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An advanced evolutionary strategy with an adaptive penalty function for mixed-discrete structural optimisation

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Abstract

Modular three-dimensional steel frames are prefabricated structures with slender elements and dominating compressive forces. Their analysis includes different non-linearities and the optimisation variables are often mixed-discrete or topological. The optimisation of these systems requires the most advanced solution techniques but also a systematic preprocessing, a sophisticated non-linear structural analysis and an automated handling of stress- and displacement-constraints as well as further side-constraints resulting from the underlying codes of practice. In this contribution, a $(\mu + 1)$-evolutionary strategy combined with an adaptive penalty function and a special selection scheme is presented to solve this optimisation problem.

Keywords: Steel structures; Optimisation; Mixed-discrete; Evolution strategy; Penalty function

1. Introduction

Modular three-dimensional steel frames are assembled from prefabricated components, the modules. The weight of the structure is an important factor to the costs of assembly and disassembly. Therefore, modular steel frames are made of metallic materials, steel or aluminium. Many examples of modular steel frames can be found in modern civil engineering, e.g. scaffoldings, regular three-dimensional frameworks, steel pallet racks or shelving assemblies.

The present contribution is limited to metal structures. Their joints are often constructed as socket joints, hinge joints or simple bolted joints for a quick and simple assembly. To allow an easy assembly, inevitable system imperfections and tolerance margins have to be considered. This leads to non-rigid joints with slack, slippage or non-linear spring stiffness which influence the results of the structural analysis.

The aim of the system analysis is a sufficiently safe construction with minimum costs. In the standard codes the safety-constraint is expressed as an inequality:

$$\frac{S_d}{R_d} \le 1.0 \tag{1}$$

where S_d is the design value of the stress and R_d is the design value of the resistance [4]. The amount by which this quotient is smaller than 1.0 can be taken as a criterion for the economic efficiency of the construction. For modular steel frames, cost optimisation is of particular importance. In general, these structures are the result of a series production. Therefore, a small reduction of weight can lead to a considerable decrease of the costs of manufacturing, transport and assembly.

As a result of the optimisation the structure becomes even more slender, or thinner. Thus, stability and local buckling problems turn out to be more important. The optimisation program has to handle all necessary calculations for global and local buckling as well as non-linear stability effects including imperfections and non-linear joint stiffness automatically. The computation of member forces and moments is based on a three-dimensional finite-element model, which contains all geometric non-linearities. The non-linear model includes the computation of the internal forces for the γ_F-loads with respect to the neighbouring states of equilibrium, the stress stiffening effects including

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lateral and flexural–torsional instability, pre-buckling displacements as prescribed in national codes, slippage and non-linear stiffness of joints [7]. With these member forces, the stresses are computed with the cross-section data from a database.

Evolutionary algorithms (EA) are especially suitable for solving the above-mentioned optimisation task. Mixed-discrete optimisation problems in structural mechanics were already solved by several authors with EAs. Some of these investigations deal with the optimisation of trusses with linear constraints using genetic algorithms (GA) [9,10,16]. Cai [2] successfully uses sequential and parallel evolutionary strategies (ES) for the optimisation of static and dynamic loaded trusses and frames with non-linear constraints. Lampinen and Zelinka [13] solve mixed-integer-discrete-continuous non-linear engineering design optimisation problems with a differential evolution algorithm, using a penalty function.

A characteristic of these optimisation problems is the high numerical effort that is necessary to compute the constraints. Hence, their processing is of special importance. Michalewicz [14] reviews methods for handling infeasible individuals in evolutionary computation and gives a survey of the investigations in this area. In many contributions on GAs constraints are dealt with by the use of penalty functions [18]. Often a simple penalty function is used, which has to be scaled by the user. Keane [12] develops a special penalty function for GAs, which is most appropriate to solve multi-peak optimisation problems. The combination of ES with penalty approaches for the solution of optimisation problems in structural mechanics is, however, not investigated well up to now.

2. Optimisation problem

The optimisation of modular steel-frames can be expressed mathematically as a constrained, non-linear optimisation problem with n_x continuous variables x and n_d discrete variables d.

Minimise

$$\min f(\underline{x}, \underline{d}) \qquad \underline{x} \in \Re^{n_x}, \;\; \underline{d} \in N^{n_d} \tag{2}$$

with the lower and upper bounds

$$x_j^l \le x_j \le x_j^u \qquad j = 1, \ldots, n_x \tag{3}$$

for a predefined number of values for each discrete variable

$$1 \le d_k \le d_k^u \qquad k = 1, \ldots, n_d \tag{4}$$

and with respect to the constraints

$$g_i(\underline{x}, \underline{d}) \ge 0 \qquad i = 1, \ldots, m \tag{5}$$

where m is the number of constraints.

The positive integer variables d_k act as a pointer on discrete values D_{jk} from the given set M_k, e.g. cross-sectional values of standard profiles

$$D_{jk} = D_{jk}(d_k) \in M_k \qquad k = 1, \ldots, n_d. \tag{6}$$

The objective function can be approximated by the structural weight. The constraints $\underline{g}(\underline{x}, \underline{d})$ must be checked for every vector $(\underline{x}, \underline{d})$. Of fundamental importance is the safety-constraint according to Eq. (1), which requires a complete structural analysis.

An example of continuous variables in the optimisation of modular steel frames are the geometrical dimensions, unless they are predefined by a discrete set of joint points. Typical discrete variables are the cross-section values and material properties, which are chosen from a given set, which is subject to the stock or to the available steel sections. Topological variables can have a value of either 0 or 1 and describe whether a construction element exists or not. In this work, they are assigned to the discrete variables. Their set of values consists of $M_{\mathrm{Topo}} = \{0; 1\}$. The number of optimisation variables in a modular steel frame is mostly limited, because of the small number of different structural elements.

