Features

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Learning Objectives

- Understand where a *feature map* sits in a machine learning pipeline.
- Understand how feature extraction can be used to extend the power of linear methods.
- Build pipelines with expanded feature spaces using the sklearn ecosystem.

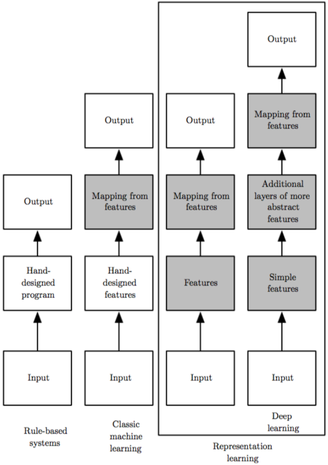

Figure 1.5: Flowcharts showing how the different parts of an AI system relate to each other within different AI disciplines. Shaded boxes indicate components that are able to learn from data.

From Goodfellow, Bengio, and Courville's *Deep Learning*.

Feature Extraction

The Input Space $\mathcal{X}$

- Our general learning theory setup: no assumptions about $\mathcal{X}$

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- But $\mathcal{X} = \mathbf{R}^d$ for the specific methods we've developed:
    - Ridge regression
    - Lasso regression
    - Linear SVM

Motivation

- Two motivations for thinking about feature extraction:
  - Motive 1 – Improving performance.

```
Boston House Prices dataset
===========================

Notes
------
Data Set Characteristics:

    :Number of Instances: 506

    :Number of Attributes: 13 numeric/categorical predictive

    :Median Value (attribute 14) is usually the target
```

Motivation

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```
from sklearn.linear_model import ElasticNetCV
```

```
en = ElasticNetCV(cv = 5)
```

```
en.fit(np.log(train_X[['LSTAT']]), train_y)
en.score(np.log(test_X[['LSTAT']]), test_y)
```

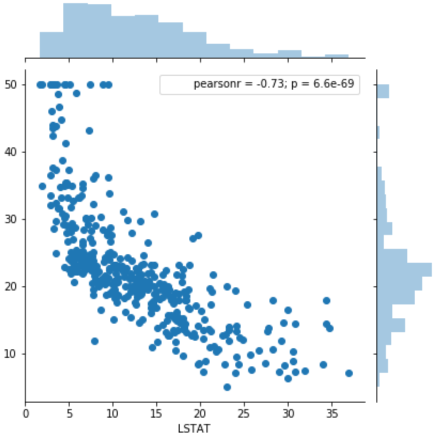
```
0.74651286928253746
```

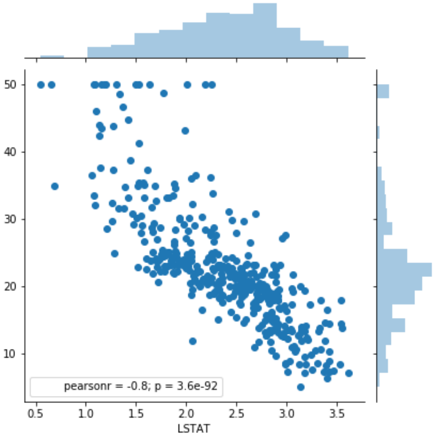
```
en.fit(train_X[['LSTAT']], train_y)
en.score(test_X[['LSTAT']], test_y)
```

