SE 102: Introduction to Computing for Engineers **Final Exam**
3/16/2021 from 8:00 am PT to 11:00 am PT

Instructions:

- *There is a total of 7* ***problems****. Tables of Finite Difference formula and Gauss quadrature are attached for reference.*
- *You have 3 hours to complete the exam. You will need to upload your solutions (in the form of one PDF file) plus any code you write (for example, the .m files) to Gradescope by 3/16/2021 at 11:00 am (PT).* <u>*Late submissions will not be accepted.*</u>
- *The exam is open-book and open-note.*
- *You are not allowed to communicate in any way about the exam or course content with anyone other than the instructor (Prof. Semnani) and TAs (Yangzi He and Izabela Batista) during the duration of the exam.*
- *If you have any questions for clarification of the problems, you may join the following Zoom link and wait in the waiting room until you are admitted. Link:*

  https://ucsd.zoom.us/j/95847937549?pwd=YkdENlNmdGMzWi9lM3BQUFd3V3FBZz09

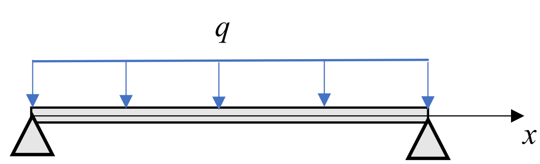
- *You must* ***complete all problems individually****, with no assistance from other people.*
- *Please note that all logic, equations, and procedures must be carefully explained. You need to include all details of your derivation and explain how you got to the answer.*

Problem 1) (solve by hand)

Consider the following beam with supported ends subject to uniform loading q. The deflection of the beam, w, is given by the differential equation:

$$\frac{d^2w}{dx^2} = \frac{S}{EI}w + \frac{qx}{2EI}(x - L) \quad , \quad 0 \le x \le L$$

At the boundaries, we have $w(0) = w(L) = 0$.



The length of the beam is L=120 in, modulus of elasticity is $E = 3 \times 10^7$ lb/in^2, intensity of uniform load is $q = 100$ lb/ft, S=1000 lb, and moment of inertia is I=625 in^4.

Divide the beam into 3 segments and use the finite difference method to solve for the deflection at $x = 40$ and $x = 80$ inches. State your procedure and all of the equations that you use in detail.

Problem 2) (Solve by hand)

Consider the following matrix:

$$\boldsymbol{A} = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$$

A) Write the characteristic equation that needs to be solved to obtain the eigenvalues of $\boldsymbol{A}$. Subsequently, solve the equation and report eigenvalues of $\boldsymbol{A}$.

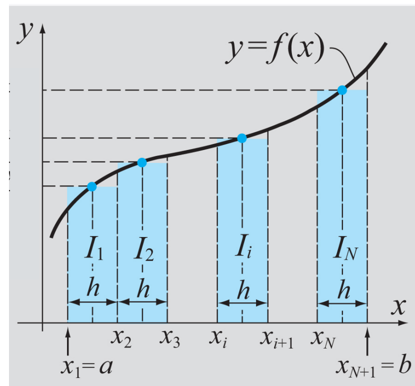
B) Decompose the matrix into an orthogonal and an upper triangular matrix.

C) Use QR iteration method (with 2 iterations) to obtain eigenvalues of $\boldsymbol{A}$. Calculate the true relative error of your results.

D) Use the smallest eigenvalue from Part (A) to derive one of the eigenvectors of $\boldsymbol{A}$ such that the magnitude (Euclidean norm) of the eigenvector is 1.

Problem 3) (MATLAB Programming)

As you have learned in the class and labs, integral of a function over an interval [a, b] can be calculated by dividing an interval [a, b] into N segments and approximating the area of each segment using a rectangle as shown in the figure below. The heights of the rectangles are determined using the midpoint of each interval. The areas of all rectangles are added to estimate the integral. Complete the following MATLAB function, so that it calculates the integral of a given function, $f(x)$, within the given interval [a, b] using the described method with N segments.




```matlab
function I = integral(a,b,N)
% Input: %
%           a, b = lower and upper limits of the integral
%           N = number of segments to use in the integration
%           I = approximate value of the integral from a to b of f(x)
%
f = @ (x) sin(x)+x.^ 2; % The function, f(x), we want to integrate

h=(b-a)/N;

x = a:h:b;
```

```matlab
end
```

Problem 4) (Solve by hand)

Consider the following system of equations:

$$x^2 - xy + y^2 = 3$$
$$x + y - xy = 0$$

A) Can point $\boldsymbol{x}^{(0)} = [-1, -1]$ be used to solve the system of equations using Newton's method? Explain why.
B) Find two initial guess vectors that **<u>cannot</u>** be used to solve the system using Newton's method.
C) Derive the Jacobian of the system. Calculate the 1-norm condition number of the Jacobian at point $\boldsymbol{x}^{(0)} = [0.5, 0]$. Is the system of equations at this initial guess ill-conditioned?
D) Use an appropriate starting point and complete the first two iterations of the Newton's method to find unknowns x and y. Report the infinity-norm of the final vector of increments, $\Delta\boldsymbol{x}$.