Both, the objective function and the constraints contain non-linearities and discontinuity points caused by the discrete variables. Also the design value of the resistance R_d is usually not in linear proportion to the weight or costs of a section. The impact of topological variables on the objective function and constraints are further discontinuity points. The actual stress distribution is computed by an iterative finite-element analysis, which also contains non-linearities, especially for slender member structures close to the limit of the loading capacity. Furthermore, from the discontinuous character of the discrete and topological variables great demands arise for the applied optimisation algorithms. With reference to the optimisation problem in Eqs. (2)–(5) non-convexity has to be assumed. This implies that beside the global optimum $(\underline{x}_{\mathrm{opt}}, \underline{d}_{\mathrm{opt}})$ some local optima could exist. To solve the described optimisation problem an advanced ES is presented in Section 3.

3. Evolution strategies

ES are stochastic search methods, which imitate biological evolution, starting with an arbitrary population of individuals. The evolutionary principles of mutation, reproduction and selection are the key operators of this method, which has proved to be efficient and robust for different problems [20].

The basic scheme of (μ, λ)-ES consists of the initialisation of a starting population with μ individuals, the build-up of λ new individuals by recombination and mutation, the calculation of the objective function value, the verification of the constraints as base of the selection of the μ best individuals for the next parent generation and the control of the termination criteria (Fig. 1). An essential property of the ES is, that they do not require the calculation of gradients.

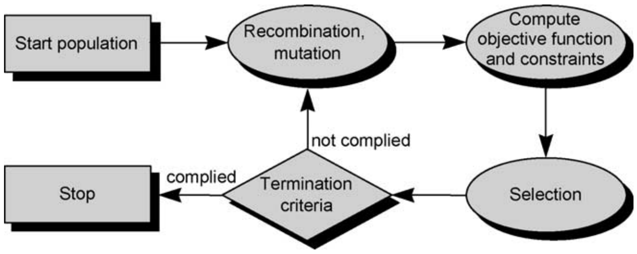

Fig. 1. Basic scheme of the ES.

The search starts with a population of different vectors and progresses exclusively through a quality evaluation of single vectors. The starting points of the ES with a number of μ individuals are distributed at random. An accelerated convergence is achieved through a preference of promising regions. The probability of reaching the global optimum under an objective function with several local optima increases with the size μ of the population, but on the other hand the speed of convergence decreases when using a larger population.

One disadvantage of the ES is the relatively high number of individuals to be created during the solution process. Checking the constraints in particular is a substantial effort. The described solution procedure therefore tries to minimise the number of these computations. The basic steps of the procedure and the necessary modifications are described below.

3.1. Recombination

A descendant is created by using a combination of single components of two or more optimisation vectors. The descendant vector of discrete recombination $(\underline{x}^R, \underline{d}^R)$ is formed by randomly controlling the choice of values out of p parent vectors $(\underline{x}^{P,a}, \underline{d}^{P,a})$:

$$x_j^R = x_j^{P,a_j}, \qquad d_j^R = d_j^{P,a_j} \tag{7}$$

where a_j is an evenly distributed integer random value with $1 \le a_j \le p$ and x_j^{P,a_j}, respectively, d_j^{P,a_j} is the jth component of the a_jth parent vector.

If a variable has adopted the same value in all parent vectors, then this value will not be changed in any further discrete recombination. For problems with only slight joint variables, this has the advantage that values of variables encountered once are no longer changed by the discrete recombination. However, this involves the risk that local optima cannot be deserted.

3.2. Mutation

The mutation of an individual produces a descendant $(\underline{x}^D, \underline{d}^D)$, which differs slightly from the parent vector by a random vector $(\underline{z}_x, \underline{z}_d)$. The result of the recombination $(\underline{x}^R, \underline{d}^R)$ is taken as parent vector for the mutation:

$$\underline{x}^D = \underline{x}^R + \underline{z}_x \qquad \underline{d}^D = \underline{d}^R + \underline{z}_d \tag{8}$$

The mutation should cause small changes on the average level. The sign of every change is equally probable into both directions and the result of the mutation operation of an individual only depends on its direct parent. The control of the step width is one of the most important details of all optimisation procedures [20]. Here, the mutation step size of discrete and topological variables is controlled by a probabilistic value, which decides whether a variable is changed or not. In this way, the mutation instalments $p_i \in [0,1]$ determine the probability for a change of the variable d_i. The number of zero components of the vector $\underline{z}_d$ grows with decreasing mutation step size.

The mutation instalments are strategy parameters, which must be stored in addition to the optimisation variables to every individual. These strategy parameters are inherited with every genetic operation and therefore are also subject to rules of recombination and of mutation. Only the strategy parameters are varied in the first stage of the mutation process, the object variables are changed then using these new values.

The inheritance and mutation of the mutation instalment is decisive for an efficient convergence of the procedure. In Ref. [15], the following mutation of the mutation instalments of discrete variables is proposed, which proved to be fairly effective:

$$p_i' = \frac{1}{1 + \dfrac{1-p_i}{p_i} \cdot \exp\left(\dfrac{-1}{\sqrt{2\sqrt{n_d}}} \cdot N_i(0,1)\right)}, \tag{9}$$

where $N_i(0,1)$ is a (0,1)-Gaussian random value.

Furthermore, we have to determine here which value to take for the individual components of $\underline{z}_d$. Evenly distributed or binomially distributed integer values of one range or geometrically distributed random values are possible. According to Cai [2], a Poisson distribution is employed. A general solution for this problem does not exist since the distribution of the components of the mutation vector is strongly dependent on the special application. With the implementation of the mutation probability p_i and the coincidence distribution of the component z_i a differentiated control of the mutation process becomes possible [7]. The mutation of the continuous variables is done with a Gaussian probability function as proposed in Ref. [20].

