Generating of Music Variations: Dynamical Systems Approach

Rajamangala University of Technology Thanyaburi
Kanatsanun Sub-udom
Wannasa Rianthong
Patipan Somwong

April 24, 2024

Abstract

This research introduces a novel approach to music composition aimed at mitigating composer burnout. Leveraging chaotic systems, specifically the Lorenz Equation known for its sensitivity to initial conditions, the proposed method maps musical data onto this system. By introducing variations in initial values, the method generates unpredictable compositions, fostering creativity and fresh perspectives.

1 Introduction

This research aims to address the prevalent issue of composer's burnout by introducing a novel approach to music composition utilizing chaotic systems. The proposed method leverages the Lorenz Equation, a chaotic system renowned for its sensitivity to initial conditions. By mapping musical data onto the Lorenz Equation and introducing variations in initial values, the method generates novel and unpredictable musical compositions, stimulating creativity and fostering fresh perspectives in music composition.

In reality, there are now many AI technologies that can create music. However, these technologies often require high computational resources, making them unable to run on devices with low processing power. Additionally, some AI music composition tools may produce music in limited styles. Well-known AI music generation platforms [1] today include:

- Mubert: An online AI music composition platform that allows users to create music easily without musical knowledge.
- Soundraw: An AI music creation tool that helps users quickly create background music for videos or games.
- Boomy: An AI music composition app that helps users create and distribute music on streaming platforms.
- AIVA: An AI assistant for music composition that helps users create music in various styles.
- Musicity: An AI music creation tool that utilizes deep learning models to generate music. This research project proposes a new approach that reduces the use of computational resources and creates music in a more diverse range of styles.

2 Literature Review

2.1 Chaotic System

Chaotic systems are like puzzles with clear rules, but tiny mistakes at the start (think a single misplaced piece) lead to wildly different solutions later on. Even though the rules are defined, the outcomes are unpredictable due to their sensitivity to initial conditions. This butterfly effect is seen in weather, fluid flow, and even some financial markets.

An example used in a Chaotic Systemg such as Modeling Natural Phenomena: Chaotic systems help model and understand various natural phenomena such as weather patterns, turbulent fluid flows, ecological systems, and population dynamics.[2] Finance and Economics: Chaotic dynamics are studied in finance and economics to model stock market fluctuations, economic cycles, and price dynamics, providing insights into the inherent unpredictability and nonlinear behavior of financial systems [3] Biomedical Systems: Chaotic systems are used to model and understand complex biological systems, including neural networks, cardiac rhythms, and gene regulatory networks, aiding in diagnosis, treatment, and understanding of diseases [4].

2.2 Lorenz Equation

The Lorenz equation is commonly defined as three coupled ordinary differential equation like:

$$\begin{aligned}
\dot{x} &= \sigma(y - x), \\
\dot{y} &= rx - y - xz, \\
\dot{z} &= xy - bz,
\end{aligned}$$

where x, y and z are the variables, and $\sigma > 0$, $r > 0$ and $b > 0$ are parameters.
When the Lorenz equation's parameters are set to $\sigma = 10$, $r = 28$ and $b = \frac{8}{3}$, it shows chaotic behavior, as illustrated in Figure 1. The Lorenz equations [5] have been applied across various domains such as Modeling Fluid Flows: A study by J.C. Sprott in Chaos and Stability in Nonlinear Analog Circuits (2003) [6] explores how the Lorenz system can be applied to model specific fluid flows. This demonstrates its use in understanding fluid dynamics beyond just atmospheric convection.[7]. Cryptography: The chaotic nature of the Lorenz system makes it a potential candidate for secure communication. A research paper by S. Baptista et al. titled "Lorenz System Parameter Determination and Application to Break the Security of Two-channel Chaotic Cryptosystems" (2006) explores this application. While this paper discusses limitations of specific implementations, it highlights the potential of the Lorenz system in cryptography [8] Engineering Applications: The Lorenz equations can be used to model certain electrical and mechanical systems that exhibit chaotic behavior. For instance, a paper by A.A. Fathy in "Chaos, Solitons Fractals" (2009) investigates the application of the Lorenz system to brushless DC motors [9]. This showcases the potential for the Lorenz system in analyzing and potentially controlling chaotic behavior in engineering systems.

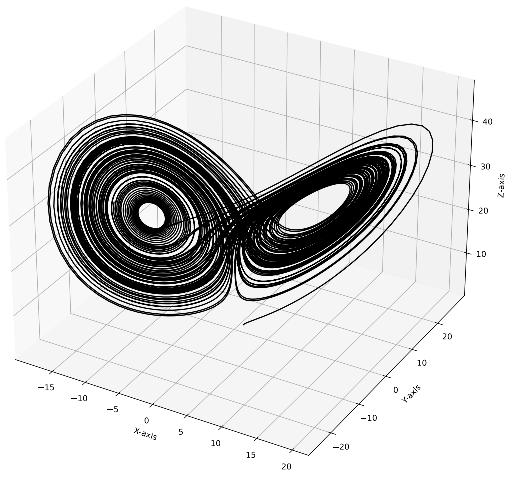


Figure 1: Chaotic behavior of the lorenz equation.

