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2/13/2018

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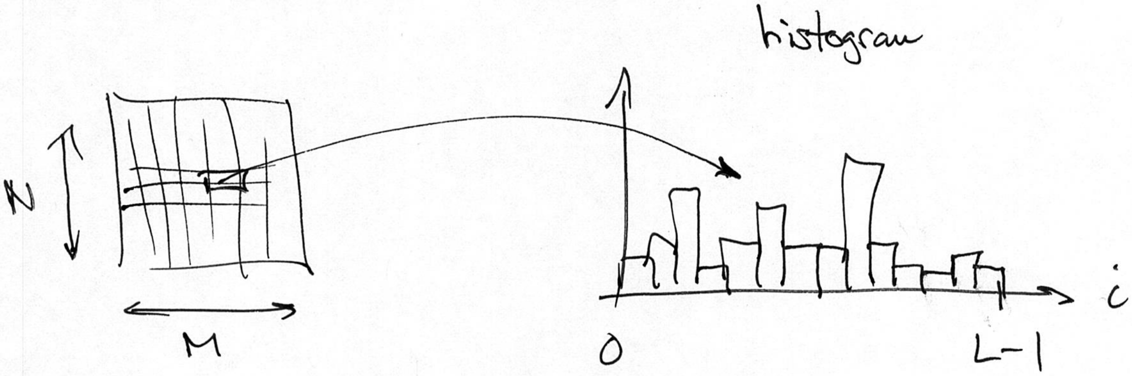
Histogram



$h(i) = n_i$ | number of pixels with intensity i

$\int hist(i)\, di = ?$

$$\sum_{i=0}^{L-1} hist(i) = \underbrace{M \cdot N}_{\text{image size}} \text{ pixels}$$

Normalization: $p(i) = \dfrac{hist(i)}{M \cdot N}$

$$\sum_{i=0}^{L-1} p(i) = \frac{M \cdot N}{M \cdot N} = 1$$

$p(i)$: what is probability to find pixel with intensity i in image $M \cdot N$

Cumulative Distribution Function (cdf)

$$F(x) = \sum_{i=-\infty}^{x} p(i)$$

maximum: $F(max_i) = F(L-1) = \sum_{i=-\infty}^{\infty} p(i) = 1$

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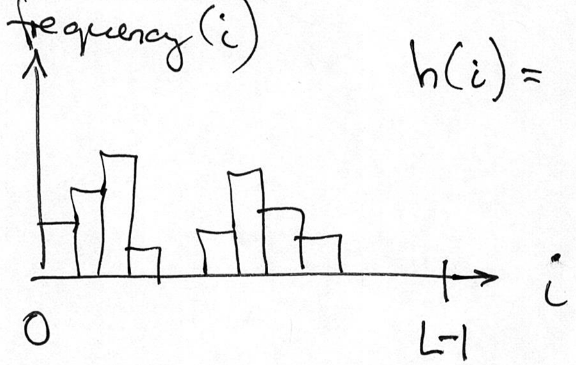
Histogram Equalization