3.3. Selection

The selection determines which of the descendants, constituted by recombination and mutation, will be accepted to the new parent population. This requires a calculation of the objective function and an FE-analysis to verify the constraints.

To reduce the high numerical effort of the constraint evaluation a $(\mu + 1)$-ES is used. This denotes that for every individual which is generated from recombination and mutation, the selection determines immediately, whether or

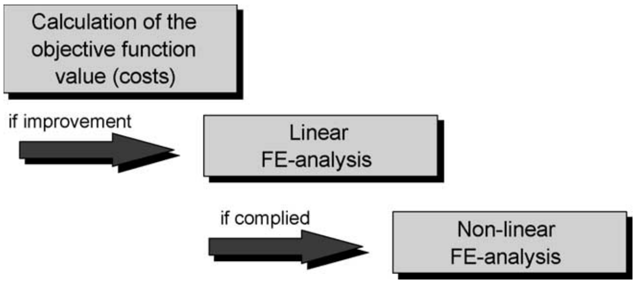

Fig. 2. Examination of the fitness of an individual.

not the individual is to be admitted to the population. This variant leads to a speedup of convergence, because good individuals will improve the parent population immediately [6], but at the same time the reliability of the algorithm tends to decrease.

To reduce the amount of costly non-linear FE-analyses to a minimum, a selection scheme as shown in Fig. 2 is proposed. If a new individual does not improve the objective function, no FE-analysis is necessary. The complete non-linear calculation is performed only if the results of the linear analysis are acceptable.

3.4. Penalty function

Using ES the constraints (5) are often considered only by strictly applying the selection where infeasible individuals are replaced by new ones. Thus, the population always consists of feasible individuals. As the optimal solution of a structural optimisation problem is often found at the boundary of at least one constraint with this method the population can approach the solution only from one side (see Fig. 3a).

Alternatively the constraints $\underline{g}(\underline{x}, \underline{d})$ can be considered using a penalty function $p(\underline{g}(\underline{x}, \underline{d}))$. If a constraint is violated, the objective function is penalised multiplicatively or additively:

$$f_p(\underline{x}, \underline{d}) = f(\underline{x}, \underline{d}) p^*(\underline{g}(\underline{x}, \underline{d})) \quad \text{where} \quad p^*(\underline{g}(\underline{x}, \underline{d})) \geq 1.0 \tag{10a}$$

or

$$f_p(\underline{x}, \underline{d}) = f(\underline{x}, \underline{d}) + p^+(\underline{g}(\underline{x}, \underline{d})) \quad \text{where} \quad p^+(\underline{g}(\underline{x}, \underline{d})) \geq 0.0. \tag{10b}$$

This ensures that those individuals that violate constraints are unfavourable compared to feasible individuals. The implementation of a penalty function can be of advantage for the optimisation process. Using an exterior penalty function, the optimisation problem stays continuous and the approximation of the optimum is possible with individuals from both sides (see Fig. 3b). This particularly improves the effect of the recombination operator. Furthermore, as the population is allowed to contain infeasible individuals, it is avoided that a feasible initial population has to be generated.

Exterior penalty functions are used herein which are inactive until at least one constraint is violated and therefore have no influence on the value of the objective function of feasible individuals. One disadvantage of exterior penalty functions is the possibility of a marginal violation of the constraints that means the optimisation may lead to a slightly infeasible approximation of the optimum. Many optimisation algorithms try to solve this problem by raising the penalty value during the progress of the optimisation process so that at the end of the optimisation the constraints are satisfied completely. Using ES this approach is not necessary because this problem can be solved by the selection procedure. The stringent observation of the constraints is possible without any problem and if constraint violations are tolerated only for a part of the population it is ensured that there are always some feasible individuals

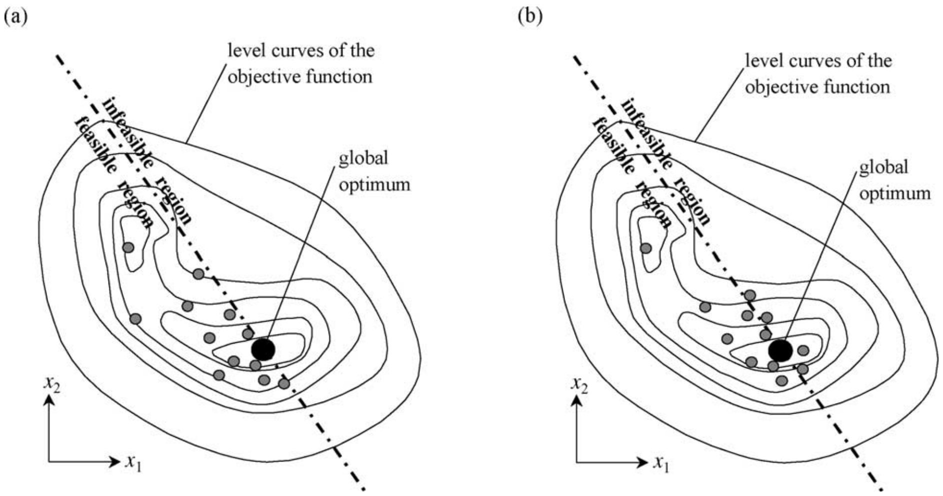

Fig. 3. Distribution of the population in the vicinity of the optimum (a) under stringent satisfaction of the constraints and (b) under the use of a penalty function.

within the population. The minimum rate of feasible individuals $e_{p,\min}$ is predetermined by the user. The result of the optimisation is the best feasible individual.

