Attainable exhaust velocity limited by available power on-board**



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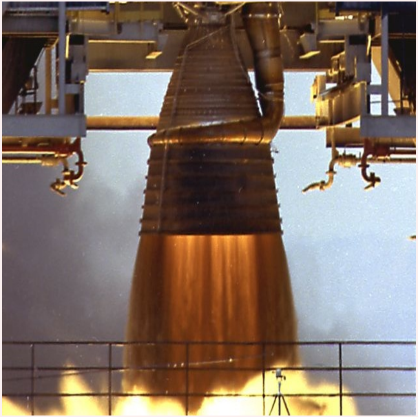

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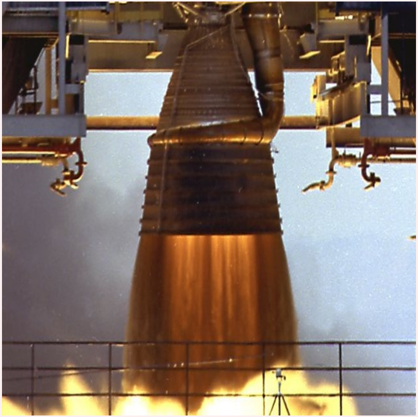

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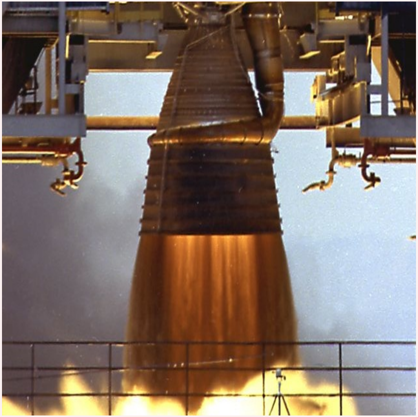

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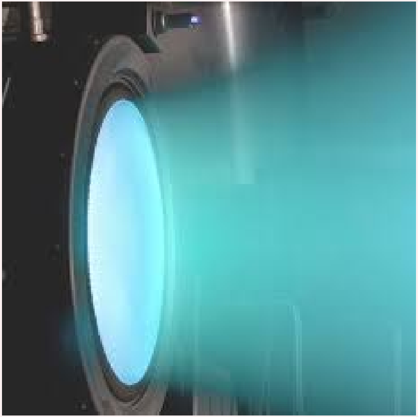

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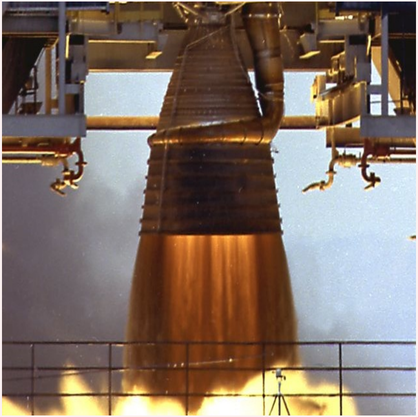

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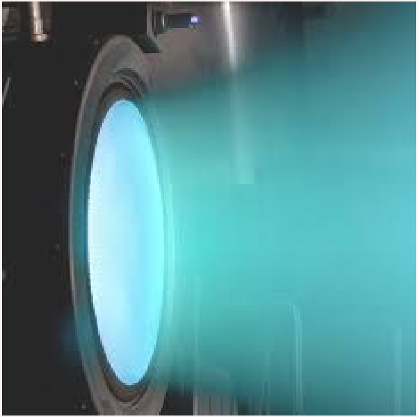

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Introduction: why electric propulsion?

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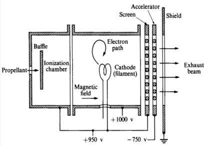
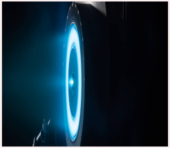
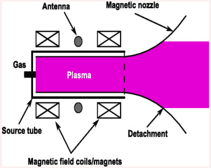
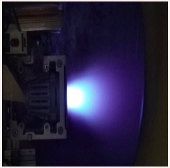




HELICON PLASMA THRUSTER





All electric thrusters require separate energy source. One should consider also the power plant weight!

Optimum I_{sp}

On a mission of given thrusting time Δt and constant thrust F

$$\Delta m = \dot{m}\Delta t = \frac{F\Delta t}{u_e} = \frac{F\Delta t}{I_{sp} g_e} \qquad (1)$$

assuming power plant mass with linear dependence on power and a constant conversion efficiency to thrust power η:

$$m_p = \alpha P = \frac{\alpha F u_e}{2\eta} = \frac{\alpha F I_{sp} g_e}{2\eta} \qquad (2)$$

The optimum I_{sp} that maximise the deliverable payload fraction is:

$$\frac{d(\Delta m + m_p)}{I_{sp}} = 0 \rightarrow \widehat{I_{sp}} = \frac{1}{g_e}\sqrt{\frac{2\eta\Delta t}{\alpha}}$$

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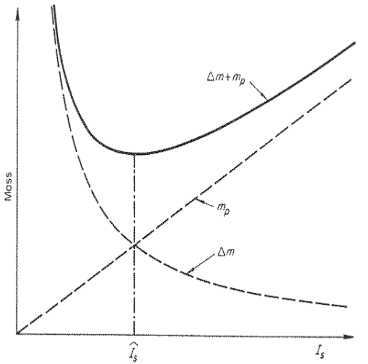

Fig. 1-1 Dependence of propellant mass Δm and power supply mass m_p on specific impulse I_s for a given constant-thrust mission.

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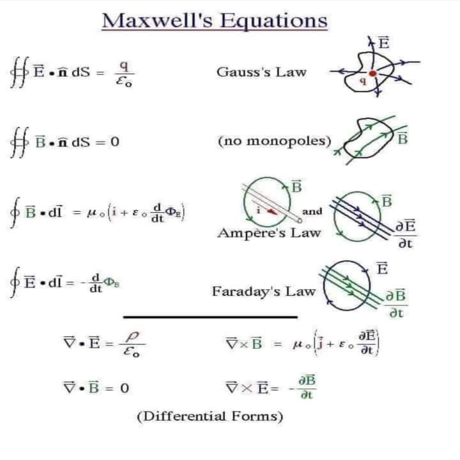
So why plasma in space propulsion?

