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Pyramid Solitaire in VDM++

Mestrado Integrado em Engenharia Informática e Computação

Métodos Formais em Engenharia de Software

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1. Informal system description and list of requirements

1.1 Informal system description

```

      PYRAMID
      3C
    9S  AH
  KC  2H  4S
5D  KH  3S  9H
JC  2D  9C  QH  7H
8C  6C  6D  6S  QS  2S
6H  10C  4C  4H  KD  QD  7D

JH

```

Select the card coordinates to move (ex: "3,3"). To select card in hand insert "0,0". To get next card in hand type "1":

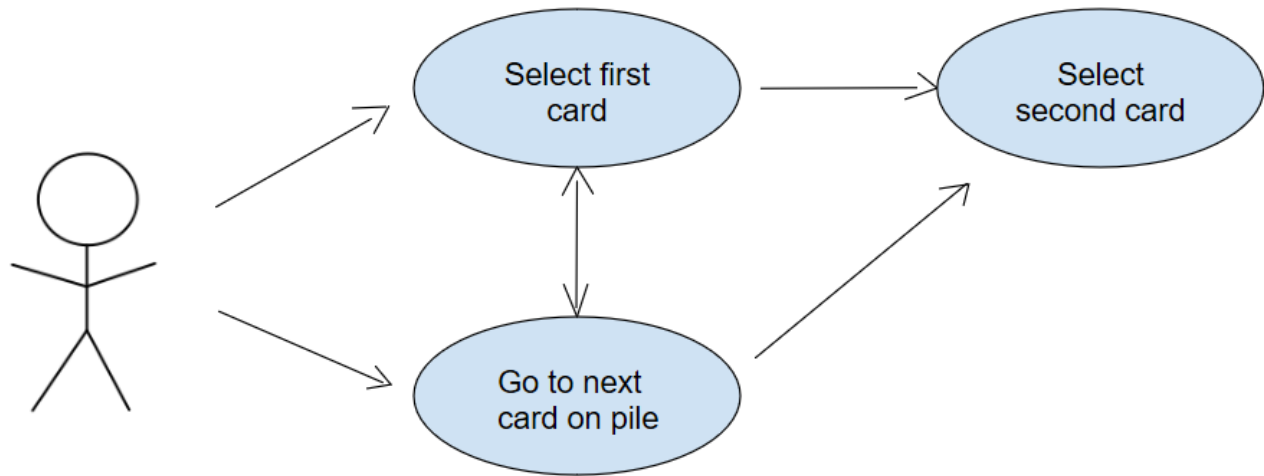
Displayed above, is the pyramid and bellow it the card in the users hand.

1.2 List of requirements

Id	Priority	Description
R1	Mandatory	The user can visualize the cards in the pyramid and in his card pile.
R2	Mandatory	The user can switch to the next card in the card pile.
R3	Mandatory	The user can combine two separate cards, which value amounts to 13, and remove them from the game.
R4	Mandatory	The user should be able to complete the game by combining all the cards from the table/pyramid.

