Robotics
Programming Exercises 1: Basic Motor Units
CS 603, Spring 2020

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1 Exercise 4.1.1: Oculomotor Control

1.1 Describe a procedure for searching for critically damped gains, report gains you end up with and describe any issues you encounter

For a harmonic oscillation, the equation is [2]:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = f(t)/m = \tilde{f(t)}$$

where,
$\tilde{f(t)}$ is the "specific" applied force,
$\omega_n = (K/m)^{1/2}[rad/sec]$ is the natural frequency, and
$\zeta = B/2(Km)^{1/2}$ is the damping ratio.

For a theoretically critically damped solution, ζ will be 1. Also, we are dealing with moment of inertia instead of just mass. So we can find $I = ml^2$ from Roger's configuration. *roger.h* file has **mass of eyes** as $0.05kg$ and **length of eyes** as $0.04m$. *Moment of inertia I* will be

$$I = ml^2 = (0.05) \cdot (0.04)^2 = 8e^{-5}kg \cdot m^2$$

Then we start with small value of B, say 0.01 and calculate K according to the damping ratio. This B moves the eyes very slowly, not at a satisfactory speed for a vital part of the robot. After $B = 0.05$, I tried to guess the value of K and change B accordingly.

Around $K = 16.0\ Nm$ and a critically damped $B = 0.0716\ kg \cdot m^2/s$, the eye appears to oscillate with satisfactory speed quickly. One issue I encountered while following the equation was that I didn't consider it to be applicable to an isolated system having only 1 degree of freedom. Roger has 9 degrees of freedom and hence, changes in one motor activates base movements (rotation and translation), inertia and torque in other motor units, leading to effects that cannot be fully expressed in a mathematical equation. That's why I had to change the values a little bit which might or might not be conforming the the damping ratio. But practically, it gave great results.

$$K = 16Nm$$
$$B = 0.0716kg \cdot m^2/s$$

1.2 Hold K fixed and change B to generate plots for under-, over-, and critically-damped responses for the eye

Holding $K = 16$ and varying B, we get critically damped response at $B = 0.072$, with satisfactory eye movements and speed.

The higher values of B can be seen to oscillate, being under-damped. The lower values of B are over-damped and moves much slower.

