

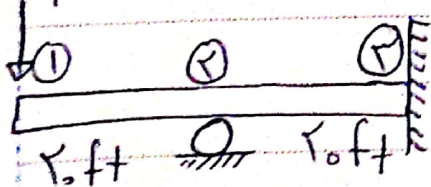
$$[K] = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \quad \text{الان. ماتريس سختی}$$

element 1: $L = 2 \text{ ft}$ $E = 3.0 \times 10^4 \text{ psi}$ $I = 200 \text{ in}^4$

element 2: $L = 2 \text{ ft}$ $E = 3.0 \times 10^4 \text{ psi}$ $I = 200 \text{ in}^4$

$$[K] = \begin{bmatrix} u_1 & \theta_1 & u_2 & \theta_2 & u_3 & \theta_3 \\ 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 12 & -6L & -12 & 6L \\ 6L & 2L^2 & 0 & 4L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \quad \begin{matrix} 6 \times 6 \\ 6 \times 6 \\ 6 \times 6 \\ 6 \times 6 \\ 6 \times 6 \\ 6 \times 6 \end{matrix}$$

kip



شرایط مرزی: $M_1 = 0$, $M_3 = 0$
 $u_1 = u_3 = 0$, $\theta_3 = 0$

$$\begin{Bmatrix} -10 \\ 0 \\ 0 \end{Bmatrix} = \frac{3.0 \times 10^4 \times 200}{(2 \times 12)^3} \begin{bmatrix} 12 & 6L & 6L \\ 6L & 4L^2 & 2L^2 \\ 6L & 2L^2 & 4L^2 \end{bmatrix} \begin{Bmatrix} u_1 \\ \theta_1 \\ \theta_2 \end{Bmatrix}$$

Matlab: $\theta_1 = 0.0054 \text{ rad}$ $\theta_2 = 0.0017 \text{ rad}$ $u_1 = -0.4 \text{ in}$

$F_1 = -500 \text{ lbf}$ $F_2 = 1500 \text{ lbf}$ $F_3 = -750 \text{ lbf}$

$M_2 = 0 \text{ ft.kip}$

element 1:
$$\begin{Bmatrix} f_1 \\ m_1 \\ f_r \\ m_r \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} u_1 \\ \theta_1 \\ u_r \\ \theta_r \end{Bmatrix}$$

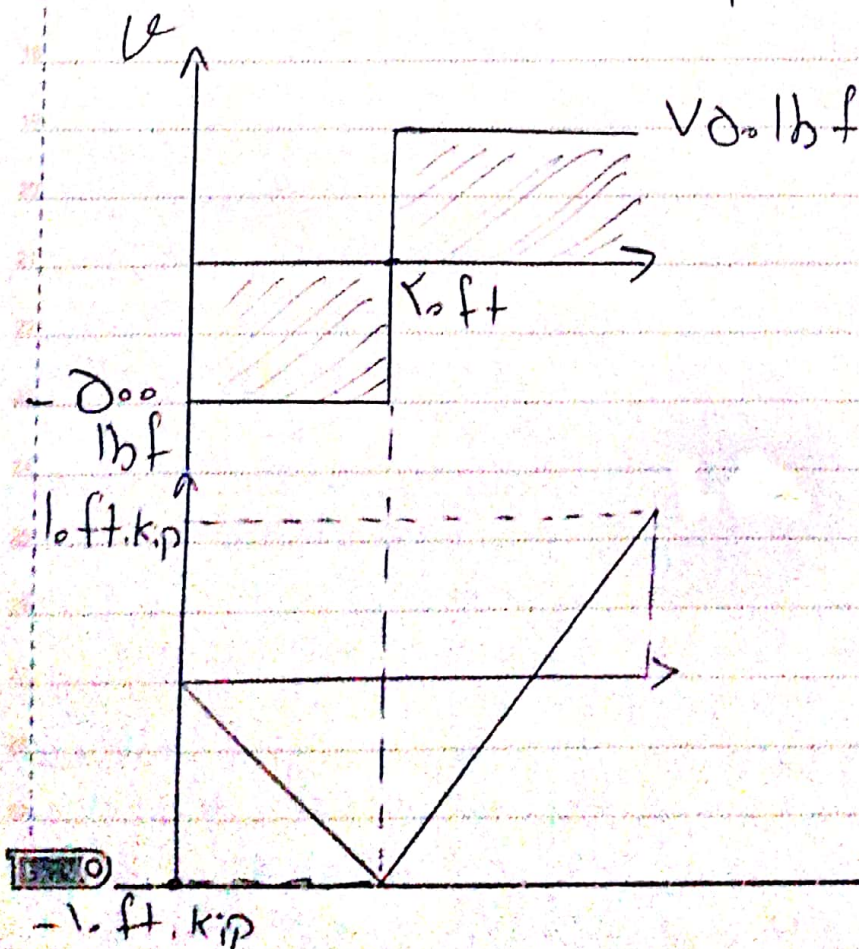
matlab: $f_1 = -200 \text{ lbf}$ $m_1 = 0$ $f_r = 200 \text{ lbf}$

