ASSIGNMENT #2

A1) Linear Shift Invariant System

Let $f_1(x) = [0,0,0,a,b,c,d,0,0,0]$

$w(x) = \frac{1}{3}[1,1,1]$

$g_1(x) = w(x) * f_1(x)$
$= \frac{1}{3}[1,1,1] * 0[0,0,0,a,b,c,d,0,0,0]0$

$g_1(x) = [0, 0, a/3, \frac{a+b}{3}, \frac{a+b+c}{3}, \frac{b+c+d}{3}, \frac{c+d}{3}, \frac{d}{3}, 0, 0]$ — (1)

Consider $x_0 = 2$

$f_2(x) = f_1(x - x_0)$
$f_2(x) = f_1(x-2)$
$f_2(x) = [0,0,0,0,0,a,b,c,d,0]$

$g_2(x) = g_1(x - x_0)$
$= g_1(x-2)$
$= [0,0,0,0,\frac{a}{3}, \frac{a+b}{3}, \frac{a+b+c}{3}, \frac{b+c+d}{3}, \frac{c+d}{3}, \frac{d}{3}]$ — (2)

$w(x) * f_2(x) = \frac{1}{3}[1,1,1] * 0[0,0,0,0,0,a,b,c,d,0]0$

$= [0,0,0,0,\frac{a}{3}, \frac{a+b}{3}, \frac{a+b+c}{3} + \frac{b+c+d}{3}, \frac{c+d}{3}, \frac{d}{3}]$

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$g_2(x) = w(x) * f_2(x)$

Hence, a smoothing filter with $\frac{1}{3}[1,1,1]$ filter kernel is shift invariant.

Let $A = [r, s, t]$

$B = [x, y, z]$

$Op = \frac{1}{3}[1,1,1]$

$$Op(A+B) = Op[r+x, s+y, t+z]$$

$$Op(A+B) = \left[\frac{r+x+s+y}{3}, \frac{r+x+s+y+t+z}{3}, \frac{s+y+t+z}{3}\right] \quad \text{— (3)}$$

$$Op(A)+Op(B) = Op[r,s,t] + Op[x,y,z]$$

$$= \left[\frac{0+r+s}{3}, \frac{r+s+t}{3}, \frac{s+t+0}{3}\right] + \left[\frac{0+x+y}{3}, \frac{x+y+z}{3}, \frac{y+z+0}{3}\right]$$

$$= \left[\frac{r+s+x+y}{3}, \frac{r+s+t+x+y+z}{3}, \frac{s+t+y+z}{3}\right] \quad \text{— (4)}$$

From (3) & (4),

We get a smoothing filter with $\frac{1}{3}[1,1,1]$ filter kernel is linear.

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Now applying a median filter using a 3 neighborhood.

Let $f_1(x) = [2, 4, 1, 6, 7, 5, 8]$
applying a median filter on $f_1(x)$, we get

$f_1(x) = 2\,[2, 4, 1, 6, 7, 5, 8]\,8$

$g_1(x) = [$ median $(2, 2, 4)$, median $(2, 4, 1)$,
median $(4, 1, 6)$, median $(1, 6, 7)$,
median $(6, 7, 5)$, median $(7, 5, 8)$,
median $(5, 8, 8)$ $]$

$g_1(x) = [2, 2, 4, 6, 6, 7, 8]$ —— ①

Let $x_0 = 2$

$\therefore f_1(x - x_0) = f_1(x-2) = 0\,[0, 0, 2, 4, 1, 6, 7, 5, 8]\,8$

$\therefore g_1(x-x_0)$ after applying median filter, we get

$g_1(x-x_1) = [$ median $(0, 0, 0)$, median $(0, 0, 2)$,
median $(0, 2, 4)$, median $(2, 4, 1)$, median $(4, 1, 6)$,
median $(1, 6, 7)$, median $(6, 7, 5)$, median
$(7, 5, 8)$, median $(5, 8, 8)$ $]$

$= [0, 0, 2, 2, 4, 6, 6, 7, 8]$ —— ②

From ①, $g_1(x - x_0) = [0, 0, 2, 2, 4, 6, 6, 7, 8]$ — ③

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from (2) and (3),

$g_1(x - x_0) = g_1(x - x_1)$

$\therefore$ A median filter with 3 neighborhood is shift invariant.