Problem 5) (MATLAB Programming)

Complete the following MATLAB function so that it takes two vectors X and Y as input, which represent the coordinates of the data points, and returns a vector with the **values of the first derivative at All** data points (including the first and last points) **with a truncation error of *O*(*h*)**. The data points can have **unequal spacing**.

```
function [deriv1] = FirstDerivative(X,Y)
% INPUTS:
% X and Y are vectors containing the coordinates of given data points
% OUTPUT:
% deriv1 :vector of first derivative of the function described by the data
set, at ALL given data points

n = length(X);
deriv1=zeros(n, 1);



end
```

Problem 6) (solve by hand)

A) We want to approximate the integral of a given function using Gauss quadrature method with **one** Gauss point. Write the equation(s) that needs to be solved to find the weight(s) and the Gauss point for integration using Gauss quadrature method with one integration point. Solve the equations(s) you derived and report the values of the weight(s) and the Gauss point.

B) Use Gauss quadrature method with 3 Gauss points to calculate the following integral:

$$\int_1^2 (x^2 + \sin x)\, dx$$

Problem 7) (solve by hand)

Consider the following system of equations in the form **Ax** = **B**.

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 3 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

A) Decompose the matrix of coefficient, **A**, into a lower-triangular matrix, **L, with ones on the diagonal** and **U, an upper-triangular matrix**. If necessary, also compute the **permutation matrix, P**. Make sure to state all of the steps.

B) Use your results from Part (A) to calculate the determinant of matrix **A**. Is matrix **A** singular?

C) Write down the first two iterations of Gauss-seidel method and the Jacobi method for solving the system of equations given above with the initial guess of your choice. Which method will converge faster? Why?

First Derivative		
Method	**Formula**	**Truncation Error**
Two-point forward difference	$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h}$	$O(h)$
Three-point forward difference	$f'(x_i) = \frac{-3f(x_i) + 4f(x_{i+1}) - f(x_{i+2})}{2h}$	$O(h^2)$
Two-point backward difference	$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h}$	$O(h)$
Three-point backward difference	$f'(x_i) = \frac{f(x_{i-2}) - 4f(x_{i-1}) + 3f(x_i)}{2h}$	$O(h^2)$
Two-point central difference	$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$	$O(h^2)$
Four-point central difference	$f'(x_i) = \frac{f(x_{i-2}) - 8f(x_{i-1}) + 8f(x_{i+1}) - f(x_{i+2})}{12h}$	$O(h^4)$

Second Derivative		
Method	**Formula**	**Truncation Error**
Three-point forward difference	$f''(x_i) = \frac{f(x_i) - 2f(x_{i+1}) + f(x_{i+2})}{h^2}$	$O(h)$
Four-point forward difference	$f''(x_i) = \frac{2f(x_i) - 5f(x_{i+1}) + 4f(x_{i+2}) - f(x_{i+3})}{h^2}$	$O(h^2)$
Three-point backward difference	$f''(x_i) = \frac{f(x_{i-2}) - 2f(x_{i-1}) + f(x_i)}{h^2}$	$O(h)$
Four-point backward difference	$f''(x_i) = \frac{-f(x_{i-3}) + 4f(x_{i-2}) - 5f(x_{i-1}) + 2f(x_i)}{h^2}$	$O(h^2)$
Three-point central difference	$f''(x_i) = \frac{f(x_{i-1}) - 2f(x_i) + f(x_{i+1})}{h^2}$	$O(h^2)$
Five-point central difference	$f''(x_i) = \frac{-f(x_{i-2}) + 16f(x_{i-1}) - 30f(x_i) + 16f(x_{i+1}) - f(x_{i+2})}{12h^2}$	$O(h^4)$

n (Number of points)	Coefficients C_i (weights)	Gauss points x_i
2	$C_1 = 1$ $C_2 = 1$	$x_1 = -0.57735027$ $x_2 = 0.57735027$
3	$C_1 = 0.5555556$ $C_2 = 0.8888889$ $C_3 = 0.5555556$	$x_1 = -0.77459667$ $x_2 = 0$ $x_3 = 0.77459667$
4	$C_1 = 0.3478548$ $C_2 = 0.6521452$ $C_3 = 0.6521452$ $C_4 = 0.3478548$	$x_1 = -0.86113631$ $x_2 = -0.33998104$ $x_3 = 0.33998104$ $x_4 = 0.86113631$
5	$C_1 = 0.2369269$ $C_2 = 0.4786287$ $C_3 = 0.5688889$ $C_4 = 0.4786287$ $C_5 = 0.2369269$	$x_1 = -0.90617985$ $x_2 = -0.53846931$ $x_3 = 0$ $x_4 = 0.53846931$ $x_5 = 0.90617985$
6	$C_1 = 0.1713245$ $C_2 = 0.3607616$ $C_3 = 0.4679139$ $C_4 = 0.4679139$ $C_5 = 0.3607616$ $C_6 = 0.1713245$	$x_1 = -0.93246951$ $x_2 = -0.66120938$ $x_3 = -0.23861919$ $x_4 = 0.23861919$ $x_5 = 0.66120938$ $x_6 = 0.93246951$