2. Visual UML model ¹

2.1 Use case model ²

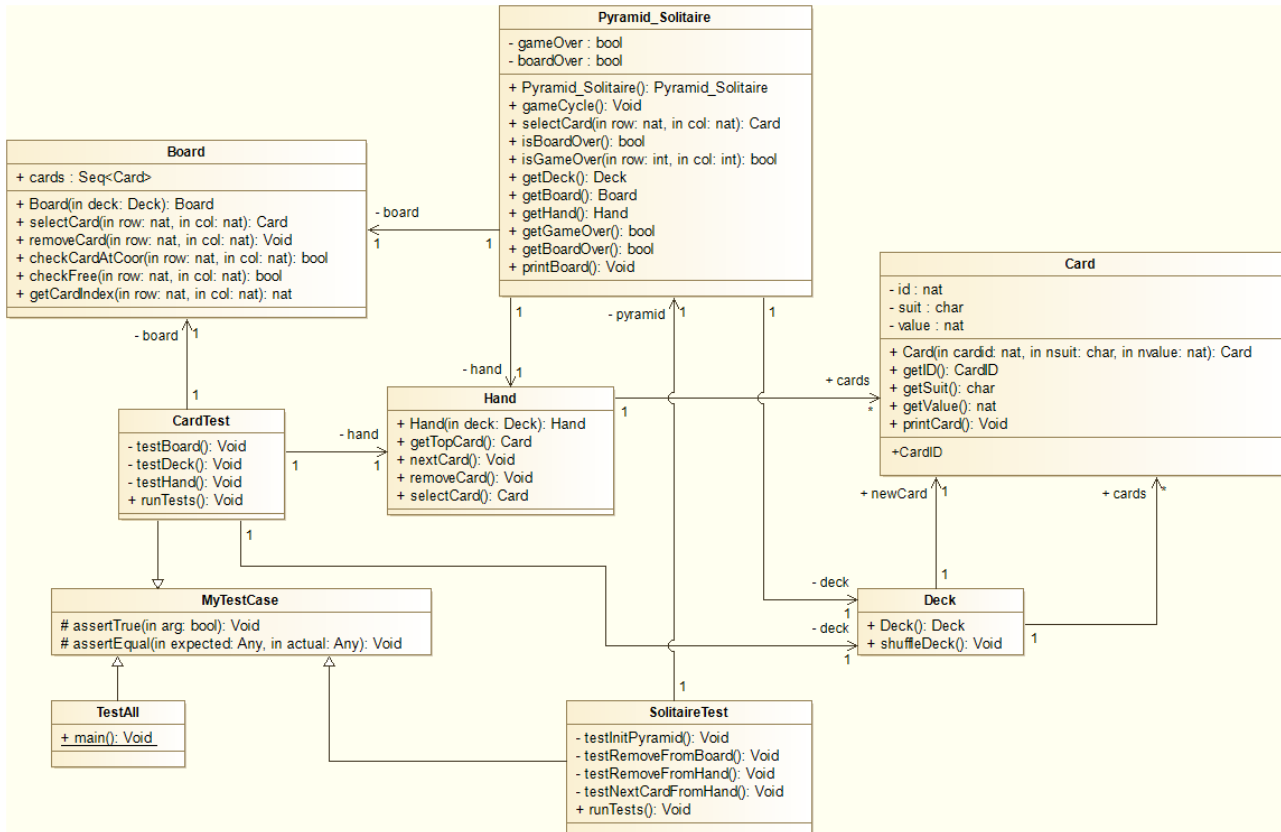


Main Use cases:

Scenario	Select first card
Description	Normal scenario for user to pick his first card to pair.
Pre-conditions	<ol style="list-style-type: none"> 1. Row must be greater than 0 and smaller than 8. 2. Column must be greater than 0 and smaller than 8.
Post-conditions	(none)
Steps	<ol style="list-style-type: none"> 1. The user is prompted with a text to insert the column and row. 2. The user inserts the desired values.
Exceptions	(none)

Scenario	Go to next card on pile
Description	Normal scenario where user cycles throw the cards on his pile.
Pre-conditions	<ol style="list-style-type: none"> 1. The pile mustn't be empty.
Post-conditions	<ol style="list-style-type: none"> 1. The pile size must continue the same after this action.
Steps	<ol style="list-style-type: none"> 1. The user is prompted with a text to insert the column and row. 2. The user selects the coordinates (0,0).
Exceptions	(none)

2.2 Class model ^{3 4}



Class	Description
Board	Core model; defines the board and all its elements and functions.
Pyramid_Solitaire	Responsible for calling all the major game functions present in other classes.
Card	Core model; defines element Card and all its elements and functions.
Hand	Core model; defines the Cards in user hand and all its elements and functions.
MyTestCase	Superclass for test classes; defines <code>assertEquals</code> and <code>assertTrue</code> .
CardTest	Defines the test/usage scenarios and test cases for the card element.
BoardTest	Defines the test/usage scenarios and test cases for the board.
SolitaireTest	Defines the test/usage scenarios and test cases for the game and its initialization.
TestAll	Calls all the test functions inside all the other Test Classes

3. Formal VDM++ model

3.1 Class Pyramid_Solitaire

class Pyramid_Solitaire

types

values

instance variables

```

private deck: Deck;
private board: Board;
private hand: Hand;
private gameOver: bool := false;
private boardOver: bool := false;

