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1 Introduction

In a world reliant on computer systems, the correctness of those systems is vital. Indeed, simple programming errors can lead to major incidents; examples of these include an

automated trader losing \$460 million [8] and the inaugural Ariane 5 flight breaking up after launch due to an overflow error [5].

In order to achieve better performance, concurrency can often be introduced to improve the performance of programs or systems, especially those with semi-independent tasks or components. Concurrent systems, be this on a single computer or distributed across a network, achieve these performance improvements at the cost of additional complexity; the design now needs to consider the interactions between threads. Each of possible interactions between threads could potentially lead to a *race condition* in a poorly designed system. A race condition is where two non-independent actions can occur in an order which produces an incorrect or unwanted outcome. Race conditions can be very damaging in practice: the Therac-25 radiation therapy machine killed three of its patients by radiation overexposure as a result of a race condition [27]. This was caused by the operator quickly changing from the high radiation beam mode to select the lower radiation beam instead; the race condition resulted in the machine erroneously still using the high radiation beam. This highlights the importance of thorough system validation; had more thorough system verification been completed, these deaths would have likely been avoided [2].

There are two main approaches to developing correct software: testing and verification. Testing can only establish the presence of a bug, not the absence of any; this can be somewhat addressed by writing exhaustive tests to cover every possible edge case, however this imposes severe restrictions on the complexity of such systems. Though thorough unit testing can be effective in reducing software bugs [15], this form of testing is significantly less effective in concurrent contexts. This is due to the inherent non-determinism of event scheduling; this makes accurately testing all possible interactions between threads a significant challenge.

Linearization testing is an effective alternative approach. This treats each operation

as having occurred (been *linearized*) at some point in time between the start and finish of the operation. It then checks that performing the operations sequentially in the same order returns the same results. This also relies on the nondeterministic testing of edge cases so is also not exhaustive either [26].

By contrast, formal verification can be used to show that systems satisfy some desired properties [3]. This, however, is a complex process and it is often impossible to model check the complete behaviour of a large system simply due to the size of the resulting state space; we instead focus on modelling the concurrent interactions between threads via a variety of synchronisation objects. Synchronisation objects are used to control the potential interaction between threads, simplifying the design of concurrent systems and helping to achieve race conditions.

There are two main benefits of this. Firstly, we prove that the synchronisation objects function correctly; debugging concurrent programs is a complex task and bugs due to issues with synchronisation objects can often be particularly challenging to resolve. The second benefit is that we can significantly optimise the model checking of systems that use these primitives. If we can prove that some synchronisation object fulfils a simpler specification which has the same properties, we can then use the specification to simplify future model checks but without changing the system behaviour. This allows us to significantly reduce the possible state space of the system, making formal verification of complex systems significantly more feasible.

We choose to use the process algebra Communicating Sequential Processes (CSP) [34] as our tool for modelling these interactions and the accompanying FDR model checker [37]. CSP is a language for describing processes that can interact both with their environment and other parallel processes. Since CSP is covered in the Part B Concurrency course [9], we assume familiarity with it (as described in [34]) throughout.

1.1 Contributions

The contributions of this project are as follows:

- We examine a variety of locks from [29] and a number of desirable properties of locks, using CSP refinement checks to determine which properties each lock satisfies. We also discuss the feasibility of modelling infinite liveness properties using CSP, presenting a novel proof that starvation-freedom cannot be captured in CSP before considering alternative approaches.
- We model both the standard JVM monitor and the Scala Concurrency Library (SCL) [20] implementation of a monitor, proving that the SCL monitor provides the same mutual exclusion and correctness properties as the JVM monitor but without some unwanted behaviours.
- We produce a CSP model of a 2-thread synchronisation object and show how multiple 2-thread synchronisation objects can be organised into an arbitrary n -thread barrier synchronisation object. We then use the CSP model to prove the correctness of the barrier synchronisation object.

2 Related Work

Current literature highlights three main approaches to formal verification of software: via verification tests written during development, through automated translation of code or alternatively through modelling by hand.

Writing verification proofs throughout development allows developers to ensure that their code meets their specification during development. Amazon Web Services (AWS) provide an example of this approach, using formal verification methods in code development in order to ensure that their code always fulfils its specifications [1, 36].

This process involves writing a specification for each function in the form of pre- and post-conditions, with these properties validated automatically whenever changes are made. Though similar in style to standard unit-tests, this approach proves the correctness of the system instead of relying on the testing of edge cases to highlight errors. Though this approach requires more effort during the development process, finding system design issues during development is preferable to discovering them at the end of development. To aid with this, some languages such as Dafny [17] have been developed with built-in model checkers as an extension of this approach; these however are yet to reach mainstream adoption.

Automated code translation is similar but tends to focus on more generic properties such as detecting deadlocks. NASA developed the Java PathFinder (JPF) as one of the first examples of automated verification of code [4]. This was able to detect and alert on deadlocks, unhandled exceptions and assertions, but could not check for correctness against some specification [13]. This limited its utility significantly; code not deadlocking is a required by, but not sufficient for, system correctness. More modern tools such as Stainless can verify some system properties when given additional information by the developer [7]. These approaches tend to be inefficient; automated translation has no inherent insight about the usage of parameters or datatypes, resulting in a potentially very inefficient model.

The final approach to formal verification is to write a model of the code by hand and then verify that model. This approach is more involved and limited in scope than the two alternative methods, but can lead to a larger range of results and additional proofs that the code fulfils certain properties beyond just correctness. It also has the benefit that it is less affected by the state space explosion problem than automated tools.

The state space explosion problem is that even small systems can have very large, or

indeed infinite, state spaces with this rendering model checking infeasible [28]. Indeed, a single process with a parameter $t : S$ and k states can have k^t possible states; a network of N of these processes can have N^{k^t} total states. This quickly becomes infeasible to check for even relatively small values of N , k and t . Additionally, an unbounded queue has an infinite number of possible permutations, rendering model checkers such as FDR or SPIN [11] incapable of generating the state space. Insights and smart design can be used to create a smaller, but still correct, model than automated translation methods. This allows for model checking of system with more threads or a wider range of parameter values, leading to greater confidence in their correctness. We therefore focus on this approach.

We choose to use CSP for this task. CSP is very suitable for modelling concurrent systems with tightly restricted communications between threads [16] and allows for natural modelling of system behaviour. Through this, we can effectively use CSP and FDR to model and then check a system's behaviours and properties. This approach has yielded significant verification results previously, most prominently being Lowe's detection of a man-in-the-middle attack on the Needham-Schroeder protocol [18]. It has also been used previously to model software running on the International Space Station, proving that such systems were free of deadlocks [6].

Indeed, CSP has also been successfully used to find bugs in concurrency primitives. Lowe used CSP to model an implementation of a concurrent channel, with FDR returning that the implementation was not deadlock free [21]. This bug was a very niche edge case that required a trace of 37 separate events to occur. The produced trace allowed for a straightforward fix to the code to made to remove the deadlock; this makes it well suited to our needs as we can both accurately model datatypes and then easily interpret any resulting error traces.

There are three main styles of concurrent programming as highlighted in [29]: lock-

based, message passing and datatype-based concurrent programming. There exists literature on formal verification of the latter two; Lowe has previously proved the correctness of a lock-free queue [19] and also an implementation of a generalised alt operator [22]. There is also more general work on the verification of lock-free algorithms, such as Schellhorn and Bäumlér [35]. Their work uses an extended form of linear temporal logic (LTL) and the rely-guarantee paradigm (introduced by [14]) to prove linearizability and lock-free properties. In this paper we will focus on lock-based concurrency primitives instead.

3 Modelling and analysing implementations of locks

The primary purpose of locks is to provide *mutual exclusion* between threads; that is to avoid two threads from operating concurrently on the same section of code, referred to as the *critical region*. A good lock should also fulfil some *liveness* requirements, essentially that something good will eventually happen assuming no thread holds the lock indefinitely. We examine how we can model certain liveness and safety properties using CSP, present a number of different locks and test if the expected properties hold.

3.1 The interface

The most straightforward interface of a lock can be seen in Listing 1. It provides a `lock` function which allows a thread to attempt to gain the lock (blocking if some other thread currently holds the lock) and an `unlock` function for the holder of the lock to release it.

Listing 1: A Scala interface for a simple lock

```
1 trait Lock{  
2   /** Acquire the Lock. */  
3   def lock : Unit  
4   /** Release the Lock. */
```



```

5  def unlock : Unit
6  ...
7  }

```

There are four main actions a thread can take that we will need to consider when we model the locks:

- A thread calling the `lock` function.
- A thread acquiring the lock; this is the linearization point of the `lock()` function.
- A thread releasing the lock; this is the linearization point of the `unlock()` function.
- A thread terminating or reaching a state where it will make no further attempts to acquire the lock.

These events can be modelled via events on the CSP channels `callLock`, `lockAcquired`, `lockReleased` : `LockID` . `ThreadID` and `end` : `ThreadID` respectively. We assign each thread an identity of the datatype `ThreadID = T.{0 .. NTHREADS - 1}` and each lock is given an identity of the datatype `LockID = L.{0..NLOCKS-1}`.

Throughout the paper, we will use the convention of `callX` to represent a thread calling function `X` and `end.t` to represent a thread that will no longer perform any operations on the corresponding synchronisation object

3.2 Properties of locks

We now specify some ideal properties of locks using the aforementioned channels.

3.2.1 Mutual exclusion

Mutual exclusion is a safety property which states that at most one thread can hold the lock at any points. Mutual exclusion is specified by the following trace refinement

which specifies that the thread holding the lock must release the lock before some other thread can acquire it.

```

1 Mutex = lockAcquired.l?t → lockReleased.l.t → Mutex
2 assert Mutex  $\sqsubseteq_T$ 
3       X \ (Σ - [|lockAcquired.l, lockReleased.l|])
4

```

3.2.2 Deadlock-freedom

This is a liveness property which specifies that if a thread attempts to acquire the lock then some thread will succeed in acquiring the lock [10].

We can capture this in the stable-failures model by ensuring that `lockAcquired.l` is always available to be communicated when some thread has called the lock but not yet obtained it.

```

1 AcquireLock :: (LockID, {ThreadID}, {ThreadID}) → Proc
2 AcquireLock(l, ts, TS) =
3   end?t:(diff (TS, ts)) → AcquireLock(l, ts, diff (TS, {t}))
4   □ callLock.l?t:(diff (TS, ts)) → AcquireLock(l, union(ts, {t}), TS)
5   □ lockAcquired.l?t:ts → AcquireLock(l, diff (ts, {t}), TS)
6
7 assert AcquireLock(l, {}, ThreadID)  $\sqsubseteq_F$ 
8       X \ (Σ - {callLock.l, lockAcquired.l, end})
9

```

`AcquireLock` takes three parameters: `l` is the identity of the lock, `ts` is the set of threads which have communicated a `callLock` but haven't yet acquired the lock. Finally, `TS` is the set of all threads that have not communicated an `end.t`; this is so that we can accurately restrict the specification to block any terminated threads from calling the lock.

