Recitation 1

Gradients and Directional Derivatives

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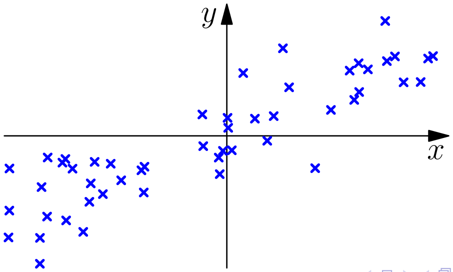
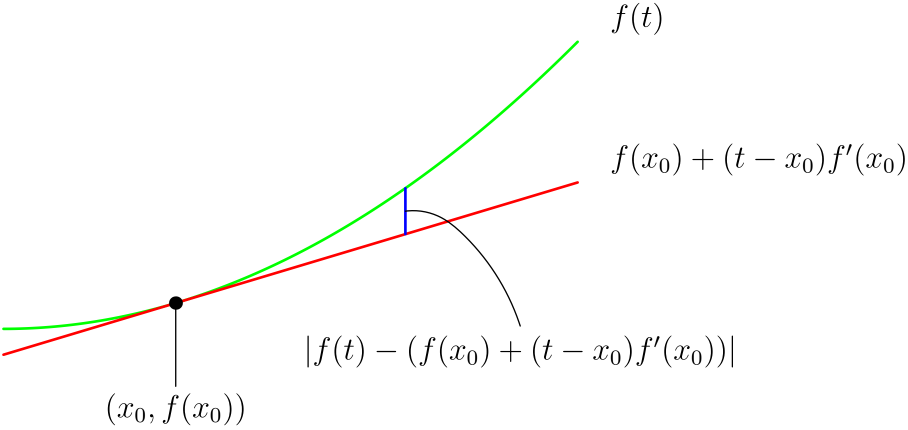
Intro Question

Question

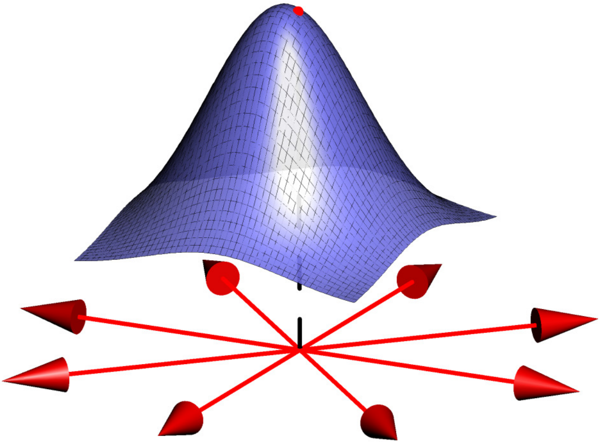
We are given the data set $(x_1, y_1), \ldots, (x_n, y_n)$ where $x_i \in \mathbb{R}^d$ and $y_i \in \mathbb{R}$. We want to fit a linear function to this data by performing empirical risk minimization. More precisely, we are using the hypothesis space $\mathcal{F} = \{f(x) = w^T x \mid w \in \mathbb{R}^d\}$ and the loss function $\ell(a, y) = (a - y)^2$. Given an initial guess $\tilde{w}$ for the empirical risk minimizing parameter vector, how could we improve our guess?



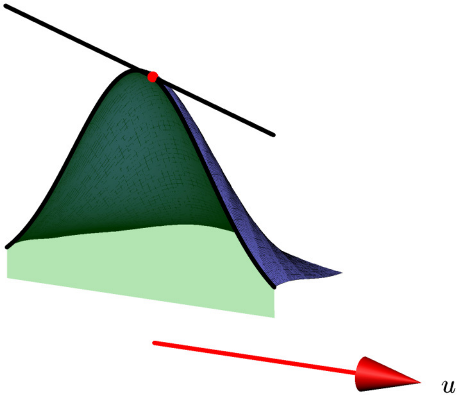
1D Linear Approximation By Derivative



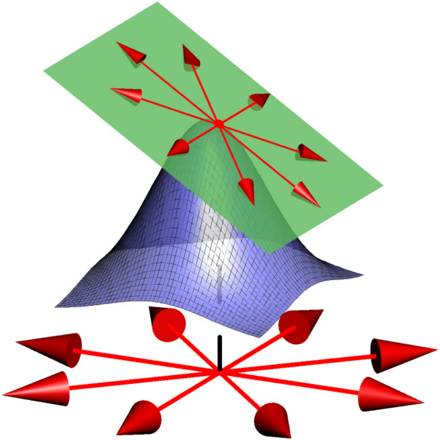
Multiple Possible Directions for $f : \mathbb{R}^2 \to \mathbb{R}$



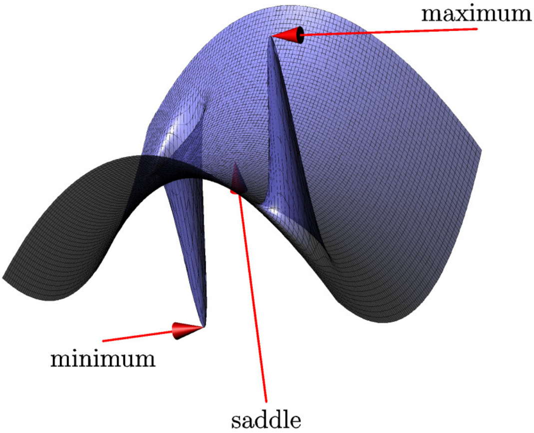
Directional Derivative as a Slope of a Slice



Tangent Plane for $f : \mathbb{R}^2 \to \mathbb{R}$



Critical Points of $f : \mathbb{R}^2 \to \mathbb{R}$



Computing Gradients

Question

For each of the following functions, compute the gradient.

1. $f : \mathbb{R}^3 \to \mathbb{R}$ is given by
$$f(x_1, x_2, x_3) = \log(1 + e^{x_1+2x_2+3x_3}).$$
2. $f : \mathbb{R}^n \to \mathbb{R}$ is given by
$$f(x) = \|Ax - y\|_2^2 = (Ax - y)^T(Ax - y) = x^T A^T Ax - 2y^T Ax + y^T y,$$
for some $A \in \mathbb{R}^{m\times n}$ and $y \in \mathbb{R}^m$.