To prove median filter is not linear:

Let $A = [15, 1, 8, 6, 22, 30]$
$B = [5, 1, 16, 18, 3, 7]$
$A+B = 20\,[20, 2, 24, 24, 25, 37]\,37$

$Op(A+B) = [$ median$(20,20,2)$, median$(20,2,24)$, median$(2,24,24)$, median$(24,24,25)$, median$(24,25,37)$, median$(25,37,37)]$

$Op(A+B) = [20, 20, 24, 24, 25, 37]$ —— (5)

$Op(A) = [$median$(15,15,1)$, median$(15,1,8)$, median$(1,8,6)$, median$(8,6,22)$, median$(6,22,30)$, median$(22,30,30)]$

$= m[15, 15, 6, 8, 22, 30]$

$Op(B) = [$median$(5,5,1)$, median$(5,1,16)$, median$(1,16,18)$, median$(16,18,3)$, median$(18,3,7)$, median$(3,7,7)]$
$= [5, 5, 16, 16, 7, 7]$

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Op(A) + Op(B) = [20, 20, 22, 24, 29, 37] — (6)

from (5) & (6), we get

$Op(A+B) \neq Op(A) + Op(B)$

∴ median filter is not linear

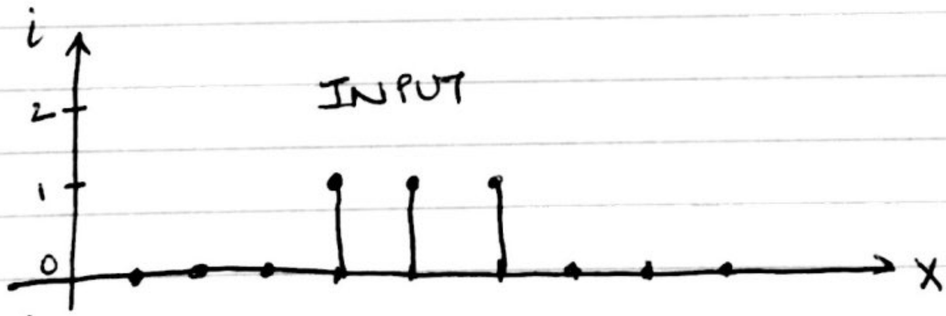
∴ median filter using 3 neighborhood is not an LSI system.

(A2) 1-D Convolution

Convolve [1,1,1] filter with [0,0,0,1,1,1,0,0,0] and graph/plot the result.

$[1, 1, 1] * [0,0,0,1,1,1,0,0,0]$

$= [0, 0, 1, 2, 3, 2, 1, 0, 0]$



(A3) ~~Median filtering~~

A4)

a) Decompose the 2D Gaussian into the two components

① Seperability

2-D gaussian – $\frac{1}{2\pi\sigma^2} \exp^{-}\left(\frac{x^2+y^2}{2\sigma^2}\right)$

$$= \frac{1}{2\pi\sigma^2}\left(\exp^{-x^2/2\sigma^2}\right)\left(\exp^{-y^2/2\sigma^2}\right)$$

[∵ the exponent term can be seperated because they are independent function of x^2 and y^2 dependent only on x and y respectively]

$$= \frac{1}{\sqrt{2\pi}\,\sigma}\left(\exp^{-x^2/2\sigma^2}\right) \cdot \frac{1}{\sqrt{2\pi}\,\sigma}\left(\exp^{-y^2/2\sigma^2}\right)$$

b) Discuss how this helps to speedup the filtering operation by making an example of a m×m filter size covering the Gaussian and then use the two 1-D filters.

The m×n filters requires m×n multiplications

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and the sum. Seperating a filter into two 1-D only requires ~~m×n~~ m+n multiplications.

Eg. an 11×11 filter would require 121 multiplication whereas running it with 1-D seperable filters would be 11 + 11 = 22, more than 5 times less.

(A6) Canny non-maximum suppression

Non-Maximum filtering - After getting gradient magnitude and direction, a full scan of image is done to remove any unwanted pixels which may not constitute the edge. For this, at the every pixel, value is checked for being the local maximum in its neighborhood in the direction of gradient.