The specific energy that can be deposited into a plasma beam is orders of magnitude larger than the specific chemical energy of known fuels

Maxwell's equations

Plasma strongly depends on electromagnetism, so let's firstly review that:



These equations are accompanied by the Lorentz's equation: $\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{u} \wedge \boldsymbol{B})$

It is said that 99% of the matter in the universe is in the plasma state; that is, in the form of an electrified gas with the atoms dissociated into positive ions and negative electrons

Saha equation

"The Saha equation tells us the amount of ionization to be expected in a gas in thermal equilibrium"

$$\boxed{\frac{n_i}{n_n} \simeq 2.4 * 10^{21} \frac{T^{\frac{3}{2}}}{n_i} e^{-\frac{U_i}{K_b T}}} \tag{4}$$

where:

- n_i = number density of ionized atoms $[m^{-3}]$
- n_n = number density of neutral atoms $[m^{-3}]$
- T = gas temperature [K]
- K_b = Boltzmann's constant
- U_i = ionization energy of the gas [eV]

Ex: for ordinary air at room temperature:

$$T \simeq 300K,\, n_n \simeq 3 * 10^{25} m^{-3},\, U_i = 14.5eV\,(Nitrogen) \rightarrow \boxed{\frac{n_i}{n_n} \simeq 10^{-122}}$$

Definition of plasma

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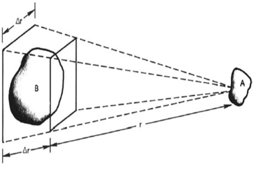

Fig. 1.1 Illustrating the long range of electrostatic forces in a plasma

Collective behaviour:
Motion depends on local and remote conditions as well

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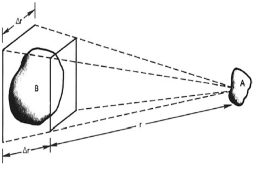

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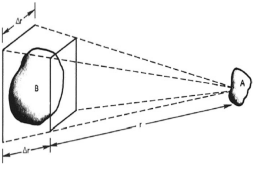

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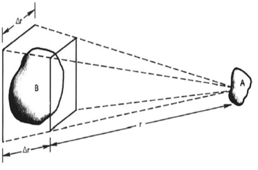

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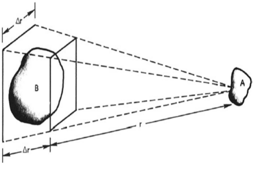

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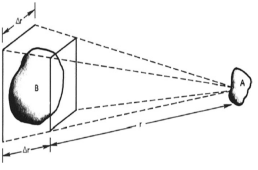

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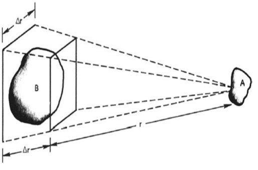

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Therefore, elements of plasma exert a force on one another even at large distances!

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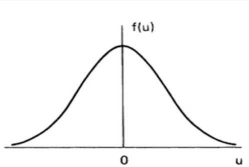

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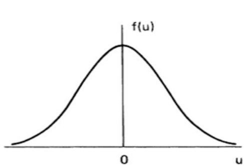

Figure: $1D$ Maxwellian distribution

In one dimension this reads:

$$\boxed{f(u) = n\sqrt{\frac{m}{2\pi K_b T}}e^{\frac{-\frac{1}{2}mu^2}{K_b T}}} \tag{5}$$

And the particle density n is given by:

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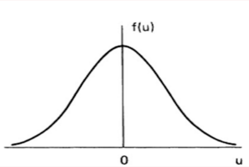

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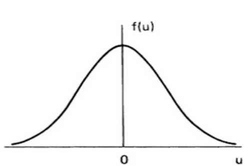

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The width of the distribution is characterized by the constant T, which we call temperature.

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To see the exact meaning of T, let's compute the average kinetic energy of particles in the distribution:

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and the average kinetic energy becomes:

$$E_{av} = \frac{\frac{1}{2} m A v_{th}^3 \int_{-\infty}^{\infty} [e^{-y^2}] y^2 dy}{A v_t h \int_{-\infty}^{\infty} e^{-y^2} dy} = \frac{\frac{1}{2} m A v_{th}^3 \frac{1}{2}}{A v_{th}} = \frac{1}{4} m v_{th}^2 = \frac{1}{2} K_b T \tag{9}$$

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Concept of temperature

To see the exact meaning of T, let's compute the average kinetic energy of particles in the distribution:

$$E_{av} = \frac{\int_{-\infty}^{\infty} \frac{1}{2} m u^2 f(u) du}{\int_{-\infty}^{\infty} f(u) du} \tag{7}$$

Defining $v_{th} = \sqrt{\frac{2K_bT}{m}}$, $y = \frac{u}{v_{th}}$ and $A = n\sqrt{\frac{m}{2\pi K_b T}}$, we can rewrite the distribution function:

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Thus the average kinetic energy is:

$$E_{av} = \frac{1}{2} K_b T \tag{10}$$

Concept of temperature

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$$\boxed{E_{av} = \frac{3}{2} K_b T} \tag{11}$$

which tells us that $E_{av} = \frac{1}{2} K_b T$ per degree of freedom.

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For $KT = 1eV = 1.6x10^{-19} J$, we have

$$T = \frac{1.6x10^{-19}}{1.38x10^{-23}} = 11600$$

Thus the conversion factor is

$$1eV = 11600K$$

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In a plasma

1. several temperatures can establish at the same time: ions and electrons have separate Maxwellian distributions with different temperatures T_i and T_e

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   - collision rate among ions or electrons themselves is larger than the rate between ions and electrons
2. when there is a magnetic field **B**, even single species (e.g. ions) can have two temperatures
   - Lorentz forces acting on an ion along and perpendicular to **B** are different, and so velocities along and perpendicular to **B** belong to different Maxwellian distribution with temperatures $T_{||}$ and $T_{\perp}$

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Plasma has the characteristic of shielding out electric potentials that are applied to it.

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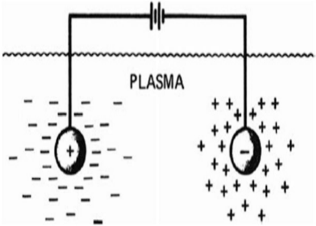

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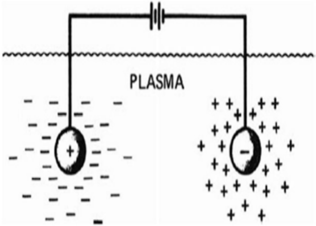

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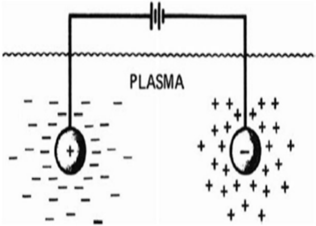

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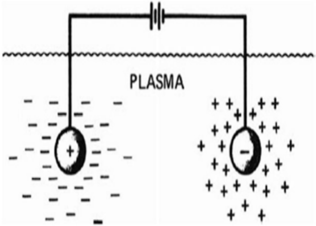

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  - perfect shielding: no electric field is felt in the portion of plasma outside the clouds
- if the plasma presents finite temperature, particles at the edge of the cloud have enough thermal motion to escape the electrostatic potential well

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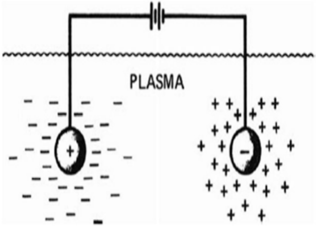

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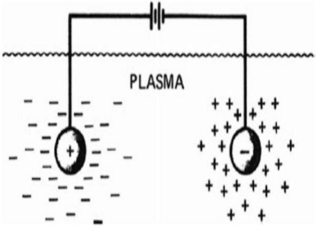

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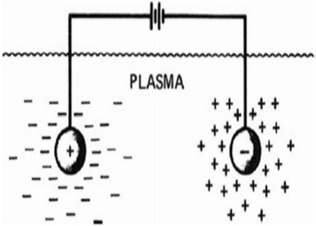