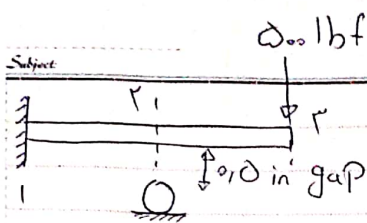
$m_r = -10 \text{ ft.kip}$

element 2:
$$\begin{Bmatrix} f_r \\ \theta_r \\ f_r \\ \theta_r \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} u_r \\ \theta_r \\ u_r \\ \theta_r \end{Bmatrix}$$

matlab: $f_r = 200 \text{ lbf}$ $m_r = 10 \text{ ft.kip}$ $f_r = -200 \text{ lbf}$

$m_r = 20 \text{ ft.kip}$





گره ← 2 درجه آزادی
ماتریس 4×2

$$\begin{bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{bmatrix} = \begin{bmatrix} 12 & 4L & -12 & 4L & 0 & 0 \\ 4L & 4L^2 & -4L & 2L^2 & 0 & 0 \\ -12 & -4L & 12 & 0 & -12 & 4L \\ 4L & 2L^2 & 0 & 4L^2 & -4L & 2L^2 \\ 0 & 0 & -12 & -4L & 12 & -4L \\ 0 & 0 & 4L & 2L^2 & -4L & 4L^2 \end{bmatrix} \begin{Bmatrix} u_1 \\ \theta_1 \\ u_2 \\ \theta_2 \end{Bmatrix}$$

شرایط مرزی $\rightarrow u_1 = 0, \theta_1 = 0, u_2 = -0.010$

$$\rightarrow \begin{bmatrix} F_2 \\ 0 \\ -500 \\ 0 \end{bmatrix} = \frac{500 \times 10 \times 200}{(200 \times 12)^3} \begin{bmatrix} 12 & 0 & -12 & 4L \\ 0 & 4L^2 & -4L & 2L^2 \\ -12 & -4L & 12 & -4L \\ 4L & 2L^2 & -4L & 4L^2 \end{bmatrix} \begin{bmatrix} -0.010 \\ \theta_2 \\ u_2 \\ \theta_2 \end{bmatrix}$$

matlab: $\theta_2 = -0.001918 \text{ rad}$ $\theta_2 = -0.001918 \times 180$
 $\theta_2 = -0.3453^\circ$

$F_1 = -0.001918 \text{ lbf}$ $M_1 = 20.93 \text{ in.kip}$ $F_2 = 1.000 \text{ lbf}$

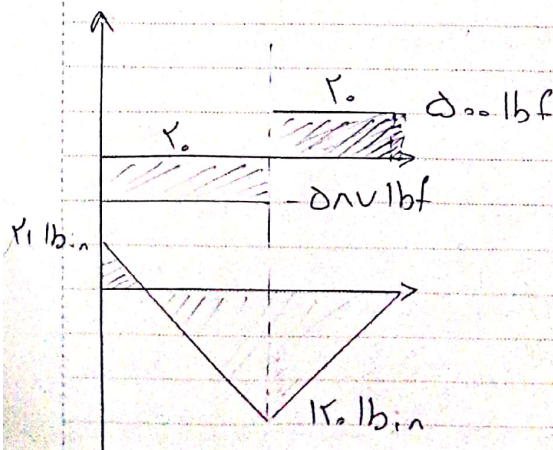
element 1:

