Magic Squares

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Abstract

Magic squares are ancient thing, dating back to 650 BC in China. They have many fantasy features which make it full of aesthetics, so they are always used as decorations of ancient architecture and appear in some famous works of art. Many mathematicians go deep into its properties in the past centuries and found many interesting things of them. Today, we will discuss some features of magic squares and the method for constructing them.

1 Introduction

A *magic square* is a $n \times n$ square grid filled with distinct positive integers in the range $1, 2, \ldots, n^2$ such that each cell contains a different integer and the sum of the integers in each row, column and diagonal is equal. The sum is called the *magic constant* or *magic sum*. There are some non-normal square which integers are not restricted in $1, 2, \ldots, n^2$. Because some magic squares can be generated by multiplying or adding some integers to the normal magic squares, We only focus on the normal magic square.

In the last thousands of years, people found many properties of magic square, like the existence of magic with any order, the formula of magic constant and the method for constructing a magic square. In this article, we will focus mainly on the constructing method and talk a little about other properties of magic square.

2 The Properties of Magic Squares

2.1 The Magic Constant

The formula of the magic constant of a normal magic square with order n is easily to get. $1, 2, \ldots, n^2$ is arithmetic sequence, so the sum of them is

$$S = \frac{n^2(n^2+1)}{2}.$$

After divided it by the order n, we can get the magic constant

$$M = \frac{n(n^2+1)}{2}.$$

2.2 The Existence of Magic Squares

The 1-by1 magic square with the only integer 1 is exactly a magic square by the definition, although it is typically not under consideration when discussing magic squares.

The magic square with order 2 doesn't exist. Suppose for sake for contradiction that there exists a magic square of order 2 and its formula is

$$\begin{matrix} a & b \\ c & d \end{matrix}$$

where a, b, c and d are not equal to each other. By the definition of magic square, we have $a + b = a + c$, then $b = c$, an contradiction. Thus a 2-by-2 magic square cannot be constructed.

Any magic squares with order $n (n \geq 3)$ can be constructed, and we will discuss the method after a while.

2.3 The Number of a Magic Square with a Given Order

Any magic square can be rotated and reflected to produce 8 trivially distinct but equivalent squares. Excluding rotations and reflections, there is exactly one 3-by-3 magic square, exactly 880 4-by-4 magic squares, and exactly $275, 305, 224$ 5-by-5 magic squares. So the number of 3-by-3 and 4-by-4 magic suqares are respectively 8 and 7040. But there is not a exact number of magic squares with order 6. We can only know that there are estimated to be approximately 1.8×10^{19} squares.

3 The Method for Constructing Magic Squares

Magic squares can be classified to three types: odd, doubly even and singly even. Constructing these squares have their corresponding methods.

3.1 The Method for Constructing