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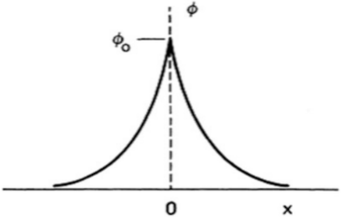
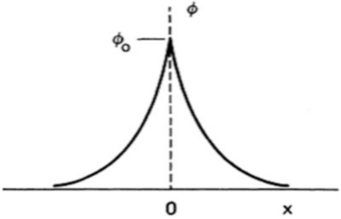
- shielding is not complete
- potentials of order $K_b T / e$ "leaks" into the plasma $\rightarrow$ finite electric fields can establish



Debye shielding

Assumption: $\frac{M_{ion}}{m_{electron}} \simeq \infty \rightarrow$ ions don't move in electron's dynamic time-frame;



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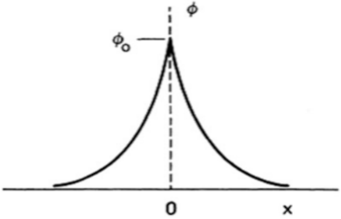

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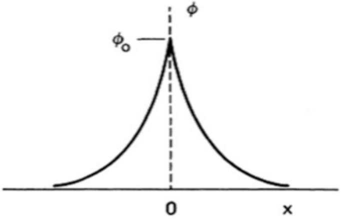

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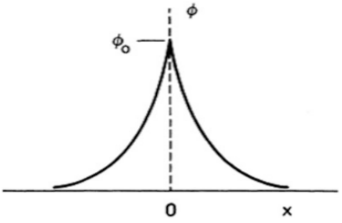

Where the potential energy is large there are fewer particles: not all of theme have enough energy to get there!

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- integrating over u, and setting $q = -e$:

$$n_e = n_\infty e^{\frac{e\phi}{K_b T_e}} \quad (14)$$

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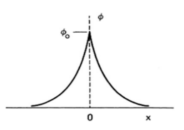
Debye shielding

- substituting for n_i and n_e in the Poisson eq. we get:

$$\epsilon_0 \frac{d^2\phi}{dx^2} = en_\infty\{[e^{\frac{e\phi}{K_b T_e}}] - 1\} \tag{15}$$



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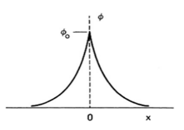

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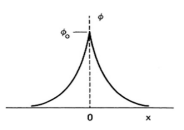

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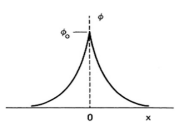

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- keeping only the linear term we get:

$$\epsilon_0 \frac{d^2\phi}{dx^2} = \frac{n_\infty e^2 \phi}{K_b T_e} \tag{17}$$

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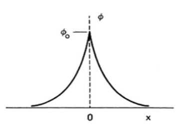
Debye shielding

- solving the equation we get:

$$\phi = \phi_0 e^{-\frac{|x|}{\lambda_D}} \tag{18}$$



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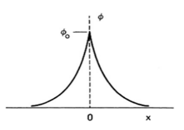

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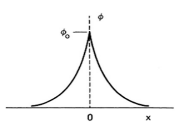

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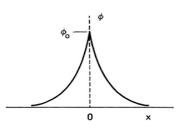

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- the Debye length is a measure of the shielding distance or thickness of the cloud, which is called the *sheath* region
- some useful relations:
  $\lambda = 69\sqrt{T_e/n}$ $[m]$, T in $[K]$
  $\lambda = 7430\sqrt{K_b T_e/n}$ $[m]$, T in $[eV]$

Debye shielding: further considerations

Debye length

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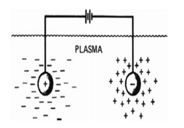

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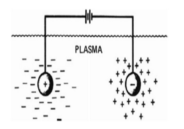

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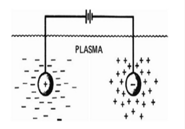

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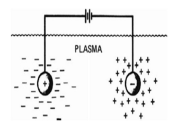

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We can define the "quasineutrality" concept more properly:

- if the dimension L of a system is much larger than λ_D, then whenever local concentrations of charges or external potentials arise, these are shielded in a distance short compared with L
- Outside of the sheath, $\nabla^2\phi$ (i.e. $\nabla \cdot E = -\nabla^2\phi$) is very small, and $n_i \simeq n_e \simeq n$, where n is a common density called *plasma density*.

A criterion for an ionized gas to be considered a plasma is that $\lambda_D \ll L$

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Debye shielding: further considerations

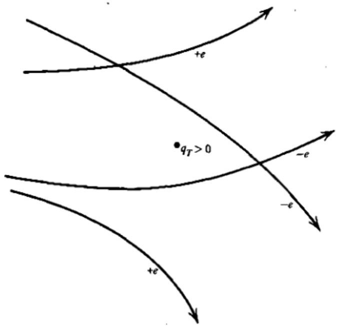

Fig. 1.1 A test charge in a plasma attracts particles of opposite sign and repels particles of like sign, thus forming a shielding cloud that tends to cancel its charge.

Debye length:

$$\boxed{\lambda_D = \sqrt{\frac{\epsilon_0 K_b T_e}{n_\infty e^2}}}$$

The plasma parameter

The Debye shielding is a valid concept only if there are many particles in the sheath. The number of particles in a Debye sphere is given by

$$N_D = \frac{4}{3}\pi n \lambda_D^3 = 1.38 * 10^6 \sqrt{T_e^3/n^2} \tag{20}$$

Moreover the condition of "*collective behavior*" requires

$$\boxed{N_D \ggg 1}$$

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Plasma frequency

Consider an hypothetical slab of plasma with L thickness

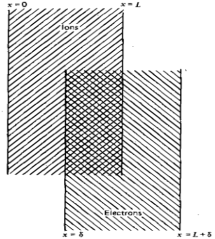

Fig. 1.2 Plasma slab model used to calculate the plasma frequency.

- $\frac{M_{ion}}{m_{electron}} \simeq \infty \rightarrow$ ions still, electrons mobile

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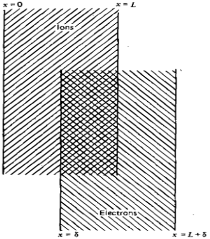

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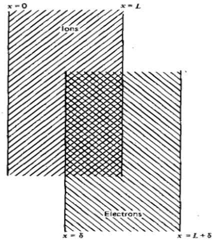

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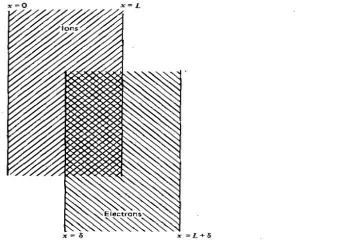

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- $\delta \ll L$

Plasma frequency

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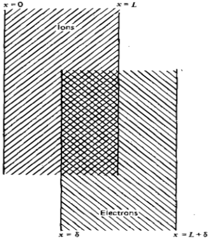

Fig. 1.2 Plasma slab model used to calculate the plasma frequency.

- $\frac{M_{ion}}{m_{electron}} \simeq \infty \rightarrow$ ions still, electrons mobile
- $n_i \simeq n_e \simeq n$
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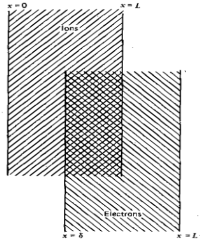

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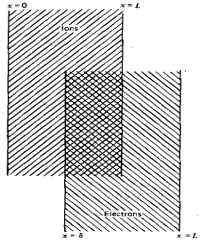

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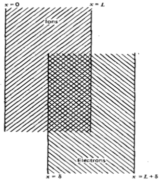
An electric field will generate causing the electron slab to be pulled toward the ions. When the electrons overlap the ions, the net force is zero, but the momentum gained make the slab overshoot. The global effect is an *harmonic oscillation*.