2.3 Fourth-Order Runge–Kutta Implementation

Let $\dot{y} = f(t, y)$. The approximation of y_{i+1} is given by:

$$\begin{aligned} y_{i+1} &= y_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4), \\ k_1 &= f(t_i, y_i), \\ k_2 &= f\left(t_i + \frac{h}{2}, y_i + \frac{h}{2}k_1\right), \\ k_3 &= f\left(t_i + \frac{h}{2}, y_i + \frac{h}{2}k_2\right), \\ k_4 &= f(t_i + h, y_i + hk_3), \end{aligned}$$

where $i = 0, 1, 2, ...$ and h is the step size, y is the variable and t is time.
The Runge-Kutta method [10] is a powerful and popular family of numerical methods for approximating solutions to ordinary differential equations. These equations describe how a quantity changes with respect to another variable, but often cannot be solved exactly. The Runge-Kutta method tackles these problems by breaking down the interval of interest into smaller subintervals and iteratively calculating the solution at each subinterval.

2.4 Musical variations from a chaotic mapping: Diana S. Dabby

Diana S. Dabby's research [11] explores using chaotic systems to create musical variations. By leveraging chaotic trajectories' sensitivity to initial conditions, the study devises a method to introduce variability into a musical piece's pitch sequence. This allows for diverse musical variations while preserving the original composition's coherence. Dabby emphasizes the dynamic nature of this approach, offering composers a flexible tool for crafting a wide range of musical variations. Additionally, the study suggests extending this method beyond music to sequences of context-dependent symbols in various fields. Overall, the research showcases an innovative application of chaotic dynamics in music composition, yielding dynamic and versatile musical outcomes.

2.5 Melodic Variation

Melodic variation is essential in songwriting, adding depth and diversity to compositions. This article emphasizes its role in enhancing musicality and emotional depth. It cautions against monotony in melodies, stressing the importance of dynamic variation to engage listeners. Additionally, melodic variation contributes to a song's structure by creating contrast between sections. Two main methods are discussed: variations on a theme and countermelodies. Lastly, the article highlights the emotional impact of melodic variation and its ability to evoke a range of feelings in listeners. In conclusion, skillful use of melodic variation enables songwriters to craft compelling compositions that resonate deeply with audiences.

3 Main Result

This section explores the application of three techniques for generating musical variations, using the Ah vous dirai-je, Maman melody as a starting point and Lorenz Equation for chaotic trajectory.

3.1 Musical Variations from a Chaotic Mapping (Diana S. Dabby)

To begin, let's introduce the definition for a clearer understanding of Diana S. Dabby's approach to generating musical variations using chaotic mappings:

Definition 1. *Musical note is a symbol on sheet music that represents the pitch (highness or lowness of sound). Musical notes use the standard letters: A (La), B (Ti), C (Do), D (Re), E (Me), F (Fa), and G (Sol). (If you're interested in musical notes, see Music Theory [12] for more pitch symbols or pitch notation system like sharps and flats.)*

Definition 2. *Musical note sequence (also known as pitch sequence in Diana S. Dabby Musical Variations from a Chaotic Mapping paper) is a sequence of symbols on sheet music that represents the pitch. For convenience, we can define $P = \{C, D, F, G\}$ be a musical note sequence.*

Definition 3. *Let $P = \{p_1, p_2, \ldots, p_n\}$ be a musical note sequence. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a x-value sequence of a trajectory corresponding one-to-one with the elements in P. Then X is sorted in ascending order. Here, x_i denotes the i^{th} element in X and p_i denotes the pitch and the duration of the i^{th} musical note in P.*

We define another sequence $X^* = \{x_1^*, x_2^*, \ldots, x_n^*\}$ with a different trajectory (not necessarily sorted) compared to X. Here, x_i^* denotes the i^{th} element in X^*.

The Dabby criteria model [11], denoted by $M(X, x_i^*)$, is a function that assigns a new pitch value to the i^{th} element x_i^* in sequence X^*, based on the comparison between a sorted sequence X and a element in sequence X^* with a different trajectory.

$$M(X, x_i^*) = \begin{cases} n_j, & \text{if there exists a smallest index } j \in \{1, 2, \ldots, i^*\} \text{ such that } x_j = x_i^* \\ n_{j-1}, & \text{if there exists a smallest index } j \in \{1, 2, \ldots, i^*\} \text{ such that } x_j > x_i^* \\ n_{i^*}, & x_{i^*} < x_i^* \\ .n_{i'}, & \text{otherwise.} \end{cases}$$

Here, i^* denotes the last index of the sequence and i' denotes the first index of the sequence.

Example 1. *Let $P = \{C, F, D, A\}$ be a musical note sequence and $X = \{0.9, 1, 1.02, 1.2\}$ be a sorted sequence of a trajectory corresponding one-to-one with the elements in N and $X^* = \{1.01, 1.05, 1.12, 1.2\}$ be a sequence of a different trajectory. By Dabby criteria model*

p_i	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}
Note	C4	C4	G4	G4	A4	A4	G4	F4	F4	E4	E4

Table 1: The musical notes created with the pitch notation system [14] of the p-component, denoted by p_i, for $i = 1, 2, \ldots, 11$.