If a point is on the edge, gradient direction is normal to the edge, then that point is checked with other points to see if it forms a local maximum. If so, it is considered for next stage, otherwise, it is suppressed.

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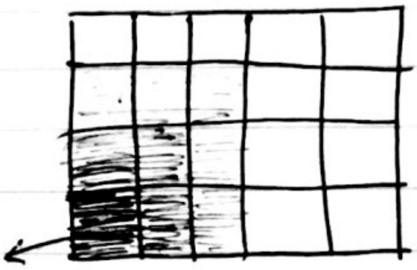
(A)

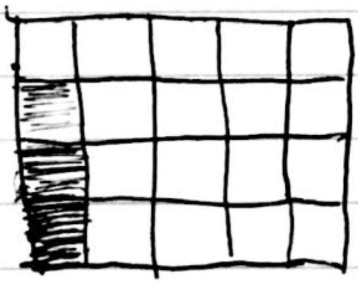


x ⟶ Gradient direction

Here, the intensity of pixel 'x' is compared with the other neighbouring pixels for local maximum in the direction of the gradient.

If the intensity of 'x' is found to be local maxima, it is preserved ~~considered for the next stage~~. Otherwise, it will be discarded.

So, here intensity of 'x' is local maximum. Hence A becomes,



The other lower intensities are suppressed.

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(A3) Median filtering

1) We use a 3x3 Median filter, for median filtering sort the elements of original image.

0	0	1	0	0
0	$\boxed{0}$	1	0	0
0	0	1	0	0
0	0	1	0	0
0	0	1	0	0

[0, 0, 0, 0 0, 0, 1, 1, 1]

Median = 0

∴ first element is 0

Shift the 3x3 filter, element after sorting = [0, 0, 0, 0, 0, 0, 1, 1, 1]

Median = 0

∴ Second element is 0

After completion of sliding –

0	0	0
0	0	0
0	0	0

2) Using the 3x3 median filter elements in sorted order

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

[0, 0, 0, 0, 0, 0, 0, 0, 1]

∴ first element = 0 (Median)

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After sliding completion $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(3) Using 3X3 Median filter elements in sorted order: [1, 1, 1, 1, 1, 1, 1, 1, 1]

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

first element = 1
Second element = 1

After sliding completion = $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

LINEAR FILTERING

Using the following filter

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

first element = $\frac{1}{9}\left[1\times 0 + 1\times 0 + 1\times 1 + 0\times 1 + 0\times 1 + 1\times 1 + 0\times 1 + 0\times 1 + 1\times 1\right]$

$= 3/9 = 1/3$

Second element = $\frac{1}{9}\left[1\times 0 + 1\times 0 + 1\times 0 + 1\times 1 + 1\times 1 + 1\times 1 + 0\times 1 + 0\times 1 + 0\times 1\right]$

$= 1/3$

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After Completing Sliding $= \frac{1}{3}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$

b) Using the filtering linear -

$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$ $\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&1&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}$

first element $= 0\times1+0\times1+0\times1+0\times1+0\times1+0\times1$
$+ 0\times1 + 0\times1 + 1\times1$
$= 1/9$

Similarly after completing sliding $1/9\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$

c) Using linear filter -

$1/9\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$ $\begin{bmatrix}1&1&1&0&0\\1&1&1&0&0\\1&1&1&0&0\\1&1&1&0&0\\1&1&1&0&0\end{bmatrix}$

first Element = 1
Second Element = 6/9
Third Element = 3/9

Similarly after sliding $= \frac{1}{9}\begin{bmatrix}9&6&3\\9&6&3\\9&6&3\end{bmatrix}$

(A5)

In the give two options, the first one is ~~the~~ better and more optimal for the following reasons.

1. When we take the partial derivative of the Gaussian filter the degree of the filter is reduced.
We assumed y to be constant, with taking the partial derivate and the dimensionality of the filter is reduced.

2. Performing the convolution after the derative, will ~~the~~ reduce the number of operations.

3. The first partial derative of the 2-D Gaussian is linearly seperable and hence there would be a reduction in the total overall number of operations performed during convolution.

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