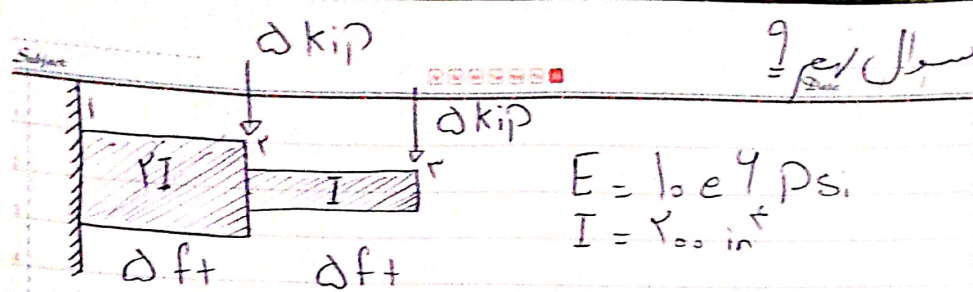
$$\begin{Bmatrix} f_1 \\ m_1 \\ f_2 \\ m_2 \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 12 & 4L & -12 & 4L \\ 4L & 4L^2 & -4L & 2L^2 \\ -12 & -4L & 12 & -4L \\ 4L & 2L^2 & -4L & 4L^2 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ -0.010 \\ -0.001918 \text{ rad} \end{Bmatrix}$$

matlab $f_1 = -0.001918 \text{ lbf}$ $m_1 = 20.93 \text{ in.kip}$ $f_2 = 0.001918 \text{ lbf}$
 $m_2 = -12. \text{ in.kip}$

element 2: $\begin{Bmatrix} f_2 \\ m_2 \\ f_1 \\ m_1 \end{Bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 12 & 4L & -12 & 4L \\ 4L & 4L^2 & -4L & 2L^2 \\ -12 & -4L & 12 & -4L \\ 4L & 2L^2 & -4L & 4L^2 \end{bmatrix} \begin{Bmatrix} -0.010 \\ -0.001918 \\ -0.001918 \\ -0.001918 \end{Bmatrix}$

$f_2 = 0.001918 \text{ lbf}$ $f_1 = -0.001918 \text{ lbf}$ $m_2 = 12.0 \text{ lbf.in}$ $m_1 = 0$





درجه آزادی \rightarrow درجه آزادی $\times 3$ گره

$$\begin{Bmatrix} F_1 \\ M_1 \\ F_r \\ M_r \\ F_r \\ M_r \end{Bmatrix} = \frac{EI}{L^2} \begin{bmatrix} 24 & 12L & -24 & 12L & 0 & 0 \\ 12L & 4L^2 & -12L & 2L^2 & 0 & 0 \\ -24 & -12L & 24 & -12L & -12 & 4L \\ 12L & 2L^2 & -12L & 4L^2 & -4L & 2L^2 \\ 0 & 0 & -12 & -4L & 12 & -4L \\ 0 & 0 & 4L & 2L^2 & -4L & 4L^2 \end{bmatrix} \begin{Bmatrix} u_1 \\ \theta_1 \\ u_r \\ \theta_r \\ u_r \\ \theta_r \end{Bmatrix}$$