Figure 2: The original of 12 variations on Ah vous dirai-je Maman in the first 3 bars.

in definition 3, we can assign a new pitch value of the musical notes in P respectively as follows:

$$\begin{aligned} M(X, 1.01) &= F, \\ M(X, 1.05) &= D, \\ M(X, 1.12) &= D, \\ M(X, 1.05) &= A. \end{aligned}$$

For convenience, we can define $P^* = \{F, D, D, A\}$ be a new musical note sequence.

Let's explore Diana S. Dabby's method for creating musical variations with chaotic mappings. We'll break it down step-by-step (See Figure 14 for flowchart of musical variations from a chaotic mapping (Diana S. Dabby)).

3.1.1 Chaotic system Selection

In Diana S. Dabby Musical Variations from a Chaotic Mapping paper, the Lorenz equation is used for simulate chaotic trajectory by fourth-order Runge–Kutta implementation.

3.1.2 Sheet Music Selection

Consider the music piece 12 variations on Ah vous dirai-je Maman [13], illustrated in Figure 2. We can simplify the representation of its first 11 musical note to musical note sequence using a Table 1, following these criteria:

1. Read the notes from left to right
2. Each step in the Table 1 corresponds to a single musical note symbol

3.1.3 Simulate Chaotic Trajectory

Give an initial condition of $(1, 1, 1)$ with a step size of $h = 0.01$. Using the fourth-order Runge–Kutta implementation, we approximate the solution of the Lorenz equation, the generated chaotic trajectory does not retrace its previous path. This characteristic, where each point is unique, ensures that the sequence of values obtained after generating the trajectory will not contain any duplicates. This property makes it suitable for generating music in the next phase.

Let x_i denote the ith x-value in the reference trajectory. This allows us to find the following first 11 x-components in the corresponding x-values sequence in Table 2.

Next, Let Consider the values in Table 2 and Table 1. If we establish the following pairings:

x_i	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x-value	1.00	1.29	2.13	3.74	6.54	11.04	16.69	19.56	15.37	7.55	1.20

Table 2: The values of the x-component, denoted by x_i, for $i = 1, 2, ..., 11$, correspond to the x-values of the reference trajectory obtained using the fourth-order Runge-Kutta implementation

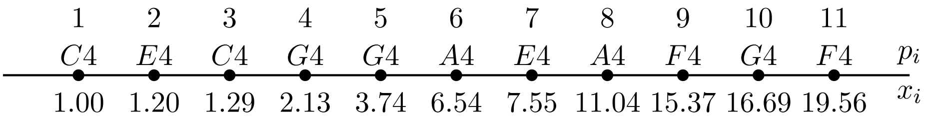


Figure 3: Number line from pairing pattern for subsequent pitches and x-values with new index.

- The first musical note p_1 from Table 1 is paired with x_1 from Table 2, representing the first x-value of the reference trajectory.
- The second musical note p_2 from Table 1 is paired with x_2 from Table 2.
- We continue this pairing pattern for musical note sequent and x-value sequent.
- Last, we sorted all the x-values, and this resulted in a new ascending index.

Then, we obtain the following number line in Figure 3 with new index and Table 3 (the main x-value sequence for the Dabby criteria model is provided in Table 4) With the establishment of a number line in Figure 3 (the number line is the same as Dabby criteria model in definition 3).

3.1.4 Simulate New Chaotic Trajectory

Let generate a new trajectory using the fourth-order Runge–Kutta implementation, but with an initial condition of $(1.01, 1, 1)$ instead of $(1, 1, 1)$. then proceed to create Table 5 for processing the Dabby criteria model.

3.1.5 Dabby Criteria Model

First, consider Table 4. Here, we define $P = \{C4, E4, C4, G4, G4, A4, E4, A4, F4, G4, F4\}$ as a musical note sequence and $X = \{1.00, 1.20, 1.29, 2.13, 3.74, 6.54, 7.55, 11.04, 15.37, 16.56, 19.56\}$ as a sorted sequence of a trajectory corresponding one-to-one with the elements in P.

Next, consider Table 5. Similarly, we define $X^* = \{1.01, 1.30, 2.15, 3.76, 6.58, 11.10, 16.73, 19.55, 15.30, 7.48, 1.15\}$ as a sequence of a different trajectory.