3.2.3 Initial Livelock-Freedom

We also check that the lock can be obtained for the first time in a finite number of internal operations. We do this by hiding the internal events and use the failures-divergences model to assert that a divergence can only occur after some `lockAcquired` event.

```

1 Live(l) = lockAcquired.l?t → DIV
2           □ STOP
3
4 assert Live(l) ⊆FD
5           X \ (Σ − [|lockAcquired.l|])

```

We include the nondeterministic choice to `STOP` to cover the case where all threads terminate via an `end`.

These are three important properties of useful locks; we will consider starvation-freedom later.

3.3 Test-and-Set lock

The Test-and-Set (TAS) lock implementation (from [29]) can be seen in Listing 2. Internally, it uses an `AtomicBoolean` variable `state` to capture whether the lock is currently held, with `true` indicating that some thread holds the lock and `false` otherwise. The `AtomicBoolean`, has atomic `get` and `set` operations to read and write values respectively. It also has a `getAndSet` operation which atomically sets the value of the Boolean and returns the old value. A thread can obtain the lock via a `getAndSet(true)` where `state` was previously `false`; this is the linearization point of the `lock()` function.

Listing 2: Test-and-set lock from [29]

```

1 import java.util.concurrent.atomic.AtomicBoolean
2
3 /** A lock based upon the test-and-set operation
4   * Based on Herlihy & Shavit, Chapter 7. */

```

```

5 class TASLock extends Lock{
6   /** The state of the lock: true represents locked */
7   private val state = new AtomicBoolean(false)
8
9   /** Acquire the Lock */
10  def lock = while(state.getAndSet(true)){ }
11
12  /** Release the Lock */
13  def unlock = state.set(false)
14 }
15

```

In order to model the TAS lock, we first need a process that acts as an `AtomicBoolean` to model the `state` variable. Listing 3 introduces a process `AtomicVar` than takes an initial value of type `T`, channels `get, set : ThreadID.T` and `getAndSet : ThreadID.T.T` for some arbitrary type `T`.

Listing 3: A process encapsulating an Atomic variable with `get`, `set` and `getAndSet` operations

```

1 AtomicVar(value, get, set, gAS) =
2   get?_!value → AtomicVar(value, get, set, gAS)
3   □ set?_?value' → AtomicVar(value', get, set, gAS)
4   □ gAS?_!value?value' → AtomicVar(value', get, set, gAS)
5

```

We can therefore represent the `state` variable from the Scala implementation by the following CSP:

```

1 channel get, set: ThreadID . Bool
2 channel getAndSet: ThreadID . Bool . Bool
3 State = AtomicVar(false, get, set, getAndSet)
4 InternalChannels = {get, set, getAndSet}

```

A communication on any of these channels is equivalent to a thread calling the corresponding operation in Scala. We use `false` to indicate that no thread holds the lock initially.

We next model the operations of the lock; both operations are trivial to convert to

CSP.

```
1  Unlock(t) = setState.t! False → SKIP
2
3  Lock(t) = getAndSet.t?v!True → if v = False then SKIP
4                                else Lock(t)
```

We model the threads that are attempting to obtain the lock by a process `Thread(x)` with thread identity `x`. Each thread can non-deterministically chooses to either terminate or attempt to obtain the lock, release the lock and repeat. Here we use `L.0` as the identity of the lock.

```
1  Thread(t) = callLock .L.0.t → Lock(t); Unlock(t); Thread(t)
2              □ end.t → SKIP
```

Finally, we construct the lock from its components. We first synchronise all the threads over the `get`, `set` and `getAndSet` channels with the `State` process. Since `getAndSet.t.False.True` and `setState.t.False` represent the linearization points of the `lock()` and `Unlock()` operations, we can rename these communications to `lockAcquired.L.0.t` and `lockReleased.L.0.t` respectively to produce `ActualSystemR`. Finally, to obtain a system that only visibly communicates the four previously identified events, we hide the internal channels of the lock to produce `TASLock`.

```
1  -- All initially do not hold the lock
2  AllThreads = ||| t : ThreadID • Thread(t)
3  -- Allow all threads to perform actions on the state variable
4  ActualSystem = (AllThreads [|InternalChannels|] State)
5  -- Rename lock acquisition and releasing and hide internal events
6  ActualSystemR = ActualSystem
7                      [getAndSet.t.False.True \ lockAcquired.L.0.t,
8                      set.t.False \ lockReleased.L.0.t | t ← ThreadID]
9  TASLock = ActualSystemR \ InternalChannels
```

3.3.1 Analysis

We firstly examine whether this model fulfils the aforementioned properties. The mutual exclusion, deadlock-freedom and initial-livelock-freedom tests from section 3.2 pass as expected. The model can, however, diverge whenever the lock is held. This occurs when a thread (or threads) attempting to obtain the lock perform an infinite number of `getAndSet` operations. An example trace of this behaviour follows

```
1  <callLock .l .T.0, callLock .l .T.1, getAndSet .T.0.False.True>^  
2  <getAndSet .T.1.True.True>^ $\omega$ 
```

This is expected; thread `T.1` is trying to obtain the lock and is being blocked by `T.0` which holds the lock. This behaviour is, however, problematic for low-level performance. Any `getAndSet` operation causes a broadcast on the shared memory bus between the processors, delaying all processors [10]. This also forces each thread to invalidate the value of `state` from the caches, regardless of whether the value has been changed. Since the above trace never results in thread `T.1` successfully setting `state`, it is preferable to limit the number of `getAndSet` operations without delaying a thread from obtaining the lock. As a result, we use less costly `get` operations in order to reduce the usage of `getAndSet` operations; the `getAndSet` operations are now limited to situations where they are likely to obtain the lock. Since `get` does not change the underlying value of a variable, the read will result in at most one cache-miss per `set/getAndSet` on `state`; this is a marked improvement. This change yields the Test-and-Test-and-Set (TTAS) lock.

3.4 Test-and-Test-and-Set lock

Listing 4: TTAS lock from [29]

```
1 class TTASLock extends Lock{  
2   ...  
3   /** Acquire the lock */  
4   def lock =
```

```

5   do{
6     while(state.get()){ } // spin until state = false
7   } while(state.getAndSet(true)) // if state = true, retry
8   ...
9 }

```

The TTAS lock makes use of this improvement, whilst otherwise remaining similar to the TAS lock. The sole change is to the `lock` function, as can be seen in Figure 4, which we then specify in our CSP model as the following:

```

1 Lock(t) = get.t?s → if s = True then Lock(t)
2           else getAndSet.t?v!True → if v = False then SKIP
3           else delay ! t → Lock(t)

```

The TTAS lock can still produce traces with threads performing an unbound number of consecutive operations, however these are now `get` operations instead of `getAndSet` operations. The TTAS lock has performs at most one `getAndSet` operation per thread when the lock becomes available. This is the case when the thread's last communication was `get.t.False`, indicating that the lock is available and hence the thread attempts to acquire the lock via a `getAndSet`.

We now have a linear bound on the number of unsuccessful `getAndSet` operations, resulting in much more efficient usage of caching and shared memory. This has been verified by synchronising with a regulator process which outputs on an error channel if `n` `getAndSets` occur in one locking cycle; this regulator acts as a watchdog. Since the trace refinement is satisfied, we have that no `error` event has been communicated and hence at most `card(ThreadID)` `getAndSet` events can occur every time the lock is released.

```

1 channel error
2 Reg(x) =  gASState?_ → if (x < card(ThreadID)) then Reg(x+1)
3           else error → STOP
4           □ lockReleased?_ → Reg(0)
5
6 assert LockSpec(L.0, {}, ThreadID)
7       ⊆T ActualSystemR [|{gASState, lockReleased}|] Reg(0)

```

3.5 Peterson lock

The Peterson lock is a lock implementation for two threads that provides mutual exclusion, deadlock-freedom and starvation-freedom between threads [32].

Listing 5: The Peterson lock code, adapted from [29]

```

1 import ox.cads.util.ThreadID
2 import java.util.concurrent.atomic.AtomicIntegerArray
3
4 class PetersonLock extends Lock{
5   private val flag = new AtomicIntegerArray(2)
6   @volatile private var victim = 0
7
8   def lock = {
9     val me = ThreadID.get; val other = 1-me
10    assert(me==0 || me==1,
11      "ThreadID needs to be 0 or 1. Try calling ThreadID.reset first")
12    flag.set(me, 1); victim = me
13    while(flag.get(other) == 1 && victim == me){ } // spin
14  }
15
16  def unlock = { val me = ThreadID.get; flag.set(me, 0) }
17 }

```

We now construct the CSP model of the Peterson Lock. We first introduce `IntArray`; this is similar to the `AtomicVar` process earlier but has an index allowing it to store multiple values simultaneously. We also have parameters `l` and `u` which specify the lower and upper bounds of the integers stored by `IntArray`; without this each `Entry` could theoretically store 2^{32} values, which results in FDR generating an infeasibly large state space. We do not include the `getAndSet` operation here as we do not use it in the Peterson lock; it could however be modelled similarly to `AtomicVar`.

```

1 IntArray(lnd, init , getl , setl , l , u) =
2   let Entry(index, value) =
3     get! index?_!value → Entry(index, value)
4     □ setl! index?_?value': {l..u} → Entry(index, value')
5   within ||| index : lnd • Entry(index, init )

```

We also make a couple of changes to the CSP model compared to the direct translation. Firstly, we store all three variables in the `IntArray` with `victim` is at index 2; this allows us to use the same `get` and `set` channels throughout for clarity. We also bound the `IntArray` to store values in the range $\{0..1\}$. The rest of the model is a fairly natural translation of the Scala code and we model threads as before.

Listing 6: The CSP implementation of the Peterson Lock

```

1 channel get, set: Index.ThreadID.{0,1}
2
3 Variables = IntArray(Index, 0, get, set, 0, 1)
4 InternalChannels = {get, set}
5
6 Lock :: (ThreadID) → Proc
7 Lock(T.x) = set.l.x.T.x.1 → set.l.2.T.x.x → WhileLock(T.x)
8
9 WhileLock(T.x) =
10   get.l.1-x.T.x.0 → SKIP — Hold lock
11   □ get.l.1-x.T.x.1 → get.l.2.T.x.y →
12     if x = y then WhileLock(T.x) — Spin
13     else SKIP — Hold lock
14
15 Unlock :: (ThreadID) → Proc
16 Unlock(T.x) = set.l.x.T.x.0 → SKIP
17
18 Thread(T.x) = callLock.L.0.T.x → Lock(T.x); Unlock(T.x); Thread(T.x)
19   □ end.t → SKIP
20
21 PetersonLock = ((AllThreads [|InternalChannels|] Variables )
22   [|get.l.(1-t).T.t.0 \ lockAcquired.L.0.T.t,
23   get.l.2.T.t.(1-t) \ lockAcquired.L.0.T.t,
24   set.l.t.T.t.0 \ lockReleased.L.0.T.t | t ← {0, 1}])

```

We can then interleave the two threads, synchronise with the `IntArray`, rename the lock acquisition and released events, and hide all internal communications. Similarly to before we use the refinements to show that this lock satisfies the properties of mutual exclusion and deadlock- and initial-livelock-freedom. We now consider the property of starvation-freedom.