Boundary Condition : $u_1 = 0$, $\theta_1 = 0$, $M_r = M_r = 0$

matlab $\rightarrow u_r = -0.110 \text{ in}$

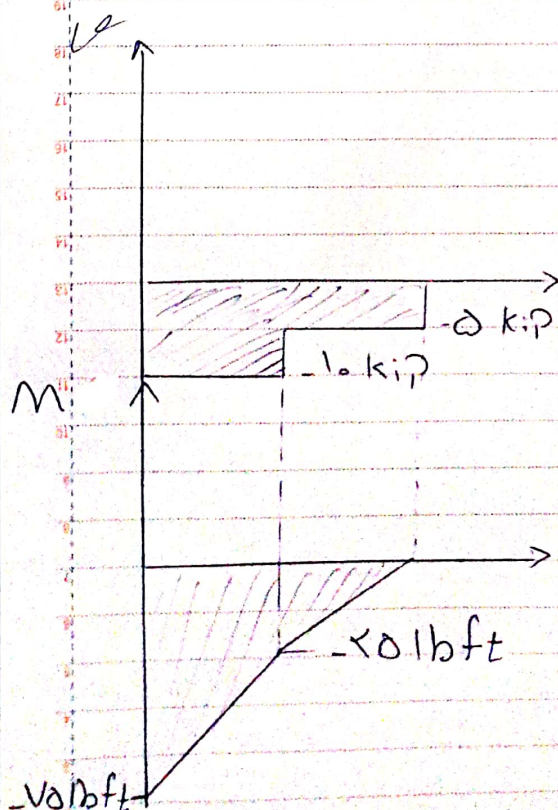
$$\theta_r = -0.009 \text{ rad}$$

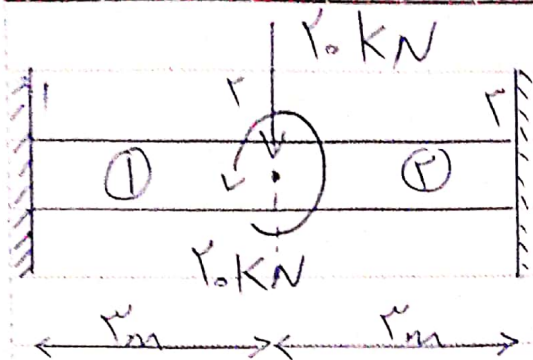
$$u_r = -1.102 \text{ in}$$

$$\theta_r = -0.0120 \text{ rad}$$

$$M_r = M_r = 0$$

$$\rightarrow F_1 = 10 \text{ kip} , M_1 = 0 \text{ kip.ft}$$





$$E = 21.0 \text{ GPa}$$

$$I = 1.5 \times 10^{-4} \text{ m}^4$$

$$K_1 = K_2 = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

ماتريس 4×4 \rightarrow 9×9 ماتريس

$$\begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \\ F_3 \\ M_3 \end{Bmatrix} = \frac{21.0 \times 10^9 \times 1.5 \times 10^{-4}}{3^3} \begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 12 & -6L & 0 & 0 \\ 6L & 2L^2 & -6L & 4L^2 & 0 & 0 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \theta_1 \\ \Delta_2 \\ \theta_2 \\ \Delta_3 \\ \theta_3 \end{Bmatrix}$$

بنايه

$$\Delta_1 = \Delta_3 = 0, \theta_1 = \theta_3 = 0$$

$$\begin{Bmatrix} -P_0 \\ 0 \\ P_0 \\ 0 \end{Bmatrix} = \frac{21.0 \times 10^9 \times 1.5 \times 10^{-4}}{3^3} \begin{bmatrix} 24 & 0 \\ 0 & 12L^2 \end{bmatrix} \begin{Bmatrix} \Delta_2 \\ \theta_2 \end{Bmatrix}$$

$$\omega_r = 0.00054 \text{ rad/s} \quad \theta_r = 1.93 \times 10^{-8} \text{ rad}$$

$$F_1 = 1 \times 10^4 \left[(1 \times 10^4 \times 1.93 \times 10^{-8}) + (1 \times 10^4 \times 1.93 \times 10^{-8}) \right] = 1.9905 \times 10^5 \text{ N}$$

$$M_1 = 1 \times 10^4 \left[1 \times 10^4 \times 1.93 \times 10^{-8} + 1 \times 10^4 \times 1.93 \times 10^{-8} \right] = -1.995 \times 10^5 \text{ N.m}$$

$$F_r = 1 \times 10^4 \left[1 \times 10^4 \times 1.93 \times 10^{-8} - 1 \times 10^4 \times 1.93 \times 10^{-8} \right] = 1.9917 \times 10^5 \text{ N}$$

