UNIVERSITY OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF APPLIED COMPUTER SCIENCE
AND MODELLING

FUNDAMENTALS OF OPTIMIZATION

Optimization of multi-variable problems using non-gradient methods

1. Aim of the exercise.

The aim of the exercise is using non-gradient optimization methods to find minimum of the given test function and to solve simple optimization problem.

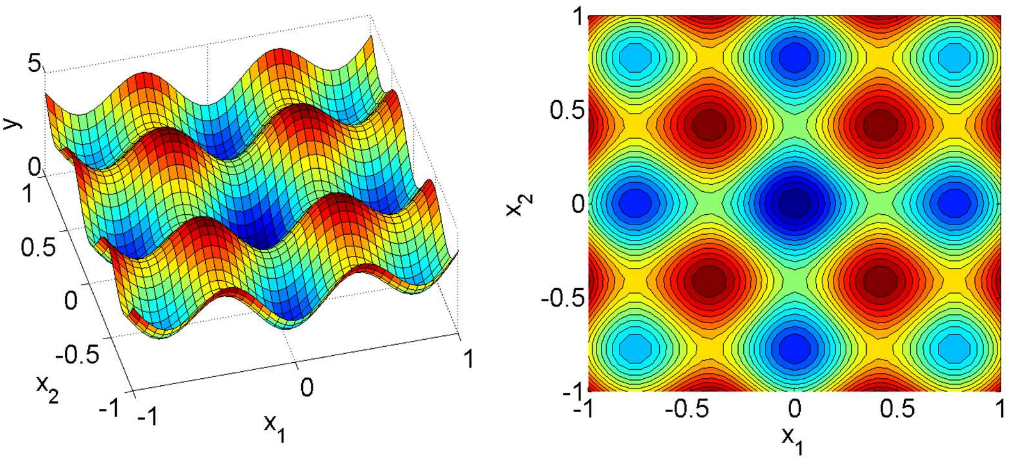
2. Test function.

The test function is given by following equation:

$$f(x_1, x_2) = x_1^2 + x_2^2 - cos(2.5\pi x_1) - cos(2.5\pi x_2) + 2$$

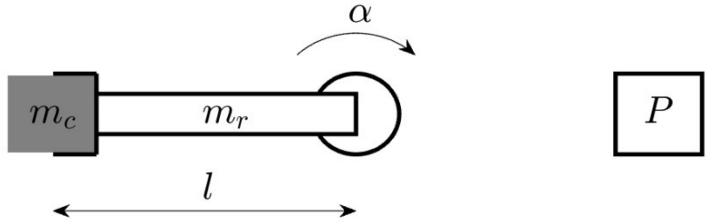
where: $x_1 \in [-1, 1], x_2 \in [-1, 1]$.

The test function is shown in the figure below.



3. Robot arm control.

A robot arm with a length of $l = 0.5\ m$ and mass $m_r = 1\ kg$ should place the container with a mass $m_c = 10\ kg$ on the platform P. For this purpose, the arm must rotate by an angle of $\pi\ rad$ and stop.



The equation describing the arm movement is as follows:

$$I\frac{d^2\alpha}{dt^2} + b\frac{d\alpha}{dt} = M(t),$$

where: $b = 0.5\ Nms$ is friction coefficient, I is moment of inertia of the arm with the container:

$I = \frac{1}{3} m_r l^2 + m_c l^2$.

The moment of force $M(t)$ is determined by the formula:

$$M(t) = k_1\left(\alpha_{ref} - \alpha(t)\right) + k_2\left(\omega_{ref} - \omega(t)\right),$$

where: $\omega = \frac{d\alpha}{dt}$ is the arm velocity, $\alpha_{ref} = \pi\ rad$, $\omega_{ref} = 0\ {}^{rad}/_{s}$, k_1 and k_2 are controller gain factors.

The aim of the optimization is to find the values of controller gain factors k_1 and k_2 which minimized the following function:

$$Q(k_1, k_2) = \int_0^T \left(10\left(\alpha_{ref} - \alpha(t)\right)^2 + \left(\omega_{ref} - \omega(t)\right)^2 + \left(M(t)\right)^2\right) dt.$$

where: $T = 100s$ and $dt = 0.1s$.

The initial values of controller gain factors should be in the range $k_1^{(0)} \in [0,10]\ Nm$ and $k_2^{(0)} \in [0,10]\ Nms$.

4. Optimization methods.

To perform optimization use **fminsearch** function. Starting point should be chosen randomly from given range.

5. Realization of the exercise.

During the exercise four m-files should be written:

- **start.m** – a script which runs all computations. It should:
  - display the names of the Authors of the code,
  - display the optimum found for the test function (x, y and the number of objective function calls),
  - plot the figure showing the test function (as a contour plot), starting point and found optimum,
  - display the optimum found for the robot arm problem (k1, k2, Q and the number of objective function calls),
  - plot two figures showing the position and the velocity of the arm as the function of time.
- **ff_test.m** – a function which calculates and returns the test function value at given point: **y=ff_test(x)**
- **ff_arm.m** – a function which calculates and returns the objective function value in the robot arm control problem: **Q=ff_arm(x)**. To compute the integral **trapz** function should be used.
- **sim_arm.m** – a function which returns the vectors containing samples of time, position and velocity of the arm: **[t,alpha,omega]=sim_arm(k1,k2)**.

To validate if the **ff_arm** and **sim_arm** functions are written correctly compute the integral Q for $x = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$. The integral Q should be about 141.82.

6. Report.

As the report, four m-files should be sent via UPeL platform (by one Author only).