

Supplementary material for Omobot: a low-cost mobile robot for autonomous search and fall detection

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APPENDIX A

HOMOGRAPHY FROM EQUATION OF A PLANE

Proof. From pinhole camera projection equations for camera 1 and 2 we have [1],

$$\lambda_1 \mathbf{u}_1 = K_1 \mathbf{x}_1, \quad \lambda_2 \mathbf{u}_2 = K_2 \underbrace{({}^2R_1 \mathbf{x}_1 + {}^2\mathbf{t}_1)}_{\mathbf{x}_2}, \quad (1)$$

where $\lambda_1, \lambda_2 \in \mathbb{R}$ are scalars that represent the depth of \mathbf{x}_1 from the respective camera origins and $\mathbf{x}_2 \in \mathbb{R}^3$ are the coordinates of the point \mathbf{x}_1 in the frame of camera 2. We can solve for λ_1 by computing the intersection of the ray $\mathbf{x}_1 = \lambda_1 K_1^{-1} \mathbf{u}_1$, with the plane \mathcal{P} .

$$\hat{\mathbf{n}}^\top (\lambda_1 K_1^{-1} \mathbf{u}_1) = h \implies \lambda_1 = \frac{h}{\hat{\mathbf{n}}^\top K_1^{-1} \mathbf{u}_1} \quad (2)$$

Now, substituting λ_1 in (1) and rearrange the equation to form the H matrix:

$$\begin{aligned} \lambda_2 \mathbf{u}_2 &= \frac{h}{\hat{\mathbf{n}}^\top K_1^{-1} \mathbf{u}_1} K_2 {}^2R_1 K_1^{-1} \mathbf{u}_1 + K_2 {}^2\mathbf{t}_1 \\ (\hat{\mathbf{n}}^\top K_1^{-1} \mathbf{u}_1) \lambda_2 \mathbf{u}_2 &= h K_2 {}^2R_1 K_1^{-1} \mathbf{u}_1 + K_2 {}^2\mathbf{t}_1 (\hat{\mathbf{n}}^\top K_1^{-1} \mathbf{u}_1) \\ \underbrace{(\hat{\mathbf{n}}^\top K_1^{-1} \mathbf{u}_1 \lambda_2)}_{\alpha} \mathbf{u}_2 &= \underbrace{(h K_2 {}^2R_1 K_1^{-1} + K_2 {}^2\mathbf{t}_1 \hat{\mathbf{n}}^\top K_1^{-1})}_{{}^2H_1} \mathbf{u}_1. \end{aligned} \quad (3)$$

In the last step, we can absorb $\hat{\mathbf{n}}^\top K_1^{-1} \mathbf{u}_1 \lambda_2$ in α as a scaling factor because α can absorb an arbitrary non-zero scalar function of \mathbf{u}_1 .

To justify this possibility of α , let us rewrite $\alpha \mathbf{u}_2 = {}^2H_1 \mathbf{u}_1$ with α as a multiple of arbitrary non-zero scalar function $f(\mathbf{u}_1) : \mathbb{P}^2 \rightarrow (\mathbb{R} \setminus \{0\})$ of \mathbf{u}_1 with $\alpha = \gamma f(\mathbf{u}_1)$ where $\gamma \in \mathbb{R}$ is a scale factor,

$$\gamma f(\mathbf{u}_1) \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_1^\top \\ \mathbf{h}_2^\top \\ \mathbf{h}_3^\top \end{bmatrix} \mathbf{u}_1 = {}^2H_1 \mathbf{u}_1. \quad (4)$$

Solving for γ we can equate, $\gamma f(\mathbf{u}_1) = \mathbf{h}_3^\top \mathbf{u}_1$ which gives, $\gamma = \frac{\mathbf{h}_3^\top \mathbf{u}_1}{f(\mathbf{u}_1)}$. Substitute γ in (4), to get the original definition of homography transform,

$$\frac{\mathbf{h}_3^\top \mathbf{u}_1}{f(\mathbf{u}_1)} f(\mathbf{u}_1) \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = \underbrace{\mathbf{h}_3^\top \mathbf{u}_1}_{\alpha'} \begin{bmatrix} u_2 \\ v_2 \\ 1 \end{bmatrix} = {}^2H_1 \mathbf{u}_1. \quad (5)$$

When we implement the homography on the robot and in practice use (3), we made these assumptions:

- All the points of detected human lies on a plane
- The plane is parallel to x-y axis of the camera, then $\hat{\mathbf{n}}^\top = [0, 0, 1.]$
- The person distance is in the range of 1 to 3 meters and we transform the image based on the different expected distance range from LiDAR estimation, for instance, the H matrix for $h = 3$ is:

$${}^2H_1 = \begin{bmatrix} 0.896 & -0.219 & 13.76 \\ 0.000 & 0.683 & 16.55 \\ 0.000 & -0.002 & 1.000 \end{bmatrix} \quad (6)$$

REFERENCES

- [1] R. Hartley and A. Zisserman, *Multiple View Geometry in Computer Vision*, 2nd ed. New York, NY, USA: Cambridge University Press, 2003.

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