Appendix B: Formulation of the Benchmark Problems

Table 8
Unconstrained bi-objective problems.

Problem	Objective functions	n	Variable bounds	Comments
Schaffer	$f_1(x) = x^2$ $f_2(x) = (x-2)^2$	1	$-10^5 \leq x \leq 10^5$	*convex*
Fonseca	$f_1(\vec{x}) = 1 - e^{-\sum_{i=1}^{n}(x_i - \frac{1}{\sqrt{n}})^2}$ $f_2(\vec{x}) = 1 - e^{-\sum_{i=1}^{n}(x_i + \frac{1}{\sqrt{n}})^2}$	3	$-4 \leq x_i \leq 4$	*nonconvex*
Kursawe	$f_1(\vec{x}) = \sum_{i=1}^{n-1}\left(-10e^{\left(-0.2*\sqrt{x_i^2+x_{i+1}^2}\right)}\right)$ $f_2(\vec{x}) = \sum_{i=1}^{n}(\lvert x_i\rvert^a + 5\sin(x_i)^b)$	3	$-5 \leq x_i \leq 5$	*nonconvex*
ZDT1	$f_1(\vec{x}) = x_1$ $f_2(\vec{x}) = g(\vec{x})[1 - \sqrt{x_1/g(\vec{x})}]$ $g(\vec{x}) = 1 + 9\left(\sum_{i=2} x_i\right)/(n-1)$	30	$0 \leq x_i \leq 1$	*convex*
ZDT2	$f_1(\vec{x}) = x_1$ $f_2(\vec{x}) = g(\vec{x})\left[1 - (x_1/g(\vec{x}))^2\right]$ $g(\vec{x}) = 1 + 9\left(\sum_{i=2}^{n} x_i\right)/(n-1)$	30	$0 \leq x_i \leq 1$	*nonconvex*
ZDT3	$f_1(\vec{x}) = x_1$ $f_2(\vec{x}) = g(\vec{x})\left[1 - \sqrt{\frac{x_1}{g(\vec{x})}} - \frac{x_1}{g(\vec{x})}\sin(10\pi x_1)\right]$ $g(\vec{x}) = 1 + 9\left(\sum_{i=2}^{n} x_i\right)/(n-1)$	30	$0 \leq x_i \leq 1$	*convex* *disconnected*
ZDT4	$f_1(\vec{x}) = x_1$ $f_2(\vec{x}) = g(\vec{x})[1 - (x_1/g(\vec{x}))^2]$ $g(\vec{x}) = 1 + 10(n-1) +$ $\sum_{i=2}^{n}[x_i^2 - 10\cos(4\pi x_i)]$)	10	$0 \leq x_1 \leq 1$ $-5 \leq x_i \leq 5$ $i = 2, ..., n$	*nonconvex*
ZDT6	$f_1(\vec{x}) = 1 - e^{-4x_1}\sin^6(6\pi x_1)$ $f_2(\vec{x}) = g(\vec{x})[1 - (f_1(\vec{x})/g(\vec{x}))^2]$ $g(\vec{x}) = 1 + 9[(\sum_{i=2}^{n} x_i)/(n-1)]^{0.25}$	10	$0 \leq x_i \leq 1$	*nonconvex* *nonunformly* *spaced*

Table 9
Constrained test bi-objective problems.

Problem	Objective functions	Constraints	n	Variable bounds
Osyczka2	$f_1(\vec{x}) = -(25(x_1-2)^2+$ $(x_2-2)^2+$ $(x_3-1)^2(x_4-4)^2+$ $(x_5-1)^2)$ $f_2(\vec{x}) = x_1^2+x_2^2+$ $x_3^2+x_4^2+x_5^2+x_6^2$	$g_1(\vec{x}) = 0 \leq x_1+x_2-2$ $g_2(\vec{x}) = 0 \leq 6-x_1-x_2$ $g_3(\vec{x}) = 0 \leq 2-x_2+x_1$ $g_4(\vec{x}) = 0 \leq 2-x_1+3x_2$ $g_5(\vec{x}) = 0 \leq 4-(x_3-3)^2-x_4$ $g_6(\vec{x}) = 0 \leq (x_5-3)^3+x_6-4$	6	$0 \leq x_1, x_2 \leq 10$ $1 \leq x_3, x_5 \leq 5$ $0 \leq x_4 \leq 6$ $0 \leq x_6 \leq 10$
Tanaka	$f_1(\vec{x}) = x_1$ $f_2(\vec{x}) = x_2$	$g_1(\vec{x}) = -x_1^2-x_2^2+1+$ $0.1\cos(16\arctan(x_1/x_2)) \leq 0$ $g_2(\vec{x}) = (x_1-0.5)^2+$ $(x_2-0.5)^2 \leq 0.5$	2	$-\pi \leq x_i \leq \pi$
Constr_Ex	$f_1(x) = x_1$ $f_2(x) = (1+x_2)/x_1$	$g_1(\vec{x}) = x_2+9x_1 \geq 6$ $g_2(\vec{x}) = -x_2+9x_1 \geq 1$	2	$0.1 \leq x_1 \leq 1.0$ $0 \leq x_2 \leq 5$
Srinivas	$f_1(\vec{x}) = (x_1-2)^2+$ $(x_2-1)^2+2$ $f_2(\vec{x}) = 9x_1-(x_2-1)^2$	$g_1(\vec{x}) = x_1^2+x_2^2 \leq 225$ $g_2(\vec{x}) = x_1-3x_2 \leq -10$	2	$-20 \leq x_i \leq 20$
Golinski	$f_1(\vec{x}) = 0.7854x_1x_2^2(10x_3^2/3+$ $14.933x_3-43.0934)$ $-1.508x_1(x_6^2+x_7^2)+$ $7.477(x_6^3+x_7^3)$ $+0.7854(x_4x_6^2+x_5x_7^2)$ $f_2(\vec{x}) = \frac{\sqrt{(\frac{745.0x_4}{x_2x_3})^2+1.69*10^7}}{0.1x_6^3}$	$g_1(\vec{x}) = \frac{1.0}{x_1x_2^2x_3}-\frac{1.0}{27.0} \leq 0$ $g_2(\vec{x}) = \frac{1.0}{x_1x_2^2x_3}-\frac{1.0}{27.0} \leq 0$ $g_3(\vec{x}) = \frac{x_4^3}{x_2x_3^2x_6^4}-\frac{1.0}{1.93} \leq 0$ $g_4(\vec{x}) = \frac{x_5^3}{x_2x_3x_7^4}-\frac{1.0}{1.93} \leq 0$ $g_5(\vec{x}) = x_2x_3-40 \leq 0$ $g_6(\vec{x}) = x_1/x_2-12 \leq 0$ $g_7(\vec{x}) = 5-x_1/x_2 \leq 0$ $g_8(\vec{x}) = 1.9-x_4+1.5x_6 \leq 0$ $g_9(\vec{x}) = 1.9-x_5+1.1x_7 \leq 0$ $g_{10}(\vec{x}) = f_2(\vec{x}) \leq 1300$ $a = 745.0x_5/x_2x_3$ $b = 1.575*10^8$ $g_{11}(\vec{x}) = \frac{\sqrt{a^2+b}}{0.1x_7^3} \leq 1100$	7	$2.6 \leq x_1 \leq 3.6$ $0.7 \leq x_2 \leq 0.8$ $17.0 \leq x_3 \leq 28.0$ $7.3 \leq x_4 \leq 8.3$ $7.3 \leq x_5 \leq 8.3$ $2.9 \leq x_6 \leq 3.9$ $5.0 \leq x_7 \leq 5.5$

