Evolutionary Many-tasking Based on Biocoenosis through Symbiosis: A Framework and Benchmark Problems

Rung-Tzuo Liaw and Chuan-Kang Ting
Department of Computer Science and Information Engineering and
Advanced Institute of Manufacturing with High-tech Innovations
National Chung Cheng University
Chia-Yi 621, Taiwan
Email: {lrt101p, ckting}@cs.ccu.edu.tw

***Abstract*—Evolutionary multitasking is an emergent topic in evolutionary computation area. Recently, a well-known evolutionary multitasking method, the multi-factorial evolutionary algorithm (MFEA), has been proposed and applied to concurrently solve two or three problems. In MFEA, individuals of different tasks are recombined in a predefined random mating probability. As the number of tasks increases, such recombination of different tasks becomes very frequent, thereby detracting the search from any specific problems and limiting the MFEA's capability to solve many-tasking problems. This study proposes a general framework, called the evolution of biocoenosis through symbiosis (EBS), for evolutionary algorithms to deal with the many-tasking problems. The EBS has two main features: the selection of candidates from concatenate offspring and the adaptive control of information exchange among tasks. The concatenate offspring represent a set of offspring used for all tasks. Moreover, this study presents a test suite of many-tasking problems (MaTPs), modified from the CEC 2014 benchmark problems. The Spearman correlation is adopted to analyze the effect of the shifts of optima on the MaTPs. Experimental results show that the effectiveness of EBS is superior to that of single task optimization and MFEA on the four MaTPs. The results also validate that EBS is capable of exploiting the synergy of fitness landscapes.**

***Index Terms*—Evolutionary multitasking, many-tasking, framework, CMAES, benchmark problems.**

I. Introduction

Evolutionary algorithms (EAs) have demonstrated their great power in search and optimization. In searching for the optima, EAs simulate natural evolution by evolving a population of individuals, which represents a set of candidate solutions to the problem. Several EAs have been proposed from different inspiration and purposes, e.g., genetic algorithm (GA), evolution strategy (ES), particle swarm optimization (PSO), and so forth [1], [2], [3].

Evolutionary multitasking is an emergent topic in evolutionary computation, which aims to deal with multiple problems concurrently using a single run of EA. The multi-factorial evolutionary algorithm (MFEA) has recently been proposed to achieve evolutionary multitasking. By working on the synergy of problem landscapes, MFEA can improve efficiency and solution quality, in comparison to handling only one problem in the conventional EAs. More specifically, MFEA assigns each problem as a task and utilizes a unified design space for all tasks. A solution in the unified design space is transformed for different tasks, which can have different types of genes and dimensions. In addition, the MFEA adopts GA with one population for all tasks. The information exchange is adjusted according to two factors: skill factor and random mating probability (rmp). The former gives the most talented task of an individual; the latter indicates the probability of mating between different tasks.

The MFEA has shown its effectiveness in evolutionary multitasking. However, there exist some issues in the MFEA. First, the appropriate setting of rmp depends upon the properties or landscapes of tasks. It is difficult to determine an rmp suitable for all problems. This issue is severe as the number of tasks increases since the probability of reproduction across different tasks is proportional to the number of tasks in the multitasking problem. Second, the effectiveness of MFEA was validated with two and three tasks; nonetheless, its performance on a larger number of tasks is still open. Furthermore, as the number of tasks increases, the balance between exploitation within one task and exploration among multiple tasks will become more significant. Third, the MFEA is hardly applied to the model-based EAs, such as ES, ant colony optimization (ACO), and estimation of distribution algorithm (EDA), due to the lack or seldom use of recombination in these EAs.

This study proposes a novel framework, called the evolution of biocoenosis through symbiosis (EBS), to address the above issues at evolutionary many-tasking. By contrast to the MFEA using only one EA, the EBS utilizes multiple EAs, each of which corresponds to a task. The populations of all tasks form the biocoenosis, while the interaction among tasks constitutes the symbiosis. Hence, the EBS is applicable to different dialects of EA such as GA, ES, and PSO. In the EBS, the symbiosis involves the interaction among tasks and serves as a form of information exchange. The candidate pool for evaluation collects the concatenate offspring generated by each EA in EBS for exchanging the information among all tasks. To control the information exchange, the proposed EBS adaptively adjusts the probability of evaluation among

978-1-5090-4601-0/17/$31.00 ©2017 IEEE

concatenated offspring or among the offspring generated by itself for each task. In addition, this study proposes a set of many-tasking benchmark problems, called the MaTPs, which are modified from the CEC 2014 test suite for performance evaluation.

The major contributions of this study include the following:

- a general framework EBS for evolutionary many-tasking;
- two operators of EBS, composed of the information exchange among concatenate offspring and an adaptive control mechanism;
- a set of benchmark problems MaTPs for evolutionary many-tasking;
- experimental study on the performance of EBS on the MaTPs, compared to single task optimization and MFEA.

The rest of this paper is organized as follows. Section II reviews the related work of MFEA. Section III describes the proposed EBS framework and the many-tasking benchmark problems. Section IV presents the experimental results. Section V draws conclusions and suggests future extension.

II. Related Work

Evolutionary multitasking establishes a new class of evolutionary algorithm for tackling multiple problems at the same time. Multitask learning is a sub-field of machine learning that aims at solving multiple machine learning tasks at the same time, such as classification, regression, and prediction [4]. In [5], [6], the Bayesian optimization with multitask Gaussian processes optimizes the acquisition function for tuning hyperparameters. Yet the tasks in the tuning of hyperparameters are overlapped and highly correlated, where the learning is done in sequential manner.