```
0.57894475666257272
```

Motivation

- Key idea: instead of using more flexible models, use better features.





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    - Image files
    - Sound recordings
    - DNA sequences

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  - But everything in a computer is a sequence of numbers?

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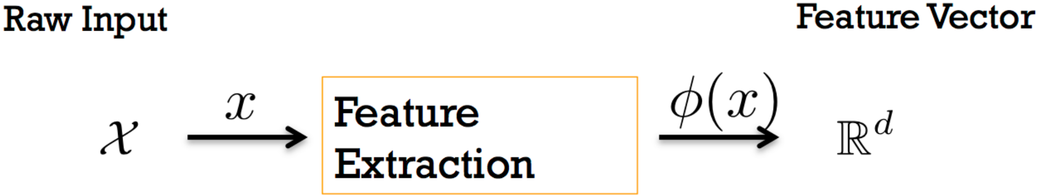
  - Motive 2 – consuming inputs that are not natively in $\mathbf{R}^d$ – examples?
    - Text documents
    - Image files
    - Sound recordings
    - DNA sequences
  - But everything in a computer is a sequence of numbers?
    - The ith entry of each sequence should have the same "meaning"
    - All the sequences should have the same length

Definition

Mapping an input from $\mathcal{X}$ to a vector in $\mathbf{R}^d$ is called **feature extraction** or **featurization**.



Feature Templates

Example: Detecting Email Addresses

- Task: Predict whether a string is an email address

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

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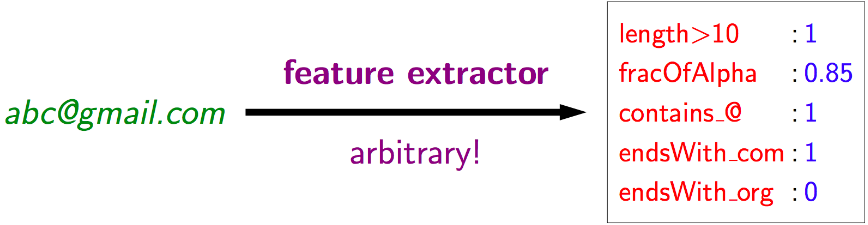

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Example: Detecting Email Addresses

- Task: Predict whether a string is an email address
- Could use domain knowledge and write down:

abc@gmail.com $\xrightarrow[\text{arbitrary!}]{\textbf{feature extractor}}$

length>10	: 1
fracOfAlpha	: 0.85
contains_@	: 1
endsWith_com	: 1
endsWith_org	: 0

- But this was ad-hoc, and maybe we missed something.
- Could be more systematic?

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Feature Templates

Definition (informal)

A **feature template** is a group of features all computed in a similar way.

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Feature Templates

- Length greater than ___
- Last three characters equal ___
- Contains character ___

Based on Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Feature Template: Last Three Characters Equal ___

- Don't think about which 3-letter suffixes are meaningful...
- Just **include them all.**

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Feature Template: Last Three Characters Equal _ _ _

- Don't think about which 3-letter suffixes are meaningful...
- Just **include them all.**

abc@gmail.com $\longrightarrow$

endsWith_aaa : 0
endsWith_aab : 0
endsWith_aac : 0
...
endsWith_com : 1
...
endsWith_zzz : 0

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Feature Template: Last Three Characters Equal ___

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- With regularization, our methods will not be overwhelmed.

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Feature Template: One-Hot Encoding

Definition

A **one-hot encoding** is a set of features (e.g. a feature template) that always has **exactly one** non-zero value.

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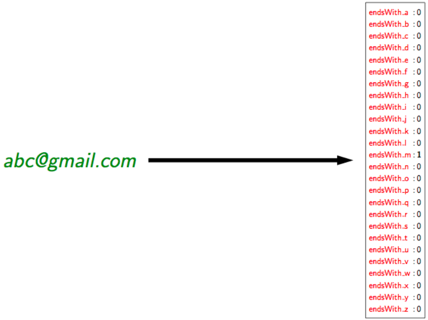

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Feature Template: Count-based

- Imagine you are trying to validate a credit card transaction.
- One feature in raw data consumed by pipeline: zip code of transaction.
- There are approximately 43,000 zip codes in the USA.
- What might be a problem with one-hot encoding this feature?

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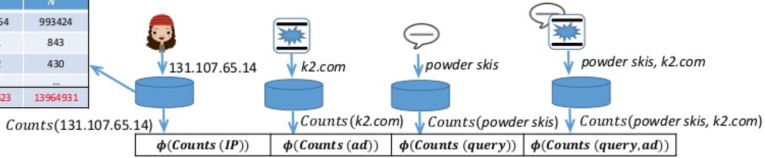
- One feature in raw data consumed by pipeline: zip code of transaction.
- There are approximately 43,000 zip codes in the USA.
- What might be a problem with one-hot encoding this feature?
- Even worse – think about internet scale problems.

Learning with counts

IP	N^+	N^-
173.194.33.9	46964	993424
87.250.251.11	31	843
131.107.65.14	12	430
...	...	...
REST	745623	13964931



- Features are per-label counts [+odds] [+backoff]

$$\phi = [N^+ \quad N^- \quad \log(N^+)-\log(N^-) \quad \text{IsRest}]$$

- $\log(N^+)-\log(N^-) = \log\frac{p(+)}{p(-)}$: log-odds/Naïve Bayes estimate
- N^+, N^-: indicators of confidence of the naïve estimate
- **IsFromRest:** indicator of back-off vs. "real count"

From Misha Bilenko/Microsoft Azure ML's "Many Shades of Scale: Big Learning Beyond Big Data"

Feature Vector Representations

fracOfAlpha	: 0.85
contains_a	: 0
...	
contains_@	: 1
...	

Array representation (good for dense features):