Table 10
Problems with more than two objectives.

Problem	Objective functions	Constraints	n	Variable bounds
Viennet2	$f_1(\vec{x}) = \frac{(x_1-2)^2}{2} + \frac{(x_1+1)^2}{13} + 3.0$ $f_2(\vec{x}) = \frac{(x_1+x_2-3)^2}{36} + \frac{(-x_1+x_2+2)^2}{8} - 17$ $f_3(\vec{x}) = \frac{(x_1+2x_2-1)^2}{175} + \frac{(2x_2+x_1)^2}{17} - 13$		2	$-4.0 \leq x_i \leq 4.0$
Viennet3	$f_1(\vec{x}) = 0.5x_1^2 + x_2^2 + sin(x_1^2 + x_2^2)$ $f_2(\vec{x}) = \frac{(3x_1-2x_2+4)^2}{8} + \frac{(x_1-x_2+1)^2}{27} + 15$ $f_3(\vec{x}) = \frac{1}{x_1^2+x_2^2+1} - 1.1exp(-x_1^2 - x_2^2)$		2	$-3.0 \leq x_i \leq 3.0$
Viennet4	$f_1(\vec{x}) = \frac{(x_1-2)^2}{2} + \frac{(x_2+1)^2}{13} + 3$ $f_2(\vec{x}) = \frac{(x_1+x_2-3)^2}{175} + \frac{(2x_2-x_1)^2}{17} - 13$ $f_3(\vec{x}) = \frac{(3x_1-2x_2+4)^2}{8} + \frac{(x_1-x_2+1)^2}{27} + 15$	$g_1(\vec{x}) = -x_2 - 4x_1 + 4 \geq 0$ $g_2(\vec{x}) = x_1 + 1 \geq 0$ $g_3(\vec{x}) = x_2 - x_1 + 2 \geq 0$	2	$-4.0 \leq x_i \leq 4.0$
Water	$f_1(\vec{x}) = 106780.37(x_2 + x_3) + 61704.67$ $f_2(\vec{x}) = 3000x_1$ $f_3(\vec{x}) = \frac{305700*2289x_2}{(0.06*2289)^{0.65}}$ $f_4(\vec{x}) = 250 * 2289x_2 exp(-39.75x_2 + 9.9x_3 + 2.74)$ $f_5(\vec{x}) = 25\frac{1.39}{(x_1x_2)+4940*x_3-80}$	$g_1(\vec{x}) = 1 - \frac{0.00139}{(x_1x_2)} + 4.94x_3 - 0.08 \geq 0$ $g_2(\vec{x}) = 1 - \frac{0.000306}{(x_1x_2)} + 1.082x_3 - 0.0986 \geq 0$ $g_3(\vec{x}) = 5000 - \frac{12.307}{(x_1x_2)} + 4.9408x_3 + 4051.02 \geq 0$ $g_4(\vec{x}) = 16000 - \frac{2.09}{(x_1x_2)} + 8046.33x_3 - 696.71 \geq 0$ $g_5(\vec{x}) = 10000 - \frac{2.138}{(x_1x_2)} + 7883.39x_3 - 705.04 \geq 0$ $g_6(\vec{x}) = 2000 - \frac{0.417}{(x_1x_2)} + 1721.26x_3 - 136.54 \geq 0$ $g_7(\vec{x}) = 550 - \frac{0.164}{(x_1x_2)} + 631.13x_3 - 54.48 \geq 0$	3	$0.01 \leq x_1 \leq 0.45$ $0.01 \leq x_2 \leq 0.10$ $0.01 \leq x_3 \leq 0.10$