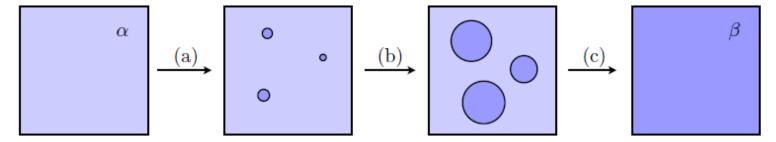




Nucleation theory

- process leading to the formation of a new phase (solid, liquid) within metastable original phase (undercooled melt, supersaturated vapor or solution)
- first step in crystallization process; plays a decisive role in determining the crystal structure and the size distribution of nuclei



- homogeneous nucleation (HON)
 (at random sites in the bulk of a parent phase)
- heterogeneous nucleation (HEN)
 (on foreign substrate, impurities, defects, active centres)

nucleus: the smallest observable "particle" (often $\approx 1 \mu m$)

Clusters of a new phase are formed on **nucleation sites** due to fluctuations and after overcoming some critical size (< 1nm) become **nuclei** (overcritical clusters).



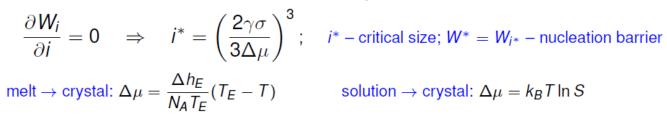
Nucleation theory

Homogeneous nucleation:

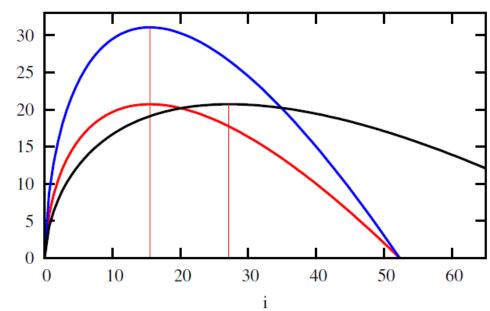
Capillarity approximation

$$W_i = -i\Delta\mu + \gamma i^{2/3} = -\frac{4\pi r^3}{3V_1}\Delta\mu + 4\pi r^2\sigma$$
$$W^S = \sum_k A_k \sigma_k \quad \text{- surface energy}$$

- \bullet i cluster size (number of molecules within cluster)
- r cluster radius; σ interfacial energy; V_1 molecular volume
- $\Delta\mu$ difference of chemical potentials
- A_k surface areas; σ_k corresponding interfacial energies



 Δh_E – heat of fusion; N_A – Avogadro constant; T_E – equilibrium temperature; T – temperature k_B – Boltzmann constant; S – supersaturation;





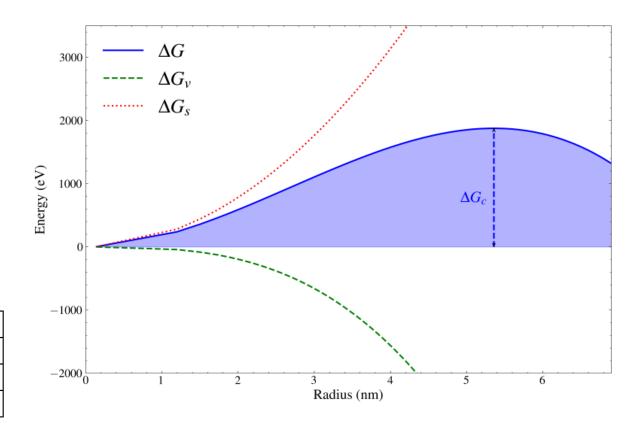
Metallic impurities: Fe

Parameter	Symbol	Value (Unit)
Temperature	T	668 K
Molar Mass	M	55.85 g/mol
Mass Density	ρ	7.87 g/cm^3
Melting Point	T_m	1538 K
Heat of Fusion	ΔH_f	11.7 kJ/mol
Surface Tension	σ	$2.5 \mathrm{J/m^2}$