The multi-factorial evolutionary algorithm (MFEA) has been proposed to achieve evolutionary multitasking [7], [8], [9]. Algorithm 1 presents the pseudocode of MFEA. The MFEA first initializes the population and evaluates the population on all tasks (line 1–4). The skill factor (line 5) and scalar fitness (line 6) of individuals in the population are then computed according to the ranking at each task. The skill factor is determined by the index of the most talented task for a given individual, and the scalar fitness is the inverse of the ranking on the most talented task. In each generation, the MFEA generates offspring by assortative mating operator (line 8) which confines the crossover operation to the parents with different skill factors in a predefined random mating probability (rmp) and to those with the same skill factor in a probability $1 - rmp$. The mutation operator is performed on all offspring in the assortative mating operator. An offspring inherits the skill factor from one of its parents. The evaluation operator (line 9) evaluates an offspring only on the task corresponding to its skill factor. The MFEA then updates the skill factor and the scalar fitness over the union of population and offspring for survival selection (line 10–12). Noteworthily, the information among all tasks is exchanged by assortative mating in the MFEA. As the number of tasks increases, the frequency of information exchange will fully depend upon the predefined rmp due to the decreasing ratio of individuals with the same task over the population.

Algorithm 1 Pseudocode of MFEA

τ_i: Task i
m: Number of tasks
1: Initialize (Pop)
2: **for** $i \in 1$ to m **do**
3: Evaluate (Pop, τ_i)
4: **end for**
5: $Skill$ ←Update skill factor (Pop)
6: $Scalar$ ←Update scalar fitness (Pop)
7: **while** Not terminated **do**
8: $Ofsp$ ←Assortative mating (Pop, $Skill$)
9: Evaluate ($Ofsp$, $Skill$)
10: $Skill$ ←Update skill factor ($Pop \cup Ofsp$)
11: $Scalar$ ←Update scalar fitness ($Pop \cup Ofsp$)
12: Pop ←Survival selection ($Pop \cup Ofsp$, $Scalar$)
13: **end while**

Regarding the fitness landscapes of multiple tasks, the moving direction beneficial to one task may also improve the fitness of other tasks. Such synergy improves the effectiveness and efficiency of MFEA in handling multiple tasks. Gupta et al. [10] analyzed the synergy of fitness landscapes on numerical benchmark functions. Several variants of MFEA have been proposed for different applications. Gupta et al. [11] further combined MFEA with a nested bi-level evolutionary algorithm to solve the bi-level optimization problems in a multitasking paradigm. Sagarna and Ong [12] adopted an MFEA to tackle the branch testing problem. Chandra et al. [13] proposed an MFEA for optimizing several feed forward neural networks with different numbers of hidden layers. Wen and Ting [14] developed a multi-factorial genetic programming to enhance the efficiency in learning the ensemble of decision trees. Zhou et al. [15] adopted the MFEA on the capacitated vehicle routing problems. The MFEA has also been adopted on multi-objective optimization problem with two tasks [8]. Two performance metrics for multi-objective optimization problem, i.e., the nondominated front and crowding distance, are treated as different tasks.

III. Evolutionary Many-tasking

This study proposes the EBS as a general framework for evolutionary many-tasking. Inspired from the symbiosis in the biocoenosis, EBS consists of multiple populations, each of which corresponds to an EA. The information exchange among all tasks constitutes symbiosis. Algorithm 2 presents the pseudocode of EBS. The EBS holds two main features: 1) adaptive control of information exchange and 2) selection of candidates for evaluation from concatenate offspring. More details are described in the following subsections.

A. Information Exchange through Concatenate Offspring

The information exchange among tasks is essential to the performance of evolutionary multitasking. The best way of

Algorithm 2 Pseudocode of EBS

EA_i: EA for the i-th task
τ_i: Task i
m: Number of tasks

for $i \in 1$ to m **do**
  Initialize EA_i
end for
while Not terminated **do**
  $Ofsp \leftarrow \bigcup_{i=1}^{m} Ofsp_i$ ▷ Concatenate offspring
  for $i \leftarrow 1$ to m **do**
    if Information Exchange **then**
      $Candidate_i \leftarrow$Random $(\lambda_i, Ofsp)$
    else
      $Candidate_i \leftarrow Ofsp_i$
    end if
    Evaluate $(Candidate_i$,$\tau_i)$
    $EA_i \leftarrow$Survival selection $(Pop_i$,$Candidate_i)$
  end for
end while

symbiosis is the mutualism, by which both tasks benefit each other. The second best way of symbiosis is the commensalism, which improves a task through the information from others. However, it is not always helpful for a task to use the information from other tasks, such as parasitism. This study proposes an adaptive control strategy to address this issue. Specifically, the probability γ_i of information exchange for the i-th task τ_i is determined by

$$\gamma_i = \frac{\mathcal{R}_i^{\text{o}}}{\mathcal{R}_i^{\text{s}} + \mathcal{R}_i^{\text{o}}}$$

with

$$\mathcal{R}_i^{\text{s}} = \frac{\text{\#improvements}_i^{\text{s}}}{\text{\#evals}_i^{\text{s}}},$$
$$\mathcal{R}_i^{\text{o}} = \frac{\text{\#improvements}_i^{\text{o}}}{\text{\#evals}_i^{\text{o}}},$$

where $\mathcal{R}_i^{\text{s}}$ and $\mathcal{R}_i^{\text{o}}$ are the proportions of times that the best-so-far solution of task τ_i is improved by the offspring of the same task and other tasks, respectively. The rate γ_i reflects the probability of using information from other tasks. If a task can be improved more times by the offspring from other tasks, the high rate of γ_i will then intensify information exchange; otherwise, it will reduce information exchange.

A concatenate offspring consisting of the offspring of all tasks is used for sharing information among tasks:

$$Ofsp \leftarrow \bigcup_{i=1}^{m} Ofsp_i.$$

The information can be shared by treating the concatenate offspring as a candidate pool for evaluation. Further, a subset of concatenate offspring are randomly selected as the candidates for evaluation to prevent increasing the number of evaluations.