However, it is difficult to formulate suitable penalty functions. Their value has to be large enough to make sure that individuals far in the infeasible region are eliminated. On the other hand, to avoid numerical problems and to enable infeasible individuals close to the boundary of the feasible region, the penalty value should not be too large. While very sophisticated penalty functions which require multiple optimisation with different penalty parameters [1] are used in gradient-based optimisation the penalty functions used in ES can be quite simple [20]. There is no generally accepted method for the design of a penalty function. The function can depend on the type of the objective function and on the topological properties of the search domain. Other parameters can be the degree of constraint violation, the optimisation progress, the size of the population or the number of design variables and constraints. An adaptive penalisation where the penalty factors are integrated into the genetic operators is also possible [14]. Especially concerning GAs, various different penalty functions can be found in literature. A suitable formulation depends on the special problem and the operators of the optimisation method [18].

A simple additive penalty function for the weight optimisation of trusses with GA was proposed by Rajan [16]. It utilises only a constant p_1 that has to be predetermined by the user, the mean value of the population f_m and the constraint (5):

$$p^+(\underset{\sim}{x},\underset{\sim}{d}) = p_1 f_m \sum_{j=1}^{m} \max(0; -g_j(\underset{\sim}{x},\underset{\sim}{d})). \tag{11}$$

Grill [11] and Keane [12] use penalty functions for structural optimisation which increase with the actual number of generations. Hence at the end of the optimisation the number of infeasible individuals in the population decreases. Keane [12] suggests an additive penalty function according to Fiacco and McCormick [8] for a GA. The function considers both violated and satisfied constraints:

$$p^+(\underset{\sim}{x},\underset{\sim}{d}) = \sum_{j=1}^{m} \frac{(g_j^v(\underset{\sim}{x},\underset{\sim}{d}))^2}{p_2^{\max\left(1;\frac{g}{\mu}-2\right)}} + \sum_{j=1}^{m} \frac{p_2^{2\cdot\max\left(1;\frac{g}{\mu}-2\right)}}{g_j^e(\underset{\sim}{x},\underset{\sim}{d})} \tag{12}$$

where

m	number of constraints
g	number of current generation
p_2	constant with $0.0 < p_2 < 1.0$
$g_i^v(\underset{\sim}{x},\underset{\sim}{d})$	violated constraints ($g_i^v(\underset{\sim}{x},\underset{\sim}{d}) < 0.0$)
$g_i^e(\underset{\sim}{x},\underset{\sim}{d})$	satisfied constraints ($g_i^e(\underset{\sim}{x},\underset{\sim}{d}) \geq 0.0$).

The smaller the constant p_2 is chosen, the larger is the influence of the current number of generations g. Both penalty functions (11) and (12) depend strongly on the factors p_1, respectively, p_2 that have to be defined by the user according to the specific problem and which have great influence on the result and on the speed of convergence.

Because it is often difficult for the user to define a proper value for the factors p_1, respectively p_2, the suggestion of Michalewicz [14] to change the value of the penalty function during the evolution process as done for the strategy parameters shall be taken up. Therefore, Eq. (11) is used with value p_1 as strategy parameter. As criterion for the control of parameter p_1 a constant target rate of feasible individuals in the population $e_{p,\mathrm{tar}}$ has to be predetermined. Every μth generation the factor p_1 is adapted depending on the actual rate of feasible individuals $e_{p,\mathrm{act}}$ in the current population.

$$p_1^g = p_1^{g-\mu}\left(\frac{1.0 - e_{p,\mathrm{act}}}{1.0 - e_{p,\mathrm{tar}}}\right)^{\frac{1}{2(n_x+n_d)}}, \quad \text{if } e_{p,\mathrm{act}} < 1.0$$

and (13)

$$p_1^g = p_1^{g-\mu}\left(\frac{1.0}{\mu(1.0 - e_{p,\mathrm{tar}})}\right)^{\frac{1}{2(n_x+n_d)}}, \quad \text{if } e_{p,\mathrm{act}} = 1.0$$

with $0.0 < e_{p,\mathrm{tar}} < 1.0$ and $0.0 < e_{p,\mathrm{act}} \leq 1.0$.

This version of a penalty function provides that the penalisation of the objective function is well balanced during the evolution process and is used in the following examples.

4. Examples

In the following, two examples are presented to demonstrate the described optimisation method. The first example is a plane truss that has been used as a standard problem by several authors and therefore allows to compare the results with other contributions. The second example is a realistic three-dimensional truss cupola that shows the use of the adaptive penalty function.

4.1. 10 bar-truss

The geometry, loads and material parameters of a plane truss with 10 members are shown in Fig. 4. For this example, imperial units are used to simplify the comparison with results of certain other contributions. The design variables are the 10 cross-sectional areas of the members A_j. The discretisation of the design variables is taken from Rajeev and Krishnamoorthy [17] according to the 'American Institute of Steel Construction Manual'. The objective function of the problem is the weight of the structure. The constraints concern the member stresses that are limited to 25.00 ksi for tension and compression.

$$g_j(\underset{\sim}{x},\underset{\sim}{d}) = 1.0 - \frac{|N_j|}{25.00 A_j} \geq 0.0 \quad \forall j \in \{1,\ldots,10\}, \tag{14}$$

where N_j is the axial force of element j.

