Signals #1: sine vs noise, sampling theorem and signal acquisition

Sines and their sampling

Pure deterministic signals have shapes defined by means of known ***basic functions***: sinusoid, exponent, gaussoid, ... or by additions and multiplications (sample-by-sample) of known functions, e.g. sinusoid by exponent. ***Mixed/complex deterministic*** signals are sums of *pure* signals. **Sinusoid** is the best known (the most popular) deterministic signal.

For example, sinusoidal signal, with **amplitude** (*strength*) $A_1 = 0.5$, **frequency** (number of repetitions per one second) $f_1 = 10$ Hz, and **phase shift (angle)** $\phi_1 = \pi/4$ radians, is defined by the following equation:

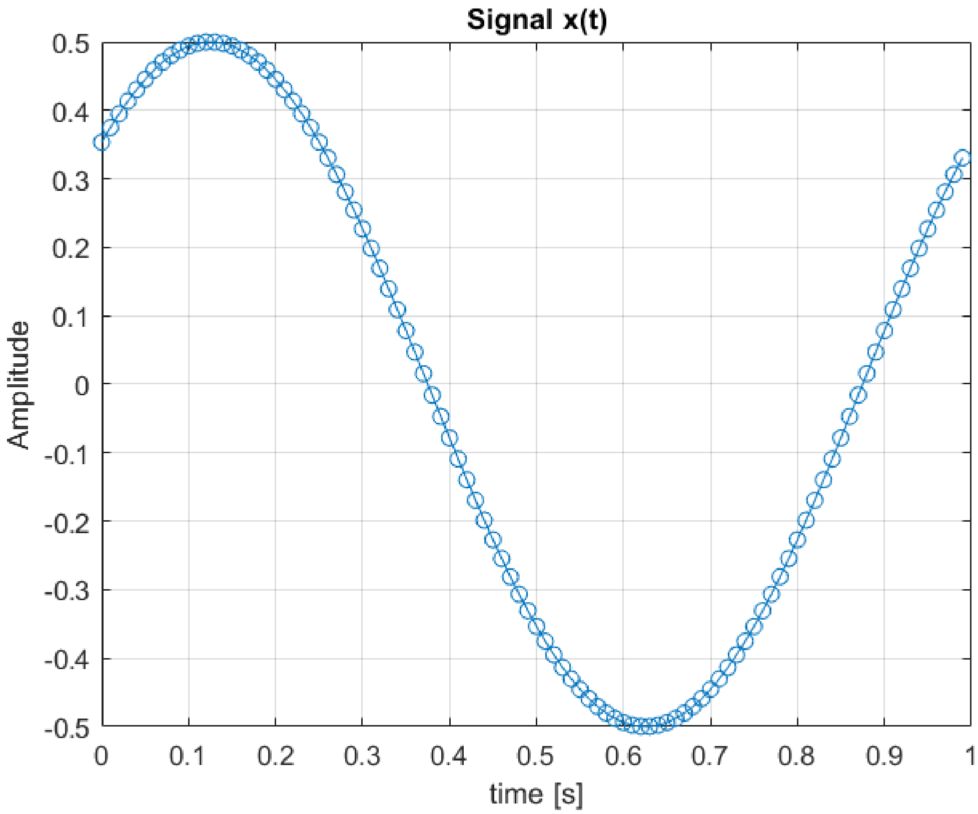
$$x_1(t) = 0.5 \cdot \sin\left(2\pi \cdot 10 \cdot t + \frac{\pi}{4}\right), \quad t \in [0, +\infty),$$

After using the following denotations: $dt = 1/f_s$ (sampling period, time interval between two signal samples, inverse of the sampling frequency f_s), $t = n \cdot dt = n/f_s$ (sampling moments), $n = 0, 1, 2, ...$ (indexes of samples), we have:

$$x_1(n) = 0.5 \cdot \sin\left(2\pi \frac{10}{f_s} n + \frac{\pi}{4}\right), \quad n = 0, 1, 2, ...$$