According to the Dabby criteria model (Definition 3), we can assign a new pitch value to the musical notes in P as follows:

x_i	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
p_i	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}
New i	x_1	x_3	x_4	x_5	x_6	x_8	x_{10}	x_{11}	x_9	x_7	x_2
x-value	1.00	1.29	2.13	3.74	6.54	11.04	16.69	19.56	15.37	7.55	1.20
Note	C4	C4	G4	G4	A4	A4	G4	F4	F4	E4	E4

Table 3: Pairing pattern of x_i and p_i for $i = 0, 1, 2, ..., 11$.

x_i	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
x-value	1.00	1.20	1.29	2.13	3.74	6.54	7.55	11.04	15.37	16.56	19.56
Note	C4	E4	C4	G4	G4	A4	E4	A4	F4	G4	F4

Table 4: The main x-value sequence for the Dabby criteria model.

x_i^*	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*	x_7^*	x_8^*	x_9^*	x_{10}^*	x_{11}^*
x^*-value	1.01	1.30	2.15	3.76	6.58	11.10	16.73	19.55	15.30	7.48	1.15

Table 5: The values of the new x-component, denoted by x_i^*, for $i = 0, 1, 2, ..., 11$, correspond to the new x-values of the reference trajectory obtained using the fourth-order Runge-Kutta implementation.

$$
\begin{aligned}
M(X, 1.01) &= C4, \\
M(X, 1.30) &= E4, \\
M(X, 2.15) &= G4, \\
M(X, 3.76) &= G4, \\
M(X, 6.58) &= A4, \\
M(X, 11.10) &= A4, \\
M(X, 16.73) &= F4, \\
M(X, 19.55) &= G4, \\
M(X, 15.30) &= A4, \\
M(X, 7.48) &= A4, \\
M(X, 1.15) &= C4.
\end{aligned}
$$

For convenience, we can define $P^* = \{C4, E4, G4, G4, A4, A4, F4, G4, A4, A4, C4\}$ be a new musical note sequence (see Table 6 for more details on the new musical note sequence).

3.1.6 Convert New Musical Note Sequence

Consider $P^* = \{C4, E4, G4, G4, A4, A4, F4, G4, A4,$
$A4, C4\}$, the new musical note sequence. This sequence can be converted to sheet music, as shown in Figure 4. Sheet music provides a much more readable representation of musical notation.

3.2 Melodic Variation with Expanded Rhythm

To begin, let's introduce the definition for a clearer understanding of Melodic Variation with Expanded Rhythm:

Definition 4. *Musical note duration is the length of time a musical note is played. This is what creates the rhythm in music. Musical note durations are typically expressed using fractions of*

x_i^*	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*	x_7^*	x_8^*	x_9^*	x_{10}^*	x_{11}^*
x^*-value	1.01	1.30	2.15	3.76	6.58	11.10	16.73	19.55	15.30	7.48	1.15
Original Note	C4	C4	G4	G4	A4	A4	G4	F4	F4	E4	E4
New note	C4	E4	G4	G4	A4	A4	F4	G4	A4	A4	C4

Table 6: Results of Applying the Dabby Criteria Model.

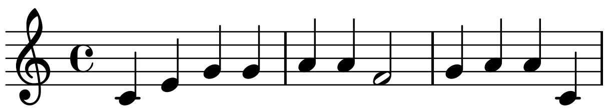


Figure 4: The new variation of Ah vous dirai-je, maman in the first 3 bars, generated by the Initial Condition $(1.01, 1, 1)$.

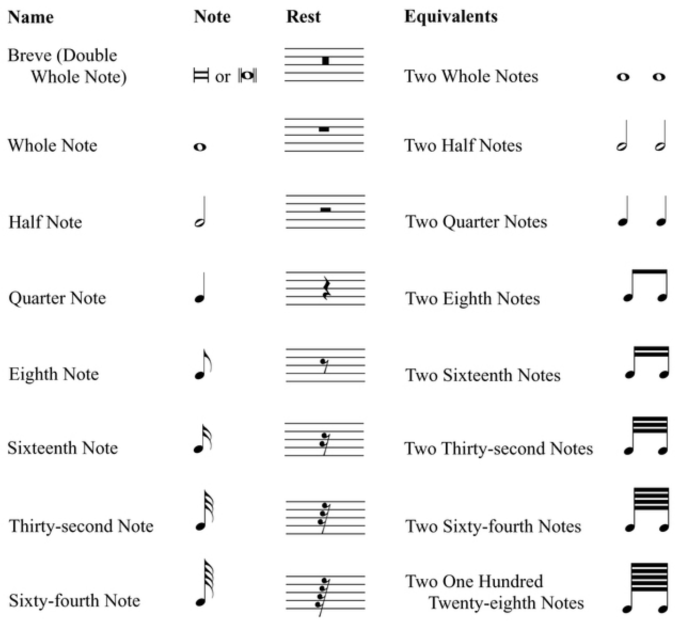


Figure 5: The musical note durations based on Essential Dictionary of Music Notation [15].

a whole note: 4 (whole note), 2 (half note), 1 (quarter note), 0.5 (eighth note), 0.25 (sixteenth note), and so on (see Music Theory [12] and Figure 5 for more details on rhythm).

Definition 5. *Given a musical note duration denoted by ϕ and divided into D equal parts, the duration, R, of each individual division is defined by the following equation:*

$$R = \frac{\phi}{D}.$$

Note: *In musical theory, equal parts refers to divisions that all have the same duration. This is distinct from concepts like scales or intervals, which define specific pitch relationships between notes.*

Example 2. *Given a musical note C that has a duration equal to 1 (quarter note) or $\phi = 1$. If we want to divide this musical note into 4 parts ($D = 4$), we can find the duration of each individual division by definition 5 as follows:*

$$R = \frac{\phi}{D} = \frac{1}{4} = 0.25.$$

From the result, we can divide a quarter note (C) into 4 sixteenth notes.