```
[0.85, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0]
```

Map representation (good for sparse features):

```
{"fracOfAlpha": 0.85, "contains_@": 1}
```

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{"endsWith_"+x[-3:]: 1}.
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  - On `"example string"` we’d get `{"endsWith_ing": 1}`.
  - Has overhead compared to arrays, so much slower for dense features.
  - Question: if we have a sparse feature vector, what are the implications for preprocessing?

From Percy Liang’s "Lecture 3" slides from Stanford’s CS221, Autumn 2014.

Handling Nonlinearity with Linear Methods

Example Task: Predicting Health

- General Philosophy: Extract every feature that might be relevant

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Example Task: Predicting Health

- General Philosophy: Extract every feature that might be relevant
- Features for medical diagnosis
    - height
    - weight
    - body temperature
    - blood pressure
    - etc...

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Issues for Linear Predictors

- For linear predictors, it's important **how** features are added

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Issues for Linear Predictors

- For linear predictors, it's important **how** features are added
- Three types of nonlinearities can cause problems:
  1. Non-monotonicity
  2. Saturation
  3. Interactions between features

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Non-monotonicity: The Issue

- Feature Map: $\phi(x) = [1, \text{temperature}(x)]$

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- Issue:
  - Health is not an affine function of temperature.
- Affine function can either say
  - Very high is bad and very low is good, or
  - Very low is bad and very high is good,
  - But here, both extremes are bad.

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Non-monotonicity: Solution 1

- Transform the input:
$$\phi(x) = \left[1, \{\text{temperature(x)-37}\}^2\right],$$
where 37 is "normal" temperature in Celsius.

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Non-monotonicity: Solution 1

- Transform the input:
$$\phi(x) = \left[1, \{\text{temperature(x)-37}\}^2\right],$$
where 37 is "normal" temperature in Celsius.
- Ok, but this requires domain knowledge
  - Do we really need that?

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Non-monotonicity: Solution 2

- Think less, put in more:

$$\phi(x) = \left[1, \text{temperature}(x), \{\text{temperature}(x)\}^2\right].$$

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- Think less, put in more:

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- **More expressive** than Solution 1.

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General Rule

Features should be simple building blocks that can be pieced together.

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Saturation: The Issue

- Setting: Find products relevant to user's query

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where $N(x) =$ number of people who bought x.

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where $N(x) =$ number of people who bought x.
- We expect a monotonic relationship between $N(x)$ and relevance, but...

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Saturation: The Issue

Is relevance linear in N(x)?

- Relevance score reflects confidence in relevance prediction.
- Are we 10 times more confident if $N(x) = 1000$ vs $N(x) = 100$?

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Saturation: The Issue

Is relevance linear in N(x)?

- Relevance score reflects confidence in relevance prediction.
- Are we 10 times more confident if $N(x) = 1000$ vs $N(x) = 100$?

- Bigger is better... but not that much better.

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Saturation: Solve with nonlinear transform

- Smooth nonlinear transformation:

$$\phi(x) = [1, \log\{1 + N(x)\}]$$

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- $\log(\cdot)$ good for values with large dynamic ranges

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- Smooth nonlinear transformation:

$$\phi(x) = [1, \log\{1 + N(x)\}]$$

- $\log(\cdot)$ good for values with large dynamic ranges
- *Does it matter what base we use in the log?*

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Saturation: Solve by discretization