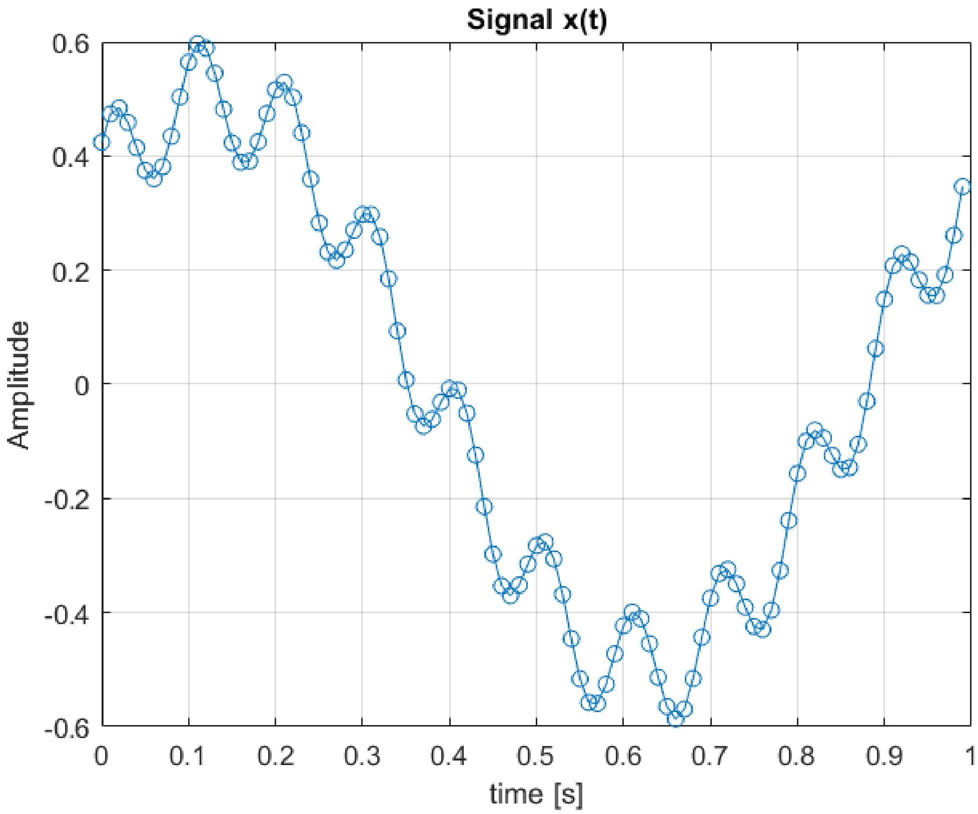
Run the code given below. Change values of **`A1,f1,p1`**: observe change of the signal shape.

```
fs=100; Nx=100;                   % sampling frequency, number of samples
dt = 1/fs;                        % sampling period
t = dt*(0:Nx-1);                  % sampling times (many moments)
A1=0.5; f1=1; p1=pi/4;            % sine #1 amplitude, frequency and phase
x1 = A1 * sin(2*pi*f1*t+p1);      % sine as a first signal component
x = x1;                           % our choice: x = x1, x1 + 0.123*x2 + 0.456*x3
figure; plot(t,x,'o-'); grid; title('Signal x(t)'); xlabel('time [s]'); ylabel('Amplitude');
```



Now generate the second signal $x_2(n)$ with different values of **`A2,f2,p2`** `and add it to` $x_1(n)$.