B. EBS using CMAES

This study adopts CMAES in the proposed EBS in view of its recognized performance in numerical optimization [16]. CMAES builds a multivariate normal distribution from its population:

$$\mathcal{N}(\boldsymbol{m}, \boldsymbol{C}) \sim \boldsymbol{m} + \mathcal{N}(0, \boldsymbol{C}),$$

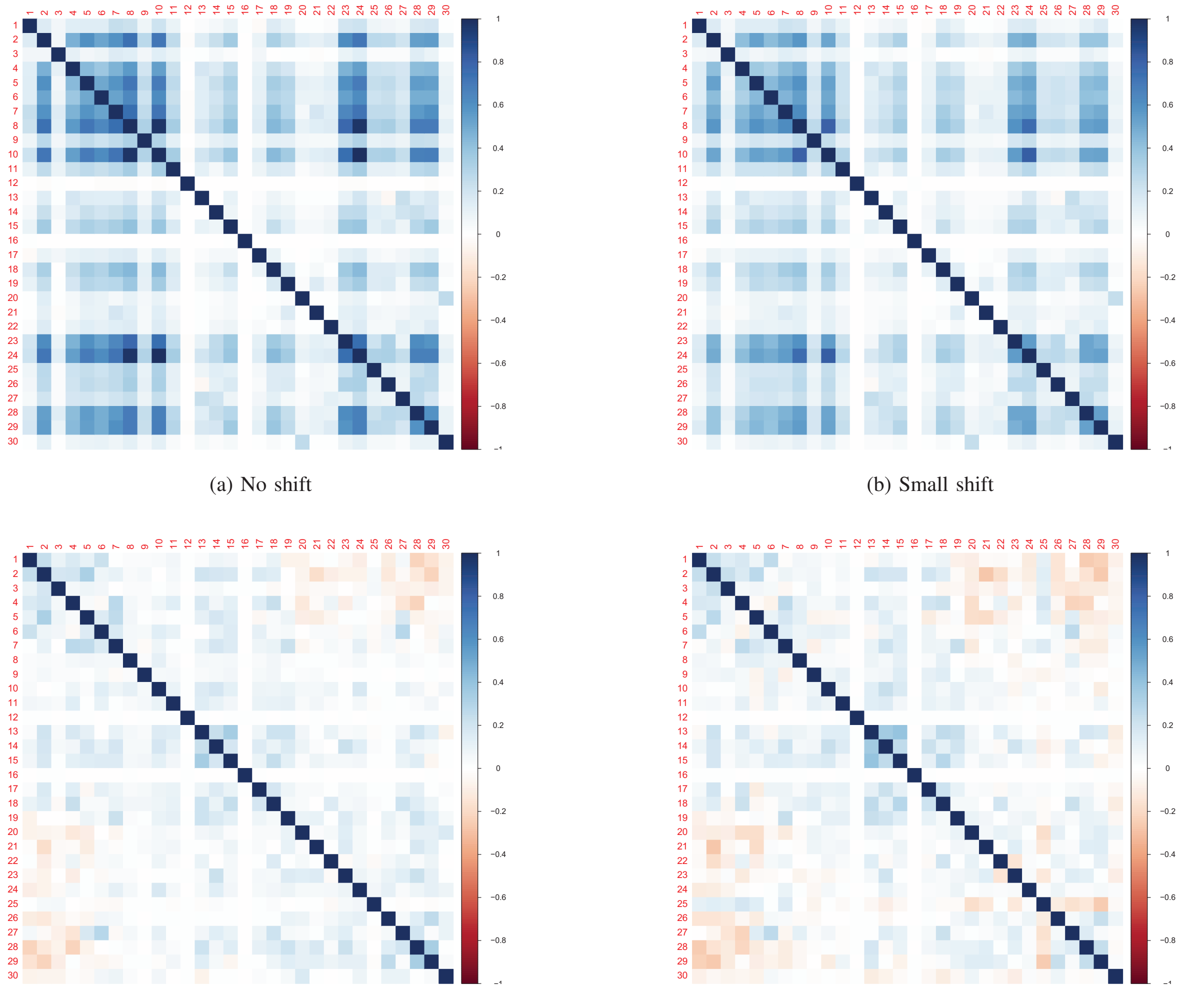
where $\boldsymbol{m}$ is the mean vector and $\boldsymbol{C}$ is the covariance matrix of population. For updating the distribution $\mathcal{N}(\boldsymbol{m}, \boldsymbol{C})$, CMAES samples several offspring and selects the best half of them as parents. Since the update of multivariate normal distribution is sensitive to the solutions considered, two distributions are applied in EBS: one is updated with the individuals from information exchange and the other is done with its generated individuals. The decision of sampling offspring from which model for a task τ_i is determined by its probability of information exchange γ_i.

C. Many-tasking Benchmark Problem

This study proposes a test suite of many-tasking problems (MaTPs) for evaluation of EA performance. The MaTPs are modified from CEC 2014 test suite [17] by adjusting the positions of optimal solutions of all tasks. The CEC 2014 test suite is composed of 30 test functions of four types: unimodal, multimodal, hybrid, and composite functions. These 30 functions are defined to be tackled at one time in the MaTPs. This study ponders four shift ranges of the optimal solutions for the MaTPs: 1) no shift, 2) small shift $U(-1, 1)$, 3) medium shift $U(-5, 5)$, and 4) large shift $U(-10, 10)$ at each dimension between any two test functions, where U denotes the uniform distribution. The four types of benchmark problems are labeled as MaTP_{N} for no shift, MaTP_{S} for small shift, MaTP_{M} for medium shift, and MaTP_{L} for large shift of the optimal solutions. Figure 1 shows the Spearman correlation coefficients within the range $[-10, 10]$ of the optima for the four MaTPs. The pairwise Spearman correlation slightly decreases from the MaTP_{N} to the MaTP_{S}. As the shift increases to medium range, the correlation drops drastically down to zero and even negative values. The correlation remains low for large shift.

IV. Experimental Results

This study tests an implementation of the EBS using CMAES (EBS-CMAES), where the CMAES is a state-of-the-art algorithm for complex numerical optimization. The experiments compare EBS-CMAES with CMAES and MFEA on the four MaTPs. The experiments also examine the effects of adaptive control over information exchange in the proposed EBS-CMAES. The synergy of fitness landscapes is further analyzed by the Spearman correlation. Table I lists the parameter setting used in the experiments. The setting of MFEA follows [7] using simulated binary crossover (SBX), polynomial mutation, and random mating probability $rmp = 0.3$. Considering the stochastic nature of EA, all experiments are performed with 30 trials, which has the minimum effect size $d_{\min} = 1.05$ obtained by power analysis under confidence level 0.99 (type-I



(a) No shift

(b) Small shift

(c) Medium shift

(d) Large shift

Figure 1. The pairwise Spearman correlation within the range [-10,10] of the optima of all tasks among the 30 test functions in the $\mathrm{MaTP_N}$, $\mathrm{MaTP_S}$, $\mathrm{MaTP_M}$, and $\mathrm{MaTP_L}$.