$$M_r = 1 \times 10^4 \left[-1 \times 10^4 \times 1.93 \times 10^{-8} + 1 \times 10^4 \times 1.93 \times 10^{-8} \right] = -9.95 \times 10^4 \text{ N.m}$$

element 1:

$$\begin{Bmatrix} f_1 \\ m_1 \\ f_r \\ m_r \end{Bmatrix} = 1 \times 10^4 \times [K_1] \times \begin{Bmatrix} 0 \\ 0 \\ -1.93 \times 10^{-8} \\ 1.93 \times 10^{-8} \end{Bmatrix}$$

$$\Rightarrow f_1 = 1.9905 \times 10^5 \text{ N} \Rightarrow f_r = -1.9905 \times 10^5 \text{ N}$$

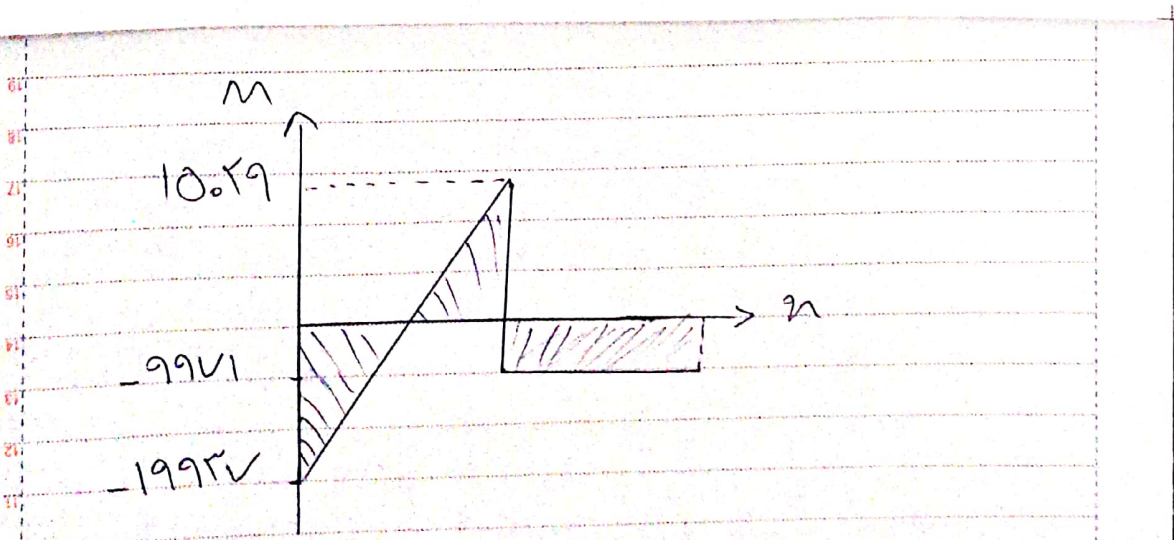
$$\Rightarrow m_1 = 1.995 \times 10^5 \text{ N.m} \quad m_r = -1.995 \times 10^5 \text{ N.m}$$

element 2:

$$\begin{Bmatrix} f_2 \\ m_2 \\ f_r \\ m_r \end{Bmatrix} = 1 \times 10^4 \times [K_2] \times \begin{Bmatrix} 1.93 \times 10^{-8} \\ 1.93 \times 10^{-8} \\ 0 \\ 0 \end{Bmatrix}$$

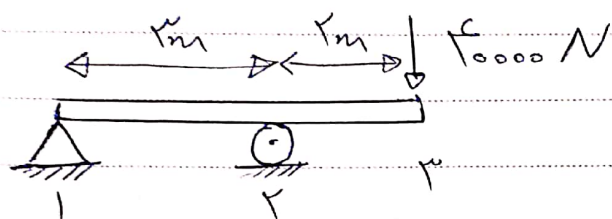
$$\begin{aligned} f_2 &= -1.9917 \times 10^5 \text{ N} \\ f_r &= 1.9917 \times 10^5 \text{ N} \\ m_2 &= -9.95 \times 10^4 \text{ N.m} \\ m_r &= -9.95 \times 10^4 \text{ N.m} \end{aligned}$$