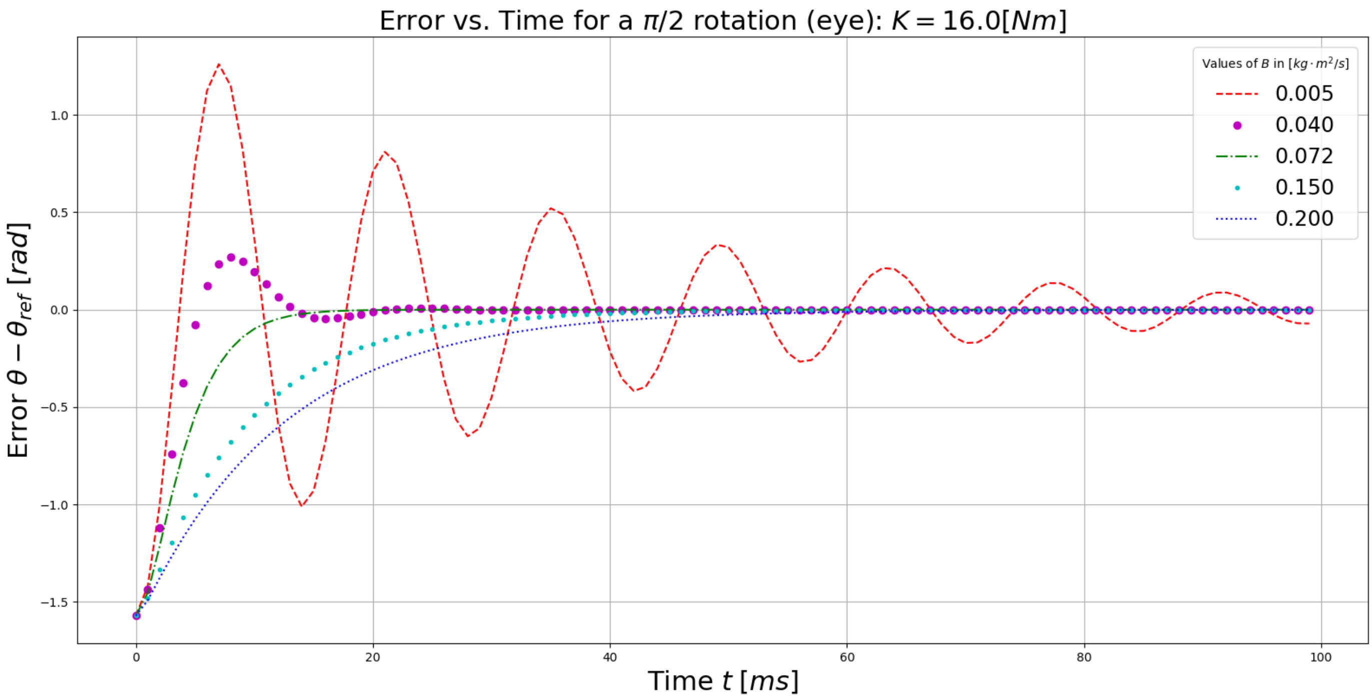

Figure 1: Eye under-, over-, and critically-damped experimental responses while holding K fixed and changing B

This experimental example considers the time-domain performance of this system subject to a particular boundary condition

$$\theta(0) = -\pi/2 \qquad \theta(\infty) = 0 \qquad \dot{\theta}(0) = 0$$

s_1 and s_2 are the roots of $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$ harmonic oscillator equation:

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

where $\zeta = B/2\sqrt{KI}$ and $\omega_n = \sqrt{\frac{K}{I}}$

We get these time-domain solutions:
Case #1: Overdamped $(\zeta > 1)$

$$\theta(t) = -\frac{\pi}{2}\frac{s_2}{s_2 - s_1}e^{s_1 t} - \frac{\pi}{2}\frac{s_1}{s_1 - s_2}e^{s_2 t}$$

Case #2: Underdamped $(\zeta < 1)$

$$\theta(t) = e^{\alpha t}\left[\frac{\alpha}{\beta} sin(\beta t) - cos(\beta t)\right]$$

where $s_{1,2} = \alpha \pm \beta i$
Case #3: Critically damped $(\zeta = 1)$

$$\theta(t) = \left(-\frac{\pi}{2} - \frac{\pi}{2}\omega_n t\right) e^{-\omega_n t}$$

2 Exercise 4.1.2: Arm Control

Plot the error versus time during the simultaneous movement of both joints from $q1 = q2 = 0.0$ to $q1 = q2 = \pi/2$. Report the gains you decided to use for these motor units and discuss your experimental results - does your plot depict the theoretical second-order response?

Each arm has two degrees of freedom (shoulder and elbow) but they have same K and B values. We calculate moment of inertia $I = ml^2$, then setting K to a small value, using critically damped B.

We must take care of setting K such that Roger will have controlled and realistic movement, that is, the base should not get rotated or moved because of the force exerted in the shoulder and elbow.

Roger has relatively heavier and longer arms, thus, larger moment of inertia. I have set up gains as follows:

$$K = 100\ Nm$$
$$B = 8.9\ kg \cdot m^2/s$$

The plot 2 shows that elbow joint starts at steeper slope. When the joints are actuated, the shoulder joints will have increased length than the elbow joints. Therefore the elbow starts at slower rate of reaching to critically dampened motion. The moment of inertia will be greater for shoulders because of the linked length which is twice that of elbow joint. [For $q1$, $l = 1[m]$; For $q2$, $l = 0.5[m]$]

They eventually reach the same set point, stabilizing at around $600\ ms$.
In the beginning of a movement, it slightly oscillates but then reaches the theoretical second-order response with critical damping, which would have a similar shape but converges faster to a set point.