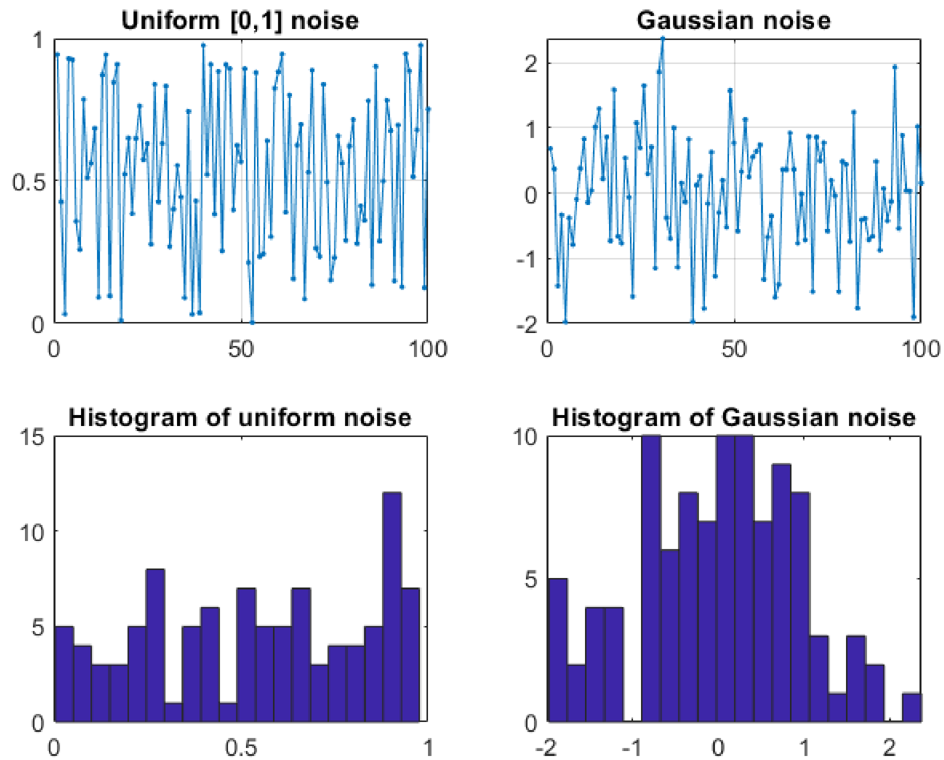
```
A2=0.1; f2=10; p2=pi/7;          % sine #2 amplitude, frequency and phase
x2 = A2 * sin(2*pi*f2*t+p1);     % sine as a second signal component
x = x1 + x2;                     % our choice: x = x1, x1 + 0.123*x2 + 0.456*x3
figure; plot(t,x,'o-'); grid; title('Signal x(t)'); xlabel('time [s]'); ylabel('Amplitude');
```



Noises

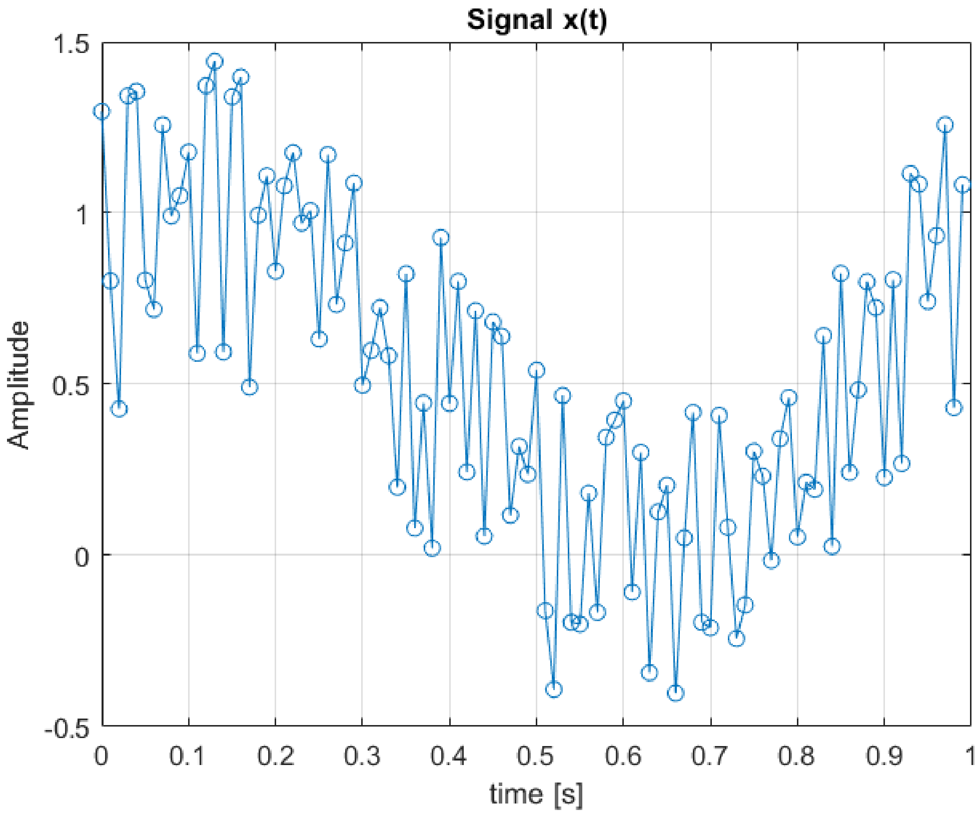
Random signals have a form of noise. They are described by probability density functions (PDFs) telling what noise values are prefered. Gaussian shape and uniform shape are typical PDFs. Run the below section. Change value of Nx. Is the Gaussian PDF becoming more "Gaussian"?

```
s1 = rand(1,Nx);    % s1 = 0.1*(2*(s1-0.5));  % uniform;  scaling to [-0.1,0.1]
s2 = randn(1,Nx);   % s2 = 0.1*s2;            % Gaussian; scaling to std=0.1
figure
subplot(221); plot(s1,'.-'); grid; title('Uniform [0,1] noise');
subplot(222); plot(s2,'.-'); grid; title('Gaussian noise');
subplot(223); hist(s1,20); title('Histogram of uniform noise');
subplot(224); hist(s2,20); title('Histogram of Gaussian noise');
```