notation: L: luminance, brightness

range: $[0 \ldots L-1]$ — 1 byte image: $[0 \ldots 255]$

image size: $M.N$

histogram



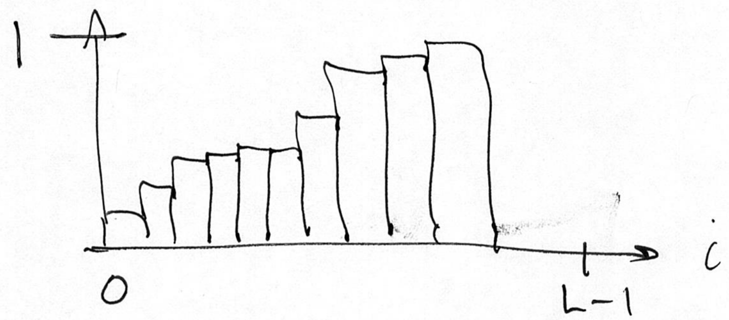
$h(i) = n_i$

normalized histogram, pdf

$p(i) = n_i / M.N$

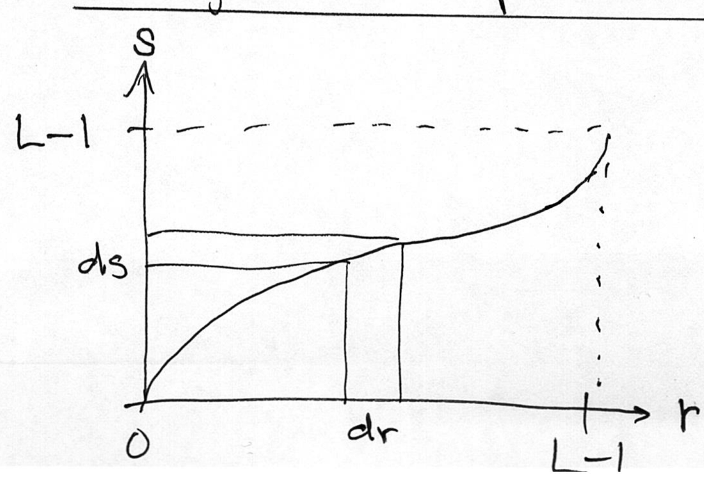
cdf: cumulative distribution function

$$cdf(x) = \sum_{i=-\infty}^{x} p(i) \qquad \sum_{i=-\infty}^{\infty} p(i) = 1$$



cdf monotonic, non-decreasing
$\Longrightarrow$ has inverse!

histogram manipulation, mapping



out, in

$s = T(r)$

T: monotonic
has inverse

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idea: use cdf as mapping function $T(r)$

why: equal sampling of frequency axis ⇒ same amount of pixels

image r: $p_r(r) = pdf(r)$

image s: $p_s(s) = pdf(s)$

} probability density function (normalized histogram)

mapping: $$p_s(s) = p_r(r) \cdot \underbrace{\left| \frac{dr(r)}{ds} \right|}_{\text{local stretching/contraction determined by } T(r)}$$

choose $T(r)$ as $cdf(r)$:

$$\frac{ds}{dr} = (L-1)\frac{d\,(cdf(r))}{dr} = \frac{d\,T(r)}{dr}$$

$(L-1)$: scaling from $[0 \ldots 1]$ to $[0 \ldots L-1]$

$$= (L-1)\,\frac{d}{dr}\left[\int_0^r p_r(w)\,dw\right] = (L-1)\;p_r(r)$$

(Leibnitz rule)*

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* Leibnitz rule: Calculus: Derivative of definite integral w.r.t upper limit is integrand evaluated at limit

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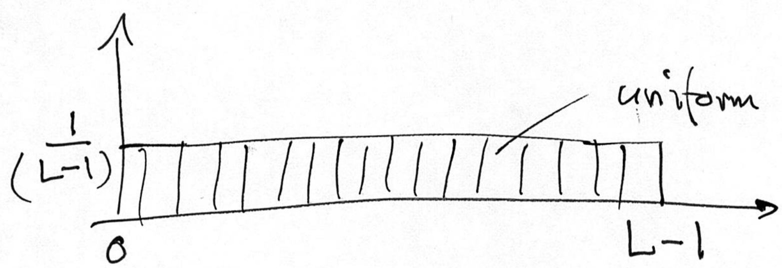
$$\Rightarrow \underline{\underline{p_s(s)}} = p_r(r)\left|\frac{dr}{ds}\right| = \cancel{p_r(r)}\,\frac{1}{(L-1)\,\cancel{p_r(r)}} = \underline{\underline{\frac{1}{(L-1)}}}$$

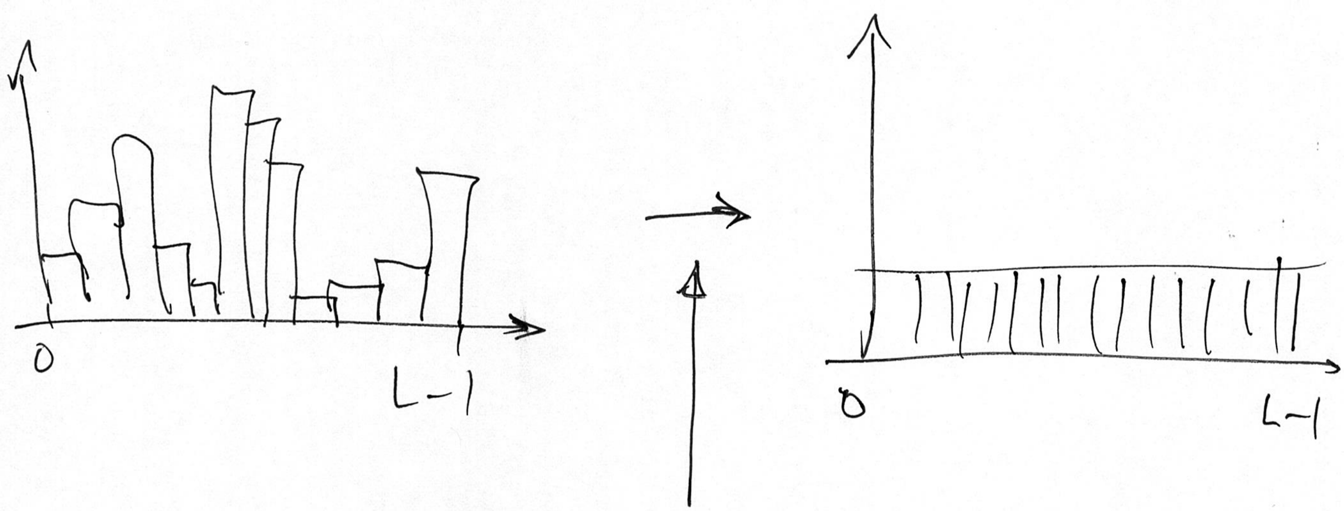
What does it mean?