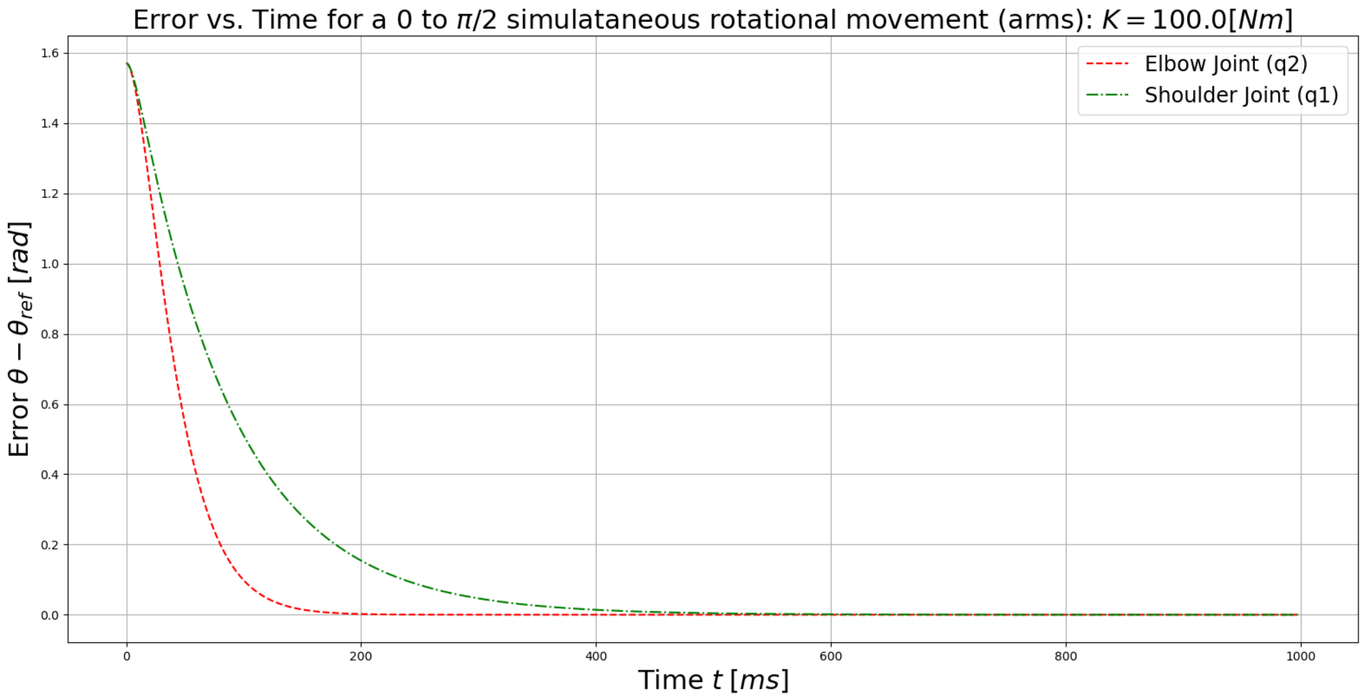

Figure 2: Error versus time during the simultaneous movement of both joints from $q1 = q2 = 0.0$ to $q1 = q2 = \pi/2$

3 Exercise 4.1.3: Base Controller

3.1 Describe your criteria for selecting control gains and your final choices.

The arms and eyes are attached to the base, so the moment of inertia is related to not only the mass and length of base but also the moment of inertia of both arms and eyes too. That's why the B values of translation and rotation controls are far greater than the theoretical ones (B_experimental_trans = 38, B_theoretical_trans = 4.41 and B_experimental_rot = 37, B_theoretical_rot = 7.83).

We had to suppress the translation errors in $\hat{y}_B$ direction when reaching the set point, that's why K_rot is much higher than K_trans. Roger aligns itself to correct angle before moving forward to the desired position. Both Bs were gradually increased until they satisfied speed and smoothness of movement requirements.

$$
\begin{aligned}
K\_trans &= 190\ Nm \\
K\_rot &= 600\ Nm \\
B\_trans &= 38\ kg \cdot m^2/s \\
B\_rot &= 37\ kg \cdot m^2/s
\end{aligned}
$$

Roger tends to oscillate around the set point for lower values of the damping coefficients. The gains set are able to quickly stabilize any perturbations occurred due to arm motions.