- Discretization (a discontinuous transformation):

$$\phi(x) = (1(5 \leqslant N(x) < 10), 1(10 \leqslant N(x) < 100), 1(100 \leqslant N(x)))$$

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- Sometimes we might prefer one-sided buckets

$$\phi(x) = (1(5 \leqslant N(x)), 1(10 \leqslant N(x)), 1(100 \leqslant N(x)))$$

  - Why?

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  - Why? Hint: What's the effect of regularization the parameters for rare buckets?
- Small buckets allow quite flexible relationship

Interactions: The Issue

- Input: Patient information x
- Action: Health score $y \in \mathbf{R}$ (higher is better)
- Feature Map

$$\phi(x) = [\text{height}(x), \text{weight}(x)]$$

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Interactions: The Issue

- Input: Patient information x
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$$\phi(x) = [\text{height}(x), \text{weight}(x)]$$

- Issue: It's the weight **relative** to the height that's important.
- Impossible to get with these features and a linear classifier.
- Need some **interaction** between height and weight.

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Interactions: Approach 1

- Google "ideal weight from height"

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Interactions: Approach 1

- Google "ideal weight from height"
- J. D. Robinson's "ideal weight" formula (for a male):

$$\text{weight(kg)} = 52 + 1.9\,[\text{height(in)} - 60]$$

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Interactions: Approach 1

- Google “ideal weight from height”
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$$\text{weight(kg)} = 52 + 1.9\,[\text{height(in)} - 60]$$

- Make score square deviation between height(h) and ideal weight(w)

$$f(x) = (52 + 1.9\,[h(x) - 60] - w(x))^2$$

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- WolframAlpha for complicated Mathematics:

$$f(x) = 3.61h(x)^2 - 3.8h(x)w(x) - 235.6h(x) + w(x)^2 + 124w(x) + 3844$$

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Interactions: Approach 2

- Just include all second order features:

$$\phi(x) = \left[1, h(x), w(x), h(x)^2, w(x)^2, \underbrace{h(x)w(x)}_{\text{cross term}}\right]$$

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General Principle

Simpler building blocks replace a single "smart" feature.

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

Predicate Features and Interaction Terms

Definition

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  - $x \mapsto d(x) = 1(\text{subject is driving})$
- For predicates, interaction terms correspond to **AND** conjunctions:
  - $x \mapsto s(x)d(x) = 1(\text{subject is sleeping AND subject is driving})$

So What's Linear?

- Non-linear feature map $\phi : \mathcal{X} \to \mathbf{R}^d$
- Hypothesis space:

$$\mathcal{F} = \left\{ f(x) = w^T \phi(x) \mid w \in \mathbf{R}^d \right\}.$$

From Percy Liang's "Lecture 3" slides from Stanford's CS221, Autumn 2014.

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Key Idea: Non-Linearity

- Nonlinear $f(x)$ is important for **expressivity**.

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Key Idea: Non-Linearity

- Nonlinear $f(x)$ is important for **expressivity**.
- $f(x)$ linear in w and $\phi(x)$: makes finding $f^*(x)$ much easier

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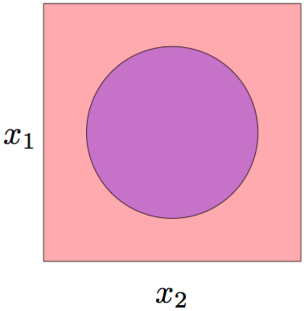
Geometric Example: Two class problem, nonlinear boundary



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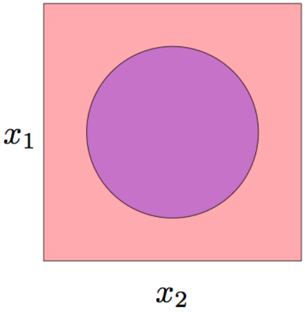

- With linear feature map $\phi(x) = (x_1, x_2)$ and linear models, no hope
- With appropriate nonlinearity $\phi(x) = \left(x_1, x_2, x_1^2 + x_2^2\right)$, piece of cake.

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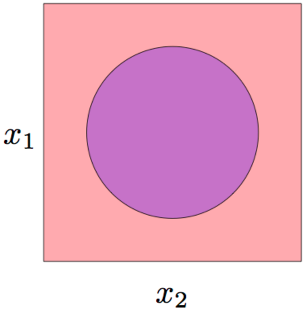

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- Video: http://youtu.be/3liCbRZPrZA

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Expressivity of Hypothesis Space

- Consider a linear hypothesis space with a feature map $\phi : \mathcal{X} \to \mathbf{R}^d$:

$$\mathcal{F} = \left\{ f(x) = w^T \phi(x) \right\}$$

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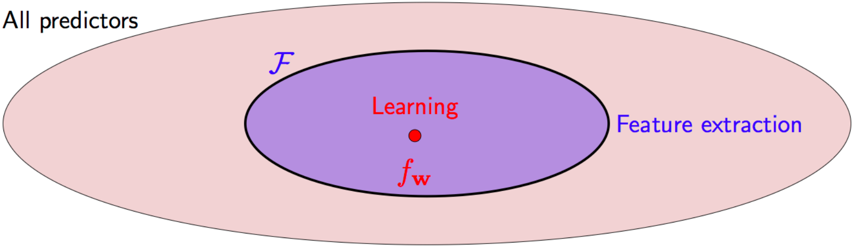

Question: does $\mathcal{F}$ contain a good predictor?

We can grow the linear hypothesis space $\mathcal{F}$ by adding more features.

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