Signals #2: different signals, signal mixtures and signal features

Signal diversity - adds and mults of sines, damped sines, amplitude modulation

Deterministic signals are generated from **concrete mathematical recipes/prescriptions** which specifies their shapes, i.e. using mathematical functions, e.g. **sin(), cos(), exp()**,...:

$$x_1(t) = A\sin(2\pi f_1 t + \varphi), \qquad x_2(t) = e^{-\alpha t}, \qquad x_3(t) = e^{-\alpha(t-t_0)^2},$$

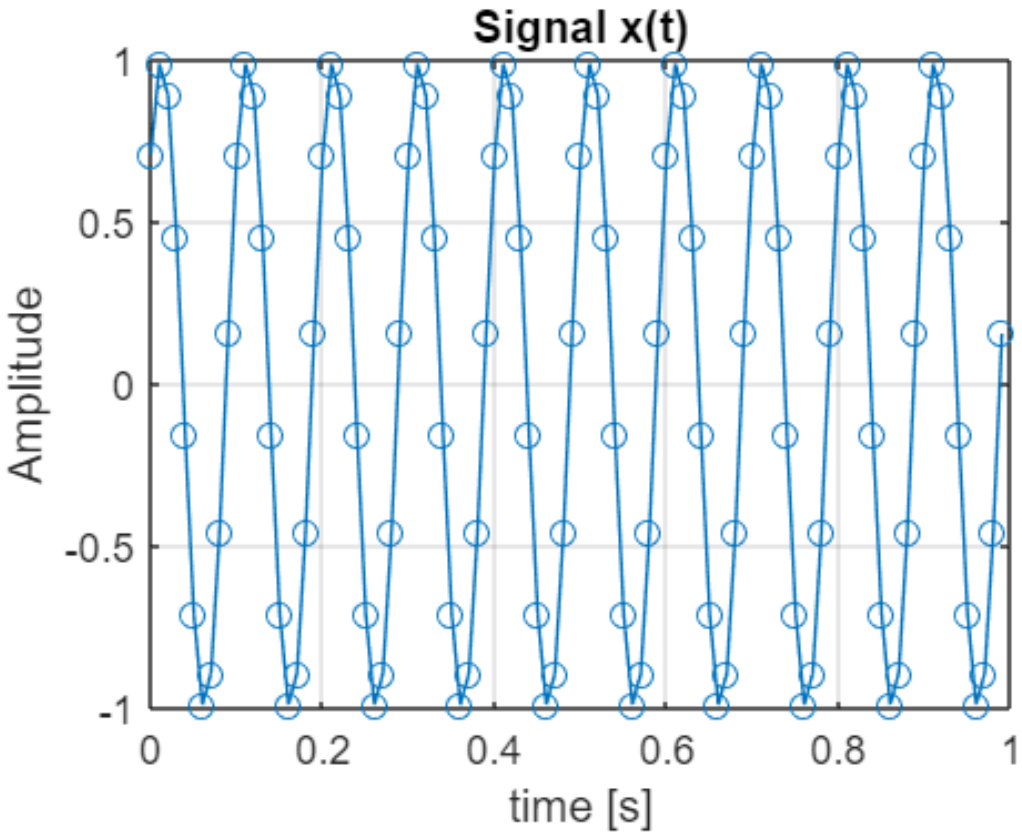
Common/simple signals can add to themselves (as additive signal components) and multiply by themselves (as multiplicative components):

$$x_4(t) = x_1(t) + x_2(t), \qquad x_5(t) = x_1(t)x_2(t)$$

. Very good example of a multiplicative signal is a damped sinusoid, i.e. a sinusoid multiplied by an exponent --- which is a solution of second order differential equations, describing damped mechanical and electrical oscillators. Let's note that sinusoid alone is a solution of differential equation describing undamped oscillators.

