Stabiity aspects of Scalable and Hybrid DDEs

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# 1 Case 1: Constant coefficients and discrete delays

$$m\ddot{x}\left(t\right)+c\dot{x}\left(t\right)+\left[k\;+g\left(t\right)\right]x\left(t\right)=H\left(x\right)\left[\int_{\tau_{min}}^{\tau_{max}}K\left(s\right)x\left(t-s\right)ds+\int_{\tau_{min}}^{\tau_{max}}C\left(s\right)\dot{x}\left(t-s\right)ds\right] \tag{1}$$

considering

$$g(t)=0,\;K\left(s\right)=\delta\left(s-\tau_1\right),\;C\left(s\right)=\delta\left(s-\tau_2\right)$$

Eq.(1) reduces to:

$$m\ddot{x}\left(t\right)+c\dot{x}\left(t\right)+k\,x\left(t\right)=H\left(x\right)\left[x(t-\tau_1)+\dot{x}(t-\tau_2)\right] \tag{2}$$

considering $t=\alpha\,T$ and setting $\alpha=\dfrac{1}{\omega_n}$, Eq(2) becomes:

$$x''\left(T\right)+2\zeta\,x'\left(T\right)+x\left(T\right)=H\left(x\right)\left[k\,x(T-T_1)+Cx'(T-T_2)\right] \tag{3}$$

where: $\omega_n=\sqrt{\dfrac{k}{m}}$, $T_1=\dfrac{\tau_1}{\alpha}$, $T_2=\dfrac{\tau_2}{\alpha}$, $C=\dfrac{1}{\sqrt{km}}$

neglecting the $x'(T-T_2)$ term, Eq.(3) becomes:

$$x''(T) + 2\zeta\, x'(T) + x(T) = H(x)\, k\, x(T-T_1) \tag{4}$$

**Problem Statement:** Stability analysis of Eq.(4) in the $k$ vs $T_1$ plane for various values of $\zeta$.