```

operations

```

public Pyramid_Solitaire : () ==> Pyramid_Solitaire
  Pyramid_Solitaire() ==
  (
    deck := new Deck();
    deck.shuffleDeck();
    board := new Board(deck);
    hand := new Hand(deck);
  );

public gameCycle : () ==> ()
  gameCycle() ==
  (
    printBoard();
    while (gameOver = false) do
    (
      boardOver := isBoardOver();
      --gameOver := isGameOver();
    )
  );

public selectCard : nat * nat ==> Card
  selectCard(row, col) == (
    if row = 0 then (
      return hand.selectCard();
    );
    if row > 0 then (
      return board.selectCard(col, row);
    );
    return new Card(0, 'N', 0)
  );

public isBoardOver : () ==> bool
  isBoardOver() == (
    if board.cards(1) = new Card(0, 'N', 0) then return true
    else return false
  )
pre len board.cards = 28;

public isGameOver : int * int ==> bool
  isGameOver(row, col) == (
    if row = -1 and col = -1 then return true
    else return false
  );

public pure getDeck : () ==> Deck
  getDeck() == return deck;

public pure getBoard : () ==> Board

```

```

getBoard() == return board;

public pure getHand : () ==> Hand
getHand() == return hand;

public pure getGameOver : () ==> bool
getGameOver() == return gameOver;

public pure getBoardOver : () ==> bool
getBoardOver() == return boardOver;

public printBoard : () ==> ()
printBoard() == (
  IO`println("          PYRAMID          ");
  IO`print("          ");
  IO`print(board.cards(1).getValue());
  IO`println(board.cards(1).getSuit());
  IO`print("          ");
  IO`print(board.cards(2).getValue());
  IO`print(board.cards(2).getSuit());
  IO`print("  ");
  IO`print(board.cards(3).getValue());
  IO`println(board.cards(3).getSuit());
  IO`print("          ");
  IO`print(board.cards(4).getValue());
  IO`print(board.cards(4).getSuit());
  IO`print("  ");
  IO`print(board.cards(5).getValue());
  IO`print(board.cards(5).getSuit());
  IO`print("  ");
  IO`print(board.cards(6).getValue());
  IO`println(board.cards(6).getSuit());
  IO`print("          ");
  IO`print(board.cards(7).getValue());
  IO`print(board.cards(7).getSuit());
  IO`print("  ");
  IO`print(board.cards(8).getValue());
  IO`print(board.cards(8).getSuit());
  IO`print("  ");
  IO`print(board.cards(9).getValue());
  IO`print(board.cards(9).getSuit());
  IO`print("  ");
  IO`print(board.cards(10).getValue());
  IO`println(board.cards(10).getSuit());
  IO`print("          ");
  IO`print(board.cards(11).getValue());
  IO`print(board.cards(11).getSuit());
  IO`print("  ");
  IO`print(board.cards(12).getValue());
  IO`print(board.cards(12).getSuit());
  IO`print("  ");
  IO`print(board.cards(13).getValue());
  IO`print(board.cards(13).getSuit());
  IO`print("  ");
  IO`print(board.cards(14).getValue());
  IO`print(board.cards(14).getSuit());
  IO`print("  ");
  IO`print(board.cards(15).getValue());
  IO`println(board.cards(15).getSuit());

```

```

IO`print(" ");
IO`print(board.cards(16).getValue());
IO`print(board.cards(16).getSuit());
IO`print(" ");
IO`print(board.cards(17).getValue());
IO`print(board.cards(17).getSuit());
IO`print(" ");
IO`print(board.cards(18).getValue());
IO`print(board.cards(18).getSuit());
IO`print(" ");
IO`print(board.cards(19).getValue());
IO`print(board.cards(19).getSuit());
IO`print(" ");
IO`print(board.cards(20).getValue());
IO`print(board.cards(20).getSuit());
IO`print(" ");
IO`print(board.cards(21).getValue());
IO`println(board.cards(21).getSuit());
IO`print(board.cards(22).getValue());
IO`print(board.cards(22).getSuit());
IO`print(" ");
IO`print(board.cards(23).getValue());
IO`print(board.cards(23).getSuit());
IO`print(" ");
IO`print(board.cards(24).getValue());
IO`print(board.cards(24).getSuit());
IO`print(" ");
IO`print(board.cards(25).getValue());
IO`print(board.cards(25).getSuit());
IO`print(" ");
IO`print(board.cards(26).getValue());
IO`print(board.cards(26).getSuit());
IO`print(" ");
IO`print(board.cards(27).getValue());
IO`print(board.cards(27).getSuit());
IO`print(" ");
IO`print(board.cards(28).getValue());
IO`println(board.cards(28).getSuit());

IO`println("");
IO`print(hand.getTopCard().getValue());
IO`println(hand.getTopCard().getSuit());
);