Run the below section. Let the final signal x be a different mixture of simpler components (adds and mults of them). When one signal is multiplied by another signal, it is plays role of its changing mplitude, i.e. modulet it in amplitude (**AM - amplitude modulation**). Damped sinusoid is the simplest example.

```
fs=100; Nx=100;              % sampling frequency, number of samples
dt = 1/fs;                   % sampling period
t = dt*(0:Nx-1);             % sampling times (many moments)
x1=sin(2*pi*10*t+pi/4);      % sine 10 Hz as a first signal component
x2=sin(2*pi*1*t);            % sine 1 Hz
x3=exp(-5*t);                % exponent decaying in time
x4=exp(-25*(t-0.5).^2);      % Gaussian hat
x = x1;                      % our choice: x1,...,x4, 0.23*x1 + x2, x1.*x3, x1.*x4,...
plot(t,x,'o-'); grid; title('Signal x(t)'); xlabel('time [s]'); ylabel('Amplitude');
```



Instantaneous frequency and frequency modulation

Instantaneous frequency of sinusoidal signal:

$$x(t) = \sin(\alpha(t))$$

is defined as derivative of the signal angle $\alpha(t)$, changing in time, divided additionally by 2π. Therefore for the sine one obtains:

$$f_{inst}(t) = \frac{1}{2\pi}\frac{\mathrm{d}\alpha(t)}{\mathrm{d}t}$$

For this reason, in order to obtain a sinusoidal signal having required instantaneous frequency $f_{inst}(t)$ it is necesary to set:

$$\alpha(t) = 2\pi \int_0^t f_{inst}(t)dt.$$

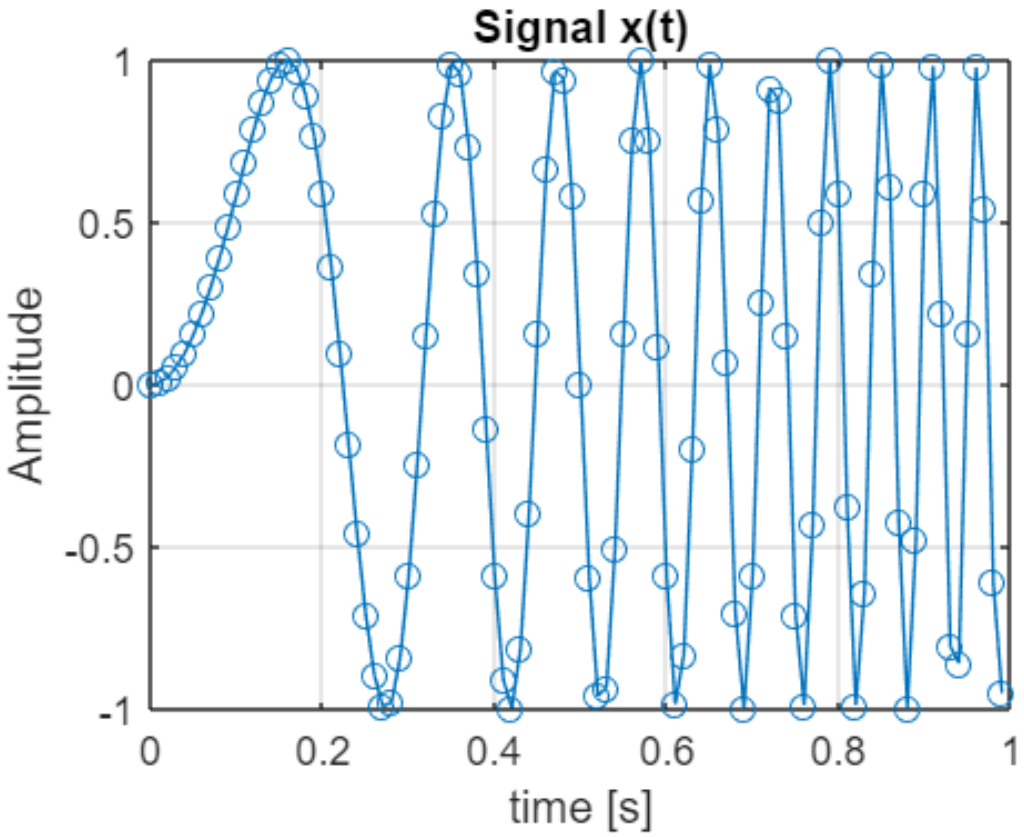
In Matlab:

```
alpha = 2*pi*cumsum( finst )*dt.
```

Now we will test this. Change modulation parameters. Add amplitude modulation.

