CS570
Analysis of Algorithms
Spring 2016
Exam III

Name: _____

Student ID: _____

Email Address:_____

_____Check if DEN Student

	Maximum	Received
Problem 1	20	
Problem 2	15	
Problem 3	20	
Problem 4	20	
Problem 5	10	
Problem 6	15	
Total	100	

Instructions:

1. This is a 2-hr exam. Closed book and notes
2. If a description to an algorithm or a proof is required please limit your description or proof to within 150 words, preferably not exceeding the space allotted for that question.
3. No space other than the pages in the exam booklet will be scanned for grading.
4. If you require an additional page for a question, you can use the extra page provided within this booklet. However please indicate clearly that you are continuing the solution on the additional page.

1) 20 pts
Mark the following statements as **TRUE**, **FALSE,** or **UNKNOWN**. No need to provide any justification.

[TRUE/FALSE/UNKNOWN]
If X≤p Y, and X is NP-complete, then Y is NP-hard.

[TRUE/FALSE/UNKNOWN]
If X≤p Y, and X is NP-complete, then Y is NP-complete.

[TRUE/FALSE/UNKNOWN]
If X≤p Integer Programming, then X is NP-hard.

[TRUE/FALSE/UNKNOWN]
If X≤p Linear Programming, then X is in P.

[TRUE/FALSE/UNKNOWN]
3-SAT cannot be solved in polynomial time.

[TRUE/FALSE/UNKNOWN]
If graph G has no cycles, then the independent set problem in G can be solved in polynomial time.

[TRUE/FALSE]
Although the general Travelling Salesman Problem is NP-complete, in class, we presented a 2-approximation algorithm for it that runs in polynomial time.

[TRUE/FALSE]
Breadth first search is an example of a divide-and-conquer algorithm.

[TRUE/FALSE]
Memoization requires memory space which is linear in size with respect to the number of unique sub-problems.

[TRUE/FALSE]
The smallest element in a binary max-heap of size *n* can be found with at most *n/2* comparisons.

2) 15 pts

Ted and Marshall are taking a road trip from Somerville to Vancouver. Because they are going on a 52-hour drive, Ted and Marshall decide to switch off driving at each rest stop they visit; however, because Ted has a better sense of direction than Marshall, he should be driving both when they depart and when they arrive (to navigate the city streets). Given a route map represented as a weighted undirected graph G = (V, E, w) with positive edge weights, where vertices represent start point, end point, and all rest stops, and edges represent routes between rest stops, (edge weights representing the route distance), devise an efficient algorithm to find a route (if possible) of minimum distance between Somerville and Vancouver such that Ted and Marshall alternate edges and Ted drives the first and last edge.

3) 20 pts
Suppose you have access to a function VALID that returns true if its input is a valid English word, and false otherwise. You are given a sentence where the punctuation has been stripped, for example, “dynamicprogrammingispowerful”. Assuming calls to VALID take constant time, give an $O(n^2)$ time algorithm to determine whether the input can be split into a sequence of valid words.

4) 20 pts
Suppose we have a variation on the 3-SAT problem called Min-3-SAT, where the literals are never negated. Of course, in this case it is possible to satisfy all clauses by simply setting all literals to true. But, we are additionally given a number k, and are asked to determine whether we can satisfy all clauses while setting at most k literals to be true. Prove that Min-3-SAT is NP-Complete.

5) 10 pts

A manufacturer produces two products, X and Y, with two machines, A and B.

- The cost of producing each unit of X is:
  a) for machine A: 50 minutes,
  b) for machine B: 30 minutes.
- The cost of producing each unit of Y is:
  a) for machine A: 24 minutes,
  b) for machine B: 33 minutes.
- Working plans for a particular week are:
  a) 40 hours of work on machine A,
  b) 35 hours of work on machine B.
- The week starts with:
  a) A stock of 30 units of X and 90 of Y, (in other words, at the start of the week we already have 30 units of X and 90 units of Y)
  b) A demand of 75 units of X and 95 of Y. (in other words, this many units of X and Y need to delivered to clients at the end of the week)

Plan the production (i.e. how many units of X and Y to produce), in order to end the week with the maximum combined stock of X and Y, while satisfying the above time constraints and demand constraints. Formulate the problem using Linear programming

6) 15 pts

There are n people, $p_1, p_2, \ldots, p_n$ and k sets. Each set consists of several people and a person can be in more than one set. We need to select one person from each set and the maximum times a person is selected should be less than m. Give a polynomial time algorithm to find such a selection if there is one, or to indicate that such a selection is not possible. Prove that your solution is correct.

Additional Space

Additional Space