3.5.1 Starvation-Freedom

Starvation-freedom is a liveness property that states that, under the assumption that no thread holds the lock indefinitely, every thread that attempts to acquire the lock eventually succeeds [10]. It requires that any thread attempting to gain the lock must can only be bypassed by other threads a finite number of times.

One common approach to checking infinite properties in CSP is to hide some (in this case internal) channels and then check that the model does not diverge. This approach does not work here: consider a starvation-free lock which uses busy waiting (repeatedly testing if the lock is available). We have that hiding the internal communications results in a divergence, however the lock is starvation-free. An example of such a lock is the Peterson lock which we examine in section 3.5.

Roscoe and Gibson-Robinson showed that every infinite traces property that can be captured by CSP refinement can also be captured by a finite-traces refinement check when combined with the satisfaction of a deterministic Büchi automaton [33]. A Büchi automaton is an automaton that takes infinite inputs and accepts an input if an accepting state is visited infinitely often [31]. We can, however, show that no deterministic Büchi automaton can capture starvation-freedom.

We will show by contradiction that no such deterministic automaton B can capture starvation-freedom. Let us without loss of generality consider 2-threaded locks.

By the definition of starvation-freedom, the automaton should be satisfied if **T.0** acquires and never releases the lock and **T.1** attempts to acquire the lock. The automaton therefore must have some accepting state which is visited an infinite number of times when **T.0** holds the lock and **T.1** attempts to acquire it; we will call this state q_a .

Now we consider the TAS lock from before. This is clearly not starvation-free as the first thread to communicate on **getAndSet** once the lock becomes available will acquire

it; this allows some thread to infinitely bypass some other waiting thread. Consider an execution where **T.0** repeatedly acquires and releases the lock, bypassing **T.1** which is repeatedly attempting and failing to obtain the lock. This execution, however, passes through the state q_a of B every time **T.0** acquires and later releases the lock. This can occur an infinite number of times, resulting in B accepting this execution. This is a contradiction as this execution is not starvation-free (**T.1** never obtains the lock) however it still satisfies B .

We hence have that no deterministic Büchi automaton can capture starvation-freedom and hence we have that starvation-freedom is impossible to verify through a standard FDR refinement check.

3.6 First-come-first-served

We can instead consider capturing the stronger property of first-come-first-served. Locks that satisfy this property can be split into a *doorway* section of a bounded number of steps and a following *waiting* section of a potentially unbounded number of execution steps. This property states that once thread **X** has completed its doorway section it cannot be overtaken by a thread **Y** that has not yet started its doorway section; i.e. **X** must acquire the lock before **Y** acquires the lock. We note that first-come-first-served implies starvation-freedom.

Checking this property requires manual renaming of some communication(s) to a new channel `doorwayComplete : LockID . ThreadID`. The implementation here requires the user to identify the doorway section manually; an automated doorway detector is left as future work.

The CSP specification of first-come-first-served is defined in Listing 7. This is split into four main processes:

- `FCFSNotStarted(l, t, b, bn)` indicates that thread **t** has not called the lock yet. **t** can

either terminate via an `end` or call the lock.

- `FCFSStarted(l, t, b, bn)` indicates that thread `t` has called the lock and started, but not completed, its doorway section.
- `FCFSCompleted(l, t, b, bn)` indicates that thread `t` has completed its doorway section and is ready to obtain the lock. When it obtains the lock, it transitions back to `FCFSNotStarted(l, t, b, bn)`.
- `FCFSTerminated(l, t)` is used when a thread has terminated. We no longer need to keep track of the actions of the other threads as thread `t` will not obtain the lock again.

At each point in time `b` (*before*) is the set of states that have completed their doorway section before `t` started its doorway section; these threads must acquire the lock before `t` can acquire it under the definition of first-come-first-served. `bn` (*before next*) is used to store the set of threads that completed their doorway section after `t`; once `t` acquires the lock all threads in `bn` must acquire the lock before `t` can reacquire it. This specification only enforces that `t` cannot bypass other threads that came first. When the specification is synchronised using alphabetised parallel across all threads it ensures that no thread can bypass another thread that came first; this therefore ensures that no thread can be bypassed.

Listing 7: The CSP specification for first-come-first-served

```

1  FCFSNotStarted(l, t, b, bn) =
2    end.t → FCFSTerminated(l, t)
3    □ callLock.l.t → FCFSStarted(l, t, lineariz , bn)
4    □ doorwayComplete.l?t':diff(ThreadID, union({t}, union(b, bn))) → FCFSNotStarted(l, t,
5      union(b, {t'}), bn)
6    □ lockAcquired.l?t':diff (union(b, bn), {t}) → FCFSNotStarted(l, t, diff (b, {t'}), bn)
7
8  FCFSStarted(l, t, b, bn) =
```

```

9     doorwayComplete.l.t → FCFSCompleted(l, t, b, bn)
10   □ doorwayComplete.l?t':diff(ThreadID, union({t}, union(b, bn))) → FCFSStarted(l, t, b,
    union(bn, {t'}))
11   □ lockAcquired.l?t':diff (union(b, bn), {t}) → FCFSStarted(l, t, diff (b, {t'}), diff (bn,
    {t'}))
12
13   FCFSCompleted(l, t, b, bn) =
14     empty(b) & lockAcquired.l.t → FCFSNotStarted(l, t, bn, {})
15   □ doorwayComplete.l?t':diff(ThreadID, union({t}, union(b, bn))) → FCFSCompleted(l, t,
    b, union(bn, {t'}))
16   □ lockAcquired.l?t':diff (union(b, bn), {t}) → FCFSCompleted(l, t, diff(b, {t'}),
    diff (bn, {t'}))
17
18   FCFSTerminated(l, t) =
19     doorwayComplete.l?t':diff(ThreadID, {t}) → FCFSTerminated(l, t)
20   □ lockAcquired.l?t':diff (ThreadID, {t}) → FCFSTerminated(l, t)
21
22   FCFSCheck(l) =
23     || t ← ThreadID • [union({callLock.l.t, end.t}, {doorwayComplete.l, lockAcquired.l})]
24     FCFSNotStarted(l, t, {}, {})

```

We can now verify the Peterson lock implementation from section 3.5 against this property.

```

1   PetersonLockDoorway =
2   (PetersonLock
3     [set .l.2.T.0.x \ doorwayComplete.L.0.T.x | x ← {0, 1}])
4     \ InternalChannels
5
6   assert FCFSCheck(L.0) ⊆F PetersonLockDoorway \ {lockReleased}

```

This refinement check passes, indicating that the Peterson lock is indeed first-come-first-served and therefore starvation-free. Testing with the TAS and TTAS lock returns that they are not first-come-first-served; this is as expected as they are not starvation-free.

4 Monitors

A monitor can be used to ensure that certain operations on an object can only be performed under mutual exclusion. Here we first consider the implementation of a monitor used by the Java Virtual Machine (JVM), before considering an alternative implementation that addresses some of the limitations of the JVM monitor.

4.1 The JVM monitor

Mutual exclusion between function calls is provided inside the JVM via `synchronized` blocks. Only one thread is allowed to be active inside the `synchronized` blocks of an object at any point; a separate thread trying to execute a `synchronized` expression will have to wait for the former to release the lock before proceeding. Inside a `synchronized` block, a thread can also call `wait()` to suspend and give up the lock. This waits until a separate thread executes a `notify()`, which wakes the waiting thread and allow it to proceed once the notifying thread has released the lock. `notifyAll()` is similar to `notify()` but wakes all waiting threads.

It is important to note that the implementation of `wait()` is buggy. Sometimes a thread that has called `wait()` will wake up even without a `notify()`; this is called a *spurious wakeup*.

4.1.1 Modelling the JVM monitor

For our model of a monitor in CSP we have extended the `JVMMonitor` provided by Lowe [23].

Lowe’s module previously provided a single monitor; this is problematic in case we have multiple objects which each require their own monitor. We instead introduce a datatype `MonitorID`, with this type containing all possible ‘objects’ that require their

own monitors. The `JVMMonitor` module is then changed to be parameterised over some subset of `MonitorID`, with this parameter used to indicate which ‘object’s’ monitor we are performing operations on. The internal channels and processes now also take some `MonitorID` value to identify which object is being referred to at any point.

Internally, the model uses one lock per `MonitorID`, with these also having an additional parameter storing the identities of any waiting threads. So that it is a faithful model of an actual `JVMMonitor` we also model spurious wakeups via the `spurious` channel. We therefore run the lock process in parallel with a regulator process `Reg = CHAOS({spurious})`, synchronising on `{spurious}`; this nondeterministically allows or blocks spurious wakeups. This is important as when we hide `spurious` events we have that every state in FDR that allows a `spurious` has a pair which blocks the `spurious`. This allows us to perform refinement checks in the stable-failures model instead of just the failures-divergences model, allowing us to write more natural specification processes.

Our model of the monitor provides the following exports:

Listing 8: The interface of the `JVMMonitor` module; changes from [23] are underlined

```

1 module JVMMonitor(MonitorID)
2   ...
3   Reg = CHAOS({spuriousWakeup})
4 exports
5
6   -- All events except spuriousWakeup
7   events = { acquire, release , wait, notify , notifyAll }
8
9   channel spuriousWakeup : MonitorID . ThreadID
10
11  InitialiseAll =
12    ||| mon ← MonitorID • (Unlocked(mon, { }) || {spuriousWakeup} || Reg)
13
14  runWith(obj, P) = P || events || (Unlocked(obj, { }) || {spuriousWakeup} || Reg)
15
16  runWithMultiple(objs, P) =
17  P || events || (|| obj ← objs • (Unlocked(obj, { }) || {spuriousWakeup} || Reg))
18
19  -- Interface to threads.
```

```

20
21  -- Lock the monitor
22  Lock(obj, t) = ...
23
24  -- Unlock the monitor
25  Unlock(obj, t) = ...
26
27  -- Perform P under mutual exclusion
28  Synchronized(obj, t, P) = ...
29
30  -- Perform P under mutual exclusion, and apply cont to the result.
31  -- MutexC :: (ThreadID, ((a) → Proc) → Proc, (a) → Proc) → Proc
32  SynchronizedC(obj, t, P, cont) = ...
33
34  -- Perform a wait(), and then regain the lock.
35  Wait(obj, t) = ...
36
37  -- perform a notify()
38  Notify(obj, t) = ...
39
40  -- perform a notifyAll()
41  NotifyAll (obj, t) = ...
42
43  endmodule

```

Each monitor provides `Wait(obj, t)`, `Notify(obj, t)` and `Synchronized(o, t, Proc)` operations to model the equivalent functions/blocks in SCL. The `Proc` parameter is used to specify the process that will be run inside the `Synchronized` block; the intended usage of this is of the form `callFunc.o.t → Synchronized(o, t, syncFunc)` where the process communicates that it is calling the model of function `Func` before completing the rest of the function whilst holding the monitor lock. `runWith(obj, threads)` and `runWithMultiple(objs, threads)` are used to initialise a single monitor with identity `obj` or multiple monitors with identities `objs` respectively; these synchronise `threads` with the initialised monitors.