Plasma frequency

We wish to derive the frequency of this oscillator:



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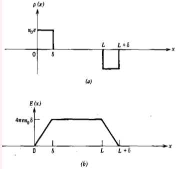

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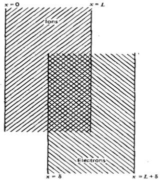

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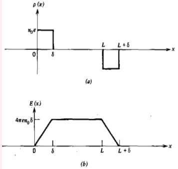

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$$(nm_e L)\ddot{\delta} = -4\pi n^2 e^2 \delta L \tag{21}$$

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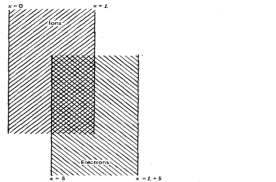

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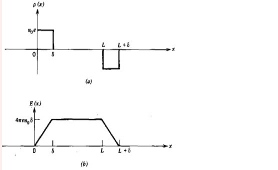

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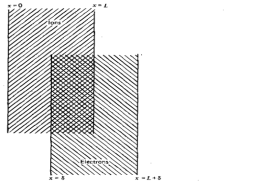

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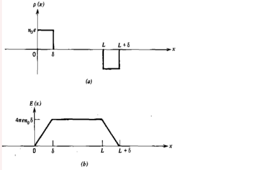

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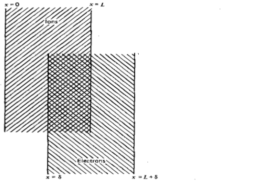

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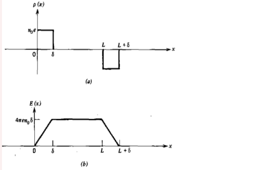

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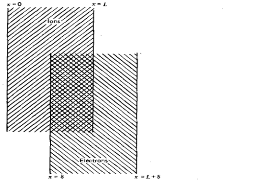
$$\boxed{\omega_e = \sqrt{\frac{4\pi ne^2}{m_e}}} \tag{23}$$

called the *electron plasma frequency*. And in the same way the *ion plasma frequency* can be found for ions (charge = Ze)

$$\boxed{\omega_i = \sqrt{\frac{4\pi n_i Z^2 e^2}{M_i}}} \tag{24}$$

Plasma frequency



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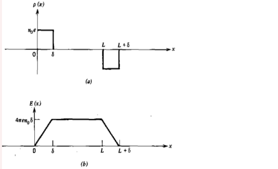

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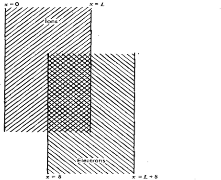

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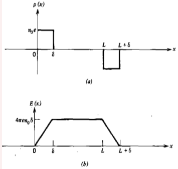

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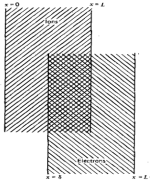

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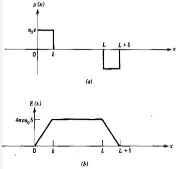

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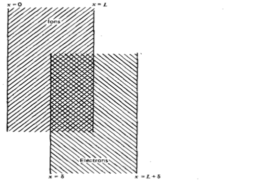

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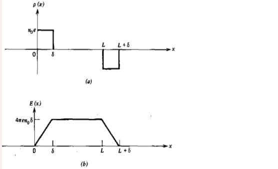

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For a gas to behave like a plasma rather than a neutral gas, if ω is the frequency of typical plasma oscillations and τ the mean time between collisions with neutral atoms, we require:

$$\boxed{\omega\tau > 1} \tag{26}$$

CRITERIA FOR PLASMAS

The conditions for an ionized gas to be considered a plasma are therefore:

$$\lambda_D \ll L$$

$$N_D \ggg 1$$

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Plasmas can be characterized by two parameters, n and $K_b T_e$:

- n varies from 10^6 to 10^{34} $[m^{-3}]$
- $K_b T_e$ varies from 0.1 to $10^6 eV$

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2. Ionosphere: $n = 10^{11}$, $K_b T_e = 0.05$
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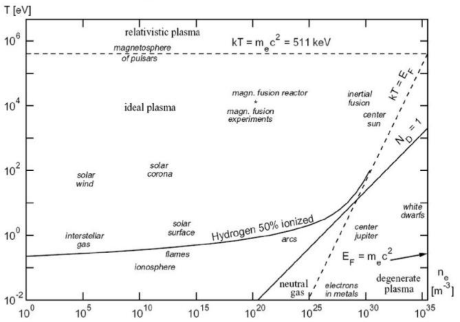

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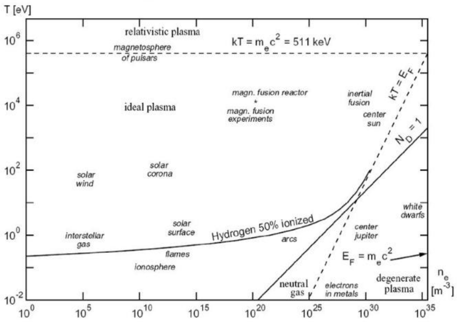

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Convince yourself that these are plasmas!

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The motion of individual charged particles in electromagnetic fields is referred as the plasma orbit theory and provides useful tools in handling problems involving plasmas

Single particle motion

A particle of mass m and charge q moving in an electromagnetic field satisifes the equation of motion:

$$m\frac{d\boldsymbol{u}}{dt} = q(\boldsymbol{E} + \boldsymbol{u} \wedge \boldsymbol{B}) \tag{27}$$

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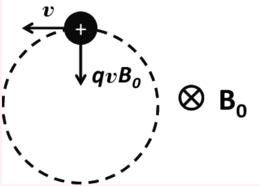
- there may be present external magnetic fields imposed on the plasma
- this equation shall be corrected for accounting the well-known phenomenon of *radiation reaction*: a charged particle when accelerated emits electromagnetic radiation, which involves an energy loss, and thus an effective 'drag' term should be added in the above equation. For most applications this phenomenon happens to be unimportant and so we can neglect it

Cyclotronic motion

Consider a uniform magnetic field $\boldsymbol{B}$ and a particle moving with velocity $\boldsymbol{u}_\perp$ perpendicular to $\boldsymbol{B}$

The equation of motion reads:

$$m\frac{d\boldsymbol{u}_\perp}{dt} = q\boldsymbol{u}_\perp \wedge \boldsymbol{B} \tag{28}$$



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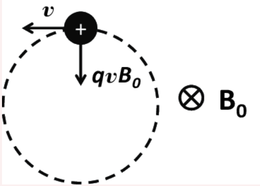