Subject



Subject:

شکل کامل مقارن است. در نتیجه بررسی نصف هندسه کافی است



$$E = 10 \text{ GPa}$$

$$I = 10 \text{ cm}^4$$

$$\rightarrow EI = 10^4$$

$$K_1 = \frac{V}{\text{mm}} \times 10^4 \begin{bmatrix} 12 & 12 & -12 & 12 \\ 12 & 24 & -12 & 12 \\ -12 & -12 & 12 & -12 \\ 12 & 12 & -12 & 24 \end{bmatrix}$$

$$K_2 = \frac{V}{\text{mm}} \times 10^4 \begin{bmatrix} 12 & 12 & -12 & 12 \\ 12 & 12 & -12 & 12 \\ -12 & -12 & 12 & -12 \\ 12 & 12 & -12 & 12 \end{bmatrix}$$

$$\text{شرایط مرزی} \quad \theta_1 = \theta_2 = \phi_2 = 0$$

$$\rightarrow \begin{cases} M_1 = 0 \\ M_2 = 0 \\ F_2 = -10000 \end{cases} = 10^4 \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ 0 & -\frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \phi_2 \end{Bmatrix}$$

$$\rightarrow \text{matlab} \rightarrow \phi_2 = -1.419 \times 10^{-3} \text{ rad} \rightarrow \theta_1 = 1.905 \times 10^{-3} \text{ rad}$$

$$\rightarrow F_1 = 10^4 \left[\frac{1}{3} \times 1.905 \times 10^{-3} + \frac{-1}{3} \times 1.419 \times 10^{-3} \right]$$

$$\rightarrow F_1 = 1119 \text{ N} \rightarrow F_2 = 1119 \text{ N}$$

$$M_2 = 0 \text{ N.m}$$

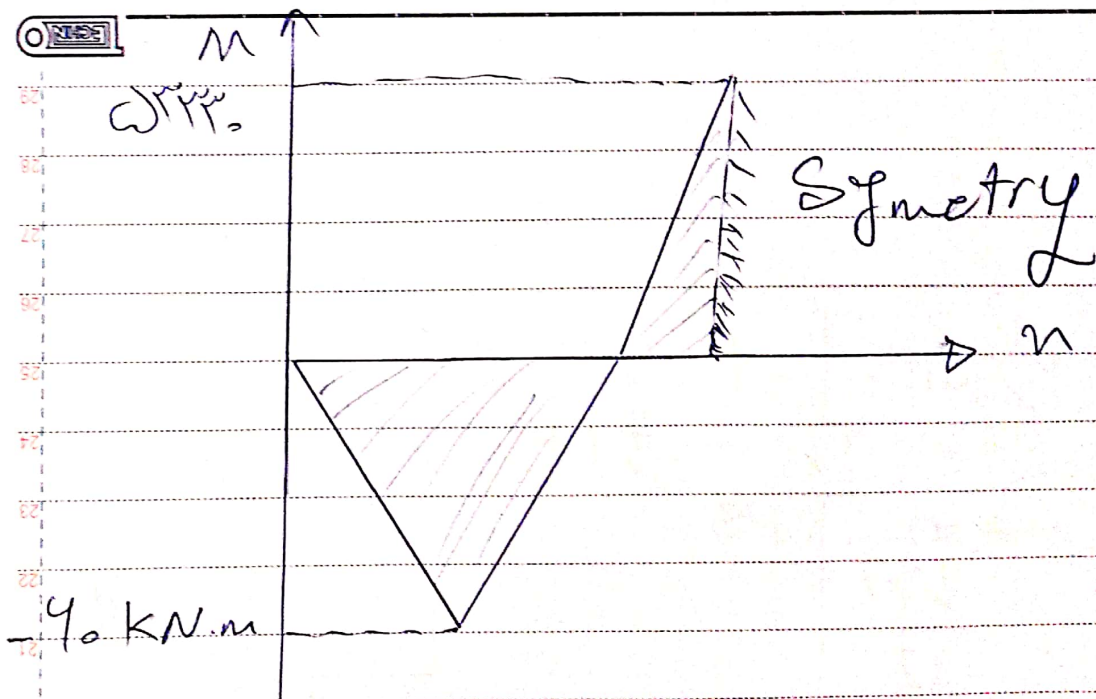
element 1: $\begin{Bmatrix} f_1 \\ m_1 \\ f_r \\ m_r \end{Bmatrix} = \frac{V \times 10^4}{2V} \times [K_1] \times \begin{Bmatrix} 0 \\ 1,905 \times 10^{-3} \\ 0 \\ -2,109 \times 10^{-3} \end{Bmatrix}$