```

functions

traces

end Pyramid_Solitaire

3.2 Class Board

class Board

types

values

instance variables


```
public cards: seq of Card;
```

operations

```
public Board: Deck ==> Board
  Board(deck) == (
    cards := [];
    for i = 1 to 28 do
      (
        cards := cards ^ [deck.cards(i)];
      );
    return self;
  )
post len cards = 28;

public selectCard : nat * nat ==> Card
  selectCard(row, col) ==
  (
    if checkCardAtCoor(row, col) = true then (
      if checkFree(row, col) = true then (
        dcl cardIndex : nat := getCardIndex(row, col);
        return cards(cardIndex);
      );
    );
    return new Card(0, 'N', 0);
  )
pre row > 0 and row < 8 and col < 8 and col > 0;

public removeCard : nat * nat ==> ()
  removeCard(row, col) == (
    dcl index : nat := getCardIndex(row, col);
    cards(index) := new Card(0, 'N', 0);
  )
post len cards = len cards~;

public checkCardAtCoor : nat * nat ==> bool
  checkCardAtCoor(row, col) ==
  (
    dcl index : nat := getCardIndex(row, col);
    if cards(index).getValue() > 0 then return true
    else return false;
  )
pre row > 0 and row < 8 and col < 8 and col > 0;

public checkFree : nat * nat ==> bool
  checkFree(row, col) ==
  (
    if row = 7 then return true;
    if checkCardAtCoor(row + 1, col) = false then (
      if checkCardAtCoor(row+1, col+1) = false then return true;
    );
    return false
  )
pre row > 0 and row < 8 and col < 8 and col > 0;

public getCardIndex : nat * nat ==> nat
  getCardIndex(row, col) ==
  (
    dcl index : nat := 0;
    for i = 1 to row do
```

```

    (
        for j = 1 to i do
        (
            index := index + 1;
            if i = row and j = col then return index;
        )
    );
    return index;
)
pre row > 0 and row < 8 and col < 8 and col > 0;

```

functions

traces

end Board

3.3 Class Deck

class Deck

types

values

instance variables

```

public cards : seq of Card;
public newCard : Card;

```

operations

```

public Deck: () ==> Deck
Deck() == (
    cards := [];

    for nvalue = 1 to 13 do
    (
        newCard := new Card(nvalue, 'S', nvalue);
        cards := cards ^ [newCard];
        newCard := new Card(nvalue + 13, 'C', nvalue);
        cards := cards ^ [newCard];
        newCard := new Card(nvalue + 26, 'H', nvalue);
        cards := cards ^ [newCard];
        newCard := new Card(nvalue + 39, 'D', nvalue);
        cards := cards ^ [newCard];
    );
    return self;
)
post len cards = 52;

public shuffleDeck : () ==> ()
shuffleDeck() ==
(
    dcl newIndex : nat1;
    dcl tempCard : Card;

    for index = 1 to 52 do
    (
        tempCard := cards(index);
        newIndex := MATH`rand(52) + 1;

```

```

        cards(index) := cards(newIndex);
        cards(newIndex) := tempCard;
    );
)
post len cards = 52;

```

functions

traces

end Deck

3.4 Class Card

class Card

types

```
public CardID = nat;
```

values

instance variables

```
id : nat;
suit : char;
value : nat;
```

operations

```
public Card : nat * char * nat ==> Card
Card(cardid, nsuit, nvalue) ==
(
    id := cardid;
    suit := nsuit;
    value := nvalue;

    return self
)
pre (nvalue >= 0 and nvalue <= 13);

public pure getID : () ==> CardID
getID() == return id;

public pure getSuit : () ==> char
getSuit() == return suit;

public pure getValue : () ==> nat
getValue() == return value;

public printCard : () ==> ()
printCard() ==
(
    if value < 11 and value > 1 then
    (
        IO`print(value);
        IO`printf("%s", [suit]);
        return;
    );
    if value = 1 then
    (
        IO`printf("%s", ['A']);
    );
)

