Generating of Music Variations: Dynamical Systems Approach

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Abstract

This paper introduces a new method for diversifying musical compositions to address the issue of composer burnout. By utilizing the characteristics of chaotic dynamical systems, well-known for their sensitivity to initial conditions, this approach combines melodic variation with an extended rhythmic structure. The proposed technique entails the mapping of musical data onto a chaotic attractor, which generates a new variation as the system's trajectories evolve. This expansion of rhythm is achieved by prolonging the duration of musical notes, resulting in a natural blending of melodic and rhythmic elements. The objective is to offer composers a systematic and creative tool for exploring innovative musical concepts, relieving creative fatigue, and invigorating the compositional process.

1 Introduction

Music variation serves as a catalyst for creative thinking in the songwriting process. It offers flexibility, capable of generating patterns ranging from close replicas to entirely different ones. The outcome depends on the composer's desires. When applied to compositions, it's like creating another version of the same song, making the music open to change every time it's heard. In the past, composers often employed techniques like inversion, retrograde or sections of music to expand upon the original musical content. However, these techniques gradually lost their appeal and were seen as tiresome. Music variation steps in to fill this gap. This technique opens up possibilities for composers to create entirely new musical patterns without being tied to the original framework. The written notes can transform with every listen, resulting in dynamic and fresh music akin to a rocket launching pad propelling composers into an unrestricted musical universe.

Nowadays, artificial intelligence (AI) technologies have significantly advanced, enabling them to create music with ever-increasing proficiency [1]. Well-known AI music generation platforms such as Mubert [2] and Musicity [3] empower users with real-time music generation capabilities, enabling them to effortlessly select their preferred genre or mood and promptly receive a personalized soundtrack tailored to their preferences. On the other hand, Soundraw [4] and Boomy [5] function as AI-driven music creation tools, furnishing a diverse array of features to aid users in sculpting their musical opuses with ease. Meanwhile, AIVA [6] harnesses the power of deep learning to craft original music closely resembling the distinctive style of a particular artist or genre. Users can furnish reference tracks or articulate their desired musical aesthetics, prompting AIVA to generate fresh compositions that align precisely with their specifications. However, these technologies often require high computational resources, making them unable to run on devices with low processing power. Additionally, some AI music composition tools may produce music in limited styles.

If the limitations of AI music composition technology remain unaddressed, it will have repercussions. Firstly, aspiring artists and musicians will miss out on the opportunity to use these tools due to limited access, as most people lack high-performance equipment. Furthermore, all music generated by AI may start to sound similar, potentially leading to a lack of musical diversity.The paper thus aims to address the aforementioned issue by employing a multi-step process. Initially, it utilizes melodic variation with expanded rhythm, which is then translated into numerical values. These numerical values are then input into a chaotic dynamical system, resulting in a new set of numbers different from the original. Finally, these numbers are mapped back to musical notes, resulting in the creation of a new piece of music. This method requires fewer computing resources compared to using AI music composition technology and allows for the creation of diverse musical compositions depending on the original song, initial values, and equations used.

In this work, We explore the theory behind chaotic system equations and the basic music theory in Section 2. In Section 3, we will commence by explaining the theory and examples from [7]. Following that, we will utilize melodic variation with expanded rhythm in Subsection 3.2 to illustrate our subsequent workflow. In Section 4, we will present and exchange ideas, opinions, and relevant information. Finally, we summarize the solutions and insights we have derived to address the issues discussed in this paper succinctly and conclusively, in Section 5

2 Literature Review

This literature review explores the intersection of chaotic systems, known for their unpredictable behavior, with music composition. We will examine the core principles of chaotic systems, including the "Butterfly Effect," and delve into specific examples like the Lorenz equations. The Runge-Kutta method, crucial for analyzing these systems, will also be introduced. Shifting focus to music, we will explore melodic variation and its role in creating engaging melodies. Finally, with a foundation in musical notes and durations, we will investigate how chaotic systems can be mapped to musical elements, potentially leading to the generation of novel and intriguing compositions.

2.1 Chaotic System

A chaotic system is a complex system that exhibits unpredictable behavior over the long term, with sensitivity to tiny changes in initial conditions and non-linear changes over time.