Figure 6: The original of 12 variations on Ah vous dirai-je Maman in the first 2 bars.

Figure 7: The melodic variation of Ah vous dirai-je, maman in the first 2 bars.

3.2.1 The Approach of Melodic Variation with Expanded Rhythm

Let's explore the melodic variation with expanded rhythm method, which aims to create variations by increasing the rhythmic complexity of a musical piece. We'll break it down step-by-step in this section.

Consider the music piece 12 variations on Ah vous dirai-je Maman, illustrated in Figure 6. The Figure shows that we already have 6 quarter notes and 1 half note.

If we want to divide each musical note into 4 parts, we can find the duration of each individual division as follows (based on definition 5):

- Half note to 4 parts:
$$R = \frac{\phi}{D} = \frac{2}{4} = 0.5$$
- Quarter note to 4 parts:
$$R = \frac{\phi}{D} = \frac{1}{4} = 0.25$$

Following this calculation, a half note can be divided into 4 eighth notes, and a quarter note can be divided into 4 sixteenth notes. This division is converted to sheet music, as shown in Figure 7.

3.3 Combining Musical Variations from a Chaotic Mapping (Diana S. Dabby) and Melodic Variation with Expanded Rhythm

This section explores how to combine musical variations generated from chaotic mappings (Diana S. Dabby) with melodic variation using expanded rhythm. We will break down the process step-by-step (See Figure 14 for flowchart of combining musical variations from a chaotic mapping (Diana S. Dabby) and melodic variation with expanded rhythm).

3.3.1 Chaotic system Selection

We use the Lorenz equation for simulate chaotic trajectory by fourth-order Runge–Kutta implementation.

3.3.2 Sheet Music Selection

Consider the music piece 12 variations on Ah vous dirai-je Maman, illustrated in Figure 6. We can simplify the representation of its first 7 musical note to musical note sequence using a Table 7, following these criteria:

p_i	p_1	p_2	p_3	p_4	p_5	p_6	p_7
Note	C4	C4	G4	G4	A4	A4	G4

Table 7: The musical notes created with the pitch notation system [14] of the p-component, denoted by p_i, for $i = 1, 2, \ldots, 7$.

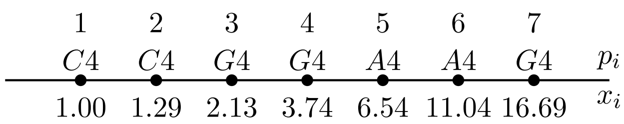


Figure 8: Number line from pairing pattern for subsequent pitches and x-values with new index.

1. Read the notes from left to right
2. Each step in the Table 7 corresponds to a single musical note symbol

3.3.3 Melodic Variation

If we use the approach of melodic variation with expanded rhythm on the first 7 musical note of music piece 12 variations on Ah vous dirai-je Maman, we obtain Figure 7. This figure can be converted into the following musical note sequence:

$$P' = \{C4, C4, C4, C4, C4, C4, C4, C4, G4, G4, G4, G4, G4, G4, G4, G4, A4, A4, A4, A4, A4, A4, A4, A4, G4, G4, G4, G4\}$$

when P' be a musical note sequence after expanded rhythm.

3.3.4 Simulate Chaotic Trajectory

This step is similar to the approach of Diana S. Dabby's musical variations from chaotic mappings. Here, we simulate a chaotic trajectory using the Lorenz equation with an initial condition of $(1, 1, 1)$ with a step size of $h = 0.01$. The fourth-order Runge-Kutta implementation is used to approximate the solution. The results are presented in the Table 8 and the number line in Figure 8.

3.3.5 Simulate New Chaotic Trajectory

Let generate a new trajectory using the fourth-order Runge–Kutta implementation, but with an initial condition of $(1.01, 1, 1)$ instead of $(1, 1, 1)$. The resulting sequence will have the same length as the P' shown in Table 9 and will be used for processing with the Dabby criteria model.

3.3.6 Dabby Criteria Model

First, consider Table 8. Here, we define $P = \{C4, C4, G4, G4, A4, A4, G4\}$ as a musical note sequence and $X = \{1.00, 1.29, 2.13, 3.74, 6.54, 11.04, 16.69\}$ as a sorted sequence of a trajectory corresponding one-to-one with the elements in N.

x_i	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x-value	1.00	1.29	2.13	3.74	6.54	11.04	16.69
Note	C4	C4	G4	G4	A4	A4	G4

Table 8: The main x-value sequence for the Dabby criteria model.