```

```

        IO`printf("%s", [suit]);
        return;
    );
    if value = 11 then
    (
        IO`printf("%s", ['J']);
        IO`printf("%s", [suit]);
        return;
    );
    if value = 12 then
    (
        IO`printf("%s", ['Q']);
        IO`printf("%s", [suit]);
        return;
    );
    if value = 13 then
    (
        IO`printf("%s", ['K']);
        IO`printf("%s", [suit]);
        return;
    );
);

```

functions

traces

end Card

3.5 Class Hand

class Hand

types

values

instance variables

```
public cards: seq of Card := [];
```

operations

```

public Hand: Deck ==> Hand
    Hand(deck) == (
        cards := [];
        for i = 29 to 52 do
        (
            cards := cards ^ [deck.cards(i)];
        );
        return self;
    )
post len cards = 24;

pure public getTopCard : () ==> Card
    getTopCard() == return hd cards
pre len cards > 0;

public nextCard : () ==> ()
    nextCard() == cards := tl cards ^ [hd cards]
pre len cards > 1

```

```

post len cards = len cards~;

public removeCard : () ==> ()
    removeCard() == cards := tl cards
pre len cards > 0
post len cards = (len cards~) - 1;

public selectCard : () ==> Card
    selectCard() == return hd cards
pre len cards > 0;

```

functions

traces

end Hand

4. Model validation

4.1 Class MyTestCase

```

class MyTestCase
/*
    Superclass for test classes, simpler but more practical than VDMUnit`TestCase.
    For proper use, you have to do: New -> Add VDM Library -> IO.
    JPF, FEUP, MFES, 2014/15.
*/

```

operations

```

-- Simulates assertion checking by reducing it to pre-condition checking.
-- If 'arg' does not hold, a pre-condition violation will be signaled.
protected assertTrue: bool ==> ()
assertTrue(arg) ==
    return
pre arg;

-- Simulates assertion checking by reducing it to post-condition checking.
-- If values are not equal, prints a message in the console and generates
-- a post-conditions violation.
protected assertEquals: ? * ? ==> ()
assertEquals(expected, actual) ==
    if expected <> actual then (
        IO`print("Actual value (");
        IO`print(actual);
        IO`print(") different from expected (");
        IO`print(expected);
        IO`println(")\n")
    )
post expected = actual

```

end MyTestCase

4.2 Class CardTest

```

class CardTest is subclass of MyTestCase

```

types**values****instance variables**

```

private deck: Deck := new Deck();
private board: Board := new Board(deck);
private hand: Hand := new Hand(deck);

```

operations

```

private testBoard: () ==> ()
testBoard() ==
(
    assertEquals(28, len board.cards);
    for index = 1 to len board.cards do
    (
        dcl tempCard: Card := deck.cards(index);
        assertTrue(tempCard.getValue() > 0);
        assertTrue(tempCard.getSuit() = 'S' or tempCard.getSuit() = 'C' or
tempCard.getSuit() = 'D' or tempCard.getSuit() = 'H');
    )
);

private testDeck: () ==> ()
testDeck() ==
(
    assertEquals(52, len deck.cards);
);

private testHand: () ==> ()
testHand() ==
(
    assertEquals(24, len hand.cards);
    for index = 1 to len hand.cards do
    (
        dcl tempCard: Card := hand.cards(index);
        assertTrue(tempCard.getValue() > 0);
        assertTrue(tempCard.getSuit() = 'S' or tempCard.getSuit() = 'C' or
tempCard.getSuit() = 'D' or tempCard.getSuit() = 'H');
    )
);

public runTests: () ==> ()
runTests() ==
(
    testBoard();
    testDeck();
    testHand();
);