4.2 The SCL monitor

There are two main limitations to the standard JVM monitor: it suffers from spurious wakeups and does not allow targeting of `notify()` calls. Spurious wakeups are obviously undesirable and are a common source of bugs where not adequately protected against. Targeting of signals can also be very beneficial to the performance of a program; take for example the one-place buffer shown below in Listing 9. Suppose we have significantly more threads wanting to `get()` a value than `put` a value. Each `notifyAll` will awake every thread waiting on a `get()`, even if `get()` is blocked by `! available`. This results in the majority of the threads immediately sleeping again, adding significant overhead. A `notifyAll` is also required since the JVM monitor makes no guarantees as to which thread is awoken by a `notify`; a thread notifying another thread waiting on the same function would cause a deadlock.

Listing 9: A single place buffer as an example of the inefficiency of untargeted signals

```
1  class OneBuff[T] {
2    private var buff = null.asInstanceOf[T]
3    private var available = false
4
5    def put(x: T) = synchronized {
6      while(!available) wait()
7      buff = x
8      available = true
9      notifyAll()
10   }
11
12   def get : T = synchronized {
13     while(!available) wait()
14     available = false
15     notifyAll()
16     return x // not overwritable as we still hold the lock
17   }
18 }
```

The SCL monitor implementation solves both of these issues with a single monitor

offering distinct **Conditions** to allow for more targeted signalling. It is worth noting that these improvements result in the SCL monitor being more computationally expensive than the JVM Monitor. Each individual **Condition** offers the following operations:

- **await()** is used to wait for a signal on the condition
- **signal()** is used to signal to a thread waiting on the condition
- **signalAll()** is used to signal to all the threads waiting on the condition

Each of these operations should be performed while holding the lock. We can note that these operations are similar to the JVM monitor **wait()**, **notify()** and **notifyAll()** respectively. This functionality is also similar to the `java.util.concurrent.locks.Condition` class; the primary difference is that the SCL monitor blocks spurious wakeups whereas they are allowing by the Java class.

Considering the single-placed buffer, we can use two conditions to separate the threads attempting to **get** and those trying to **put**. We then modify the program above to only perform a single **signal** towards the threads that waiting on the opposite condition, resulting in significant efficiency gains as no threads need to immediately sleep after being woken up.

The implementation of the SCL Monitor [25], makes use of the Java **LockSupport** class; we explore the implementations of these in the next section and present CSP models. Wording

4.2.1 The SCL monitor implementation

The SCL monitor is implemented in Scala as two classes: these are a central **Lock** and **Conditions** associated with the **Lock**. The **Lock** definition is a re-entrant lock (i.e. one thread can acquire the lock multiple times whilst holding it) with a variable indicating

the current holder. For our model we will use a simple, single-entry lock; this will reduce the state space significantly without removing functionality.

We instead focus on the `Condition` class. This internally uses a queue of `ThreadInfo` objects to store the identities of the waiting threads, with each of these `ThreadInfo` objects having a Boolean `ready` indicating if that thread has been signalled. When a thread calls `await()` it releases the `lock` and enqueues a `ThreadInfo` object into the queue of the respective condition. A signalling thread dequeues a `ThreadInfo` object from the queue (if non-empty), sets the `ThreadInfo`'s `ready` value to `true` indicating that it has been signalled and then unparks the corresponding thread. `signalAll` acts similarly to `signal` but repeats the above process for all `ThreadInfo` objects in the queue. We note that we have excluded all code regarding interruptions from the SCL code below: this is to reduce the complexity of the resulting CSP model. The interruption handling is also quite simple - it stops the thread from parking again once it has been interrupted and throws a `InterruptedException`.

Listing 10: A subset of the `Condition` class from [25]

```

1 import java.util.concurrent.locks.LockSupport
2
3 /** A condition associated with 'lock'. */
4 class Condition(lock: Lock){
5     /** Information about waiting threads. */
6     private class ThreadInfo{
7         val thread = Thread.currentThread //the waiting thread
8         @volatile var ready = false // has this thread received a signal?
9     }
10
11     /** Check that the current thread holds the lock. */
12     @inline private def checkThread =
13         assert(Thread.currentThread == lock.locker,
14             s"Action on Condition by thread ${Thread.currentThread}, but the "+
15             s"corresponding lock is held by ${lock.locker}")
16
17     /** Queue holding ThreadInfos for waiting threads.
18         * This is accessed only by a thread holding lock. */
19     private val waiters = new scala.collection.mutable.Queue[ThreadInfo]()
20

```

```

21  /** Wait on this condition. */
22  def await(): Unit = {
23      checkThread
24      // record that I'm waiting
25      val myInfo = new ThreadInfo; waiters.enqueue(myInfo)
26      val numLocked = lock.releaseAll           // release the lock
27      while(!myInfo.ready){
28          LockSupport.park()                     // wait to be woken
29      }                                           // reacquire the lock
30      lock.acquireMultiple(numLocked)
31  }
32
33  /** Signal to the first waiting thread. */
34  def signal(): Unit = {
35      checkThread
36      if (waiters.nonEmpty){
37          val threadInfo = waiters.dequeue()
38          if (!threadInfo.ready){
39              threadInfo.ready = true; LockSupport.unpark(threadInfo.thread)
40          }
41          else signal() // try next one; that thread was interrupted or timed out
42      }
43  }
44
45  /** Signal to all waiting threads. */
46  def signalAll() = {
47      checkThread
48      while(waiters.nonEmpty){
49          val threadInfo = waiters.dequeue()
50          threadInfo.ready = true; LockSupport.unpark(threadInfo.thread)
51      }
52  }
53  }

```

4.2.2 The SCL monitor model

DIAGRAM

We now consider our model of the SCL monitor. This consists of three main components:

- The monitor lock: as described above, we model the re-entrant lock of the imple-

mentation with a single-entry lock here. This is because a thread could obtain the lock an unbounded number of times, resulting in a potentially unbounded state space in FDR.

The lock we use is a simple process $\text{Lock}(m) = \text{acquire.m?t} \rightarrow \text{release.m.t} \rightarrow \text{Lock}(m)$ which specifies that only one thread can hold the lock and that same thread must release the lock before some other thread can obtain it. This provides mutual exclusion between threads as required.

- The `LockSupport` module, described in section 4.2.2.1.
- The queue of `ThreadInfo` values, described in section 4.2.2.2.

4.2.2.1 LockSupport

The Java `LockSupport` [30] class offers two main operations: `park` and `unpark`, which are used to suspend and resume a thread respectively. A thread `t` typically calls `LockSupport.park()` to suspend itself until a permit becomes available by another thread calling `LockSupport.unpark(t)`. As this is a permit-based system, a permit is stored if a thread calls `unpark(t)` and `t` is not yet parked. This allows `t` to immediately resume when it does call `park`. It is also important to note that `LockSupport` is also affected by spurious wakeups where some parked thread can resume without an `unpark`.

Threads interact with the `LockSupport` module via three main events: a thread can park itself, a thread can unpark another thread and a parked thread can wake up. We therefore introduce channels `park`, `unpark` and `wakeUp` to represent these three synchronisations. We also use the `spurious` channel to indicate that the following wakeup is spurious; this will allow us to check that `LockSupport` can only diverge when an infinite number of spurious wakeups occur.

Internally, the model stores two sets of threads: `waiting` stores the parked threads

and `permits` stores the threads with permits available. A thread that is parking is either added to the waiting set if no permit is available or it is immediately re-awoken if a permit is available. A thread that is performing an `unpark(t)` will either awake `t` if it is currently waiting or store a permit for `t`. We split our definition into two processes: `LockSupport` can either nondeterministically allow a waiting thread to spuriously wakeup, or it can act as the deterministic `LockSupport1` which initially only communicates `park` or `unpark` events. GAVIN???

Listing 11: The CSP model of the Java LockSupport module

```

1 module LockSupport
2
3   channel park: ThreadID
4   channel unpark: ThreadID.ThreadID
5   channel wakeup: ThreadID.Bool
6
7   LockSupport1(waiting, permits) =
8     park?t→ (
9       if member(t, permits)
10        then wakeup.t → LockSupport(waiting, diff(permits, {t}))
11      else LockSupport(union(waiting, {t}), permits) )
12   □
13   unpark?t?t'→ (
14     if member(t', waiting)
15      then wakeup.t' → LockSupport(diff(waiting, {t'}), permits)
16    else LockSupport(waiting, union(permits, {t'})))
17   LockSupport(waiting, permits) =
18     if waiting = {} then LockSupport1(waiting, permits)
19   else (LockSupport1(waiting, permits)
20     □ spurious → wakeup.$t':waiting → LockSupport(diff(waiting, {t'}), permits))
21
22
23   LockSupportDet :: ({ThreadID}, {ThreadID}) → Proc
24   LockSupportDet(waiting, permits) = LockSupportDet1(waiting, permits)
25   LockSupportDet1(waiting, permits) = ... -- Analogous to LockSupport1
26
27 exports
28
29   channel spurious
30
```

```

31  InitLockSupport = LockSupport({}, {})
32
33  InitLockSupportDet = LockSupportDet({}, {})
34
35  Park(t) = park.t → wakeUp.t?_ → SKIP
36
37  Unpark(t, t') = unpark.t.t' → SKIP
38
39
40  endmodule

```

4.2.2.2 The ThreadInfo Queue

As seen in listing 10, each **Condition** maintains a queue of **ThreadInfo** objects which each have a thread identity and a variable **ready** indicating whether the corresponding thread has been unparked. The natural method of modelling this queue in CSP is with a sequence of identities, one **ThreadInfo** process per identity and some co-ordinator processes.