3.2 Plot x, y, and θ errors in the base position as a function of time for a reference:

3.2.1 (a) 0.75 meters directly in front of the robot (in the $\hat{x}_B$ direction)

Plot 3 shows that for a linear translation in straight x_B direction in world space, the error is reduced in y_B direction; with x_B and θ errors remaining at zero.

3.2.2 (b) 0.75 meters to the left of the robot (in the $\hat{y}_B$ direction)

Plot 4 shows that for a linear translation in straight $\hat{y}_B$ direction in world space, Roger will rotate and translate simultaneously. That's why there's a slight curve to $\hat{y}_B$ direction. That error is induced by θ error which is due to arms changing angles as base rotates and that produces the shoulder and elbow joint torques too. The error in $\hat{x}_B$ direction is because the movement is towards $\hat{y}_B$ direction and that's why Roger has to adjust to the error in $\hat{x}_B$.

3.2.3 (c) a pure $\pi/2$ rotation around the $\hat{z}_B$ axis

Plot 5 shows that θ errors are getting stabilized as the base rotates to the desired angle. We must note that K_rot is much higher than K_trans. A small error in x and y remains due to same reasons as in previous problem.

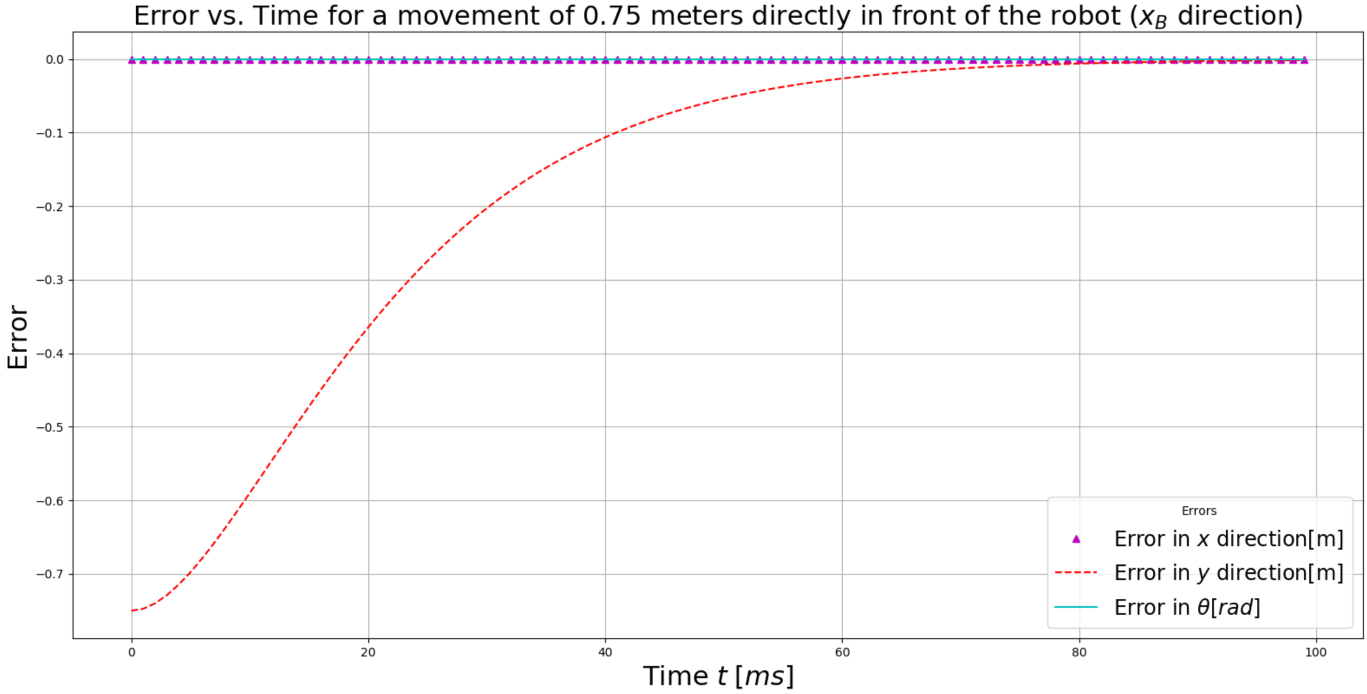

Figure 3: Error vs. Time for a movement of 0.75 meters directly in front of the robot ($\hat{x}_B$ direction); K_trans = $190\ Nm$, K_rot = $600\ Nm$, B_trans = $38\ kg{\cdot}m^2/s$, B_rot = $37\ kg{\cdot}m^2/s$

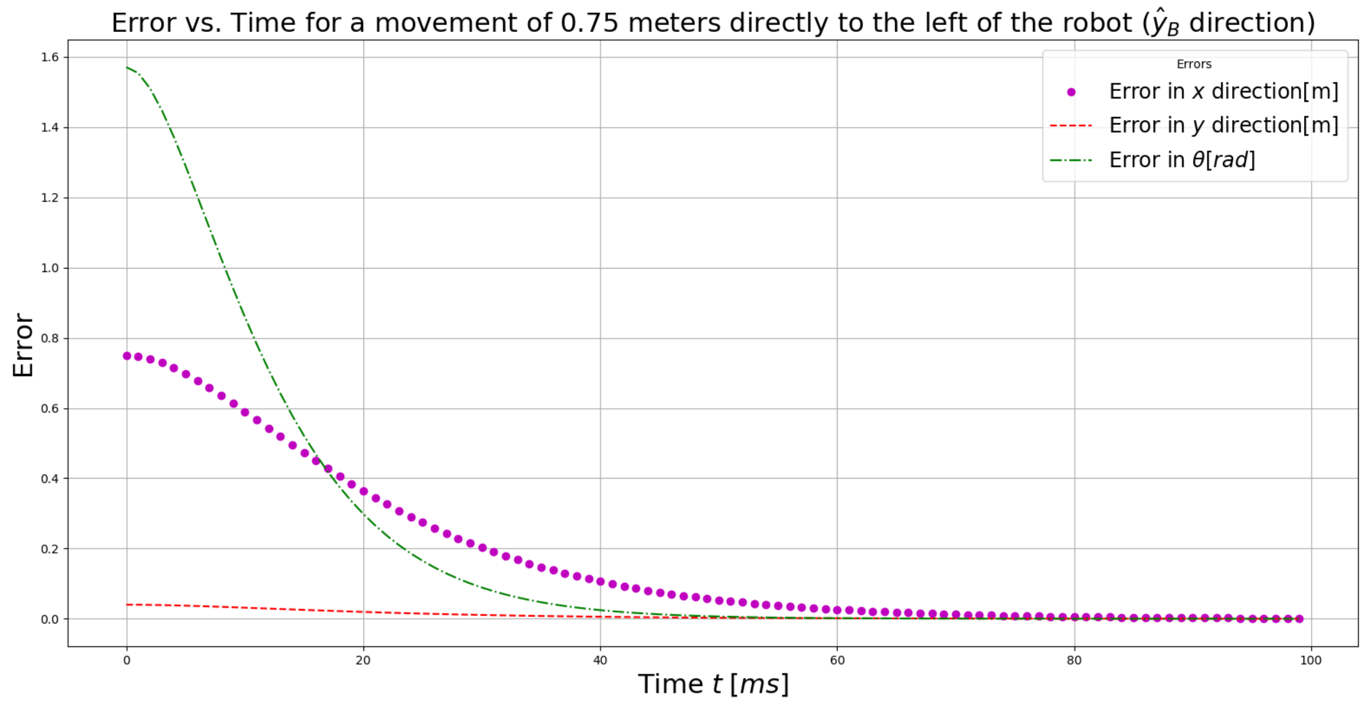

Figure 4: Error vs. Time for a movement of 0.75 meters to the left of the robot ($\hat{y}_B$ direction); K_trans = $190\ Nm$, K_rot = $600\ Nm$, B_trans = $38\ kg{\cdot}m^2/s$, B_rot = $37\ kg{\cdot}m^2/s$