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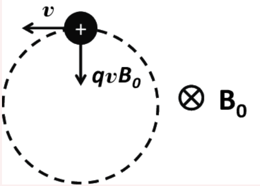
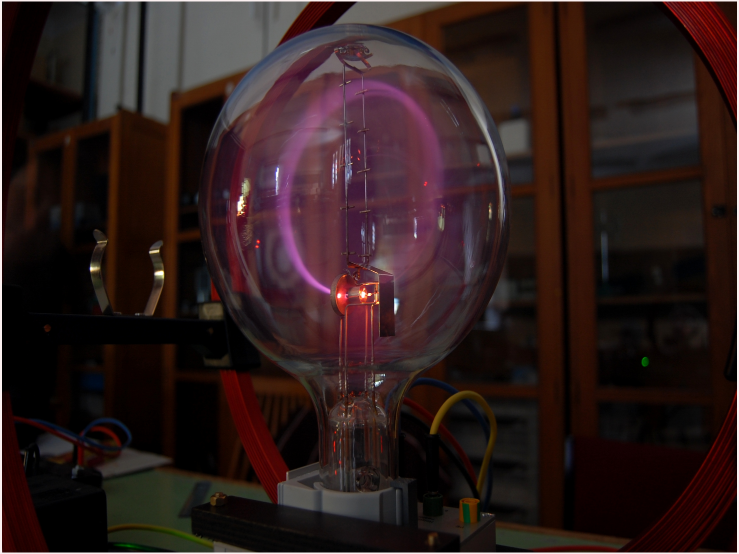


- solution is a circular motion lying on a plane transverse to $\boldsymbol{B}$
- the circular frequency, called cyclotron frequency, is:

$$\omega_c = \frac{|q|\boldsymbol{B}}{m} \quad (29)$$

Cyclotronic motion



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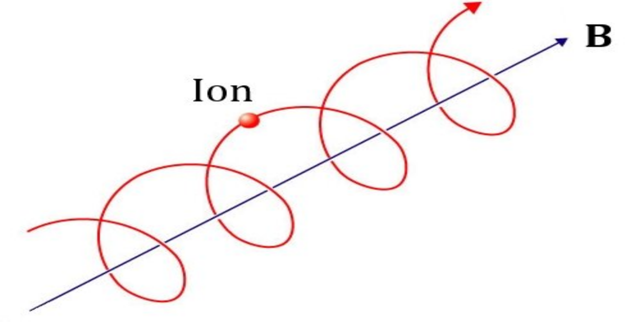
Guiding centre theory

If the particle has a component of velocity $\boldsymbol{u}_{||}$ parallel to $\boldsymbol{B}$, the solution shows that the magnetic field does not affect this component: this component leads to a uniform translation of the circular trajectory with velocity $\boldsymbol{u}_{||}$, making the path helical



Particle motion can be thought as the result of two combining motions:

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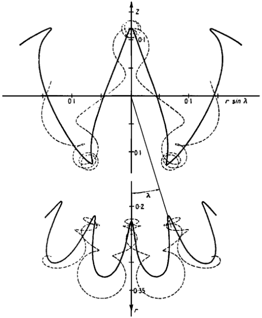
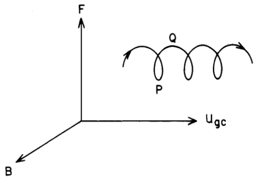
Even when a charged particle moves in a non-uniform field the motion it is possible to break up the motion in the same manner



Motion of a charged particle in a dipole field calculated by Stormer from the full equations (dashed lines) and by Alfven using the guiding centre theory (solid lines indicate g.c. trajectory)

The effect of a perpendicular force

Consider now a uniform force $\boldsymbol{F}$ acting upon a particle in a direction perpendicular to a uniform magnetic field $\boldsymbol{B}$



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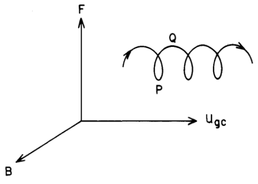

The equation of motion reads:

$$m\frac{d\boldsymbol{u}}{dt} = \boldsymbol{F} + q\boldsymbol{u} \wedge \boldsymbol{B} \tag{30}$$

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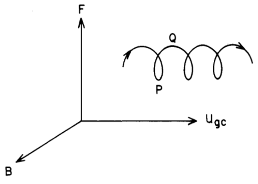

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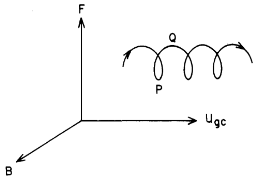

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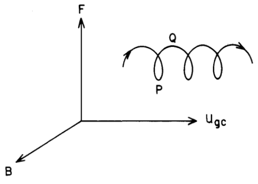

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It is clear that there is a drift $\boldsymbol{u_{gc}}$ of the guiding centre in a direction perpendicular to both $\boldsymbol{B}$ and $\boldsymbol{F}$

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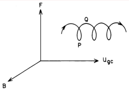
The effect of a perpendicular force

We want to derive an expression for $\boldsymbol{u_{gc}}$:

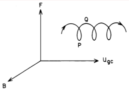


From the eq. of motion it is easy to see that there is no acceleration parallel to $\boldsymbol{B}$, so again if there is any initial velocity in that direction it remains unaffected.
We focus on the velocity on the plane perpendicular to $\boldsymbol{B}$:

$$\boldsymbol{u}_{\perp} = \boldsymbol{u}_c + \boldsymbol{u}_{gc} \tag{31}$$

The effect of a perpendicular force

We want to derive an expression for $\boldsymbol{u_{gc}}$:



From the eq. of motion it is easy to see that there is no acceleration parallel to $\boldsymbol{B}$, so again if there is any initial velocity in that direction it remains unaffected.

We focus on the velocity on the plane perpendicular to $\boldsymbol{B}$:

$$\boldsymbol{u}_\perp = \boldsymbol{u_c} + \boldsymbol{u_{gc}} \tag{31}$$

- where $\boldsymbol{u_c}$ is the velocity of the circular motion satisfying:

$$m\frac{d\boldsymbol{u_c}}{dt} = q\boldsymbol{u_c} \wedge \boldsymbol{B} \tag{32}$$

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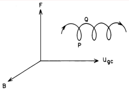

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- and $\boldsymbol{u_{gc}}$ is the drift of the guiding centre

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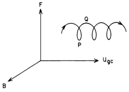
Positive and negative particles will drift in opposite direction if F does not depend on q (e.g. gravity)



$\boldsymbol{E} \perp \boldsymbol{B}$

If the perpendicular force is due to an electric field $\boldsymbol{F} = q\boldsymbol{E}$ we have:

$$\boxed{\boldsymbol{u}_{\boldsymbol{ExB}} = \frac{\boldsymbol{E} \wedge \boldsymbol{B}}{B^2}} \tag{36}$$

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The drift is the same for all charged particles depending only on $\boldsymbol{E}$ and $\boldsymbol{B}$

Gravity drift

If the perpendicular force is due to gravity $m\boldsymbol{g}$ we have:

$$\boxed{\boldsymbol{u}_{\boldsymbol{g}} = \frac{m}{q}\frac{\boldsymbol{g} \wedge \boldsymbol{B}}{B^2}} \tag{37}$$

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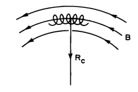
Under gravitational force ions and electrons drift in opposite directions, so there is a net current density in the plasma given by:

$$\boldsymbol{j} = n(M + m)\frac{\boldsymbol{g} \wedge \boldsymbol{B}}{B^2} \tag{38}$$

$\boldsymbol{u_g}$ is usually negligible

Curvature drift

Consider now a constant magnetic field region and a constant radius of curvature R_C.