We first consider the **ThreadInfo** process. Each **ThreadInfo** process has a corresponding **ThreadInfoID** which remains unchanged throughout; this allows us to use the **ThreadInfoID** datatype to specify which **ThreadInfo** process a thread is interacting with. The **ThreadInfo(n)** process communicates over five channels:

- **channel** `initialiseThreadInfo : ThreadInfoID . ThreadID`. A communication of `initialiseThreadInfo.n.t` is used to indicate that **ThreadInfo(n)** will act as thread **t**'s **threadInfo** object until it is released.
- **channel** `releaseThreadInfo : ThreadInfoID . ThreadID`. A communication on this will stop **ThreadInfo(n)** from acting as **t**'s **threadInfo** object, it can now be reinitialised by some thread.
- **channel** `setReady : ThreadInfoID . ThreadID . ThreadID` is used by some thread to in-

dicate that thread t has been signalled; this is only allowed to occur once per initialisation.

- `channel isReady : ThreadInfoID . ThreadID . ThreadID . Bool` is used to indicate whether or not thread t has been signalled yet. It is used by t to protect against spurious wakeups, reparking itself if it has awoken without being signalled.
- `channel threadInfoOwner : ThreadInfoID . ThreadID . ThreadID`. A communication on this channel of the form `threadInfoOwner.n.t.t2` indicates to thread $t2$ that `ThreadInfo(n)` is currently initialised by thread t .

Each of the `ThreadInfo` processes can be initialised by a thread, at which point it then acts as that thread's `ThreadInfo` object. It allows threads (including its parent thread) to check the value of `ready` via a communication on the `isReady` channel, and for other threads to set the value of `ready` to true via the `setReady` channel. Once the parent thread has reawoken legitimately (i.e. via an `Unpark` not a spurious wakeup) it releases the `ThreadInfo` object via `releaseThreadInfo`; this is valid as the `ThreadInfoID` must have been dequeued and `setReady`, hence no further communications should happen until it is reinitialised. We also introduce the channel `threadInfoOwner : ThreadInfoID . ThreadID . ThreadID`; this indicates which thread has initialised the corresponding `ThreadInfo` processes. We always allow `ThreadInfo(n)` to communicate on any of its channels, however we diverge whenever some communication occurs that should not be possible in the original Scala code. This allows us to check that the `ThreadInfo` processes are used correctly; any incorrect usage will lead to a divergence which we can discover.

Listing 12: The definition of a `ThreadInfo` process

```

1 ThreadInfo :: (ThreadInfoID) → Proc
2 ThreadInfo(n) =
3   initialiseThreadInfo . n?t → ThreadInfoF(n, t)
4   □ isReady.n?t?t2.true → DIV
5   □ setReady.n?t?t2 → DIV
```



```

6   □ threadInfoOwner.n?t?t' → DIV
7   □ releaseThreadInfo .n?t → DIV
8   ThreadInfoF(n, t) =
9       isReady.n! t?t2.false → ThreadInfoF(n, t)
10  □ setReady.n!t?t2.diff (ThreadID, t) → ThreadInfoT(n, t)
11  □ initialiseThreadInfo .n! t → DIV
12  □ threadInfoOwner.n.t?t' → ThreadInfoF(n, t)
13  □ releaseThreadInfo .n?t → DIV
14  ThreadInfoT(n, t) =
15      isReady.n! t?t2.true → ThreadInfoT(n, t)
16  □ setReady.n!t?t2 → DIV
17  □ initialiseThreadInfo .n! t → DIV
18  □ threadInfoOwner.n.t?t' → ThreadInfoT(n, t)
19  □ releaseThreadInfo .n.t → ThreadInfo(n)

```

Since there are n threads running and all of them can be waiting at the same time, we hence need n `ThreadInfo` processes; we therefore define `datatype ThreadInfoID = N.{0..n-1}`.

We then use a process called `IDAllocator` to allocate available `ThreadInfo` identities to threads and also to collect them once they have been released. We note that any thread can use any `ThreadInfoID` in this model; this is analogous to the `ThreadInfoID` being memory chunks allocated to each thread by `IDAllocator` and then garbage collected once they are no longer needed.

We note that neither the `IDAllocator` nor the `ThreadInfo` processes themselves take a `MonitorID` as a parameter; this is because each thread can only perform an `await` on one condition or monitor at any point in time. This choice allows us to use the same `ThreadInfoIDs` across multiple monitors, leading to significantly smaller FDR state spaces when modelling using multiple monitors.

Listing 13: The `IDAllocator` process

```

1   IDAllocator (ns) =
2       (not(empty(ns))) &
3       (initialiseThreadInfo $n:ns?t → IDAllocator(diff(ns, {n})))
4   □ releaseThreadInfo ?n:ns?t → DIV
5   □ releaseThreadInfo ?n:diff (ThreadInfoID, ns)?t → IDAllocator(union(ns, {n}))

```

Finally we have the **Queue** processes, which model the queues maintained inside each Condition. Each **Queue** keeps a sequence of the **ThreadInfoID** identities waiting on its condition. The **Queue** processes have three parameters: **m** is the identity of the SCL monitor and **c** is the identity of the condition that the queue belongs to; **qs** is the current state of the queue.

```

1 Queue(m, c, qs) =
2   (not(null (qs))) & dequeue.m.c?t!head(qs) → Queue(m, c, tail(qs))
3   □ (null (qs)) & isEmpty.m.c?t → Queue(m, c, qs)
4   □ enqueue.m.c?t?n:diff(ThreadInfoID, QS) → Queue(m, c, qs^<n>)

```

We have that each **Queue** is always ready to accept an **enqueue** (unless all **ThreadInfoID**s are already in that queue) but will only communicate one of **dequeue** or **isEmpty**. We note that the restriction on the values of **n** that can be enqueued is such that the queue is of finite length; this is required for efficient model checking in FDR.

4.2.3 The functions and interface of the monitor

Now we have the components of the model of the monitor, the last step is to place these processes in parallel. The majority of these processes are independent of each other; only the **IDAllocator** and the **ThreadInfo** processes need to synchronise with each other, which occurs when a **ThreadInfo** process is either initialised or released.

```

1 Queues(m, setC) = ||| c ← setC • Queue(m, c, <>)
2 ThreadInfoSystem =
3   IDAllocator (ThreadInfoID)
4   [|{initialiseThreadInfo , releaseThreadInfo }|] (|| n ← ThreadInfoID • ThreadInfo(n))
5
6 InitialiseMon (m, setC) =
7   (Lock(m) ||| Queues(m, setC) ||| ThreadInfoSystem ||| InitLockSupport)

```

The first function we export as part of the interface of our module is **runWith(P, mon,**

`setC`) which each takes a process `P` representing program threads, an identity for the monitor and a set of conditions on that monitor. This processes synchronises `P` with the `InitialiseMon(m, setC)`. This is to allow the threads to ‘call’ the various functions that act on the monitor correctly and so that mutual exclusion between threads can be obtained using the functions of the monitor. Include spurious in next?

```

1  ...
2  -- The set of events that are hidden when a thread uses the monitor
3  HideSet(m, setC) =
4    {park, unpark, wakeUp, enqueue.m.c, dequeue.m.c, initialiseThreadInfo,
5     threadInfoOwner,
6     setReady, isReady, isEmpty.m.c, releaseThreadInfo | c ← setC}
7
8  -- The set of events to synchronise on between a thread and the monitor
9  SyncSet2(m, setC) = Union({{acquire.m}, {release.m}, HideSet(m, setC)})
10
11 exports
12
13 channel spurious
14 channel acquire, release : MonitorID.ThreadID
15 channel callAcquire, callRelease : MonitorID.ThreadID
16 channel callAwait, callSignalAll : MonitorID.ConditionID.ThreadID
17 channel callSignal : MonitorID.ConditionID.ThreadID
18
19 -- Runs the monitor with internal spurious wakeups
20 runWith(P, mon, setC) =
21   ((P [|SyncSet(mon, setC)|]
22     InitialiseMon (mon, setC)) \ HideSet(mon, setC))
23
24 -- Runs the monitor without internal spurious wakeups
25 runWithDet(P, mon, setC) =
26   ((P [|SyncSet(mon, setC)|]
27     InitialiseMonDet (mon, setC)) \ HideSet(mon, setC))
28
29 ...

```

In the definitions above, `SyncSet` contains every event that we need to synchronise on between a series of threads and the monitor. We then hide all events except for those representing a thread acquiring and releasing the lock; this is the contents of `HideSet`.

We now consider the operations offered by the monitor. We have the interface given below, with each of the five operations corresponding to the function of the same name. Each process starts with a communication on the correspondingly named `callX` channel to indicate that the specified thread has just called that function; this makes examining any traces produced significantly simpler. We will refer to these `callX` communications as ‘external’ and all other channels as being ‘internal’.

```

1 export
2   ...
3
4   -- Operations on the monitor
5   Await(mon, cnd, t) = callAwait . mon . cnd . t → Await1(mon, cnd, t)
6
7   Signal(mon, cnd, t) = callSignal . mon . cnd . t → Signal1(mon, cnd, t)
8
9   SignalAll (mon, cnd, t) = callSignalAll . mon . cnd . t → SignalAll1(mon, cnd, t)
10
11  Lock(mon, t) = callAcquire . mon . t → acquire . mon . t → SKIP
12
13  Unlock(mon, t) = callRelease . mon . t → release . mon . t → SKIP
14
15 endmodule

```

Both `Lock` and `Unlock` both only require a single internal communication (either acquiring or releasing the lock) after the thread’s external communication. By contrast `Await`, `Signal` and `SignalAll` are more complex, so the initial external communication is followed by another process, in each case named `X1`. Each of these processes are natural translations of the Scala code into CSP; the main exception is using `Await2` to represent the `while` loop in the Scala `await()` function.

```

1 Signal1(mon, cnd, t) =
2   isEmpty . mon . cnd . t → SKIP
3   □ dequeue . mon . cnd . t ? n → threadInfoOwner . n ? t2 ! t → isReady . n . t2 . t ? b →
4     (if b then Signal1(mon, cnd, t)
5      else setReady . n . t2 . t → Unpark(t, t2))
6
7 SignalAll1 (mon, cnd, t) =

```

```

8      isEmpty.mon.cnd.t → SKIP
9      □ dequeue.mon.cnd.t?n → threadInfoOwner.n?t2!t → setReady.n.t2.t →
10         Unpark(t, t2); SignalAll1 (mon, cnd, t)
11
12  Await1(mon, cnd, t) =
13    initialiseThreadInfo ?n!t → enqueue.mon.cnd.t.n → release.mon.t → Await2(mon,
14      cnd, t, n)
15
16  Await2(mon, cnd, t, n) =
17    isReady.n.t.t?b → (if b then releaseThreadInfo .n.t → acquire.mon.t → SKIP
18      else Park(t); Await2(mon, cnd, t, n))

```

4.2.4 Correctness

We now consider the correctness of our model. We present a specification process for a idealised monitor with conditions and perform refinement checks against it. We show that the ordering of `awaits` is also upheld by a separate refinement check.