Now add one of one noise, s1 or s2, to one sine, x1 or x2. Check all combinations. Scale noise componets, i.e. make it stronger or weaker.

```
x = x1 + s1;                           % our choice: x = x1+s1, x1+s2, x2+s1, x2+s2
figure; plot(t,x,'o-'); grid; title('Signal x(t)'); xlabel('time [s]'); ylabel('Amplitude');
```



Sampling theorem

Every **real-value signal** has to be sampled in such way that a sinusoid with the highest frequency, present in the signal, should posses more than two samples per its period, i.e.

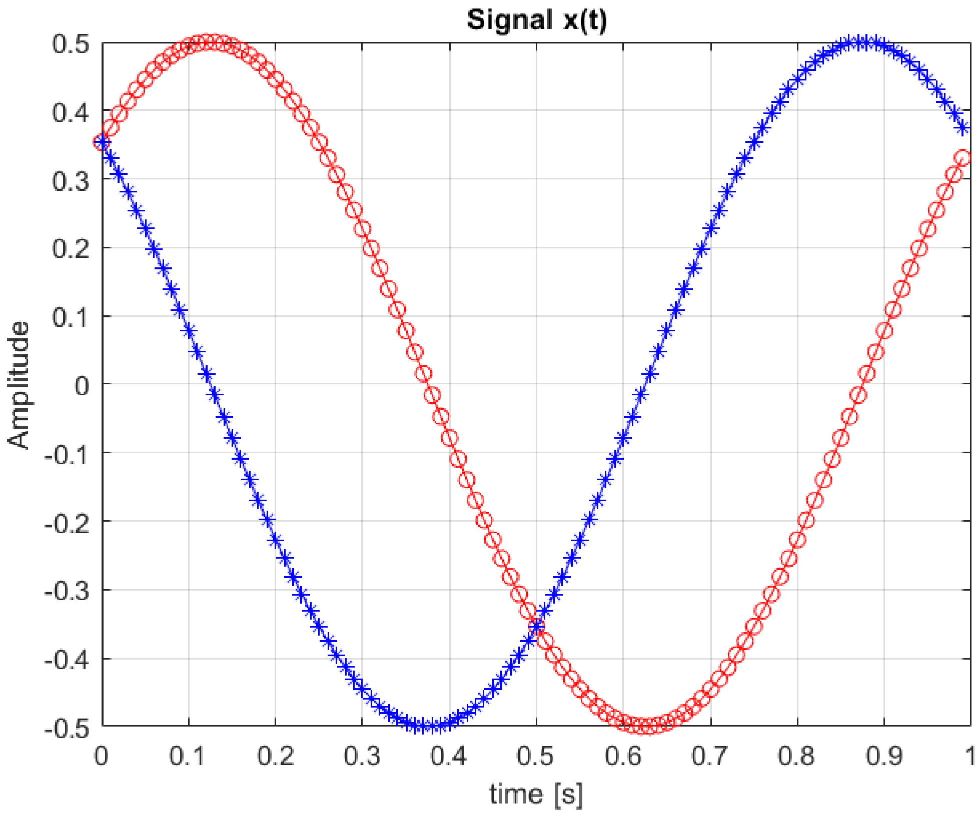
$$f_s > 2 \cdot f_{max}$$

If the **sampling theorem** is not fullfilled the high-frequency sine looks like a low-frequency one, for example if $f_1 = k \cdot f_s \pm f_x$ we have:

$$x_1(n) = A_1 \sin\left(2\pi \frac{k \cdot f_s \pm f_x}{f_s} n + \phi_1\right) = A_1 \sin\left(2\pi kn \pm 2\pi \frac{f_x}{f_s} n + \phi_1\right) = \pm A_1 \sin\left(2\pi \frac{f_x}{f_s} n + \phi_1\right)$$