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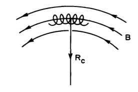

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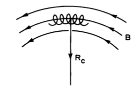

Since a charge gyrates around a field line, there should be an average centrifugal force acting on it as it moves along the field line:

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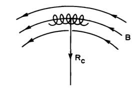

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and the guiding centre velocity is:

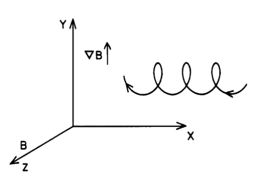
$$\boxed{\boldsymbol{u}_C = -\frac{mu_\parallel^2}{qB^2}\frac{\boldsymbol{R}_C \wedge \boldsymbol{B}}{R_C^2}} \tag{40}$$

Note that

Currents may rise as electrons and ions drift in opposite directions

Gradient drift

Consider a positive charge moving on a magnetic field $B(y)\hat{\boldsymbol{e}}_z$ of which the strength is varying along y direction.



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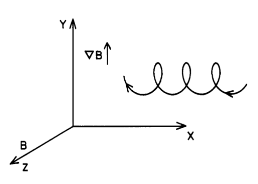

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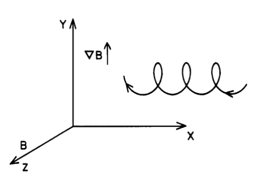

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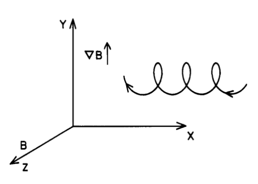

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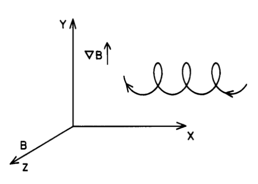

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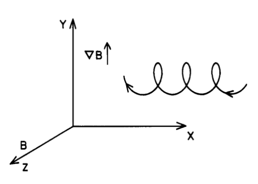

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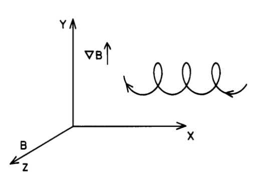

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$$B_z(y) = B_0 + y\frac{dB_z}{dy} \tag{41}$$

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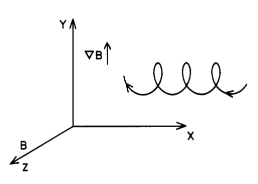

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Hence the istantaneous Lorentz force is:

$$F_y = -qu_x[B_0 + y\frac{dB_z}{dy}] \tag{42}$$

Gradient drift

We average the force:
Noticing that B_0 is constant, $\overline{u_x} = 0$ and $\overline{u_x y} = \pm\frac{1}{2}u_\perp r_L$
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which is called the gradient drift

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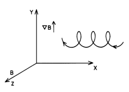

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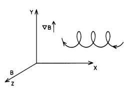

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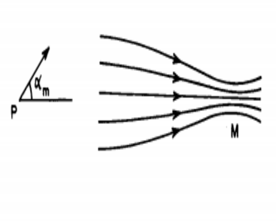
Note that

- The drift has opposite sign for opposite charges
- Ions and electrons drifting in opposite directions give rise to currents!

Magnetic mirrors

Consider a magnetic configuration to be symmetric around a field line in such a way that the field strength varies moving along the central field line.



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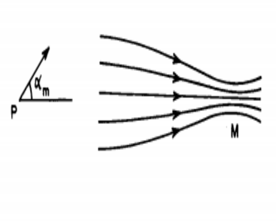

The axial component is predominant, but there is a small radial component B_r which can be found from $\nabla \cdot B = 0$

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_r) + \frac{\partial B_z}{\partial z} = 0 \tag{45}$$

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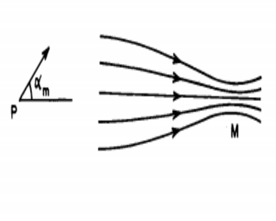

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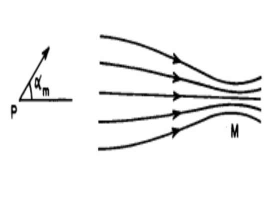

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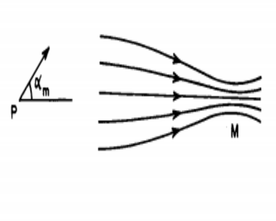

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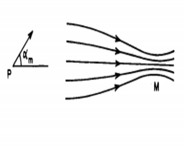
For a particle gyrating around the central line, the z-component of the Lorentz force is:

$$F_z = -q u_\theta B_r \tag{48}$$



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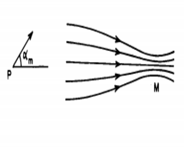

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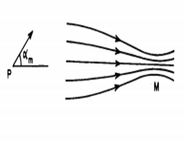

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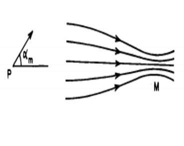

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$$\mu = \pm \frac{q}{2} u_\perp r_L = \frac{\frac{1}{2} m u_\perp^2}{B} = \pm \frac{\omega_c q}{2\pi} \pi r_L^2 \tag{50}$$

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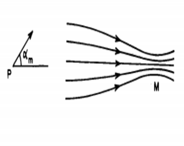
Magnetic mirrors



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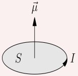
It is known that the current encircling an area multiplied by that, is the **magnetic moment**. $-\frac{\omega_c q}{2\pi}$ is the current associated with the gyrating charge q, and πr_L^2 the area, thus μ is the **magnetic moment** of the gyrating particle.



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Since we have:

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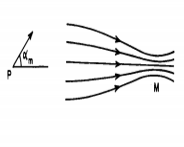

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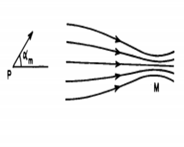
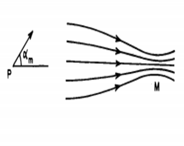


Magnetic mirrors



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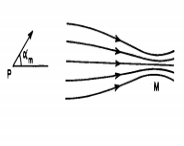

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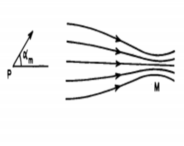

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Magnetic mirrors

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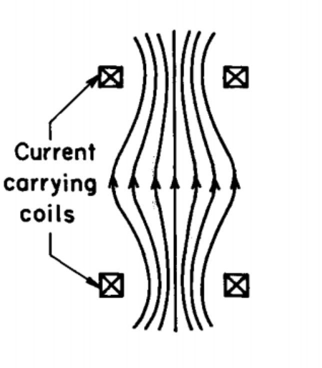
- A charged particle moving along the symmetry axis is unaffected by magnetic forces, and so is not reflected by the magnetic mirror

Magnetic mirrors: the magnetic bottle

Consider a region with two magnetic mirrors at the two ends:



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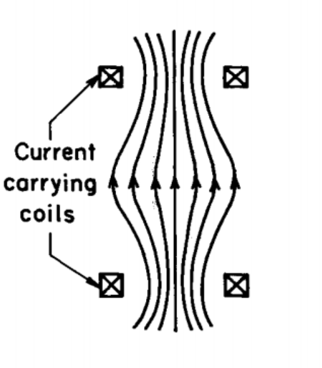