We will first consider the specification process of a monitor with multiple conditions. Each monitor process is parameterised over the identity of the monitor and a map of `ConditionID => {ThreadID}` representing the set of threads waiting on each `Condition`. We choose to use sets of waiting threads instead of queues of waiting threads to make this a more general specification of a monitor and it is also more efficient. We consider orderings in a later test. `initSet` is the initial mapping of the waiting threads, with each condition mapping to an empty set. We define `valuesSet` as a helper function which returns a set of all the threads that are currently waiting on any condition; this allows us to restrict the specification to only allow threads that aren't waiting to obtain the lock.

```

1  initSet = mapFromList(<(c, {}) | c ← seq(ConditionID)>)
2  valuesSet (map) = Union({mapLookup(map, cnd) | cnd ← ConditionID})
3  Spec2Unlocked(m, waiting) =
4    SCL::acquire .m?t:diff(ThreadID, values(waiting)) → Spec2Locked(m, t, waiting)
5
6  Spec2Locked(m, t, waiting) =

```

```

7   □ c': ConditionID •
8   (
9       (mapLookup(waiting, c') = {}) & SCL::callSignal .m.c'.t →
10      Spec2Locked(m, t, waiting)
11   □ (mapLookup(waiting, c') ≠ {}) &
12      □ t': mapLookup(waiting, c') • SCL::callSignal .m.c'.t →
13      Spec2Locked(m, t,
14      mapUpdate(waiting, c', diff (mapLookup(waiting, c'), {t'})))
15   )
16   □ SCL::callSignalAll .m?c:ConditionID!t →
17      Spec2Locked(m, t, mapUpdate(waiting, c, {}))
18   □ SCL::callRelease .m.t → SCL::release.m.t →
19      Spec2Unlocked(m, waiting)
20   □ SCL::callAwait .m?c:ConditionID!t → SCL::release.m.t →
21      Spec2Unlocked(m, mapUpdate(waiting, c, union(mapLookup(waiting, c), {t})))
22
23
24   Spec2Thread(t, m) = SCL::callAcquire.m.t → SCL::acquire.m.t → Spec2Thread2(t, m)
25   Spec2Thread2(t, m) =
26       SCL::callAwait .m?c.t → SCL::release.m.t → SCL::acquire.m.t → Spec2Thread2(t, m)
27   □ SCL::callRelease .m.t → SCL::release.m.t → Spec2Thread(t, m)
28   □ SCL::callSignalAll .m?c.t → Spec2Thread2(t, m)
29   □ SCL::callSignal .m?c.t → Spec2Thread2(t, m)
30
31   Spec2SCL = (let m = SigM.S.0 within
32       (Spec2Unlocked(m, initSet) [|SpecChans(m, ConditionID)|]
33       (|| t ← ThreadID • (Spec2Thread(t, m)))))
34

```

We first note that the specification process provided is divergence free; we choose this as an idealised monitor should never internally diverge. We use the technique of having multiple threads linear

To test against this specification, we interleave a number of **ThreadSCL(t)** processes, with each of these representing the potential (correct) usage of the monitor that thread **t** could perform. These are interleaved to form **ThreadsSCL** and then this is then synchronised with the SCL monitor, via the use of **runWith** or **runWithDet** as outlined above.

```

1   ThreadSCL(t) = SCL::Lock(SigM.S.0, t); ThreadSCL1(t)
2   ThreadSCL1(t) =

```

```

3   □ c : ConditionID •
4   (
5       (SCL::Await(SigM.S.0, c, t); ThreadSCL1(t))
6       □ (SCL::Signal(SigM.S.0, c, t); ThreadSCL1(t))
7       □ (SCL::Signal(SigM.S.0, c, t); ThreadSCL1(t))
8       □ (SCL::SignalAll(SigM.S.0, c, t); ThreadSCL1(t))
9   )
10  □ (SCL::Unlock(SigM.S.0, t); ThreadSCL(t))
11
12  ThreadsSCL = ||| t ← ThreadID • ThreadSCL(t)
13
14  SCLSystemSpur = SCL::runWith(ThreadsSCL, SigM.S.0, ConditionID)
15  SCLSystem = SCLSystemSpur \ {spurious}
16
17  assert SCLSystemSpur : [divergence free]
18  assert not SCLSystem : [divergence free]

```

We have that both the assertions pass: `SCLSystemSpur` is divergence free, but hiding the `spurious` channel introduces divergences. Since the system is divergence-free with the `spurious` channel visible, we can conclude that spurious wakeups don't lead to a state where the system can diverge without more spurious wakeups. We can therefore conclude that the divergences of the system with `spurious` hidden are only due to repeated spurious wakeups

We note that using the stable failures-model is normally inappropriate for a system that can diverge. However, this is valid here as for any state that could be unstable due to repeated hidden spurious wakeups there exists a corresponding stable state where the non-deterministic choice in the `LockSupport` model blocks the `spurious` event.

```

1  assert SpecSCL ⊆F (SCLSystem)

```

We have that this the assertion holds, indicating that the SCL monitor fulfils the specification of a monitor as required.

We next consider the fairness of the monitor with regards to individual `signal` calls. In the SCL monitor, queues are used so that each `signal` wakes the thread that has

been waiting for the longest time on the condition (if one exists). We test that this property holds using `OrderCheck`, a process which maintains a list of the threads waiting on each condition in the order that they started waiting. `OrderCheck` effectively acts as a watchdog, completing a communication over the new channel `error` if an error in ordering is detected; we therefore verify this has not occurred by a trace refinement against the specification.

We choose to use a watchdog as the left-hand side of any assertion has to be normalised by FDR. Normalisation is an expensive process for complex specifications with many states; maintaining queues of waiting threads on each condition is therefore more suited to the right-hand side of an assertion as this does not require normalisation. For further details, see [34].

```

1 valuesSeq(map) = Union({set(mapLookup(map, cnd)) | cnd ← ConditionID})
2 channel error : MonitorID
3 OrderCheck(m, waiting) =
4   SCL::acquire .m?t:ThreadID →
5     (if member(t, valuesSeq(waiting)) then error .m → STOP--DIV
6      else OrderCheck(m, waiting))
7   □ SCL::callAwait .m?c?t →
8     (if member(t, valuesSeq(waiting)) then error .m → STOP
9      else OrderCheck(m, mapUpdate(waiting, c, mapLookup(waiting, c)^(t>)))
10  □ SCL::callSignalAll .m?c?_ →
11    OrderCheck(m, mapUpdate(waiting, c, <>))
12  □ SCL::callSignal .m?c?_ →
13    (if null (mapLookup(waiting, c)) then OrderCheck(m, waiting)
14     else OrderCheck(m, mapUpdate(waiting, c,
15                               tail (mapLookup(waiting, c)))))

```

This new process only synchronises on events that indicate a thread waking, waiting or acquiring the lock; these are all the communications offered by `OrderCheck` except for `error.m`. We can then run the refinement checks by placing `OrderCheck` in parallel with `SCLSystem`.

We then check that this still trace refines `SpecSCL`, which it does. We can therefore

conclude that no **error** events occur and no new stable failures are introduced, hence the ordering within the model of the SCL monitor are maintained correctly.

```

1 assert SpecSCL  $\sqsubseteq_T$  (OrderCheck(SigM.S.0, initSeq)
2     [|{SCL::callAwait.SigM.S.0,
3       SCL::acquire.SigM.S.0,
4       SCL::callSignal.SigM.S.0,
5       SCL::callSignalAll.SigM.S.0}|] SCLSystem)

```

4.2.5 Limitations of natural model of the queue

Though the model given above is a natural model of the SCL monitor, this is quite ill-suited to refinement checking in FDR. The current implementation of the **ThreadInfo** allows any thread to obtain and use any of the **ThreadInfo**s; this leads to exponential blow up in the number of states as the number of threads increases. Considering a case where we have n threads and m are currently waiting with their **ThreadInfo**s queued, this has $\binom{n}{m}$, or $O(n^m)$ permutations.

We can instead use the same **ThreadInfo**s, but restrict them so that each **ThreadInfo** **N.x** can only be used by the respective thread **T.x**. This forces each thread to use the same **ThreadInfo** process each time, removing this source of blow up. This is most trivially done by changing **Await1** to specify the **ThreadInfo** as follows:

```

1 Await1(mon, cnd, t) = initialiseThreadInfo .N.t → ...
2
3 IDAllocator (ns) =
4     (not(empty(ns))) & (initialiseThreadInfo ?n:ns?t → IDAllocator(diff(ns, {n})))
5     □ ...

```

We will refer to this version of the queue as the ‘Simple’ model. This simplified model has one possible bijective mapping of **ThreadID** \Rightarrow **ThreadInfoID**. By contrast, the natural model has $n!$ bijective mappings of threads to **ThreadInfo**s. As a result, for each single state of the model with the simplified queue, there are upto $n!$ states of the

natural model that are identical in all facets other than the `ThreadInfoID` allocations.

For further performance improvements, we can also remove the allocator process as each `ThreadInfoID` is pre-allocated. Additionally we can change the type of `ThreadInfoID` to `N.ThreadID` and simplify many of the channels (removing `threadInfoOwner` and `releaseThreadInfo` entirely) as `ThreadInfoID` indicates which thread it corresponds to as follows:

```

1  datatype ThreadInfoID = N.ThreadID
2  channel enqueue: MonitorID.ConditionID.ThreadInfoID
3  channel dequeue: MonitorID.ConditionID.ThreadID.ThreadInfoID
4  channel setReady: ThreadInfoID.ThreadID
5  channel isReady: ThreadInfoID.ThreadID.Bool
6  channel initialiseThreadInfo : ThreadInfoID
7  channel isEmpty: MonitorID.ConditionID.ThreadID
8  channel await, signalAll : MonitorID.ConditionID.ThreadID

```

All the definitions remain the same apart from removing any `threadInfoOwner` and `releaseThreadInfo` communications and the required type changes. We keep `initialiseThreadInfo` in order to reset the `ready` value to false. We also change `InitialiseMon` and `InitialiseMonDet` to remove the `IDAllocator`. We will refer to this as the ‘optimised’ version.

We first need to check that this simplified model remains correct. The refinement checks still all pass, indicating that the monitor model with a modified queue fulfils the specifications that the natural queue did.

We then verify that the efficiency improvements occur in practice. We do this by running the FDR verification of `assert SpecSCL [F= SCLSystem` for a range of numbers of threads and conditions. We then compare the number of states generated by the different queue models, with the results presented in table 1.

