Q3 Lucas - Kanade Algorithm

Given that the equations for optical flow are (u, v) :-

$$u(x, y) = a_1 x + b_1 y + c_1$$
$$v(x, y) = a_2 x + b_2 y + c_2$$

For each pixel, we assume that optical flow (u, v) and motion field is constant within a small neighborhood 'n'.

For every point $(k, l) \in \cancel{\;} n$, we get

$$I_x(k, l)\, u + I_y(k, l)\, v + I_t(k, l) = 0$$

Let size of the neighbor n be $n \times n$

$$\underbrace{\begin{bmatrix} I_x(1,1), I_y(1,1) \\ I_x(k,l), I_y(k,l) \\ \vdots \\ I_x(n,n) \;\; I_y(n,n) \end{bmatrix}}_{X} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{P} = \underbrace{\begin{bmatrix} I_t(1,1) \\ I_t(k,l) \\ \vdots \\ I_t(n,n) \end{bmatrix}}_{\cancel{B}\, Y}$$

From above, we solve for X and Y, we get

$$X^T X B = X^T Y$$

This can be represented in matrix format as:

$$\begin{bmatrix} \sum_W I_x I_x & \sum_W I_x I_y \\ \sum_W I_x I_y & \sum_W I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -\sum_W I_x I_t \\ -\sum_W I_y I_t \end{bmatrix}$$

Known   unknown   Known

Let pixel (x, y) is displaced by $(x+v, y+u)$

$$E(u,v) = \sum \left[I(x+u, y+v) - T(x,y) \right]^2$$

$$\simeq \sum \left[I(x,y) + u I_x(x,y) + v I_y(x,y) - T(x,y) \right]^2$$

$$= \sum \left[u I_x(x,y) + v I_y(x,y) + D(x,y) \right]^2$$

Finding partial derivative & equating to 0, we get

$$\frac{\partial E}{\partial u} = \sum \left[u I_x(x,y) + v I_y(x,y) + D(x,y) \right] \times I_x(x,y) = 0$$

$$\frac{\partial E}{\partial v} = \sum \left[u I_x(x,y) + v I_y(x,y) + D(x,y) \right] I_y(x,y) = 0$$

This can be written as:

$$\sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\sum \begin{bmatrix} I_x D \\ I_y D \end{bmatrix}$$

This can be summarized as

→ Computing I_x, I_y, I_t for Images

→ If determinant of X is zero or not

→ If $\det |A| = 0$ then that pixel uses least squares.