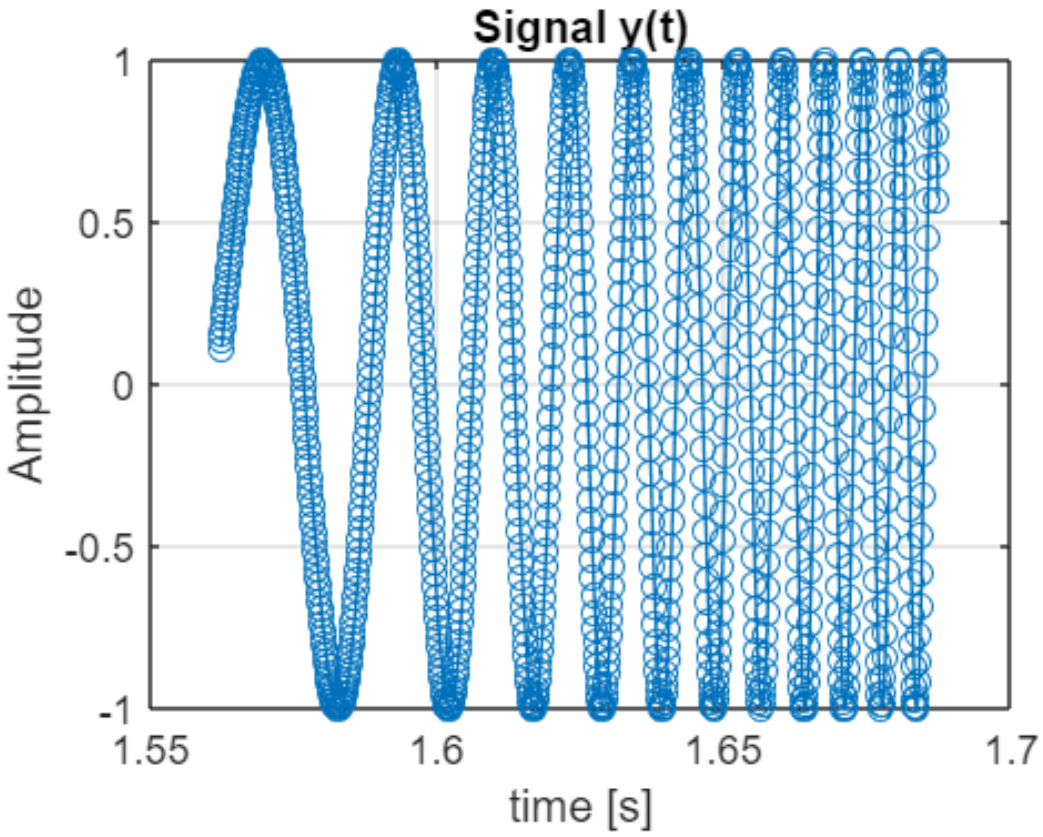
```
x5=sin(2*pi*(0*t+0.5*20*t.^2));                % linear freq. increase: start 0 Hz, +20Hz/s
x6=sin(2*pi*(10*t-(9/(2*pi*1)*cos(2*pi*1*t)))); % sinus. FM: 9Hz around 10Hz, 1 per sec
x7=sin(2*pi*(10*t+9*cumsum(x2)*dt));           % the same as x6; why?
x = x5;                                        % our choice: x5,x6,7
```

```
plot(t,x,'o-'); grid; title('Signal x(t)'); xlabel('time [s]'); ylabel('Amplitude');
```

