# Poisson Stats

$$P(x, \lambda) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$ ≡ Prob of a distribution having value $x$, given an expectation (mean) value $\lambda$

Probability of finding 0 events

$$P(0) = \frac{e^{-\lambda} \lambda^{0}}{0!} = e^{-\lambda}$$

90% confidence level is an accepted level of quoting result. Hence if we find zero KN, what is the expectation value $\lambda$, that would imply we have set at 90% confidence limit.

Or, what is the value of $\lambda$, such that $P(0) = 0.1$

$$0.1 = e^{-\lambda}$$

$$\lambda = -\ln(0.1) = 2.3$$

Hence we need to work out what the KN rate is within our volume & time limited survey, which would give us a detected number of 2.3 KN. That KN rate should be in units of number of KN $Gpc^{-3}\ yr^{-1}$

$$N_{KN} = \varepsilon V T R_{KN}$$ where

$N_{kn}$ = number of KN detected
$\varepsilon$ = efficiency of recovery
$V$ = volume sampled
$T$ = time of survey
$R_{KN}$ = true KN rate

We define $V$ and $T$ through our choices of distance limit and start and end time of the survey. We calculate $\mathcal{E}$ for those parameters, hence $\mathcal{E}(V, T)$.
We set $N_{KN} = 2.3$ and hence can determine $R_{KN}$

As you suggested, run efficiency calculation in increments of 5 or 10 Mpc from 0 to 100 Mpc.
For each of these, you will calculate $V$.
Can experiment with 2 values of $T$, e.g.
57377 to present (includes early 4KO only phase)
57800 to present (full 4KO + mKO operations)

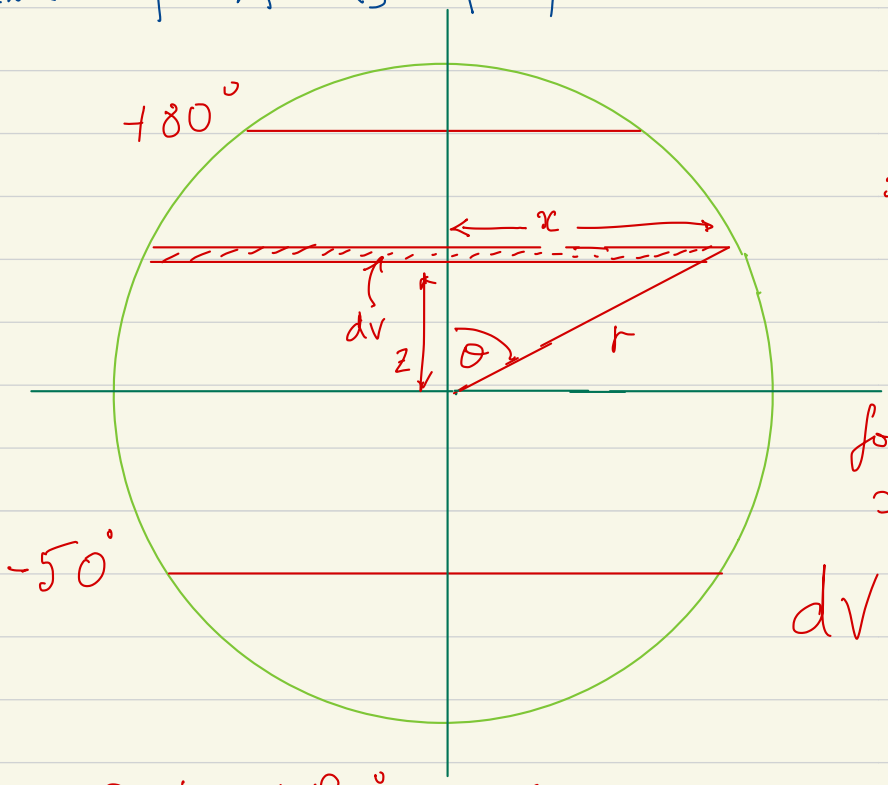
$\mathcal{E}(V, T)$ should decrease as $V$ increases. Therefore increasing $R_{KN}$. But that will be compensated by the increase in $V$ which will reduce $R_{KN}$. Produce a table and plots to see this co-dependence.

| D Mpc | V $Gpc^3$ | T yr | $\mathcal{E}$ | $R_{KN}$ ($90\%$ confidence limit i.e. $N_{KN} = 2.3$) $Gpc^{-3}\ yr^{-1}$ |
|---|---|---|---|---|
| 5 | | | | |
| 10 | | | | |
| ⋮ | | | | |
| 90 | | | | |
| 95 | | | | |
| 100 | | | | |

Volume enclosed by 4 MOST foot. survey footprint - see next page.

this column will represent the upper limits to the true KN rate. i.e. we can say the true rate of KN is $< R_{KN}\ Gpc^{-3}\ yr^{-1}$ at 90% confidence.

# Volume of ATLAS footprint




$$x^2 + z^2 = r^2$$

$$x = \sqrt{r^2 - z^2}$$

for unit sphere

$$x = \sqrt{1 - z^2}$$

$$dV = \pi (1 - z^2)\, dz$$

Vol from 0 to +80° $\quad z = 0$ to $z = \cos(90 - 80)$

$$V_{80} = \int_0^{\cos 10} \pi (1 - z^2)\, dz = \pi \left[ z - \frac{z^3}{3} \right]_0^{\cos 10^\circ}$$

Vol from 0 to -50 $\equiv$ Vol from 0 to +50

$$V_{50} = \int_0^{\cos 40} \pi (1 - z^2)\, dz = \pi \left[ z - \frac{z^3}{3} \right]_0^{\cos 40^\circ}$$