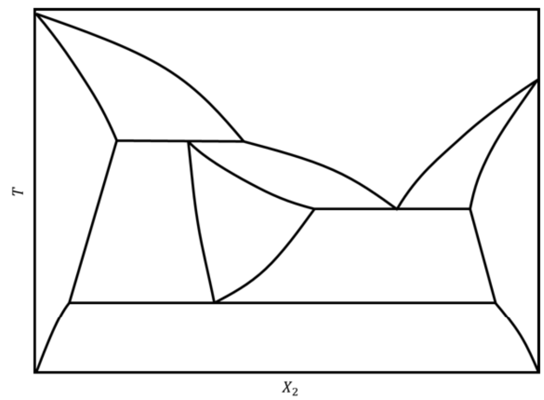
MSEN 640 Fall 2025 – Homework # 8

Instructions: Please answer the problems below either using a computer, tablet, or paper. Save your solutions as a PDF document; remember: if using paper, please **scan** your answers! Remember to **show all of your work** for a full grade.

1. Consider the blank phase diagram below



(a) Given that the phases present are α, β, γ, and L, label all of the single phase and two phase regions. Note that α is the crystal structure of pure component 1 and β is the crystal structure of pure component 2.

(b) Sketch a plausible phase diagram for the same system with T vs. a_2 (activity of component 2) axes.

2. Consider a 2D strip of silicon (conductivity $\sigma = 4.35 \times 10^{-4} (\Omega\mathrm{m})^{-1}$, Seebeck coefficient $S = 440\,\mu\mathrm{V/K}$). Using the expression for electrical current density $\mathbf{J_q} = -\sigma\nabla\phi - \sigma S \nabla T$:

(a) Determine the temperature gradient needed to maintain a potential difference of 8 V across the material, assuming that no net current is transferred.

(b) If the material is a 1 cm $\times$ 1 cm panel (with the bottom-left corner at the origin) and 8 V is maintained across the x-direction with potential function $\phi(x) = (8\,\mathrm{V/cm})x$ and a temperature difference of 10 K is maintained along the y-direction with temperature function $T(y) = 300\,\mathrm{K} + (10\,\mathrm{K/cm})y$, determine the magnitude of the current density.

3. Consider a very simple model of a pickering emulsion. A spherical drop of oil with radius $r_{\mathrm{oil}} = 50$ μm suspended in water; the interfacial free energy density of the oil-water interface is $\gamma_{\mathrm{oil-water}} = 0.05\,\mathrm{J/m^2}$. Now, microspherical particles of radius $r_\mu = 1$ μm that are functionalized to have an interfacial energy density $\gamma_{\mu-\mathrm{oil}} = 0.01\,\mathrm{J/m^2}$ for regions exposed to oil and $\gamma_{\mu-\mathrm{water}} = 0.02\,\mathrm{J/m^2}$ for regions exposed to water.

(a) If a microsphere is brought from the pure water region into the pure oil region (so that it is completely immersed in water and oil), compute the change in enthalpy of the system, assuming that the enthalpy is dominated by the interfacial cost ($H \approx \gamma A$).

(b) If a microsphere is brought from the pure water region to the oil-water interface, calculate the change in entropy, assuming that half of the particle's surface is in the oil and the other half is in the water.

(c) What is the change in the enthalpy of the oil-in-water droplet if 50% of the droplet's surface is covered with microspheres?

(d) Using the result of (c) determine the effective interfacial free energy density $\gamma_{\text{eff}} = H/A$ of the oil droplet at 50% coverage.

4. Consider a pool of molten magnesium in an open furnace (so that there is a vapor phase above the liquid phase). The melt is held at a temperature of $T = 800°\text{C}$ and has the following properties:

$$\begin{aligned}
&\text{molar volume (liquid)} : V^L = 15.283 \times 10^{-6}\,\text{m}^3/\text{mol} \\
&\Delta H^{0,L\to V}_{\text{Mg}} = 127.4\,\text{kJ/mol} \\
&T^b_{\text{Mg}} = 1087°\text{C} \;\; \text{at atmospheric pressure} \\
&\gamma^L_{\text{Mg}} = 721 \times 10^{-5}\,\text{J/cm}^2
\end{aligned}$$

(a) Calculate the vapor pressure over the pool of molten magnesium in the furnace. [Hint: consider that the liquid and vapor phases are in equilibrium]

(b) Calculate the vapor pressure of magnesium within a 10 nm bubble that is produced by cavitation.