Table 3: Parameters for Fe

Parameter	Symbol	Value (Unit)
Critical radius	r	5.34 nm
Energy barrier	ΔG_c	$3.00 \times 10^{-16} \text{ J}$
Critical number of molecules	n	54722

Table 4: Critical values for Fe





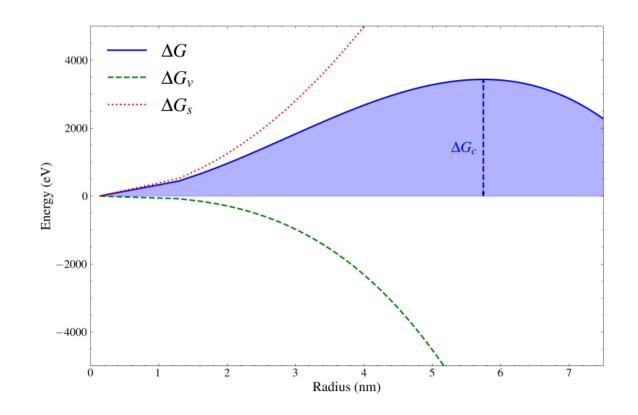
Metallic impurities: Cr

Parameter	Symbol	Value (Unit)
Temperature	T	668 K
Molar Mass	M	51.9961 g/mol
Mass Density	ρ	7.19 g/cm^3
Melting Point	T_m	2130 K
Heat of Fusion	ΔH_f	14.6 kJ/mol
Surface Tension	σ	3.98 J/m^2

Table 1: Parameters for Cr

Parameter	Symbol	Value (Unit)
Critical radius	r	5.74 nm
Energy barrier	ΔG_c	$5.50 \times 10^{-16} \text{ J}$
Critical number of molecules	n	66115

Table 2: Parameters for Cr





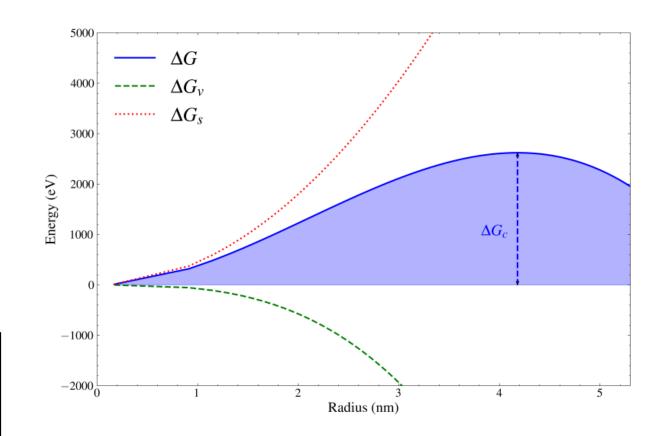
Metallic impurities: AIN

Parameter	Symbol	Value (Unit)
Temperature	T	668 K
Molar Mass	M	40.98 g/mol
Mass Density	ρ	3.26 g/cm^3
Melting Point	T_m	2200 K
Heat of Fusion	ΔH_f	49.59 kJ/mol
Surface Tension	σ	$5.74 \; J/m^2$

Table 5: Parameters for AlN

Parameter	Symbol	Value (Unit)
Critical radius	r	4.18 nm
Energy barrier	ΔG_c	$4.2 \times 10^{-16} \text{ J}$
Critical number of molecules	n	14646