x_i^*	x^*-value	x_i^*	x^*-value
x_1^*	1.01	x_{15}^*	-7.06
x_2^*	1.30	x_{16}^*	-7.87
x_3^*	2.15	x_{17}^*	-8.64
x_4^*	3.76	x_{18}^*	-9.29
x_5^*	6.58	x_{19}^*	-9.70
x_6^*	11.10	x_{20}^*	-9.73
x_7^*	16.73	x_{21}^*	-9.37
x_8^*	19.55	x_{22}^*	-8.76
x_9^*	15.30	x_{23}^*	-8.07
x_{10}^*	7.48	x_{24}^*	-8.50
x_{11}^*	1.15	x_{25}^*	-7.17
x_{12}^*	-2.70	x_{26}^*	-7.12
x_{13}^*	-4.85	x_{27}^*	-7.37
x_{14}^*	-6.13	x_{28}^*	-7.85

Table 9: The values of the new x-component with the same length as P' sequence, denoted by x_i^*, for $i = 1, 2, ..., 28$, correspond to the new x-values of the reference trajectory obtained using the fourth-order Runge-Kutta implementation.

Next, consider Table 9. Similarly, we define $X^* = \{1.01, 1.30, 2.15, 3.76, 6.58, 11.10, 16.73, 19.55, 15.30, 7.48, 1.15, -2.70, -4.85, -6.13, -7.06, -7.87, -8.64, -9.29, -9.70, -9.73, -9.37, -8.76, -8.07, -8.50, -7.17, -7.12, -7.37, -7.85\}$ as a sequence of a different trajectory.

According to the Dabby criteria model (Definition 3), we can assign a new pitch value to the musical notes in P' as follows:

$$\begin{aligned}
M(X, 1.01) &= C4, \\
M(X, 1.30) &= C4, \\
M(X, 2.15) &= G4, \\
M(X, 3.76) &= G4, \\
M(X, 6.58) &= A4, \\
M(X, 11.10) &= A4, \\
M(X, 16.73) &= A4, \\
M(X, 19.55) &= G4, \\
M(X, 15.30) &= A4, \\
M(X, 7.48) &= A4, \\
M(X, 1.15) &= C4, \\
M(X, -2.70) &= C4, \\
M(X, -4.85) &= C4, \\
M(X, -6.13) &= C4,
\end{aligned}$$

$$\begin{aligned}
M(X, -7.06) &= C4, \\
M(X, -7.87) &= C4, \\
M(X, -8.64) &= C4, \\
M(X, -9.29) &= C4, \\
M(X, -9.70) &= C4, \\
M(X, -9.73) &= C4, \\
M(X, -9.37) &= C4, \\
M(X, -8.76) &= C4, \\
M(X, -8.07) &= C4, \\
M(X, -8.50) &= C4, \\
M(X, -7.17) &= C4, \\
M(X, -7.12) &= C4, \\
M(X, -7.37) &= C4, \\
M(X, -7.85) &= C4.
\end{aligned}$$

For convenience, we can define $P^* = \{C4, C4, G4, G4, A4, A4, A4, G4, A4, A4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4\}$ be a new musical note sequence with expanded rhythm.

Figure 9: The new variation with melodic variation of Ah vous dirai-je, maman in the first 2 bars, generated by the Initial Condition $(1.01, 1, 1)$.

Figure 10: The original of Pachelbel's Canon in the first 10 bars.

3.3.7 Convert New Musical Note Sequence with Expanded Rhythm

Consider $P^* = \{C4, C4, G4, G4, A4, A4, A4, G4, A4, A4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4, C4\}$, the new musical note sequence. This sequence with expanded rhythm can be converted to sheet music, as shown in Figure 9. Sheet music provides a much more readable representation of musical notation.

4 Discussion

This section explores the potential of Combination of Musical Variations from a Chaotic Mapping and Melodic Variation with Expanded Rhythm technique for generating musical variations. It analyzes the impact of this approach on the resulting variations compared to traditional melodic variation techniques.

4.1 Strengths of the Combined Approach

The combined approach shows potential in generating interesting musical variations, as shown in the new variations with melodic variation of Pachelbel's Canon [16] (Figure 11) compared to the original melody (Figure 10).

This method appears to be particularly effective for pieces with a larger range of musical pitches, as exemplified by Pachelbel's Canon. The additional pitches provide more material for the chaotic mapping and melodic variation techniques to manipulate, leading to richer and more diverse variations.

4.2 Weaknesses of the Combined Approach

Let's consider the music sheet for Vanessa Carlton's A Thousand Miles (Figure 12) and the new variation with melodic variation (Figure 13). This result shows that using music sheets with too many musical note durations can lead to new variations with melodies that are difficult to play and listen to. This is a major weakness of this method.

4.3 Limitations and Considerations

Further evaluation with a wider range of musical pieces is necessary to determine the generalizability of these observations. The effectiveness of the combined approach might vary depending on the musical style and characteristics of the original piece.

The computational complexity of the chaotic mapping technique should also be considered. While it offers a powerful approach for variation, it might require more processing power compared to simpler melodic variation methods.

Figure 11: The new variation with melodic variation of Pachelbel's Canon in the first 10 bars (including 2 additional bars at the end of the 10th bar), generated by the Initial Condition $(0.99, 0.99, 0.99)$.