Table I
PARAMETER SETTING

Parameter	Value
Problem size (n)	30
#Evaluations	10^4n
#Trials	30
Population size (μ)	7
Offspring size (λ)	14

error rate $\alpha = 0.01$) and statistical power 0.95 (type-II error rate $\beta = 0.05$) for Student's t-test.

A. Information Exchange

We first investigate the influences of the adaptive control on information exchange in the EBS-CMAES. Table II presents the mean best fitness (MBF) for EBS-CMAES using deterministic (fixed with $\gamma_i = 1$) and adaptive control over the probability of information exchange. According to the results, EBS-CMAES using adaptive control significantly betters EBS-CMAES using fixed probability on 23 out of 30 test functions, and performs comparably on 4 functions. Regarding the function types, the adaptive control gains better results than deterministic control does on unimodal, multimodal, and hybrid functions, while the two control strategies are comparable on composite functions.

B. Solution Quality

Tables III and IV list the MBF for CMAES, MFEA and EBS-CMAES on the four MaTPs. On the $\mathrm{MaTP_N}$, EBS-CMAES performs significantly better than CMAES does on 26 out of 30 functions, and comparably on four functions (including three unimodal functions and one simple multimodal function F_4). The EBS-CMAES excels MFEA on all test functions. On the $\mathrm{MaTP_S}$, EBS-CMAES outperforms CMAES on 16 functions and the two methods are comparable on 11 functions. In addition, EBS-CMAES surpasses MFEA in terms of solution quality on all $\mathrm{MaTP_S}$ functions.

Table II

THE MBF FOR EBS-CMAES USING DIFFERENT CONTROL STRATEGIES FOR INFORMATION EXCHANGE ON $\mathrm{MaTP_L}$ AT 300000 EVALUATIONS. SIGNIFICANCE ANALYSIS IS CONDUCTED USING STUDENT'S t-TEST UNDER CONFIDENCE LEVEL $\alpha = 0.01$. THE ENTRIES WIN, LOSE, AND EQUAL MEAN THAT THE ADAPTIVE STRATEGY IS SIGNIFICANTLY BETTER THAN (+), SIGNIFICANTLY WORSE THAN (-), AND COMPARABLE TO (~) THE FIXED STRATEGY, RESPECTIVELY. BOLDFACE MARKS STATISTICAL SIGNIFICANCE.

Function	EBS-CMAES Fixed	EBS-CMAES Adaptive	p-value
F_1	1.23E+09	**0.00E+00**	9.90E-16 (+)
F_2	6.93E+10	**0.00E+00**	1.77E-20 (+)
F_3	1.26E+05	**0.00E+00**	4.83E-17 (+)
F_4	1.54E+04	**2.08E+01**	1.23E-15 (+)
F_5	2.10E+01	**1.73E+01**	1.57E-11 (+)
F_6	4.33E+01	**1.89E+01**	8.85E-21 (+)
F_7	6.74E+02	**3.14E-01**	2.20E-21 (+)
F_8	3.61E+02	**1.78E+02**	1.26E-21 (+)
F_9	4.53E+02	**1.89E+02**	1.28E-21 (+)
F_{10}	7.77E+03	**4.93E+03**	3.93E-16 (+)
F_{11}	7.77E+03	**4.98E+03**	1.06E-17 (+)
F_{12}	2.88E+00	**3.46E-01**	9.01E-24 (+)
F_{13}	7.29E+00	**2.95E-01**	1.12E-26 (+)
F_{14}	2.39E+02	**3.44E-01**	3.47E-19 (+)
F_{15}	7.37E+06	**3.56E+00**	1.16E-07 (+)
F_{16}	1.33E+01	**1.07E+01**	1.69E-16 (+)
F_{17}	7.38E+07	**1.77E+03**	1.14E-05 (+)
F_{18}	3.43E+09	**5.07E+06**	1.25E-12 (+)
F_{19}	5.87E+02	**1.38E+01**	1.43E-14 (+)
F_{20}	3.88E+06	3.09E+02	1.01E-01 (~)
F_{21}	4.88E+07	**1.07E+03**	2.77E-04 (+)
F_{22}	2.11E+03	**3.53E+02**	1.56E-09 (+)
F_{23}	**2.22E+02**	3.04E+02	1.08E-13 (-)
F_{24}	**2.02E+02**	2.29E+02	4.65E-04 (-)
F_{25}	**2.00E+02**	2.07E+02	1.43E-09 (-)
F_{26}	1.93E+02	**1.08E+02**	4.12E-16 (+)
F_{27}	2.33E+02	2.06E+02	1.56E-01 (~)
F_{28}	2.38E+02	3.61E+03	6.70E-09 (-)
F_{29}	2.63E+07	2.66E+07	4.92E-01 (~)
F_{30}	1.74E+06	**3.53E+04**	2.23E-28 (+)
Win			23
Lose			4
Equal			3

As the shift increases to medium range, EBS-CMAES performs better than CMAES does on 10 $\mathrm{MaTP_M}$ functions, but worse on 6 functions, most from composite functions. The EBS-CMAES outperforms MFEA on almost all $\mathrm{MaTP_M}$ functions. Moreover, EBS-CMAES achieves better, worse, and comparable results than CMAES does on 6, 5, and 19 $\mathrm{MaTP_L}$ functions, respectively. Compared to MFEA, the EBS-CMAES obtains significantly better performance on 29 out of 30 $\mathrm{MaTP_L}$ functions.