$p_s(s) = \frac{1}{(L-1)} =$ constant ; uniform distribution

The resulting normalized histogram after histogram normalization is a uniform pdf.



Every brightness value is equally likely!

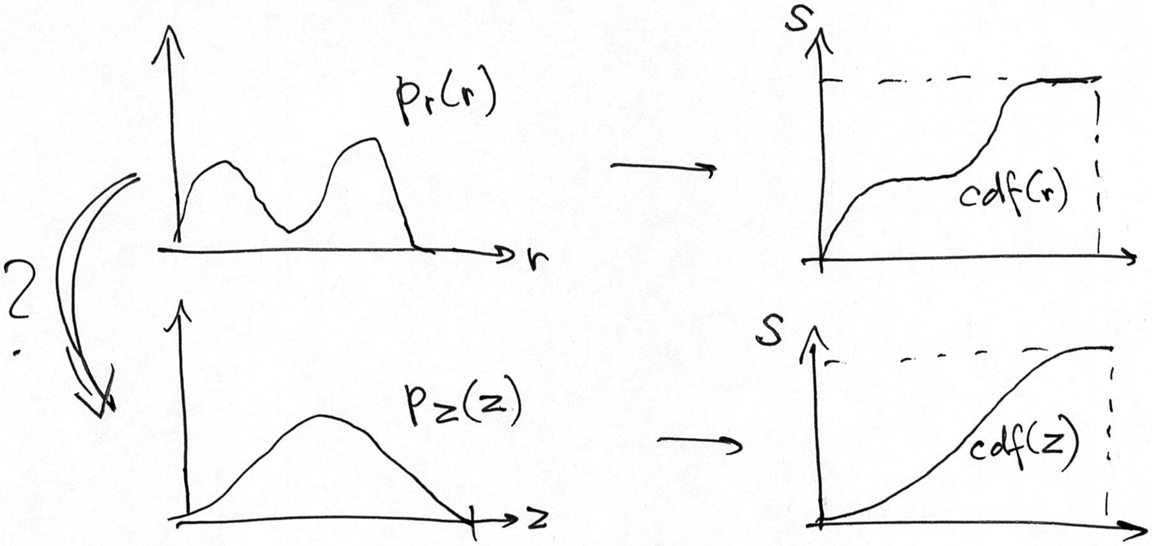


histogram equalization

Implementation: new_intensity(x,y) = (L-1) cdf (intensity(x,y))

Histogram Matching

Goal: Map intensities of source image so that new histogram matches the goal histogram.



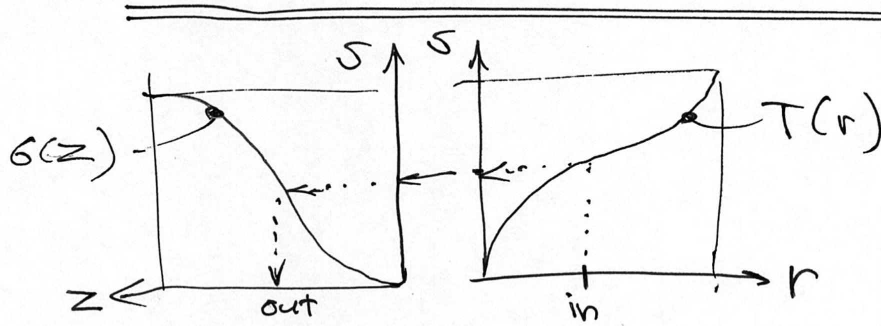
① $s = T(r)$ | $T =$ cdf(r) of $p_r(r)$

② $s = G(z)$ | $G =$ cdf(z) of $p_z(z)$

$$G(z) \stackrel{!}{=} T(r) \Rightarrow \text{cascading:}$$

$$z = G^{-1}[T(r)] = G^{-1}[s]$$

dual mapping



Implementation

Look-up table