Here we see that the restricted model with each thread is allocated a `ThreadInfo` process results in a state space reduced by a factor of over $n!$ where n is the number of threads. This is as expected due to the reduction in the mappings from threads to nodes as stated above. Though the state space still grows exponentially, it is much

Table 1: The number of states generated by FDR for the different queue implementations. The reduction value is given as $\frac{\text{Original number of states}}{\text{Reduced number of states}}$

No. threads	No. conditions	Number of states				
		Natural	Simple	Reduction	Optimised	Reduction
2	1	2288	1088	2.10	904	2.53
3	1	239428	36262	6.60	26494	9.04
4	1	3.14×10^7	1180416	26.7	792240	39.7
5	1	5.39×10^9	4.06×10^7	133	2.59×10^7	208
2	2	4932	2382	2.07	2382	2.40
3	2	686896	106672	6.44	82973	8.27
4	2	1.22×10^8	4655652	26.2	3363492	36.3
2	3	8436	4106	2.05	3634	2.32
3	3	1445008	227512	6.35	184276	7.84
4	3	3.15×10^8	1.22×10^7	25.9	9212868	34.2

more efficient and makes refinement checks for larger numbers of threads and conditions significantly more feasible.

5 Barrier synchronisation

A *barrier synchronisation* object is used to synchronise some number of threads. These are typically used in shared memory programs which progress in rounds; the barrier synchronisation is used to synchronise the threads at the end of each round ensuring the thread-safety of the shared memory. Such programs which use *global synchronisations* (synchronising all threads) typically operate by instantiating some barrier object and then having each thread call `sync` on the barrier once they have completed their current round. Each call to `sync` only returns after all threads have called `sync`, synchronising all the threads at that point in time and allowing the threads to proceed afterwards [20].

Here we present a novel model and analysis of an `n`-thread barrier synchronisation object which internally uses a binary heap of `n` two-thread signalling objects. The Scala code for the barrier object can be found at [24].

5.1 The signalling object

We first consider the signalling object `Signal`. This is used to synchronise between a parent and child thread, providing three external operations:

- `signalUpAndWait` is used by the child to signal to the parent that the child is ready to synchronise and waits until the parent signals back;
- `waitForSignalUp` is used by the parent to wait for the child to be ready;
- `signalDown` is used by the parent to indicate to the child that the synchronisation has completed.

The Scala code for the `Signal` object can be found in figure 14. Internally, the `Signal`

object makes use of a private Boolean variable `state` with `true` indicating that a child is waiting and `false` otherwise. The use of this variable is protected by a JVM monitor.

Listing 14: The Signal object from [24]

```

1 private class Signal{
2   /** The state of this object. true represents that the child has signalled,
3     * but not yet received a signal back. */
4   private var state = false
5
6   /** Signal to the parent, and wait for a signal back. */
7   def signalUpAndWait = synchronized{
8     require(!state,
9       "Illegal state of Barrier: this might mean that it is\n"+
10      "being used by two threads with the same identity.");
11     state = true; notify()
12     while(state) wait()
13   }
14
15   /** Wait for a signal from the child. */
16   def waitForSignalUp = synchronized{ while(!state) wait() }
17
18   /** Signal to the child. */
19   def signalDown = synchronized{ state = false; notify() }
20 }

```

The `signalUpAndWait` function first asserts that there currently is no other child waiting for the parent to complete a `signalDown`; this ensures that only one child thread uses a `Signal` object. It then sets `state` to `true`, indicating that the child is now waiting and it notifies the parent, waking them if they are waiting. The child thread then waits until the parent sets `state` to `false` and notifies the child; the use of the `while` loop here is to guard against spurious wakeups.

`waitForSignalUp` is used by a parent to wait for the child node to perform a `signalUpAndWait` and notify the parent; the `while` loop again guards against spurious wakeups.

The `signalDown` function is used to signal to the child that the synchronisation has been completed and that the child can return from its `signalUpAndWait` call.

5.1.1 Modelling the Signal objects

We use our model for a JVM monitor from section 4.1.1 in our modelling of the **Signal** object. We define the datatype $\text{SignalID} = S.\{0..n-1\}$ where n is the number of threads and the datatype $\text{MonitorID} = \text{SigM.SignalID}$; these allow us to specify the **Signal** object we are using and that object's monitor respectively.

For each of the three operations, a thread will first communicates a **callX** before running the **syncX** process within a **Synchronized** block (as defined in section 8).

Listing 15: The state variable(s) and the function call channels

```
1 Var(value, get, set) =
2   get?_!value → Var(value, get, set)
3   □ set?_?value' → Var(value', get, set)
4
5 channel getState, setState : SignalID . ThreadID . Bool
6 stateChannels(s) = {getState.s, setState.s}
7
8 State :: (SignalID) → Proc
9 State(s) = Var(false, getState.s, setState.s)
10
11 channel callSignalUpAndWait, callWaitForSignalUp, callSignalDown : SignalID . ThreadID
12 signalChannels = {callSignalUpAndWait, callWaitForSignalUp, callSignalDown}
13
```

We first initialise the state variable, with the **SignalID** parameter in the channels indicating the **Signal** object the variable belongs to. We also introduce the function call channels as indicated above.

Listing 16: The CSP model of the **signalUpAndWait** function of the **Signal** object

```
1 SignalUpAndWait :: (SignalID, ThreadID) → Proc
2 SignalUpAndWait(s, t) =
3   callSignalUpAndWait.s.t → Synchronized(SigM.s, t, syncSignalUpAndWait(s, t))
4 syncSignalUpAndWait(s, t) =
5   getState.s.t?val → if val = true then DIV — Required to be false
6                     else setState.s.t.true →
7                       Notify(SigM.s, t); SignalWaitingForFalse(s, t)
8
9
```

```

10 SignalWaitingForFalse :: (SignalID, ThreadID) → Proc
11 SignalWaitingForFalse (s, t) =
12   getState.s.t?val → if val = false then SKIP
13   else Wait(SigM.s, t); SignalWaitingForFalse (s, t)

```

The `SignalUpAndWait` process enters a JVM `synchronized` block and checks that `state` is not true, diverging if so. This divergence is used to model a failed assertion; the rest of the code is a more direct translation.

Listing 17: The CSP model of the `waitForSignalUp` function of the `Signal` object

```

1 WaitForSignalUp :: (SignalID, ThreadID) → Proc
2 WaitForSignalUp(s, t) =
3   callWaitForSignalUp.s.t → Synchronized(SigM.s, t, syncWaitForSignalUp(s, t))
4 syncWaitForSignalUp(s, t) =
5   getState.s.t?val → if val = true then SKIP
6   else Wait(SigM.s, t); syncWaitForSignalUp(s, t)

```

This is a natural model of the Scala code presented earlier; thread `t` communicates that it has called `waitForSignalUp` on Signal `s`, enters a `synchronized` block and then simulates the `while` loop used to guard against spurious wakeups.

Listing 18: The CSP model of the `signalDown` function of the `Signal` object

```

1 SignalDown :: (SignalID, ThreadID) → Proc
2 SignalDown(s, t) = callSignalDown.s.t → Synchronized(SigM.s, t, syncSignalDown(s, t))
3 syncSignalDown(s, t) = setState.s.t.false → Notify(SigM.s, t); SKIP

```

`SignalDown` is the simplest function to model; it obtains the monitor's lock, sets the `state` variable to false and then notifies the child that the synchronisation has completed.

We now consider the initialisation of the `Signal` objects.

Listing 19: The initialisation of the `Signal` objects

```

1 InitialiseSignal (sig, threads) =
2   runWith(SigM.sig, threads [|stateChannels(sig)|] State(sig))
3   \ union(stateChannels(sig), events) —Hides the variable channels and the monitor
   channels
4
5 allStateChannels (sigs) = {getState.s, setState.s | s ← sigs}
6 States(sigs) = ||| s ← sigs • State(s)

```



```

7 monitors(sigs ) = {SigM.s | s ← sigs}
8
9 InitialiseSignals (sigs , threads) =
10   runWithMultiple(monitors(sigs ), threads [|allStateChannels (sigs )|] States(sigs ))
11   \ union(allStateChannels(sigs ), events)

```

We define processes `InitialiseSignal` and `InitialiseSignals` to initialise a single signal and multiple signals respectively. These synchronise the threads with the interleaving of the state variables and the monitors and then hide the internal behaviour of the `Signal` objects. `runWith` and `runWithMultiple` are used to initialise the monitor(s) for the initialised `Signal` object(s).

5.2 The Barrier object

When initialised, the `Barrier(n: Int)` object creates an array of `n` `Signal` objects, with these organised in the structure of a heap. As per the trait of a barrier synchronisation, `Barrier` only provides a single function `sync(me)` which takes the thread's identity as an input:

Listing 20: The Scala definition of the `Barrier.sync` function from [24]

```

1  /** Perform a barrier synchronisation.
2   * @param me the unique identity of this thread. */
3  def sync(me: Int) = {
4    require(0 <= me && me < n,
5      s"Illegal parameter $me for sync: should be in the range [0..$n).")
6    val child1 = 2*me+1; val child2 = 2*me+2
7    // Wait for children
8    if (child1 < n) signals(child1).waitForSignalUp
9    if (child2 < n) signals(child2).waitForSignalUp
10   // Signal to parent and wait for signal back
11   if (me != 0) signals(me).signalUpAndWait
12   // Signal to children
13   if (child1 < n) signals(child1).signalDown
14   if (child2 < n) signals(child2).signalDown
15 }

```

This checks that the thread's identity is such that `signals(me)` does not cause an `ArrayIndexOutOfBoundsException`. It then waits for the thread's children (if they exist) to

signal that they are ready to synchronise, before signalling to its parent that all of its children are ready to synchronise. Once the parent signals back that the synchronisation has occurred the thread notifies its children that the synchronisation has completed before returning. The exception to this is thread 0, which has no parent to signal to. Line 11 of the `sync(0)` call can therefore be considered the linearization point of the barrier synchronisation.

5.2.1 Modelling the Barrier object

Listing 21: The CSP model of a thread interacting with the Barrier object

```

1 Thread(T.me) = beginSync.T.me → Sync(T.me) □ end.T.me → SKIP
2
3 Sync(T.me) =
4   let child1 = 2*me+1
5     child2 = 2*me+2
6   within
7     (if (child1 < n) then WaitForSignalUp(S.(child1), T.me) else SKIP);
8     (if (child2 < n) then WaitForSignalUp(S.(child2), T.me) else SKIP);
9     (if (me ≠ 0) then SignalUpAndWait(S.me, T.me) else SKIP);
10    (if (child1 < n) then SignalDown(S.(child1), T.me) else SKIP);
11    (if (child2 < n) then SignalDown(S.(child2), T.me) else SKIP);
12    endSync.T.me → Thread(T.me)
13
14 Threads = ||| t : ThreadID • Thread(t)
15
16 BarrierSystem = InitialiseSignals (Threads) \ signalChannels

```

We recall from earlier that `datatype ThreadID = T.{0..n-1}` and `SignalID = S.{0..n-1}`. The process `Thread(T.me)` models the individual behaviour of a specific thread with identity `T.me :: ThreadID`. Each thread nondeterministically either communicate an `end.T.me` and terminates or starts a synchronisation. In the latter case `beginSync.T.me` is used to indicate the start of the synchronisation. The `Sync(T.me)` definition is very straightforward; the only change to the Scala definition is that `Sync(T.me)` communicates a `endSync.T.me` event just before it terminates.