C. Synergy of Fitness Landscapes

Figure 2 shows the Spearman correlation between the rank of MBF and the rank of shift range of optima for CMAES, MFEA and EBS-CMAES on the four MaTPs. A high correlation exists between the rank of MBF and the rank of shift range of optima, indicating that the performance of such method improves as the fitness landscapes of all tasks become similar. That is, the high correlation of a method corresponds to its ability to exploit the synergy of fitness landscapes. The MBF of EBS-CMAES is positively correlated to the shift range on most of the functions. By contrast, the MBF of CMAES and MFEA is less correlated to the shift range, which implies a lower synergy of fitness landscapes in CMAES and MFEA. Table V compares the correlation in terms of four classes: high positive, low positive, zero, and negative correlation. The MBF of EBS-CMAES has a high positive correlation to the shift of optimal solutions on 25 out of 30 functions and a negative correlation on 3 functions. The MBF of CMAES and MFEA has fewer functions with high positive correlation but more with negative correlation, especially the MFEA. These results validate that EBS-CMAES can exploit and benefit from the synergy of fitness landscapes more than CMAES and MFEA.

V. CONCLUSIONS

Evolutionary multitasking is an emergent topic in evolutionary computation. Current research of evolutionary multitasking focuses on tackling the problems with few tasks. As the number of tasks increases, the information exchange between different tasks becomes a key to the performance. However, the methods such as MFEA have no control tactic for the information exchange, which will be a problem when solving the

Table III
THE MBF FOR CMAES, MFEA AND EBS-CMAES ON THE $\mathrm{MaTP_N}$ (NO SHIFT) AND $\mathrm{MaTP_S}$ (SMALL SHIFT) AT 300000 EVALUATIONS. SIGNIFICANCE ANALYSIS IS CONDUCTED USING STUDENT'S t-TEST UNDER CONFIDENCE LEVEL $\alpha = 0.01$. THE ENTRIES WIN, LOSE, AND EQUAL MEAN THAT THE LATTER IS SIGNIFICANTLY BETTER THAN (+), SIGNIFICANTLY WORSE THAN (-), AND COMPARABLE TO (~) THE FORMER, RESPECTIVELY. BOLDFACE MARKS STATISTICAL SIGNIFICANCE.

	MBF on $\mathrm{MaTP_N}$			p-value		MBF on $\mathrm{MaTP_S}$			p-value	
	CMAES	MFEA	EBS-CMAES	(1) vs. (3)	(2) vs. (3)	CMAES	MFEA	EBS-CMAES	(1) vs. (3)	(2) vs. (3)
F_1	0.00E+00	2.02E+09	9.73E-12	1.63E-01 (~)	4.79E-18 (+)	0.00E+00	2.14E+09	0.00E+00	- (~)	1.81E-15 (+)
F_2	0.00E+00	1.19E+11	1.35E-09	1.63E-01 (~)	8.49E-26 (+)	0.00E+00	1.18E+11	0.00E+00	- (~)	2.01E-28 (+)
F_3	0.00E+00	3.07E+05	0.00E+00	- (~)	2.41E-24 (+)	0.00E+00	2.74E+05	0.00E+00	- (~)	5.91E-19 (+)
F_4	1.87E+00	3.12E+04	0.00E+00	1.07E-01 (~)	2.18E-23 (+)	4.72E+00	3.04E+04	1.39E+00	8.46E-02 (~)	1.23E-18 (+)
F_5	2.00E+01	2.10E+01	**5.45E-09**	2.6E-124 (+)	1.49E-75 (+)	2.00E+01	2.10E+01	**5.10E+00**	2.31E-32 (+)	3.47E-33 (+)
F_6	3.92E+01	4.29E+01	**2.51E-06**	5.25E-26 (+)	3.84E-39 (+)	4.08E+01	4.37E+01	**5.10E+00**	1.92E-21 (+)	4.46E-38 (+)
F_7	1.97E-03	1.08E+03	**0.00E+00**	3.26E-03 (+)	1.66E-28 (+)	1.97E-03	1.12E+03	7.20E-02	1.66E-01 (~)	5.06E-27 (+)
F_8	1.97E+02	4.99E+02	**0.00E+00**	6.18E-23 (+)	6.84E-36 (+)	1.98E+02	4.85E+02	**1.05E+01**	1.53E-21 (+)	3.68E-32 (+)
F_9	3.20E+02	6.24E+02	**0.00E+00**	3.18E-23 (+)	8.39E-35 (+)	3.20E+02	6.17E+02	**2.48E+01**	4.99E-22 (+)	6.88E-31 (+)
F_{10}	5.00E+03	8.31E+03	**0.00E+00**	8.58E-29 (+)	1.06E-40 (+)	5.10E+03	8.12E+03	**4.17E+02**	1.40E-27 (+)	5.35E-30 (+)
F_{11}	5.11E+03	7.57E+03	**0.00E+00**	7.99E-29 (+)	3.27E-38 (+)	5.00E+03	7.79E+03	**5.65E+02**	8.13E-24 (+)	2.29E-33 (+)
F_{12}	2.71E-01	2.88E+00	**1.22E-09**	2.15E-09 (+)	1.31E-25 (+)	1.96E-01	2.75E+00	1.89E-01	4.20E-01 (~)	5.32E-24 (+)
F_{13}	2.45E-01	9.95E+00	**7.94E-04**	1.40E-20 (+)	4.01E-30 (+)	2.53E-01	9.62E+00	**1.26E-01**	2.38E-09 (+)	2.71E-30 (+)
F_{14}	3.36E-01	3.63E+02	**7.31E-06**	9.80E-27 (+)	1.04E-26 (+)	3.44E-01	3.55E+02	**1.10E-01**	7.02E-13 (+)	3.90E-26 (+)
F_{15}	3.47E+00	3.45E+07	**0.00E+00**	2.41E-20 (+)	3.50E-09 (+)	3.18E+00	4.65E+07	3.66E+00	1.48E-01 (~)	1.47E-09 (+)
F_{16}	1.38E+01	1.37E+01	**0.00E+00**	5.68E-39 (+)	9.67E-54 (+)	1.38E+01	1.36E+01	**7.59E+00**	1.64E-24 (+)	7.08E-32 (+)
F_{17}	1.57E+03	1.46E+08	**0.00E+00**	1.60E-20 (+)	2.40E-14 (+)	1.62E+03	1.20E+08	1.69E+03	2.86E-01 (~)	2.14E-11 (+)
F_{18}	1.33E+02	5.60E+09	**4.36E-07**	5.87E-15 (+)	2.96E-16 (+)	1.38E+02	5.80E+09	1.61E+02	2.72E-02 (~)	1.12E-15 (+)
F_{19}	1.05E+01	8.87E+02	**1.61E-07**	7.26E-20 (+)	4.71E-15 (+)	1.22E+01	1.01E+03	**6.35E+00**	5.85E-03 (+)	4.23E-14 (+)
F_{20}	1.65E+02	7.25E+06	**2.75E-06**	4.03E-14 (+)	9.99E-07 (+)	1.58E+02	4.54E+06	**6.99E+01**	9.24E-07 (+)	2.63E-05 (+)
F_{21}	9.72E+02	4.79E+07	**4.14E-07**	2.44E-17 (+)	1.80E-17 (+)	1.02E+03	5.24E+07	1.15E+03	1.01E-01 (~)	6.44E-12 (+)
F_{22}	5.04E+02	3.51E+03	**3.13E-04**	6.00E-12 (+)	1.12E-12 (+)	5.74E+02	2.88E+03	**4.57E+01**	1.79E-11 (+)	6.30E-16 (+)
F_{23}	2.00E+02	1.50E+03	**1.53E-07**	2.1E-245 (+)	1.99E-23 (+)	2.00E+02	1.66E+03	**1.23E+02**	4.51E-06 (+)	9.87E-25 (+)
F_{24}	2.00E+02	5.24E+02	**3.53E-10**	4.3E-215 (+)	1.72E-31 (+)	2.00E+02	5.09E+02	2.05E+02	2.90E-01 (~)	5.89E-19 (+)
F_{25}	2.00E+02	3.39E+02	**1.93E-08**	1.4E-271 (+)	7.43E-27 (+)	2.00E+02	3.59E+02	**1.33E+02**	4.06E-06 (+)	1.14E-16 (+)
F_{26}	2.00E+02	1.20E+02	**0.00E+00**	0.00E+00 (+)	1.15E-20 (+)	2.00E+02	1.22E+02	**7.97E+01**	6.68E-20 (+)	4.17E-07 (+)
F_{27}	2.00E+02	1.60E+03	**7.81E-05**	1.3E-189 (+)	1.26E-33 (+)	2.00E+02	1.64E+03	**4.71E+00**	1.44E-53 (+)	2.15E-30 (+)
F_{28}	2.00E+02	4.49E+03	**2.85E-07**	1.2E-237 (+)	8.39E-25 (+)	**2.00E+02**	4.65E+03	3.40E+02	1.92E-05 (-)	1.96E-20 (+)
F_{29}	2.00E+02	2.05E+08	**3.86E-02**	3.51E-89 (+)	1.05E-16 (+)	**2.00E+02**	1.89E+08	1.23E+07	9.46E-03 (-)	5.74E-14 (+)
F_{30}	2.00E+02	4.11E+06	**1.32E-03**	8.6E-132 (+)	1.71E-11 (+)	**2.00E+02**	5.35E+06	2.51E+03	3.15E-11 (-)	3.76E-13 (+)
Win				26	30				16	30
Lose				0	0				3	0
Equal				4	0				11	0