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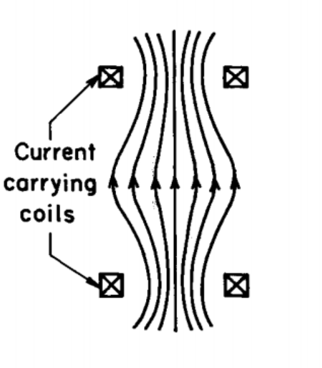

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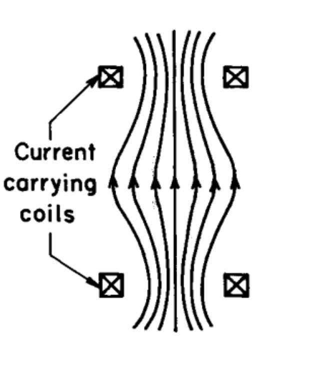

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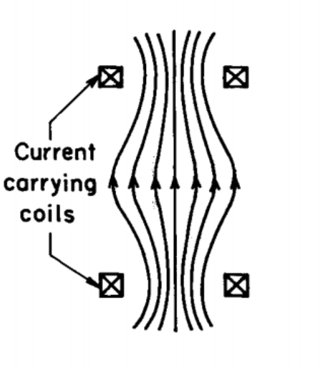

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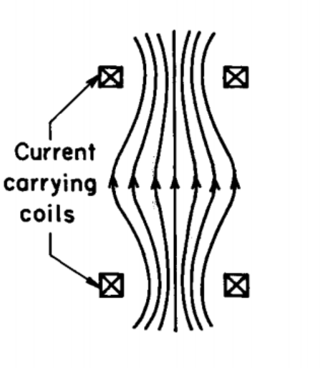

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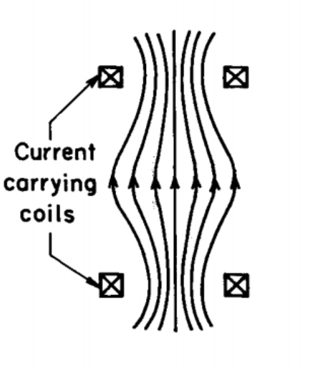

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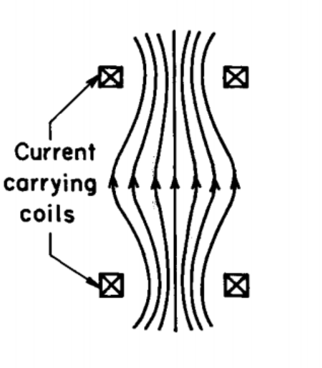

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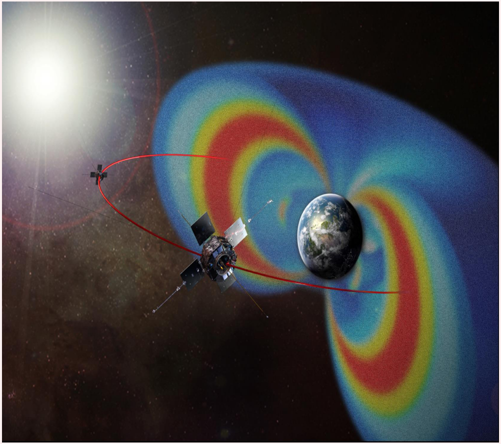
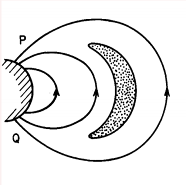
All the other charged particles, shall not pass!

Magnetic mirrors: Van Allen belt





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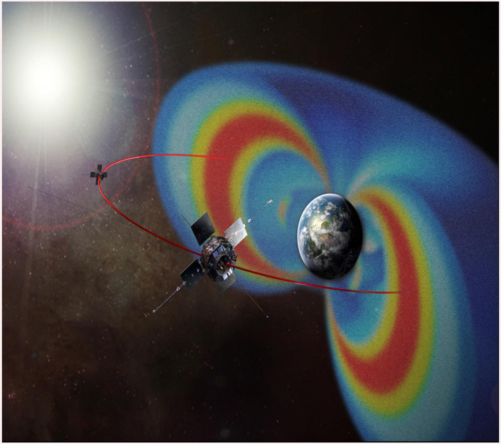
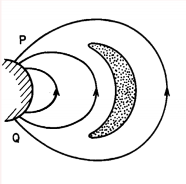
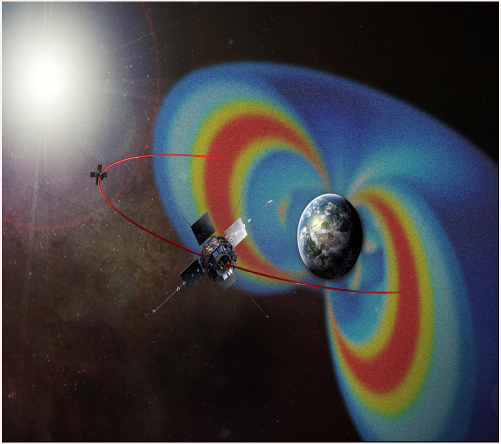
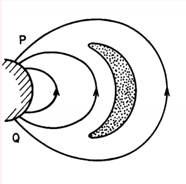




- Curvature drift

Magnetic mirrors: Van Allen belt





- Curvature drift
- Gradient drift

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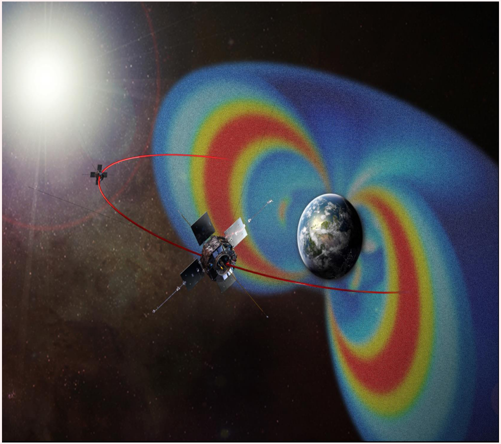
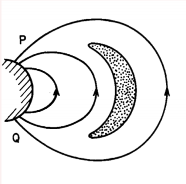
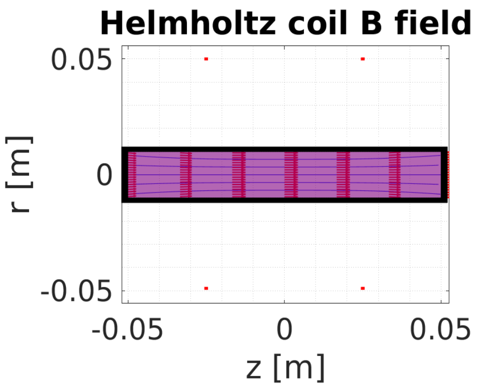
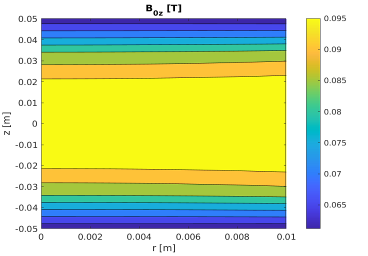


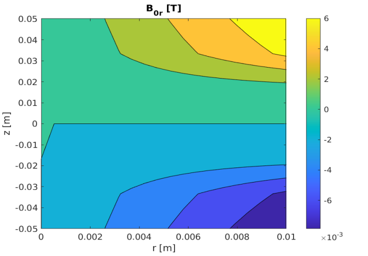


- Curvature drift
- Gradient drift
- Electrons and ions drift in opposite direction: azimuthal currents arise!