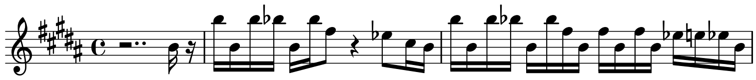


Figure 12: The original of Vanessa Carlton - A Thousand Miles in the first 3 bars.

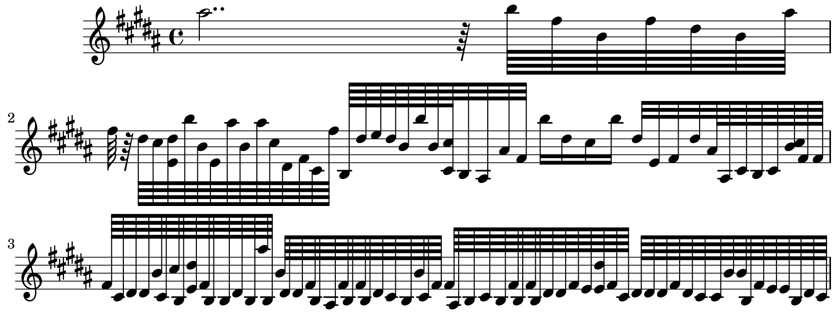


Figure 13: The new variation with melodic variation of Vanessa Carlton - A Thousand Miles in the first 3 bars, generated by the Initial Condition $(0.99, 0.99, 0.99)$.

4.4 Future Directions

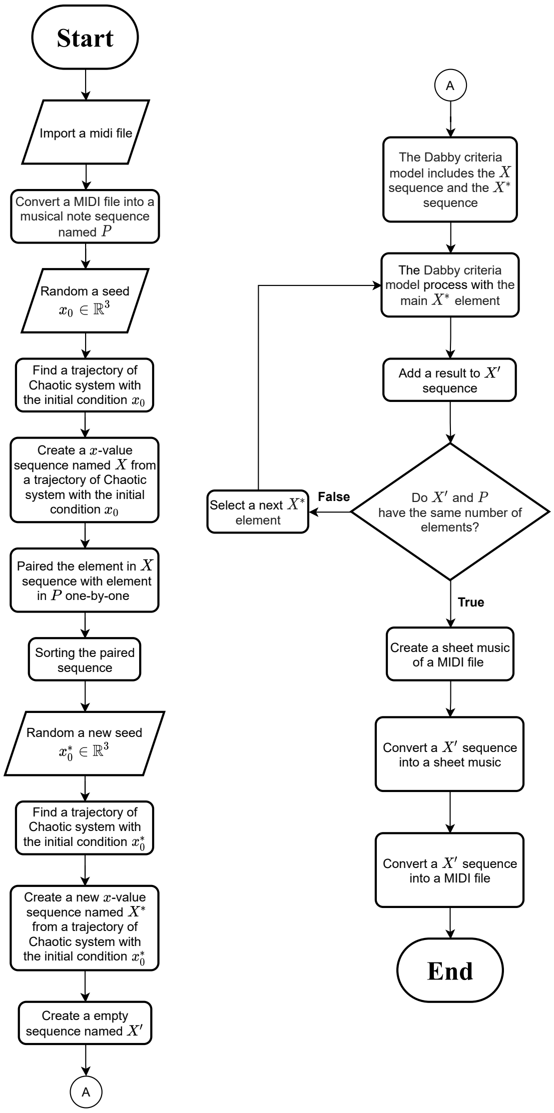
Exploring different parameters and configurations within the chaotic mapping and melodic variation techniques could potentially lead to a wider range of variation styles and outcomes.

Investigating methods for user control over the variation process would be valuable. This could allow musicians to tailor the generated variations to their specific creative goals.

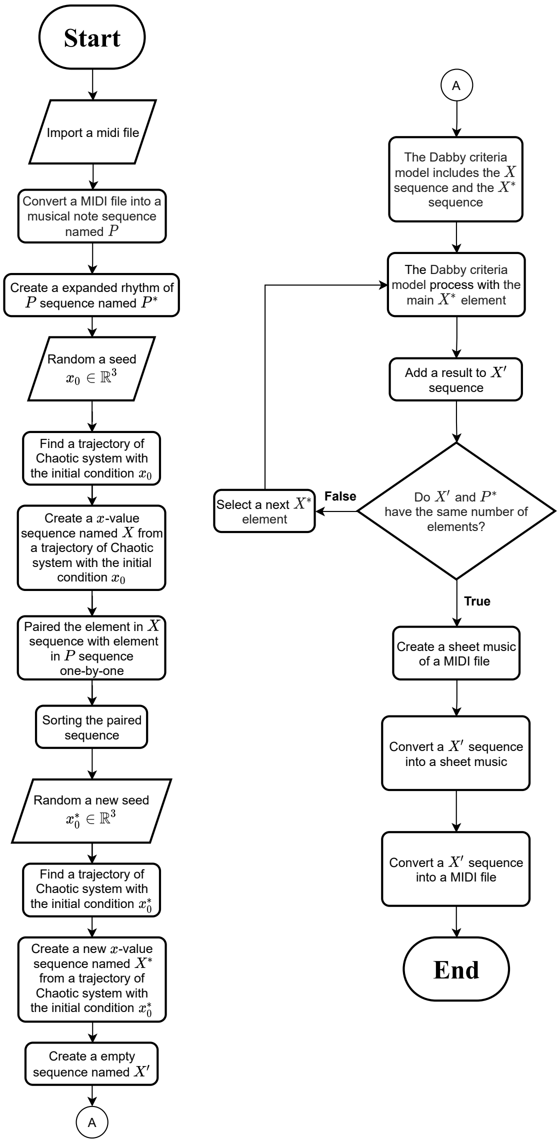
Integrating this combined approach into music composition tools could provide composers with a valuable tool for generating new musical ideas and exploring creative possibilities.