many-tasking problems. This study presents a general framework for evolutionary many-tasking based on the evolution of biocoenosis through symbiosis (EBS). The EBS framework has two main features, i.e., the adaptive control of information exchange and the selection of candidates for evaluation from concatenate offspring. This study also proposes a set of many-tasking problems MaTPs modified from CEC 2014 test suite. According to the shift range of optimal solutions, we proposed four types of MaTPs.

This study examined the EBS using CMAES. The experimental results indicate that the adaptive control of information exchange performs significantly better than the fixed control. The results also show that EBS-CMAES can outperform CMAES on MaTPs with no shift, small shift, and medium shift of optima. Compared to the MFEA, the proposed EBS-CMAES has significantly better performance on almost all test functions for the four types of MaTPs. In addition, this study analyzes the synergy of fitness landscapes by comparing the Spearman correlation between the ranks of MBF and shift range. The results reveal that EBS-CMAES can exploit better synergy of fitness landscapes than CMAES and MFEA.

Some directions remain for future study. First, this study considers only numerical optimization problems. Other types of problems, such as combinatorial optimization problems, can be taken into account for the many-tasking problems. Second, EBS allows the combination of different EAs for different problems, which is a potential topic to be explored.

ACKNOWLEDGMENT

This work was supported by Ministry of Education, Taiwan.

REFERENCES

[1] J. Holland, *Adaptation in Natural and Artificial Systems*. University of Michigan Press, 1975.
[2] D. E. Goldberg, *Genetic Algorithms in Search, Optimization and Machine Learning*. Addison Wesley, 1989.
[3] A. E. Eiben and J. E. Smith, *Introduction to Evolutionary Computing*. Natural Computing. Springer-Verlag, 2003.
[4] E. Bonilla, K. M. Chai, and C. Williams, "Multi-task Gaussian process prediction," in *Proceedings of 20th Annual Conference on Neural Information Processing System*, 2008, pp. 153–160.

Table IV

MBF FOR CMAES, MFEA AND EBS-CMAES ON THE $\mathrm{MaTP_M}$ (MEDIUM SHIFT) AND $\mathrm{MaTP_L}$ (LARGE SHIFT) AT 300000 EVALUATIONS. SIGNIFICANCE ANALYSIS IS CONDUCTED USING STUDENT'S t-TEST UNDER CONFIDENCE LEVEL $\alpha = 0.01$. THE ENTRIES WIN, LOSE, AND EQUAL MEAN THAT THE LATTER IS SIGNIFICANTLY BETTER THAN (+), SIGNIFICANTLY WORSE THAN (-), AND COMPARABLE TO (~) THE FORMER, RESPECTIVELY. BOLDFACE MARKS STATISTICAL SIGNIFICANCE.