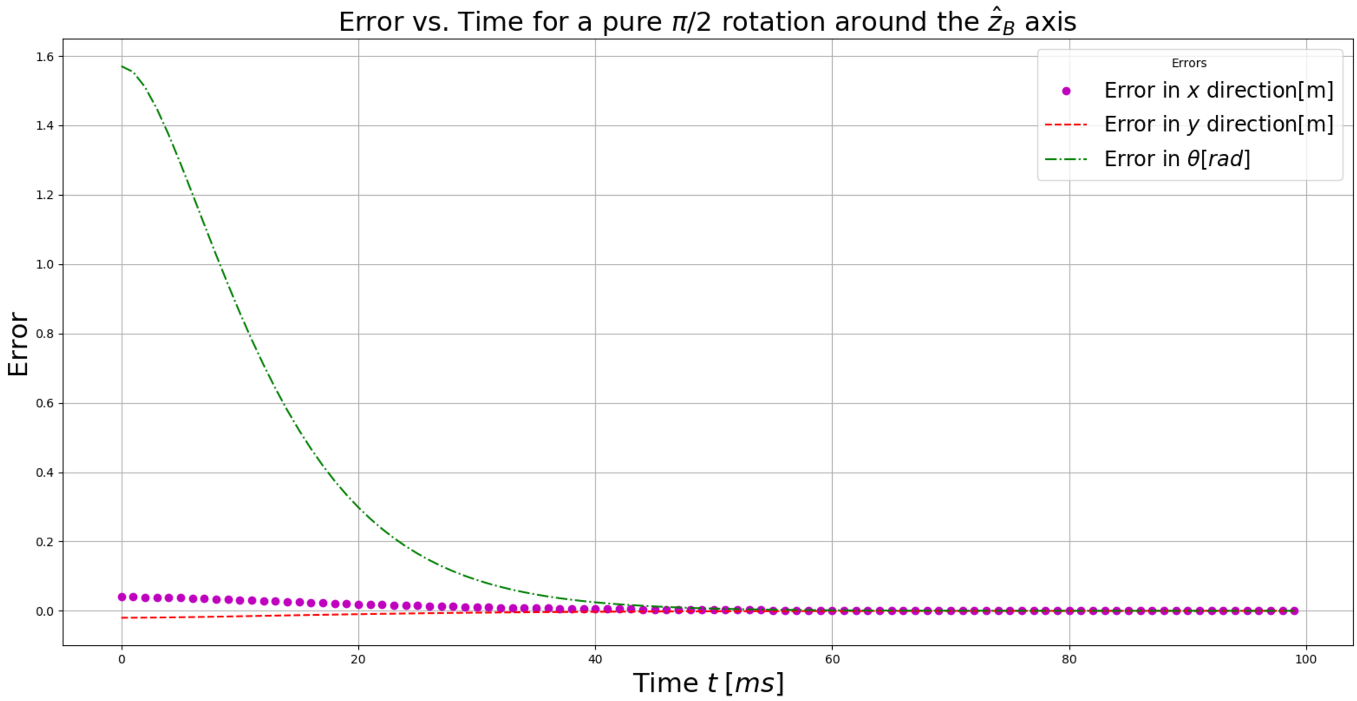

Figure 5: Error vs. Time for a pure $\pi/2$ rotation around the $\hat{z}_B$ axis; K_trans = $190\ Nm$, K_rot = $600\ Nm$, B_trans = $38\ kg \cdot m^2/s$, B_rot = $37\ kg \cdot m^2/s$

References

[1] Rod Grupen, *Actuation* http://www-robotics.cs.umass.edu/~grupen/book2020/2-Actuation.pdf

[2] Rod Grupen, *Control* http://www-robotics.cs.umass.edu/~grupen/603/slides/CONTROL.pdf