Problem 1. Suppose that the state of "fast speed" of a machine is denoted by the fuzzy set F with membership function $\mu_F(v)$. Then the state of "very fast speed", where the *linguistic hedge* "very" has been incorporated, may be represented by $\mu_F(v-v_o)$ with $v_o > 0$. Also, the state "presumably fast speed", where the linguistic hedge "presumably" has been incorporated, may be represented by $\mu_F^2(v)$.

(a) Discuss the appropriateness of the use of these membership functions to represent the respective linguistic hedges.

(b) In particular, if

$$F = \{\frac{0.1}{10}, \frac{0.3}{20}, \frac{0.6}{30}, \frac{0.8}{40}, \frac{1.0}{50}, \frac{0.7}{60}, \frac{0.5}{70}, \frac{0.3}{80}, \frac{0.1}{90}\}$$

in the discrete universe $V = \{0, 10, 20, \ldots, 190, 200\}$ rev/s and $v_o = 50$ rev/s, determine the membership functions of "very fast speed" and "presumably fast speed". Display both membership functions over the discrete Universe V.

Problem 2. Sketch the membership function $\mu_A(x) = e^{-\lambda(x-a)^n}$ for $\lambda = 2$, $n = 2$, and $a = 3$ for the support set $S = [0,6]$. On this sketch separately show the shaded areas that represent the following fuzziness measures given by M_1, M_2 and M_3:

(a) $M_1 = \int_S f(x)dx$ where $f(x) = \mu_A(x)$ for $\mu_A(x) \le 0.5$
$= 1 - \mu_A(x)$ for $\mu_A(x) > 0.5$

(b) $M_2 = \int_S \left|\mu_A(x) - \mu_{A_{1/2}}(x)\right| dx$

where $\mu_{A_{1/2}}$ is the $\alpha-$cut of $\mu_A(x)$ for $\alpha = 1/2$

(c) $M_3 = \int_S \left|\mu_A(x) - \mu_{\overline{A}}(x)\right| dx$

where $\overline{A}$ is the complement of the fuzzy set A. Evaluate the values of M_1, M_2 and M_3 for the given membership function.

i. Establish relationships between M_1, M_2 and M_3.
ii. Indicate how these measures can be used to represent the degree of fuzziness of a membership function.
iii. Compare your results with the case $\lambda = 1$, $a = 3$, and $n = 2$ for the same support set, by showing the corresponding fuzziness measures on a sketch of the new membership function.

Problem 3. The characteristic function χ_A of a crisp set A is analogous to the membership function of a fuzzy set, and is defined as follows:

$$\chi_A(x) = 1 \text{ if } x \in A$$
$$= 0 \text{ otherwise}$$

Using isomorphism of crisp sets and binary logic, show that

$$\chi_{A'} = 1 - \chi_A$$
$$\chi_{A \cup B} = \max\left(\chi_A, \chi_B\right)$$
$$\chi_{A \cap B} = \min\left(\chi_A, \chi_B\right)$$
$$\chi_{A \to B}\left(x, y\right) = \min\left[1, \left\{1 - \chi_A\left(x\right) + \chi_B\left(y\right)\right\}\right]$$

where A and B are defined in the same universe X, except in the last case (implication) where A and B may be defined in two different universes X and Y.

What are the implications of these results?

Problem 4. Show that $max[0, x + y - 1]$ is a t-norm. Also, determine the corresponding t-conorm (i.e., s-norm). *Hint:* Show that the non-decreasing, commutative, and associative properties and the boundary conditions are satisfied.