

# A Mathematical Glossary

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This is a list of words, phrases and sayings that are used by English-speaking higher mathematicians in writing or in speech. As cultures evolve and languages shift I expect this list to be fluid and ever-growing.

The goal of this project is to give a slight taster for the vernacular one might encounter in the jungle of mathematics, with the aim of providing people new to higher math a larger (and ideally more colorful) dictionary to work with. Even if you've read and/or written a lot of math already, I hope there's some terms in here that you might have never heard of, and end up sprinkling in your next mathematical conversation or exposition.

## What you WON'T find:

- Definitions of rigorous mathematical terms. This is better reserved for textbooks.
- Words or phrases that are commonly used in both math and general English texts, whose meanings don't differ in math. This includes words like "standard", "general" and "intuitive", for example. This does NOT include archaic English words, such as "whence", which is included in this glossary.

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## 1 Abstract Nonsense

**noun**

Used (light-heartedly) to refer to long, theoretical parts of a mathematical argument that use a lot of very abstract concepts. Used very frequently for category theory or homology arguments.

**EXAMPLE 1.1.** *It is a standard exercise in abstract nonsense to show left-adjoint functors commute with inverse limits.*

## 2 Abuse of Notation

**noun**

The practice of using technically incorrect or non-rigorous but convenient shortcuts in written mathematics. Also often used: **Abuse of Language**.

**EXAMPLE 2.1.** *With a slight **abuse of notation**, we write  $f^n(x)$  for the  $n$ -th derivative of  $f$ .*

**EXAMPLE 2.2.** *Let  $X$  be a set and  $\wp(X)$  denote the power set of  $X$ . Then an outer measure on  $\wp(X)$  (or on  $X$ , by **abuse of language**), is defined as...*

## 3 Ansatz

## 4 Canonical

**adjective**

A mathematical object that is conventional, or the most "standard" or "reasonable" form of that object to use. Can also be used to refer to mathematical ideas in general, like proofs, concepts or techniques.

**EXAMPLE 4.1.** *There are uncountably many bases of  $\mathbb{R}^2$ , but we usually work with the **canonical** basis  $\{[1, 0], [0, 1]\}$ .*

**EXAMPLE 4.2.** *Euclid's argument is the **canonical** proof of the infinitude of primes.*

**EXAMPLE 4.3.** *The **canonical** definition of a prime number is one with no proper, non-trivial factors.*

## 5 Characterize

## 6 Chasing

## 7 De Facto

*adjective*

A Latin phrase meaning

## 8 Deep

## 9 Degenerate

*adjective*

Refers to subclass of mathematical objects that are much simpler than other objects of the same type. In some fields (like graph theory), "degenerate" also has a more rigorous, specific definition.

*EXAMPLE 9.1.* Note any three points in the plane form a triangle, but we ignore the degenerate case of the three points all being on one line (because then the triangle itself is simply a line segment).

## 10 Elegant

## 11 Elementary

## 12 Epsilon

## 13 Folklore

*adjective*

Refers to a result that is well-known and accepted by most experts in a field, but for which there are little to no examples of a fully-written proof.

*EXAMPLE 13.1.* Most introductory topology classes will introduce the concept of a connect-sum of connected surfaces, and will implicitly assume it is well-defined independent of choice of neighborhood. However since the rigorous proof of this is highly complex, many texts exclude it entirely, making it a *folklore* result.

## 14 Hand-Wave

## 15 Identity

## 16 Iff

*abbreviation*

Shorthand for "if and only if" (recall "A if and only if B" means "A implies B AND B implies A" i.e A and B are mathematically equivalent statements). Often used in the statement of definitions.

**EXAMPLE 16.1.** We say a function  $f : X \rightarrow Y$  between topological spaces is continuous **iff** the pre-image of any open set in  $Y$  is an open set in  $X$ .

**EXAMPLE 16.2.** Recall that a real-valued matrix is invertible **iff** it has nonzero determinant.

## 17 Immediate

*adjective*

Something that is a simple consequence of, or follows easily from, a previously stated result.

**EXAMPLE 17.1.** Consider the function  $f(x) = \sqrt{x^2 + y^2}$ . Continuity of this function is immediate, and...

## 18 Inspection

## 19 Ipso Facto

## 20 Modulo

## 21 Morally

*adverb*

An object or concept is "morally X" if one should expect it to be "X"

## 22 QED

## 23 Pathological

*adjective*

Referring to an example or object that exhibits a lot of unintuitive or unexpected properties. (Noun: pathology).

**EXAMPLE 23.1.** *The rational comb is the standard example of a **pathological** space that is connected but not path-connected.*

**EXAMPLE 23.2.** *Real analysis is full of **pathologies** like the Weierstrass function.*

## 24 **Serious Fact**

A mathematical result that has important consequences and is not simple to prove. Used liberally by Brian Conrad.

**EXAMPLE 24.1.** *It's a serious fact*

## 25 **TFAE**

## 26 **Toy (Example/Theorem/Problem)**

## 27 **Up to (X)**

## 28 **Vacuous**

## 29 **Visibly**

## 30 **WLOG**

*abbreviation*

An acronym for Without Loss Of Generality. Used before making an arbitrary choice that doesn't affect the overall mathematical argument. Can also be used all in lower case.

**EXAMPLE 30.1.** *Take real numbers  $x, y$  and **wlog** let  $x \leq y$ .*

**EXAMPLE 30.2.** *Suppose  $f$  is a constant function. **WLOG** let  $f(x) = 1$  for all  $x$ .*

## 31 **Well-behaved**

## 32 **Whence**