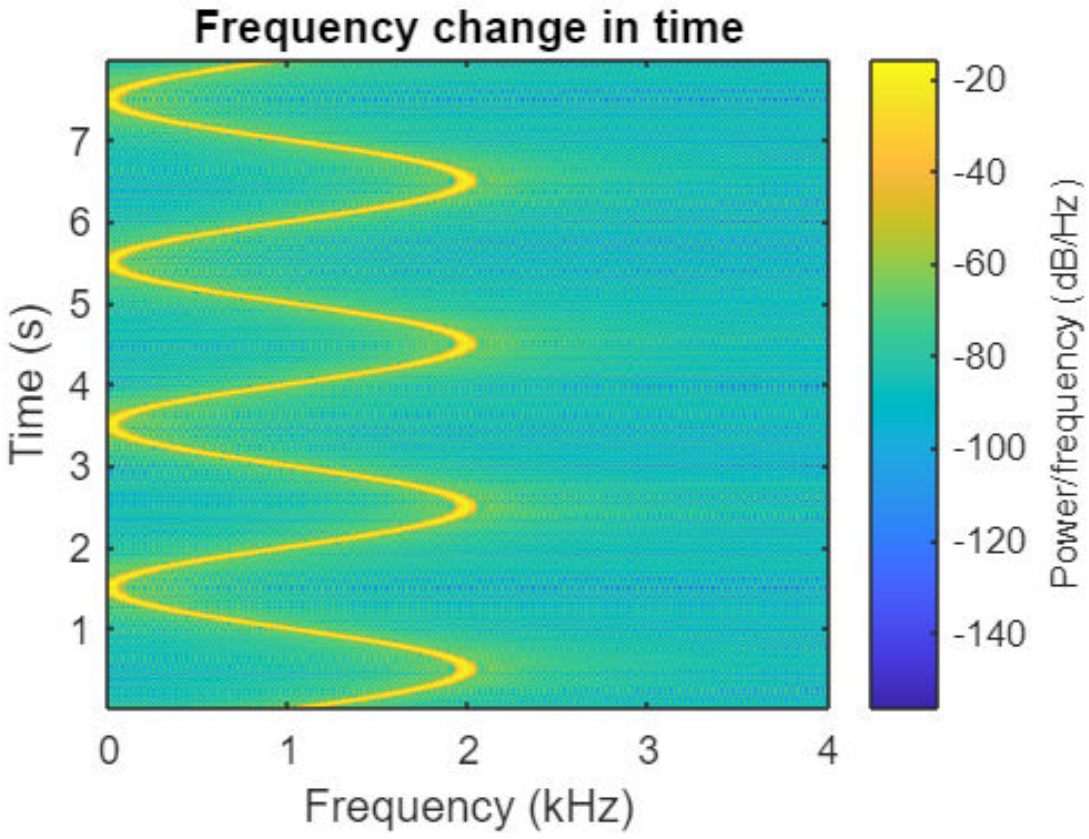


Finally, we will generate an FM modulated signal for a fire truck. Enlarge the signal figure and observe the signal shape. Check the correctness of frequency change in spectrogram plot. Modify values of signal parameters.

```
fs=8000; Nx=8*fs;                       % sampling frequency, number of samples
dt = 1/fs; t = dt*(0:Nx-1);             % sampling period, sampling times (many moments)
y1=sin(2*pi*(0*t+0.5*1000*t.^2));       % linear frequency increase: start 0 Hz, +1000Hz/s
y2=sin(2*pi*(1000*t-(990/(2*pi*0.5)*cos(2*pi*0.5*t))));  % sinusoidal FM
y3=sin(2*pi*(1000*t+990*cumsum(cos(2*pi*0.5*t))*dt));    % the same as x6; why?
y = y2;                                                  % our choice: y1,y2,y3
sound(0.1*y,fs);                                         % listening
figure; plot(t,y,'o-'); grid; title('Signal y(t)'); xlabel('time [s]'); ylabel('Amplitude');
```




```
figure; spectrogram(y,256,256-32,512,fs); title('Frequency change in time');
```



Signal features

Any signal has the following feauters/descriptors: **min / max / peak-to-peak / mean value, variance, standard deviation, energy, power, rms value, auto-correlation function**. Find their definition and calculate them for our signals. Find Matlab functions doing this (e.g. `min, max, var, std, rms, xcorr`).