5 Conclusion

The proposed method, leveraging the power of chaotic systems, offers a promising approach to music composition, addressing the limitations of existing AI techniques and providing composers with a tool to generate novel, unpredictable, and diverse musical compositions while reducing computational resource demands. This method has the potential to stimulate creativity, alleviate composer's burnout, and expand the boundaries of musical expression, paving the way for a new era of music creation driven by chaos and innovation.



(a) Flowchart of musical variations from a chaotic mapping (Diana S. Dabby)



(b) Flowchart of combining musical variations from a chaotic mapping (Diana S. Dabby) and melodic variation with expanded rhythm

Figure 14: Flowcharts of musical variations

References

[1] Alexandra Bonnici et al. *Music and AI.* Frontiers Media SA, Mar. 16, 2021. 170 pp. ISBN: 978-2-88966-602-7.

[2] Robert May. "Simple Mathematical Models With Very Complicated Dynamics". In: *Nature* 26 (July 1, 1976), p. 457. ISSN: 978-1-4419-2330-1. DOI: `10.1038/261459a0`.

[3] Benoit B. Mandelbrot. *The Variation of Certain Speculative Prices.* Thomas J. Watson Research Center, 1962. 100 pp.

[4] E.M. Izhikevich. "Neural excitability, spiking and bursting". In: *International Journal of Bifurcation and Chaos in Applied Sciences and Engineering* 10 (June 1, 2000), pp. 1171–1266.

[5] Edward N. Lorenz. "Deterministic Nonperiodic Flow". In: *Journal of the Atmospheric Sciences* 20.2 (Mar. 1, 1963). Publisher: American Meteorological Society Section: Journal of the Atmospheric Sciences, pp. 130–141. ISSN: 0022-4928, 1520-0469. DOI: 10.1175/1520-0469(1963)020<0130:DNF>2.0.CO;2.

[6] Esteban Tlelo-Cuautle et al. *Analog/Digital Implementation of Fractional Order Chaotic Circuits and Applications.* Springer Nature, Nov. 11, 2019. 223 pp. ISBN: 978-3-030-31250-3.

[7] Jiri Petrzela. "Chaos in Analog Electronic Circuits: Comprehensive Review, Solved Problems, Open Topics and Small Example". In: *Mathematics* 10 (Nov. 4, 2022), p. 4108. DOI: `10.3390/math10214108`.

[8] Amalia Orue et al. "Lorenz System Parameter Determination and Application to Break the Security of Two-channel Chaotic Cryptosystems". In: (July 11, 2006).

[9] Zheng-Ming Ge, Chang Ching Ming, and Yen-Sheng Chen. "Anti-control of chaos of single time scale brushless dc motors and chaos synchronization of different order systems". In: *Chaos, Solitons & Fractals* 27 (Mar. 1, 2006), pp. 1298–1315. DOI: `10.1016/j.chaos.2005.04.095`.

[10] Sujit Kumar Bose. *Numerical Methods of Mathematics Implemented in Fortran.* Springer, May 13, 2019. 479 pp. ISBN: 9789811371141.

[11] Diana S. Dabby. "Musical variations from a chaotic mapping". In: *Chaos: An Interdisciplinary Journal of Nonlinear Science* 6.2 (June 1, 1996), pp. 95–107. ISSN: 1054-1500. DOI: `10.1063/1.166171`.

[12] Nicolas Carter. *Music Theory: From Beginner to Expert - The Ultimate Step-By-Step Guide to Understanding and Learning Music Theory Effortlessly.* 4th edition, Aug. 11, 2016. 244 pp.

[13] Maurice Hinson. *12 Variations on Ah, Vous Dirai-Je, Maman, K. 265.* ALFRED PUBN, Nov. 1987. 16 pp. ISBN: 978-0-7390-2032-6.

[14] Vanshika Verma Khare. *Play Keyboard with Ease: Chords and Notations of Top Bollywood Songs.* Notion Press, June 21, 2020. 241 pp. ISBN: 978-1-64892-909-0.

[15] Tom Gerou and Linda Lusk. *Essential Dictionary of Music Notation.* Alfred Publishing Company, 1996. 351 pp. ISBN: 978-0-88284-730-6.

[16] Johann Pachelbel and Willard A. Palmer. *Canon in D: Variations on the Theme for Piano.* Alfred Music, May 3, 2005. 8 pp. ISBN: 978-0-7390-7134-2.