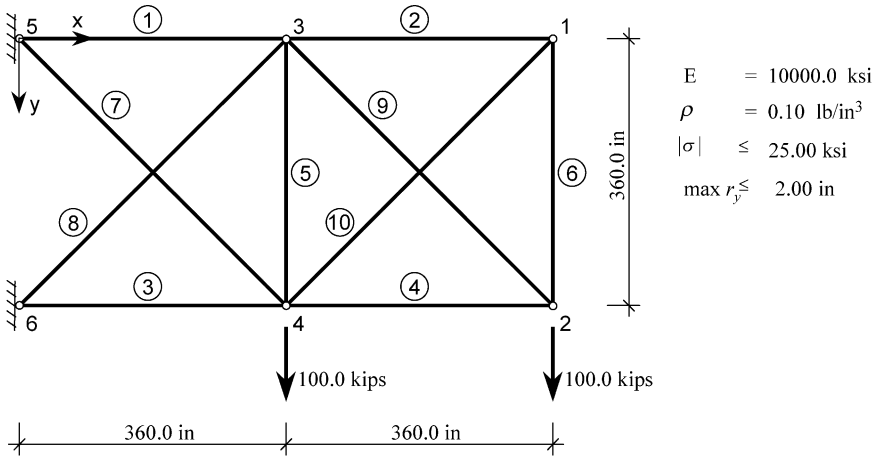

Fig. 4. Plane truss with 10 members.

Furthermore, the maximum vertical displacement is limited to 2.0 in for the unconstrained nodes:

$$g_j(\underset{\sim}{x}, \underset{\sim}{d}) = 1.0 - \frac{r_{y,(j-10)}}{2.00} \geq 0.0 \tag{15}$$

$\forall j \in \{11, 12, 13, 14\}$.

Effects resulting from non-linearity and buckling are neglected and the nodal displacements and stresses are evaluated using linear theory.

Some of the published results are summarised in Table 1 and compared to the results of the method described in this contribution. Two different optimisation problems are considered. Rows A–E show the results of the simple optimisation of the cross-sectional areas. The best-known solution has a weight of exactly 5490.74 lb and is also found by Cai [3] (see Fig. 5a). Some solutions found in the literature with a lower weight are violating the constraints. The displacement constraint is crucial in every publication.

The results for a modified optimisation problem are shown in rows F–G of Table 1 where additional to the optimisation of the cross-sectional areas a topology optimisation is carried out. To avoid singularities, non-existent members are considered to have an area of 10^{-5} in^2. This value has no influence on the result of the objective function. Individuals that are kinematically unstable lead to very large displacements and are eliminated when the selection is executed.

Furthermore, the optimisation problem is extended by three continuous design variables, the y-co-ordinates of nodes 1, 3 and 5. Their value is limited to $-640.0 \leq y_i \leq 180.0$ in., $\forall i \in \{1, 3, 5\}$ to make the result comparable to the work of Rajan [16].

The best solution found has a weight of 2705.17 lb (Fig. 5b) and is 50.7% lighter than the solution without the variation of the nodal co-ordinates and the topology optimisation. The members 2, 5, 6 and 10 were eliminated by the algorithm and the achieved result is 16.9% better compared to the result of Rajan [16].

The first optimisation problem has an optimal solution which is exactly defined, because only discrete variables are involved. For this problem, the probability of finding the global optimum was estimated for the algorithm with and without penalty function with 1000 calculations each. Without using the penalty function (i.e. constraints were only considered by a strict application of the selection) the optimum was reached in 40% of all calculations and with using the penalty function in 95%. The size of the population was $\mu = 30$. The search was terminated, when

Table 1
Results of the weight optimisation of the 10 bar-truss

	$f(\underset{\sim}{x}, \underset{\sim}{d})$ (lb)	A_1 (in.^{2})	A_2 (in.^{2})	A_3 (in.^{2})	A_4 (in.^{2})	A_5 (in.^{2})	A_6 (in.^{2})	A_7 (in.^{2})	A_8 (in.^{2})	A_9 (in.^{2})	A_{10} (in.^{2})	max r_y (in)	max$\lvert\sigma\rvert$ (ksi)
A	5613.58	33.5	1.62	22	15.5	1.62	1.62	14.2	19.9	19.9	2.62	2.0008	9.440
B	5490.74	33.5	1.62	22.9	14.2	1.62	1.62	7.97	22.9	22	1.62	1.9989	14.197
C	5458.34	33.5	1.62	22	14.2	1.62	1.62	7.97	22.9	22	1.62	2.0123	14.351
D	5452.55	33.5	1.62	22.9	14.2	1.62	1.62	7.22	22	22.9	1.62	2.0176	15.220
E	5490.74	33.5	1.62	22.9	14.2	1.62	1.62	7.97	22.9	22	1.62	1.9989	14.197
F	3254.73	9.9	9.4	11.5	1.5	0	12	11.5	3.6	0	10.4	1.9946	12.779
		$y_1 = 173.5$			$y_3 = -194.5$			$y_5 = -426.9$					
G	2705.17	11.5	0	11.5	7.22	0	0	5.74	2.88	13.5	0	2.0000	19.145
					$y_3 = -126.79$			$y_5 = -429.94$					

Sources and methods: A, Rajeev and Krishnamoorthy [17]: GA; B, Cai [3]: $(\mu + \lambda)$-ES; C, Galante [9]: GA with rebirthing; D, Ghasemi et al. [10]: GA; E, presented paper: $(\mu + 1)$-ES with penalty function (13); F, Rajan [16]: GA; G, presented paper: $(\mu + 1)$-ES with penalty function (13).