	MBF on $\mathrm{MaTP_M}$			p-value		MBF on $\mathrm{MaTP_L}$			p-value	
	CMAES	MFEA	EBS-CMAES	(1) vs. (3)	(2) vs. (3)	CMAES	MFEA	EBS-CMAES	(1) vs. (3)	(2) vs. (3)
F_1	0.00E+00	2.14E+09	0.00E+00	- (~)	1.14E-15 (+)	0.00E+00	2.31E+09	0.00E+00	- (~)	1.40E-16 (+)
F_2	0.00E+00	1.29E+11	1.17E+08	1.24E-01 (~)	6.04E-31 (+)	0.00E+00	1.22E+11	0.00E+00	- (~)	3.03E-28 (+)
F_3	0.00E+00	2.74E+05	0.00E+00	- (~)	2.38E-21 (+)	0.00E+00	3.14E+05	0.00E+00	- (~)	1.61E-10 (+)
F_4	6.83E+00	3.25E+04	3.28E+00	1.73E-01 (~)	3.76E-19 (+)	1.68E+01	3.50E+04	2.08E+01	3.19E-01 (~)	4.92E-18 (+)
F_5	2.00E+01	2.10E+01	**1.28E+01**	2.74E-15 (+)	8.49E-17 (+)	2.00E+01	2.10E+01	**1.73E+01**	1.09E-08 (+)	1.36E-11 (+)
F_6	4.05E+01	4.34E+01	**1.30E+01**	3.89E-16 (+)	8.81E-26 (+)	3.89E+01	4.34E+01	**1.89E+01**	2.40E-11 (+)	9.90E-23 (+)
F_7	9.86E-04	1.08E+03	1.64E-03	1.99E-01 (~)	1.11E-26 (+)	2.54E-03	1.06E+03	3.14E-01	1.64E-01 (~)	2.75E-26 (+)
F_8	2.02E+02	5.00E+02	**1.26E+02**	2.33E-10 (+)	1.54E-30 (+)	2.21E+02	4.92E+02	**1.78E+02**	2.17E-06 (+)	2.81E-32 (+)
F_9	3.27E+02	6.23E+02	**1.54E+02**	3.30E-16 (+)	2.93E-32 (+)	3.50E+02	6.21E+02	**1.89E+02**	5.24E-13 (+)	2.72E-25 (+)
F_{10}	5.28E+03	7.96E+03	**4.31E+03**	3.88E-06 (+)	1.97E-18 (+)	5.32E+03	7.84E+03	4.93E+03	5.19E-02 (~)	1.14E-16 (+)
F_{11}	5.16E+03	7.80E+03	**4.41E+03**	1.31E-03 (+)	1.29E-16 (+)	5.14E+03	7.78E+03	4.98E+03	1.64E-01 (~)	1.12E-15 (+)
F_{12}	2.70E-01	2.71E+00	2.07E-01	1.47E-01 (~)	1.50E-21 (+)	2.30E-01	2.72E+00	3.46E-01	6.20E-02 (~)	1.83E-22 (+)
F_{13}	2.27E-01	1.00E+01	2.29E-01	4.36E-01 (~)	1.12E-31 (+)	2.48E-01	9.84E+00	2.95E-01	2.13E-01 (~)	1.34E-28 (+)
F_{14}	3.29E-01	3.60E+02	9.05E-01	5.26E-02 (~)	6.23E-29 (+)	3.37E-01	3.81E+02	3.44E-01	4.15E-01 (~)	4.37E-26 (+)
F_{15}	3.31E+00	5.82E+07	3.79E+00	3.57E-02 (~)	4.06E-10 (+)	3.56E+00	4.08E+07	3.56E+00	4.99E-01 (~)	4.82E-09 (+)
F_{16}	1.38E+01	1.36E+01	**9.47E+00**	5.34E-20 (+)	1.36E-21 (+)	1.39E+01	1.36E+01	**1.07E+01**	3.16E-17 (+)	8.12E-17 (+)
F_{17}	1.60E+03	1.13E+08	3.21E+05	1.63E-01 (~)	1.37E-11 (+)	1.64E+03	1.28E+08	1.77E+03	1.15E-01 (~)	2.97E-09 (+)
F_{18}	**1.35E+02**	6.43E+09	1.63E+02	6.49E-03 (-)	3.43E-16 (+)	1.57E+02	7.43E+09	5.07E+06	1.63E-01 (~)	3.47E-20 (+)
F_{19}	1.24E+01	8.44E+02	1.12E+01	2.84E-01 (~)	1.66E-15 (+)	1.26E+01	1.04E+03	1.38E+01	3.41E-01 (~)	5.16E-14 (+)
F_{20}	2.16E+02	5.48E+06	2.68E+02	2.70E-02 (~)	1.18E-07 (+)	**2.03E+02**	6.60E+06	3.09E+02	5.24E-05 (-)	1.14E-05 (+)
F_{21}	1.07E+03	5.82E+07	1.10E+03	3.53E-01 (~)	1.79E-08 (+)	9.92E+02	7.27E+07	1.07E+03	1.80E-01 (~)	9.27E-09 (+)
F_{22}	5.46E+02	2.69E+03	**2.78E+02**	3.53E-04 (+)	2.53E-13 (+)	5.03E+02	3.53E+03	3.53E+02	2.49E-02 (~)	5.82E-09 (+)
F_{23}	**2.00E+02**	1.57E+03	2.92E+02	3.90E-11 (-)	3.09E-22 (+)	**2.00E+02**	1.51E+03	3.04E+02	2.44E-16 (-)	1.43E-18 (+)
F_{24}	**2.00E+02**	4.97E+02	2.43E+02	3.12E-04 (-)	2.82E-24 (+)	**2.00E+02**	4.81E+02	2.29E+02	3.06E-04 (-)	1.10E-24 (+)
F_{25}	**2.00E+02**	3.54E+02	2.09E+02	2.61E-12 (-)	4.11E-23 (+)	**2.00E+02**	3.14E+02	2.07E+02	4.58E-10 (-)	5.54E-16 (+)
F_{26}	2.00E+02	1.16E+02	**1.14E+02**	1.64E-14 (+)	3.79E-01 (~)	2.00E+02	1.23E+02	**1.08E+02**	1.30E-18 (+)	9.65E-03 (+)
F_{27}	2.00E+02	1.65E+03	**8.37E+01**	6.32E-08 (+)	8.07E-32 (+)	2.00E+02	1.67E+03	2.06E+02	4.06E-01 (~)	1.22E-28 (+)
F_{28}	**2.00E+02**	4.66E+03	1.61E+03	3.28E-07 (-)	5.84E-11 (+)	**2.00E+02**	4.56E+03	3.61E+03	5.33E-09 (-)	2.84E-02 (~)
F_{29}	**2.00E+02**	2.25E+08	2.92E+07	4.54E-03 (-)	7.11E-10 (+)	2.00E+02	2.41E+08	2.66E+07	4.19E-02 (~)	1.79E-11 (+)
F_{30}	2.00E+02	5.00E+06	5.53E+05	6.91E-02 (~)	4.08E-06 (+)	2.00E+02	4.01E+06	3.53E+04	1.47E-01 (~)	2.10E-15 (+)
Win				10	29				6	29
Lose				6	0				5	0
Equal				14	1				19	1