The `Thread` processes are then interleaved together to yield `Threads`. We then initialise the `Signal` objects. We hide the channels of the `Signal` object, leaving $\{\text{beginSync}, \text{endSync}, \text{spurious}, \text{end}\}$ as the only visible channels of `BarrierSystem`.

5.3 Correctness of the model

We will show that the barrier synchronisation is correct; here correct requires that the synchronisation can be correctly linearized and if a synchronisation is possible then it will always occur.

Correctly linearized in this context means that the barrier synchronisation can be considered to occur at some point between when all n threads have communicated `beginSync` and when the first thread communicates an `endSync` event. The requirement that a linearization must occur means that if all n threads communicate a `beginSync` then none of the threads can be blocked from communicating their respective `endSync`.

Listing 22: The linearizer specification for barrier synchronisations

```

1 Linearizer (t) = beginSync.t → sync → endSync.t → Linearizer(t)
2               □ end.t → STOP
3 Linearizers  = || t ← ThreadID • [{beginSync.t, sync, endSync.t, end.t}] Linearizer (t)
4 Spec = Linearizers \ {sync}

```

`Linearizer(t)` allows any thread to `beginSync.t` followed by an `endSync.t`, representing the call and return of `barrier.sync()`. The `sync` event can be considered to be the point at which the barrier synchronisation occurs since all threads must synchronise on this, fulfilling the requirement above. Additionally, each thread can terminate via `end.t`, indicating that it will perform no further synchronisations. This blocks all other threads from completing a barrier synchronisation, which is the intended behaviour.

We first note that `BarrierSystem` is divergence-free, but `BarrierSystem {spurious}` is not. `BarrierSystem` being divergence-free with spurious wakeups visible is relevant as this means that we never breach the assertion in the `SignalUpAndWait` function and that

no internal divergence can be caused by a finite number of spurious wakeups. This therefore means hiding the **spurious** events must be the cause of the divergences. This is expected behaviour as one thread could spuriously wakeup, check the test condition and wait again before spuriously waking again indefinitely. This is still not a particular cause for concern; in practice spurious wakeups occur infrequently within the JVM.

We now consider the traces model, where we have that the following holds:

1 `assert Spec \sqsubseteq_T BarrierSystem \ {spurious}`

This means that **BarrierSystem** fulfils the linearization requirement of **Spec** i.e. that it can be linearized and that the synchronisation between all **n** threads occurs correctly (if indeed it does occur).

Since **Spec** cannot diverge we will also consider refinement under the stable-failures model. This ensures that if a synchronisation can occur then it must occur and that all threads can then return.

Similarly to section 4.2.4 we will check refinement under the stable-failures model even though the model can diverge. This is valid as for every state that could be unstable due to a hidden **spurious** there exists a corresponding stable state where the regulator process **Reg** blocks the spurious wakeup. FDR yields that the following assertion holds for systems of up to 6 threads; this can be verified in 1280 seconds.

1 `assert Spec \sqsubseteq_F BarrierSystem \ {spurious}`

As a result, we have that the **Barrier** object presented earlier is a correct implementation of barrier synchronisation for $n \leq 6$. This gives us significant confidence that the system is correct for larger n .

5.4 Specification processes for the Signal objects

Our current model of `Signal` models all the internal workings of the object, modelling the `synchronized` blocks and the internal `state` variable. This direct model is rather complex; we can only test for correctness on models with up to 6 threads in reasonable time. We can instead construct a specification process which models the use of the `Signal` object. Though this still results in a model size exponential in the number of threads, the model will be of significantly smaller size. This will allow us to verify the `Barrier` object for larger numbers of threads.

By inspecting the usage of `Signal` we observe that there are two synchronisations between threads performed by each `Signal` object

- `waitForSignalUp` and `signalUpAndWait` synchronise to indicate that that all threads using objects in this subtree are waiting to synchronise. This synchronisation has the parent waiting on the child to signal, with the child being allowed to signal and progress immediately
- `signalDown` and `signalUpAndWait` synchronise, with the parent signalling to the child that the barrier synchronisation has occurred and that `signalUpAndWait` can return. This synchronisation has the child thread waiting on the parent signalling down to it; the child thread is always waiting first as the child starts waiting on this synchronisation immediately after the previous synchronisation occurs.

We can model this simplified `Signal` object via the following CSP:

Listing 23: The CSP model of the specification `Signal` object

```
1 channel endWaitForSignalUp, endSignalUpAndWait : SignalID . ThreadID
2 waitChannels = {endWaitForSignalUp, endSignalUpAndWait}
3
4 -- Simplified spec for a correctly used Signal object
5 SpecSig(s) =
6   callSignalUpAndWait.s?t → callWaitForSignalUp.s?t2 → SpecSig2(s, t, t2)
```

```

7   □ callWaitForSignalUp.s?t2 → callSignalUpAndWait.s?t → SpecSig2(s, t, t2)
8   SpecSig2(s, t, t2) =
9     endWaitForSignalUp.s.t2 → callSignalDown.s.t2 → endSignalUpAndWait.s.t → SpecSig(s)
10
11  -- The individual functions for the Signal object
12  SpecSignalUpAndWait(s, t) = callSignalUpAndWait.s.t → endSignalUpAndWait.s.t → SKIP
13  SpecWaitForSignalUp(s, t) = callWaitForSignalUp.s.t → endWaitForSignalUp.s.t → SKIP
14  SpecSignalDown(s, t) = callSignalDown.s.t → SKIP
15
16  -- Construct the system for each of the SpecSig objects
17  SpecSignals =
18    || s ← SignalID • [{callSignalUpAndWait.s, callWaitForSignalUp.s,
19                      callSignalDown.s, endWaitForSignalUp.s, endSignalUpAndWait.s}]
20                      SpecSig(s)
21
22  -- Method for barrier-sync to initialise the two objects
23  InitialiseSpecSignals (threads) =
24    (SpecSignals [|union(signalChannels, waitChannels)|] threads) \ waitChannels
25

```

We introduce channels `endWaitForSignalUp` and `endSignalUpAndWait` to represent the synchronisations between the child and parent, with each of the channels indicating that their respective functions are able to return. `SpecSig(s)` is used to dictate the order that communications are allowed to occur:

1. Initially, it can either communicate a `callSignalUpAndWait` from the child thread or a `callWaitForSignalUp` from the parent. It then communicates the other event.
2. It then communicates an `endWaitForSignalUp` to indicate to the parent that the first synchronisation has occurred.
3. The parent then communicates a `callSignalDown` indicating that the barrier synchronisation has occurred.
4. Finally, an `endSignalUpAndWait` is communicated to indicate to the child that they can now terminate; `SpecSig(s)` then repeats.

We also define the specification versions of the three external methods offered by a `Signal` object. `SpecSignalUpAndWait` and `SpecWaitForSignalUp` both initially communicate an event indicating that they have been ‘called’ before communicating a `endSignalUpAndWait` or `endWaitForSignalUp` respectively before terminating. By contrast, `SpecSignalDown` immediately terminates after communicating that it has been ‘called’ as it does not require a second synchronisation with another thread.

Finally for the `Signal` specifications, we let `SpecSignals` be the alphabetised parallel composition of each of the `SpecSig(s)` processes, with the parallel composition forcing each specification object to only synchronise on events with the matching `SignalID`. The individual threads and the overall system are defined similarly to the above, with the exception that all calls are to the specification processes and not the originals.

Listing 24: The implementation of the `Barrier` based on

```

1  -- sThread is the same as Thread but uses the spec Signal
2  sThread(T.me) = beginSync.T.me → sSync(T.me) □ end.T.me → SKIP
3  sSync(T.me) =
4    let child1 = 2*me+1
5      child2 = 2*me+2
6    within (if (child1 < n) then SpecWaitForSignalUp(S.(child1), T.me) else SKIP);
7      (if (child2 < n) then SpecWaitForSignalUp(S.(child2), T.me) else SKIP);
8      (if me ≠ 0 then SpecSignalUpAndWait(S.me, T.me) else SKIP);
9      (if (child1 < n) then SpecSignalDown(S.(child1), T.me) else SKIP);
10     (if (child2 < n) then SpecSignalDown(S.(child2), T.me) else SKIP);
11     endSync.T.me → sThread(T.me)
12
13  -- Initialise the spec system
14  sThreads = ||| t : ThreadID • sThread(t)
15  sBarrierSystem = InitialiseSpecSignals (sThreads)
16
17  assert Spec ⊆FD sBarrierSystem

```

Since the simplified `Signal` object does not use monitors we have that the system should be divergence-free. This is verified by FDR as the above refinement holds against our (divergence-free) linearization checker. We have that using the simpler specification

`Signals` result in model checking with 10 threads being possible in reasonable time. This is a significant increase over the natural model and significantly increases our confidence in the correctness of the `Barrier` object.

6 Conclusions and future work

In this paper, we have examined a range of concurrency primitives offered by the Java Virtual Machine, Scala Concurrency Library module and a range of different lock designs. We have examined and proved the correctness of each of these whilst also proving complexity results and examining other properties. There are, however, some limitations to our work.

Firstly, by the nature of model checking, we are only able to model a limited number of threads with restrictions on other parameters too. Though model checking with larger numbers of threads is technically possible, the exponential blow up in the number of states renders it practically infeasible. If a model is correct for small numbers of threads, we have significantly more confidence in the model remaining correct for larger numbers of threads; we do however note that this does not necessarily imply correctness.

Take, for example, the SCL monitor which we have previously proved correct for six threads and two conditions in section 4.2. The components of each individual condition are distinct with the exception of the `ThreadInfo` objects; any flaw with this would be expected to show itself with only two conditions. Likewise any issue due to interfering threads must require at least seven interacting threads; this seems remarkably unlikely due to the simplicity of the system and limited possible interactions between threads.

We have also only considered a number of specific concurrency primitives. Though our approach can be extended to many other primitives, this would still require significant work to verify the correctness of these. An automated translation system from

normal code to CSP would aid in this task. Specialised examples do exist, such as SVA for shared variable programs [12], however no more general models exist. It is worth noting that any general translator would likely suffer from additional complexity blow up due to a lack of insight. An example of this can be seen in the SCL Monitor model, where a more natural implementation of the queue lead to state space $n!$ times larger than a model with additional insight. Though a naïve translation is very feasible, the utility of such an approach is limited, though non-zero. Implementing a general automated translator, either optimised or naïve, is beyond the scope of the project and therefore left as further work.

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