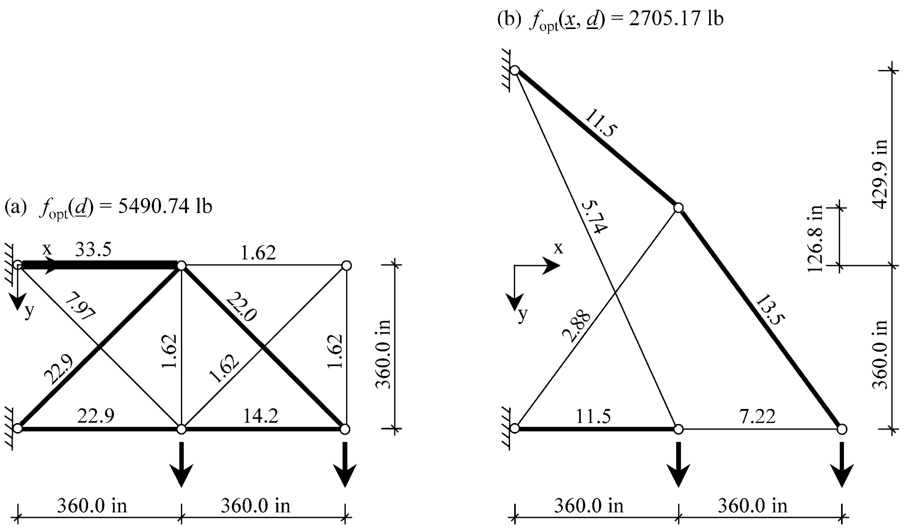

Fig. 5. Optimal solution of the 10 bar-truss with discrete cross-sectional variables: (a) optimisation of the cross-sectional area, (b) optimisation of the cross-sectional area, topology and y-co-ordinates of upper nodes.

the exact solution was found or a maximum number of 20,000 individuals was generated. For reaching the optimum an average of approximately 10,000 individuals had to be generated.

4.2. Truss cupola

The second example is a three-dimensional truss cupola with 120 members and a geometry taken from Saka and Ulker [19] as seen in Fig. 6. The cupola is quite flat and the vertical members have an angle between 41.1 and 16.2°. The nodal displacements have a major effect on the load carrying behaviour and therefore a non-linear calculation of the displacements and the internal forces is inevitable. The discrete values for the 7 groups of cross-sections are a selection of hot formed tubes according to the European code EN 10210-2 [5]. This selection contains 121 cross-sections with outer diameters between 21.3 and 355.6 mm and wall thicknesses between 2.3 and 25 mm taken from Ebenau [7]. Loads are acting on nodes 1–37:

$$R_{1,z} = -60.0\ \text{kN},$$

$$R_{i,z} = -30.0\ \text{kN}, \quad \forall i \in \{2,\ldots,13\},$$

$$R_{j,z} = -10.0\ \text{kN}, \quad \forall j \in \{14,\ldots,37\}.$$

The aim of the optimisation is the minimisation of the weight. The density is taken as $\rho = 7850\ \text{kg/m}^3$. The example is solved with different constraints as shown in Table 2. For all calculations and for every group of cross-sections the stress is limited to:

$$g_j(\underline{x}, \underline{d}) = 1.0 - \frac{|N_j|}{N_{pl,d}} \geq 0.0 \qquad \forall j \in \{1,\ldots,7\} \tag{16}$$

with the axial resistance force $N_{pl,\,d} = A_j f_{y,\,d}$, $f_{y,\,d} = 24.0/1.1 = 21.82\ \text{kN/cm}^2$ and j as number of the cross-section group. Due to the axis-symmetry the normal forces are of the same value for all members of a group of cross-sections. The calculation is carried out on the complete three-dimensional model so that unsymmetric eigenmodes are also considered in the non-linear calculation.

For the results shown in row A–F of Table 2, the maximum vertical displacement r_z is limited to 1.0 cm:

$$g_8(\underline{x}, \underline{d}) = 1.0 - \frac{\max|r_z|}{1.000} \geq 0.0. \tag{17}$$

To receive a practicable solution the buckling of the single members has to be considered (see row E–G of Table 2). Therefore, the stress-constraints (16) are replaced by:

$$g_j(\underline{x}, \underline{d}) = 1.0 - \frac{|N_j|}{\kappa_j N_{pl,d}} \geq 0.0 \qquad \forall j \in \{1,\ldots,7\}. \tag{18}$$

According to the European design code for steel structures Eurocode 3 [4] κ_j is given as:

$$\kappa_j = \frac{1.0}{\phi_j + \sqrt{\phi_j^2 - \bar{\lambda}_{K,j}^2}} \qquad \text{if } \bar{\lambda}_{K,j} > 0.2 \text{ and } N_j < 0.0, \tag{19}$$

else

$$\kappa_j = 1.0 \qquad \forall j \in \{1,\ldots,7\}$$

where

$$\phi_j = 0.5[1.0 + 0.21(\bar{\lambda}_{K,j} - 0.2) + \bar{\lambda}_{K,j}^2] \qquad \forall j \in \{1,\ldots,7\} \tag{20}$$

and the slenderness ratio

$$\bar{\lambda}_{K,j} = l_{sk,j}\sqrt{\frac{A_j f_{y,k}}{I_{y,j} E \pi^2}} \qquad \forall j \in \{1,\ldots,7\} \tag{21}$$

$l_{sk,j}$ is defined as the length of the members of the cross-section group j and the modulus of elasticity is $E = 21,000$ kN/cm^2.