Run the below code for: 1) different multiplies of f_s, 2) $\pm f_1$, 3) sine and cosine.

```
x1hf = A1 * sin(2*pi*(5*fs-f1)*t+p1);  % too high-frequency sine
figure; plot(t,x1,'ro-',t,x1hf,'b*-'); grid; title('Signal x(t)'); xlabel('time [s]'); ylabel(
```



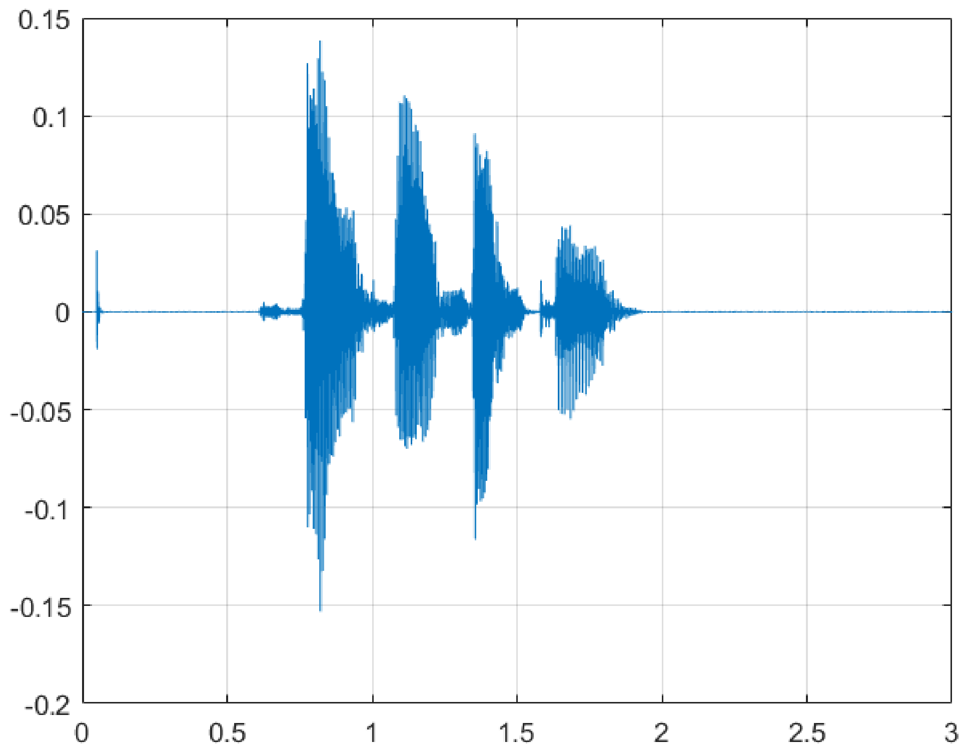
Real-world signal acquisition

Let's record our own speech: 1) run the code, 2) press any key, 3) after 1 second, 4) say the word: "San Fransisco".

```
fs=16000; Nx=3*fs;                          % sampling rate, number of recorded samples
Nbits=16; Nchans=1;                         % number of bits and audio channels
recorder = audiorecorder(fs,Nbits,Nchans);  % create the recorder
display('Press any .. say San Francisco'); pause;
```

```
Press any .. say San Francisco
```

```
recordblocking( recorder, Nx/fs );   % record Nx/fs seconds of data
x = getaudiodata( recorder );               % get the samples
plot( 1/fs*(0:length(x)-1), x ); grid;      % plot speech x(t)
```




```
soundsc(x,fs);                                    % listen to your own speech
```

Observe the signal. Does it consist of sum-of-sines like and noisy like fragments?

Inspection of different real-world signals

Download, observe and play a few sonds taken from the webpage: https://findsounds.com/