Mathematical Methods I

Assignment 3: Fourier Analysis - Part II

Total marks: 25

All questions are compulsory. Please hand in your answer sheets and submit your codes to one of the tutors no later than **5pm Friday 28 August 2015**. You may use any programming language, provided you can demonstrate that your code compiles and runs without errors. (For newbies, the recommended language is python.)

1 Discrete Fourier Transform *(25 marks)*

You are given N equally spaced samples of a function $f(t)$ in the range $-T/2 \le t < T/2$, i.e., at the values $t_n = n\Delta t$ where $\Delta t = T/N$ and $-N/2 \le n \le N/2 - 1$. (You can assume that N is even.)

Consider the Discrete Fourier Transform (DFT) of these samples given by

$$\tilde{f}_m = \sum_{n=-N/2}^{N/2-1} f(n\Delta t)\, \mathrm{e}^{-2\pi i m n/N}\,, -N/2 \le m \le N/2 - 1\,.$$

1.1 Coding

Write a numerical program to calculate $\tilde{f}_m$ for the following combinations of $f(t)$, N and T:

1. $f(t) = \mathrm{sinc}(2\pi t)$
   - $T = 2$; $N = 8$
   - $T = 10$; $N = 40$
   - $T = 40$; $N = 160$
2. $f(t) = \mathrm{e}^{-t^2/2}/\sqrt{2\pi}$
   - $T = 2$; $N = 8$
   - $T = 10$; $N = 40$
   - $T = 40$; $N = 160$

1.2 Plotting

- Plot the real and imaginary parts of $\tilde{f}_m \Delta t$ against $\omega = 2\pi m/T$. (You should have 6 curves each for the real and imaginary parts.) Make sure you understand the behaviour of the curves as T and N change.
- Are any of the imaginary parts non-zero? Why?
- Also plot the analytical expressions for the Fourier integral transforms $\tilde{f}(\omega)$ of the functions $f(t)$, which you derived in Assignment 1. Why does it make sense to compare $\tilde{f}(\omega)$ with the product $\tilde{f}_m \Delta t$ and not with $\tilde{f}_m$ alone?