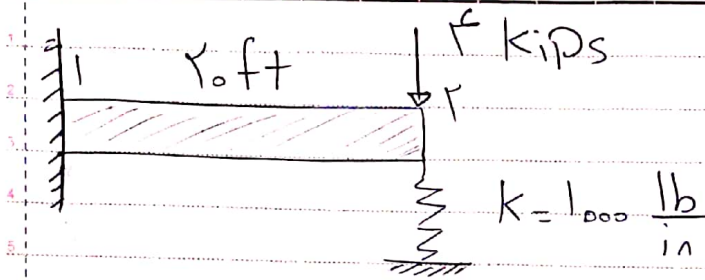
$\Rightarrow f_1 = -11190 \text{ N} \quad m_1 = 0$
 $f_r = 11190 \text{ N} \quad m_r = -244 \text{ V} \cdot \text{N} \cdot \text{m}$

element 2: $\begin{Bmatrix} f_r \\ m_r \\ f_p \\ m_p \end{Bmatrix} = \frac{V \times 10^4}{n} \times [K_2] \times \begin{Bmatrix} 0 \\ -2,109 \times 10^{-3} \\ -2,419 \times 10^{-3} \\ 0 \end{Bmatrix}$

$\Rightarrow f_r = 80000 \text{ N} \quad f_p = -80000 \text{ N}$

$m_r = 244 \text{ V} \cdot \text{N} \cdot \text{m} \quad m_p = 8222 \text{ V} \cdot \text{N} \cdot \text{m}$





$$E = 29 \times 10^6 \text{ psi}$$

$$I = 200 \text{ in}^4$$

ماتریس 4×4 → درجه آزادی 2×2 آره

$$[K] = \frac{EI}{L^3} \begin{bmatrix} 12 & 4L & -12 & 4L \\ 4L & 4L^2 & -4L & 2L^2 \\ -12 & -4L & 12 + \frac{KL^3}{EI} & -4L \\ 4L & 2L^2 & -4L & 4L^2 \end{bmatrix}$$

شرایط مرزی: $\psi_1 = \theta_1 = 0$

$$\rightarrow \begin{bmatrix} -40000 \\ 0 \end{bmatrix} = \frac{29 \times 10^6 \times 200}{(20 \times 12)^3} \begin{bmatrix} 12 + \frac{KL^3}{EI} & -4L \\ -4L & 4L^2 \end{bmatrix} \begin{bmatrix} \psi_2 \\ \theta_2 \end{bmatrix}$$

$$\rightarrow \theta_2 = -0.011054 \quad \psi_2 = -1.772 \text{ in}$$

$$\rightarrow \begin{bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{bmatrix} = \frac{29 \times 10^6 \times 200}{(20 \times 12)^3} \begin{bmatrix} 12 & 4L & -12 & 4L \\ 4L & 4L^2 & -4L & 2L^2 \\ -12 & -4L & 12 + \frac{KL^3}{EI} & -4L \\ 4L & 2L^2 & -4L & 4L^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1.772 \\ -0.011054 \end{bmatrix}$$

$$\rightarrow F_1 = 222 \text{ lbs}, \quad F_2 = -222 \text{ lbs}$$

$$M_1 = -0.34 \text{ kip}, \quad M_2 = 0$$