Plasma confinement: Helmholtz coils







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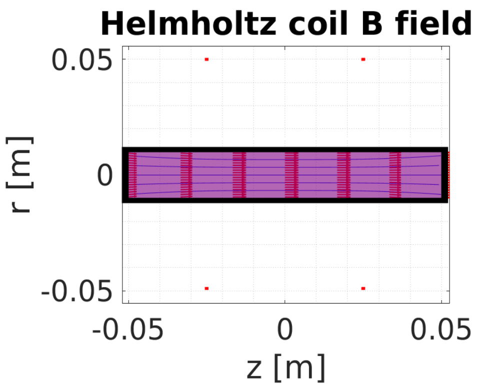

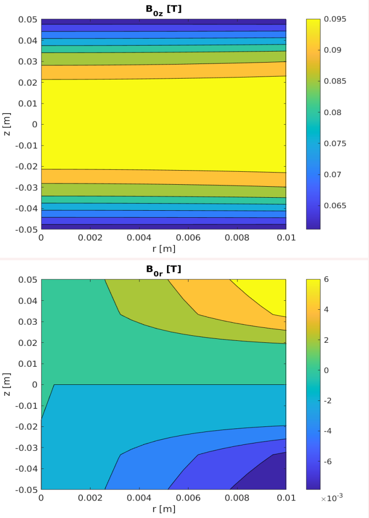

- Can be done with coils or permanent magnets

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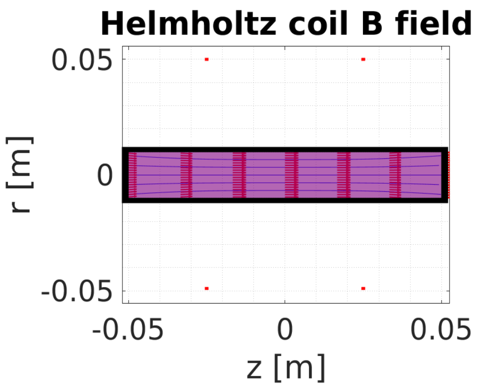
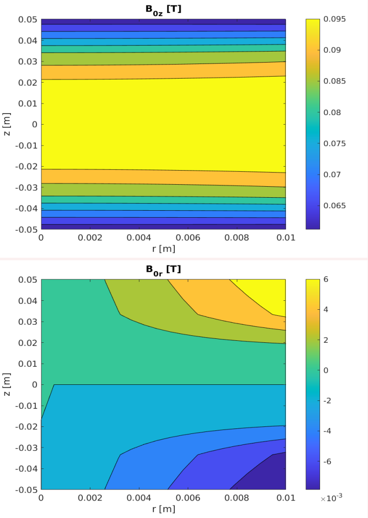
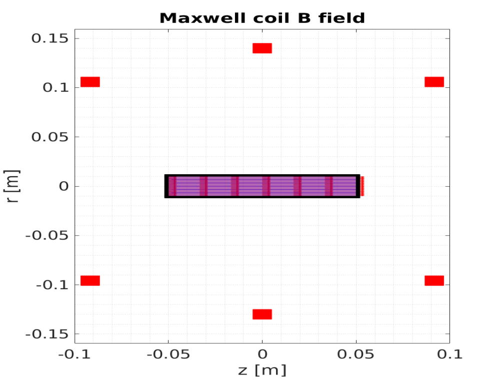
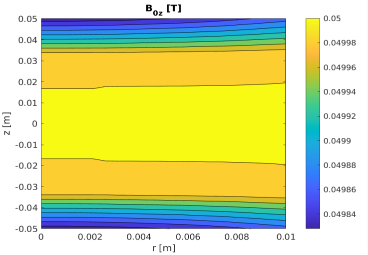


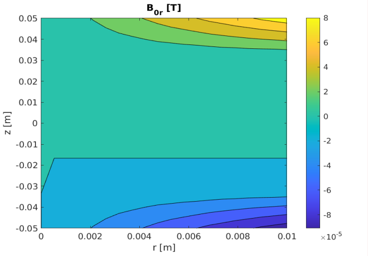


- Can be done with coils or permanent magnets
- Provides uniform axial component of ***B***

Plasma confinement: Maxwell coils







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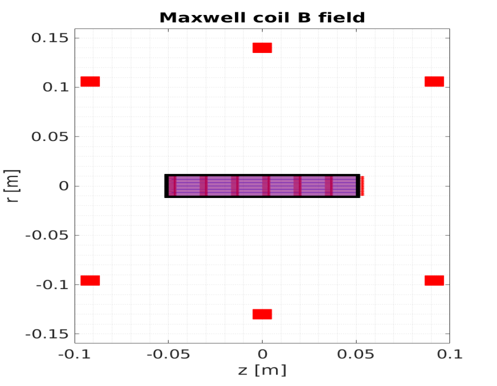
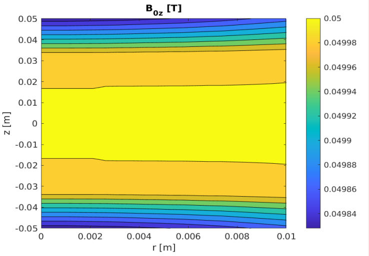
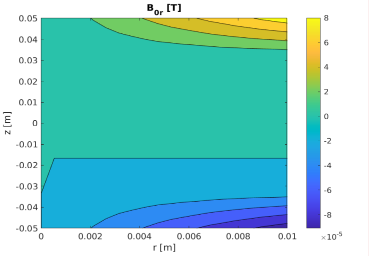
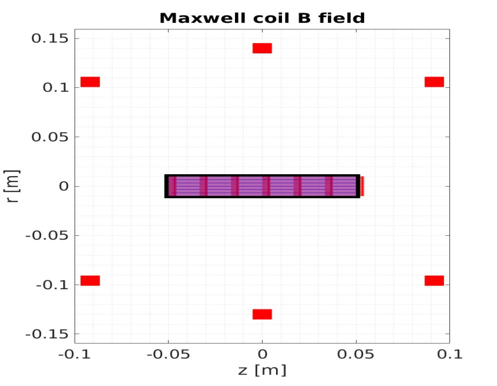


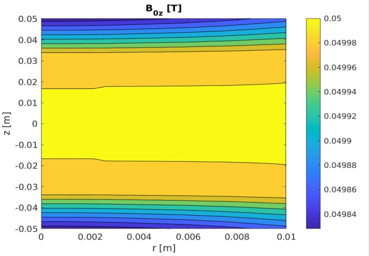


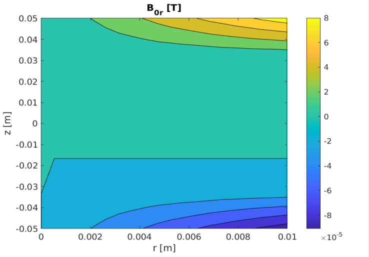


- Better than Helmholtz

Plasma confinement: Maxwell coils







- Better than Helmholtz
- Used in industrial plasma sources and in propulsion