Table 6: Critical values for AlN





Non Metallic impurities: LiH, LiD, LiT

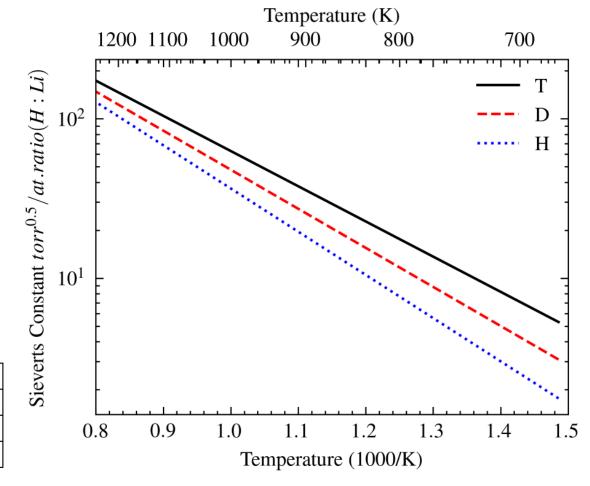
Reaction	Ratio	$\mathrm{mol/m}^{8}$	(H:Li) Ratio
(n, Xp)	0.015	6.5×10^{-7}	9.12×10^{-6}
(n, Xd)	0.15	6.34×10^{-6}	8.96×10^{-5}
(n, Xt)	1.53	6.5×10^{-5}	9.1×10^{-4}
Neutrons/Pulse	6.2×10^{21}		

Table 7: Hydrogen nuclides production

$$P^{1/2} = K \cdot C_1$$
 $\frac{p_{D_2}}{p_{H_2}} = \sqrt{2}, \quad \frac{p_{T_2}}{p_{D_2}} = \sqrt{3}$

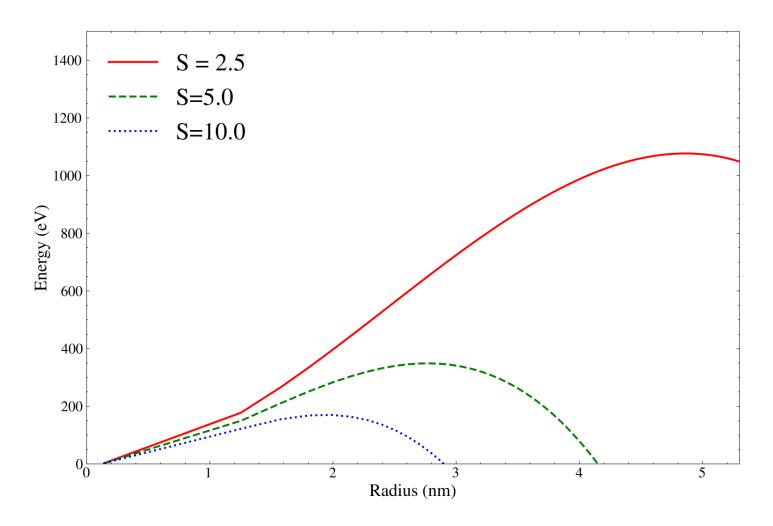
Nuclide	Sieverts Constant $(Torr^{0.5}/[(H:Li)])$	(H:Li) Ratio
Н	1.65	0.0045
D	2.90	0.00363
Τ	5.02	0.00360

Table 8: Solubility of hydrogen isotopes in lithium





Non Metallic impurities: LiH, LiD, LiT





Kinetic equations

 $k_i^+(k_i^-)$ — attachment (detachment) frequencies of molecules

$$\frac{dF_i}{dt} = k_{i-1}^+ F_{i-1} - \left[k_i^+ + k_i^-\right] F_i + k_{i+1}^- F_{i+1} = J_{i-1}(t) - J_i(t)$$

$$J_i(t) = k_i^+ F_i(t) - k_{i+1}^- F_{i+1}(t)$$

- F_i number density of clusters of size i
- J_i cluster flux density (nucleation rate for i^*)
- $k_i^+(k_i^-)$ attachment (detachment) frequencies

 F_i^0 — equilibrium number of cluster formed by i molecules. It can be shown that

$$F_i^0 = B_2 \exp\left(-\frac{W_i}{kT}\right)$$

Homogeneous nucleation, self-consistent model: $B_2 = N_1 \exp\left(-\frac{W_1}{kT}\right)$



Initial and boundary conditions

$$F_i(t=0) = F_i^0$$
 for $i \le i_0$ (usually $i_0 \approx i^*/2$)

 F_i^0 — equilibrium distribution of nuclei

$$F_i(t=0) = 0$$
 for $i > i_0$

$$F_1(t) = F_1^0 = N_n = \text{const.}$$

 N_n — number of nucleation sites $(N_n = N_1 \text{ for HON})$