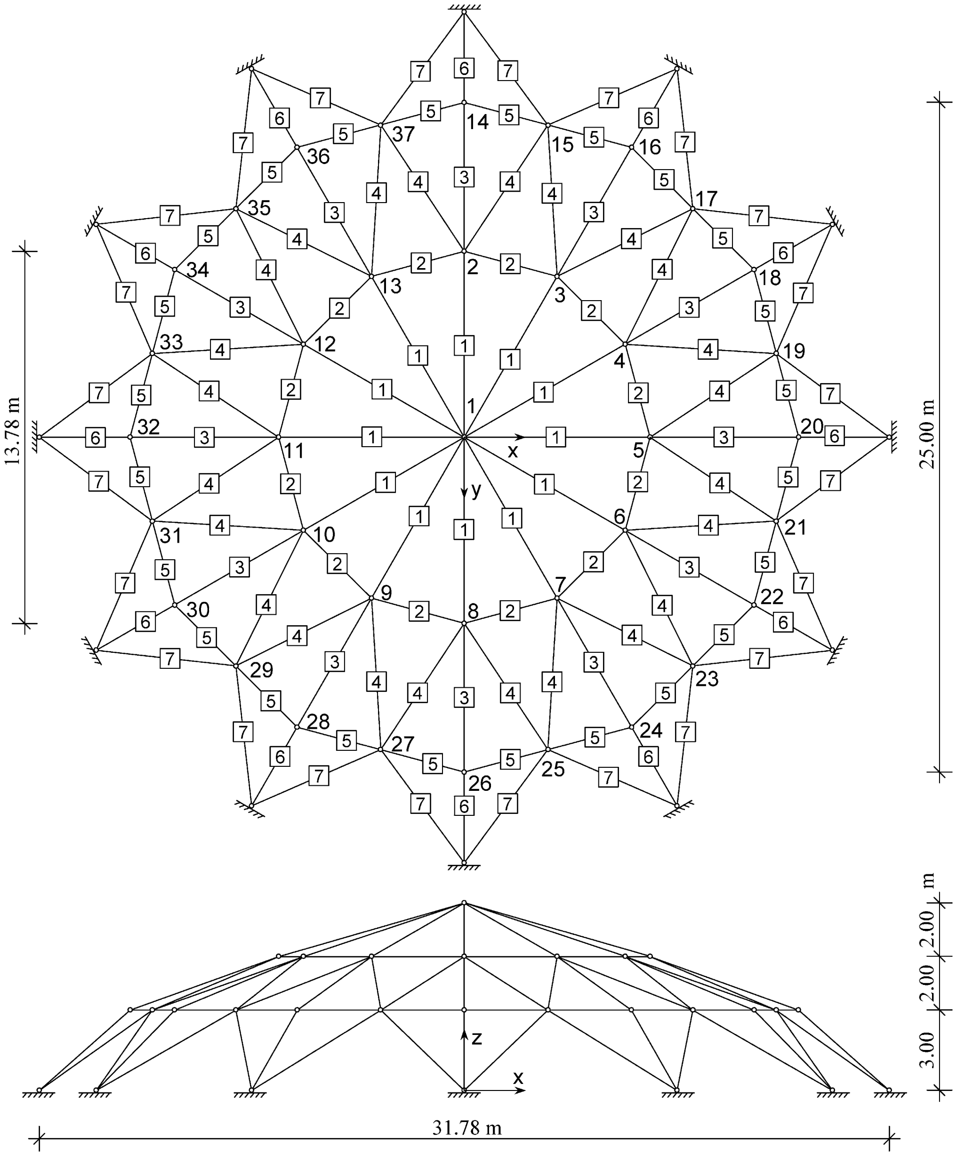

Fig. 6. Three-dimensional truss cupola with 120 members, dimensions, nodal numbers and numbers of cross-section groups.

Table 2
Results of the weight optimisation of the truss cupola with different constraints

	$f(\underline{x}, \underline{d})$ (kg)	A_1 (cm^2)	A_2 (cm^2)	A_3 (cm^2)	A_4 (cm^2)	A_5 (cm^2)	A_6 (cm^2)	A_7 (cm^2)	max$\|r_z\|$ (cm)	max$\|\sigma\|$ (kN/cm^2)	$\max \frac{\|N_{j,i}\|}{\kappa_j N_{pl,d,j}}$
A	8512.2	16.66	44.84	24.89	9.66	21.93	16.59	11.74	0.999	−3.47	
B	7588.5	17.5	45.56	25.45	8.44	22.3	15.96	3.9	0.988	−4.359	
C	5678.2	5.567	42.097	25.651	1.527	23.524	15.626	1.527	0.998	−11.133	
D	6326.6	9.892	42.117	25.651	4.534	23.524	16.348	1.373	1.000	−11.910	
E	6923.3	8.616	46.672	25.651	5.74	23.524	13.179	6.004	1.000	−4.367	0.896
F	3549.5	8.616	8.616	9.892	6.004	1.373	8.616	8.616	1.000	−7.751	1.000
		$z_1 = 267.76$		$z_2 = 547.62$		$z_3 = 666.16$					
G	4579.9	9.892	20.647	13.861	5.74	11.168	9.892	6.004	1.999	−7.330	0.904

Sources and constraints: A, Saka and Ulker [19], linear calculation; B, Saka and Ulker [19], non-linear calculation; C, presented paper, linear calculation; D, presented paper, non-linear calculation; E, presented paper, non-linear calculation, buckling of single members; F, presented paper, non-linear calculation, buckling of single members, dead weight, variation of z_1, z_2 and z_3; G, presented paper, non-linear calculation, buckling of single members, max$|r_z| \leq 2.0$ cm.

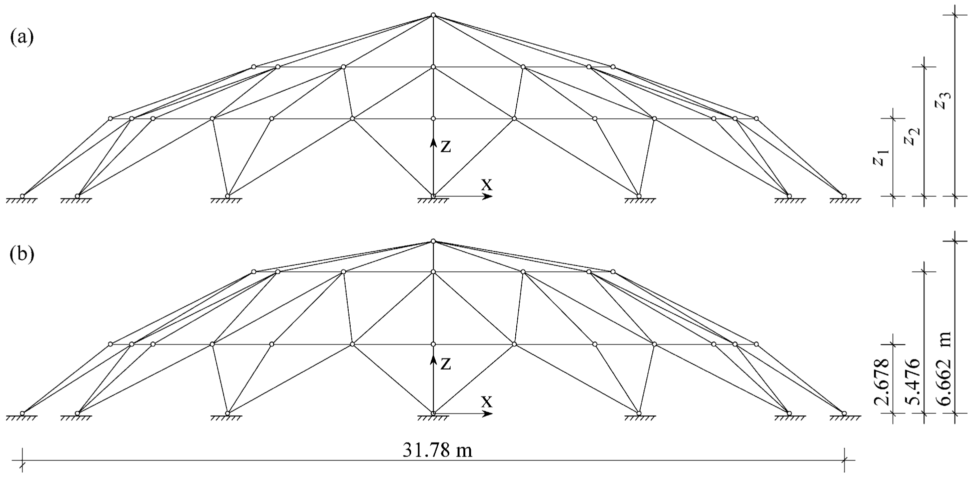

Fig. 7. Variation of the co-ordinates z_1, z_2 and z_3, (a) initial geometry. According to Saka and Ulker [19], (b) optimal geometry.