```

functions**traces**

end CardTest

4.3 Class SolitaireTest

class SolitaireTest **is subclass of** MyTestCase

types

values

instance variables

private pyramid: Pyramid_Solitaire := **new** Pyramid_Solitaire();

operations

private testInitPyramid: () ==> ()

testInitPyramid() ==

```
(
    assertEquals(false, pyramid.getGameOver());
    assertEquals(false, pyramid.getBoardOver());
);
```

private testRemoveFromBoard: () ==> ()

testRemoveFromBoard() ==

```
(
    dcl index: nat := pyramid.getBoard().getCardIndex(1, 1);
    assertTrue(pyramid.getBoard().cards(index).getValue() > 0);
    pyramid.getBoard().removeCard(1, 1);
    assertTrue(pyramid.getBoard().cards(index).getValue() = 0);
);
```

private testRemoveFromHand: () ==> ()

testRemoveFromHand() ==

```
(
    dcl tempCard: Card := pyramid.getHand().getTopCard();
    assertTrue(tempCard.getValue() > 0);
    pyramid.getHand().removeCard();
    assertTrue(pyramid.getHand().getTopCard().getValue() <> tempCard.getValue()
or pyramid.getHand().getTopCard().getSuit() <> tempCard.getSuit());
);
```

private testNextCardFromHand: () ==> ()

testNextCardFromHand() ==

```
(
    dcl tempCard: Card := pyramid.getHand().getTopCard();
    assertTrue(tempCard.getValue() > 0);
    pyramid.getHand().nextCard();
    assertTrue(pyramid.getHand().getTopCard().getValue() <> tempCard.getValue()
or pyramid.getHand().getTopCard().getSuit() <> tempCard.getSuit());
);
```

public runTests: () ==> ()

runTests() ==

```
(
    testInitPyramid();
    testRemoveFromBoard();
    testRemoveFromHand();
    testNextCardFromHand();
);
```

functions

traces

end SolitaireTest

4.4 TestAll

class TestAll is subclass of MyTestCase

types

values

instance variables

operations

```
public static main: () ==> ()
main() ==
(
  dcl boardTest : CardTest := new CardTest();
  dcl solitaireTest : SolitaireTest := new SolitaireTest();
  boardTest.runTests();
  solitaireTest.runTests();
);
```

functions

traces

end TestAll

5. Model verification

5.1 Example of domain verification

One of the proof obligations generated by Overture is:

No.	PO Name	Type
3	Board`selectCard(nat,nat)	legal sequence application

The code under analysis (with the relevant map application underlined) is:

```
public selectCard : nat * nat ==> Card
selectCard(row, col) ==
(
  if checkCardAtCoor(row, col) = true then (
    if checkFree(row, col) = true then (
      dcl cardIndex : nat := getCardIndex(row, col);
      return cards(cardIndex);
    );
  );
  return new Card(0, 'N', 0);
)
pre row > 0 and row < 8 and col < 8 and col > 0;
```

In this case the proof is trivial because the preconditions 'row > 0 and row < 8 and col < 8 and col > 0' assures that the sequence is accessed only inside its domain.

5.2 Example of invariant verification

Another proof obligation generated by Overture is:

No.	PO Name	Type
13	Deck`shuffleDeck()	state invariant holds

The code under analysis (with the relevant state changes underlined> is:

```

public shuffleDeck : () ==> ()
  shuffleDeck() ==
  (
    dcl newIndex : nat1;
    dcl tempCard : Card;

    for index = 1 to 52 do
    (
      tempCard := cards(index);
      newIndex := MATH`rand(52) + 1;
      cards(index) := cards(newIndex);
      cards(newIndex) := tempCard;
    );
  ) post len cards = 52;

```

The relevant invariant under analysis is:

```
inv len cards = 52;
```

The function only randomly selects an index (within bounds) from the sequence and swaps the card with a different one.

6. Conclusions

The model that was developed covers all the requirements.

If time permitted, as future work, it would be interesting to develop a more appealing interface for the user to play the game, a graphical one for example. This project took approximately 14 hours to develop, with an equal contribution to the final product from both elements.

7. References

1. VDM-10 Language Manual, Peter Gorm Larsen et al, Overture Technical Report Series No. TR-001, March 2014
2. Overture tool web site, <http://overturetool.org>