Table V

COMPARISON OF SPEARMAN CORRELATION η BETWEEN THE RANK OF MBF AND THE RANK OF SHIFT RANGE OF OPTIMA FOR CMAES, MFEA AND EBS-CMAES ON THE FOUR MATPS.

	CMAES	MFEA	EBS-CMAES
High Positive ($\eta \geq 0.8$)	10	6	25
Low Positive ($0 < \eta < 0.8$)	6	11	2
Zero ($\eta = 0$)	11	2	1
Negative ($\eta < 0$)	3	11	2

[5] J. Snoek, H. Larochelle, and R. P. Adams, "Practical Bayesian optimization of machine learning algorithms," in *Proceedings of 24th Annual Conference on Neural Information Processing System*, 2012.

[6] K. Swersky, J. Snoek, and R. P. Adams, "Multi-task Bayesian optimization," in *Proceedings of 25th Annual Conference on Neural Information Processing System*, 2013.

[7] A. Gupta, Y.-S. Ong, and L. Feng, "Multifactorial evolution: Toward evolutionary multitasking," *IEEE Transactions on Evolutionary Computation*, vol. 20, no. 3, pp. 343–357, 2016.

[8] A. Gupta, Y.-S. Ong, L. Feng, and K. C. Tan, "Multiobjective multifactorial optimization in evolutionary multitasking," *IEEE Transactions on Cybernetics*, 2016.

[9] S. Strasser, J. Sheppard, N. Fortier, and R. Goodman, "Factored evolutionary algorithms," *IEEE Transactions On Evolutionary Computation*, 2016.

[10] A. Gupta, Y.-S. Ong, B. Da, L. Feng, and S. D. Handoko, "Landscape synergy in evolutionary multitasking," in *Proceedings of IEEE Congress on Evolutionary Computation*, 2016.

[11] A. Gupta, J. Mańdziuk, and Y.-S. Ong, "Evolutionary multitasking in bi-level optimization," *Complex & Intelligent Systems*, vol. 1, no. 1, pp. 83–95, 2015.

[12] R. Sagarna and Y.-S. Ong, "Concurrently searching branches in software tests generation through multitask evolution," in *Proceedings of IEEE Symposium Series on Computational Intelligence*, 2016.

[13] R. Chandra, A. Gupta, Y.-S. Ong, and C.-K. Goh, "Evolutionary multitask learning for modular training of feedforward neural networks," in *Proceedings of International Conference on Neural Information Processing*, 2016.

[14] Y. W. Wen and C. K. Ting, "Learning ensemble of decision trees through multifactorial genetic programming," in *Proceedings of the IEEE Congress on Evolutionary Computation (CEC)*, 2016, pp. 5293–5300.

[15] L. Zhou, L. Feng, J. Zhong, Y.-S. Ong, Z. Zhu, and E. Sha, "Evolutionary multitasking in combinatorial search spaces: A case study in capacitated vehicle routing problem," in *Proceedings of IEEE Symposium Series on Computational Intelligence*, 2016.

[16] N. Hansen, "The CMA evolution strategy: A comparing review," in

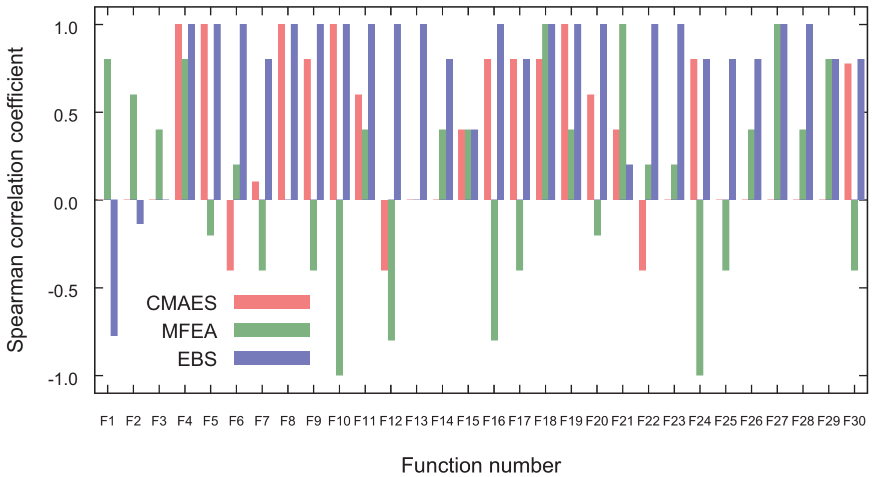


Figure 2. The Spearman correlation between the rank of MBF and the rank of shift range of optima for CMAES, MFEA, and EBS-CMAES on the four MaTPs.

Towards A New Evolutionary Computation. Advances in Estimation of Distribution Algorithms, J. A. Lozano, P. L. nga, I. Inza, and E. Bengoetxea, Eds. Springer, 2006, pp. 75–102.

[17] J. J. Liang, B.-Y. Qu, and P. N. Suganthan, "Problem definitions and evaluation criteria for the cec 2014 special session and competition on single objective real-parameter numerical optimization," Nanyang Technological University, Tech. Rep., 2013.