These modified constraints make the optimisation problem significantly more difficult because the influence of the discrete variables on the objective function is no longer proportional to their influence on the constraints. While one part of the restrictions, the calculation of the displacements and the stresses of tension members depends on the cross-sectional area only, the buckling of a single member depends on the radius of gyration of the tubes. The optimisation procedure has to overcome these discontinuities to reach the global optimum.

Another variant of the optimisation problem is the introduction of three continuous design variables for the z-co-ordinate of the nodes (Fig. 7a). These are allowed to move ± 0.80 m based on their initial value: $2.20 \le z_1 \le 3.80$, $4.20 \le z_2 \le 5.80$ and $6.20 \le z_3 \le 7.80$.

Compared to the results of Saka and Ulker [19] that were achieved with continuous design variables and with an optimality criteria method the linear, respectively, non-linear calculation leads to another reduction of the weight of 33.3% respectively 16.6% (rows C and D of Table 2). In this example, the high robustness of the used (30 + 1)-ES leads to significantly better results. If the buckling of single members is included in the constraints the weight is increased by 9.4% (row E of Table 2). The initial geometry of the truss has a very flat angle of 176.6° between the members of the cross-section groups 1 and 3. Hence there are large normal forces resulting from the vertical loads. With the introduction of the nodal z-co-ordinates as continuous design variables a mixed-discrete optimisation problem with numerous local optima arises. Under consideration of the dead weight of the members, the optimal solution results to a weight of 3549.5 kg (Table 2, row F). Compared to the optimisation problem without variation of the nodal co-ordinates the weight can be reduced another 48.7%. The optimal geometry is illustrated in Fig. 7b. The improvement of the angle between the members of cross-section groups 1 and 3 is obvious.

A mathematical proof that the achieved optima are global ones is impossible. The only possibility to verify this is the multiple calculation of the problem starting from different initial points. For each listed result 100 calculations were carried out so that the global validity of the reached optima is very likely.

The effect of single components of the developed ES shall be examined on the example with only discrete variables and therefore an exact solution. Therefore, the optimisation is carried out alternatively with and without the adaptive penalty function. Furthermore the application of the discrete mutation operator with constant variance that is controlled via the mutation rate is used. The variance is constantly chosen to $\gamma = \gamma_j = 0.8, \forall j \in \{1, \ldots, 7\}$. For each variant 100 calculations are carried out with a maximum of 20,000 generated individuals.

The comparison is done with two different constraints for the nodal displacements, $\max|r_z| \le 1.0$ cm and $\max|r_z| \le 2.0$ cm. In the first case, the limitation of the displacements is crucial for the dimensioning of most members. For $\max|r_z| \le 2.0$ cm the buckling of single members gains in influence and with them the discontinuities. Fig. 8 shows the probability of finding the global optimum for the different variants. It is obvious that the ES combined with a penalty

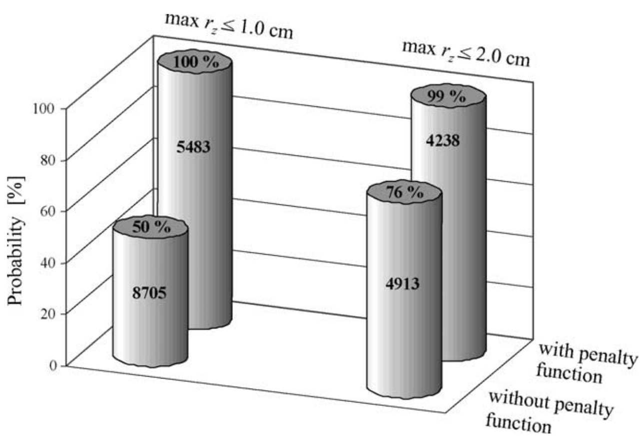

Fig. 8. Probability of finding the optimum and number of generated individuals using different variants of the ES.

function are superior. The reliability is improved substantially for both displacement constraints and the average number of generated individuals is less than for the variant without penalty function.

5. Summary and conclusions

In this contribution, the application of an advanced ES with an adaptive penalty function was used on structural optimisation problems with mixed-discrete variables.

Because the verification of the constraints usually requires very expensive FE-computations a $(\mu + 1)$-ES was used to reduce the numerical effort. Within the ES a mutation procedure was implemented which computes the actual mutation vector according to a Poisson distribution for the discrete variables and a Gaussian distribution for the continuous variables. The mutation step-size is controlled by the mutation probability of single components. With this mutation procedure the probability of finding the global optimum for discontinuous variables and objective functions with local optima is improved.

The application of the algorithm was demonstrated for two well-known examples taken from literature. The examples show that the speed of convergence and robustness of the used $(\mu + 1)$-ES mainly originates from the combination with a penalty function. A penalty function depending not only on the constraint value, but also adapted depending on the actual rate of feasible individuals in the current population, turned out to be most suitable.

The achieved results show that the utilised mutation operator and the combination with a penalty function is a robust and efficient variant of the $(\mu + 1)$-ES for optimisation with mixed-discrete variables. Even with non-linear and discontinuous constraints that arise in real life applications of structural analysis the global optimum is reached with a fairly high probability.

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