A chaotic system is one that exhibits high sensitivity to initial conditions. In other words, minute variations in the starting state of the system can lead to drastically different outcomes. This paper leverages this property to generate musical patterns by establishing a correspondence between chaotic trajectories and the pitch sequence of a musical piece. While chaotic trajectories diverge over time, the mapping employed guarantees that the resulting variations retain a connection to the original composition.

A chaotic system consists of numerous interconnected subsystems, forming a highly complex network. Even small changes in the initial state of the system can lead to vastly different outcomes over time, making long-term predictions highly uncertain. This phenomenon is known as the "Butterfly Effect," where small changes in initial conditions lead to nonlinear changes in behavior over time. Graphs depicting the relationships between variables in chaotic systems often exhibit patterns resembling a spiral or randomness.

Instances of chaotic systems, such as those employed in modeling natural phenomena [8], are exemplified by their capacity to elucidate various natural phenomena, including weather patterns [9], turbulent fluid flows [10], ecological systems [11], and population dynamics [12]******. In the domain of finance and economics [13], scholars investigate chaotic dynamics to model stock market fluctuations [14], economic cycles [15], and price dynamics [16], thereby providing insights into the intrinsic unpredictability and nonlinear behavior inherent in financial systems. Likewise, within biomedical systems [17], chaotic systems assume a pivotal role in the modeling and comprehension of intricate biological systems [18], encompassing neural networks [19], cardiac rhythms [20], and gene regulatory networks [21]. This facilitates advancements in diagnosis, treatment, and understanding of diseases.The Belousov-Zhabotinsky Reaction (BZ Reaction) [22], The BZ reaction is a complex system that can be chaotic depending on the initial concentrations of the reactants and the temperature such as Spiral waves in the Belousov-Zhabotinsky reaction [23], The reaction can exhibit beautiful spiral wave patterns that emerge due to its chaotic nature, Cellular automata [24],The BZ reaction can be used to create cellular automata, which are simple computational models that can exhibit complex behavior.

One of the most famous and paradigmatic examples of chaotic systems is the celebrated Lorenz equations. They are typically formulated as a set of three coupled ordinary differential equations, like so:

$$\begin{aligned} x_1 &= \sigma(y - x), \\ x_2 &= rx - y - xz, \\ x_3 &= xy - bz, \end{aligned}$$

where x, y and z are the variables, and $\sigma > 0$, $r > 0$ and $b > 0$ are parameters. When the Lorenz equation's parameters are set to $\sigma = 10$, $r = 28$ and $b = \frac{8}{3}$, it shows chaotic behavior, as illustrated in Figure 1. The Lorenz equations [25] have been applied across various domains such as Modeling Fluid Flows, A study by J.C. Sprott in Chaos and Stability in Nonlinear Analog Circuits (2003) [26] explores how the Lorenz system can be applied to model specific fluid flows. This demonstrates its use in understanding fluid dynamics beyond just atmospheric convection.[27]. Cryptography, The chaotic nature of the Lorenz system makes it a potential candidate for secure communication. A research paper by S. Baptista et al. titled "Lorenz System Parameter Determination and Application to Break the Security of Two-channel Chaotic Cryptosystems" (2006) explores this application. While this paper discusses limitations of specific implementations, it highlights the potential of the Lorenz system in cryptography [28] Engineering Applications, The Lorenz equations can be used to model certain electrical and mechanical systems that exhibit chaotic behavior. For instance, a paper by A.A. Fathy in "Chaos, Solitons Fractals" (2009) investigates the application of the Lorenz system to brushless DC motors [29]. This showcases the potential for the Lorenz system in analyzing and potentially controlling chaotic behavior in engineering systems.

2.2 Fourth-Order Runge–Kutta Method

While analytical solutions exist for some differential equations, many require numerical approaches to approximate their behavior. The field of mathematics offers a numerical methods for solving both single and systems of linear and nonlinear differential equations. Popular examples include the Euler method and Taylor series methods. However, when it comes to achieving a balance between accuracy and efficiency, the Runge-Kutta method reigns supreme for approximating solutions.

The Runge-Kutta method [30] is a numerical methods for approximating solutions to ordinary differential equations. These equations describe how a quantity changes with respect to another variable, but often cannot be solved exactly. The Runge-Kutta method tackles these problems by breaking down the interval of interest into smaller subintervals and iteratively calculating the solution at each subinterval.

Let $\dot{y} = f(t, y)$. The approximation of y_{i+1} by fourth-order Runge–Kutta method is given

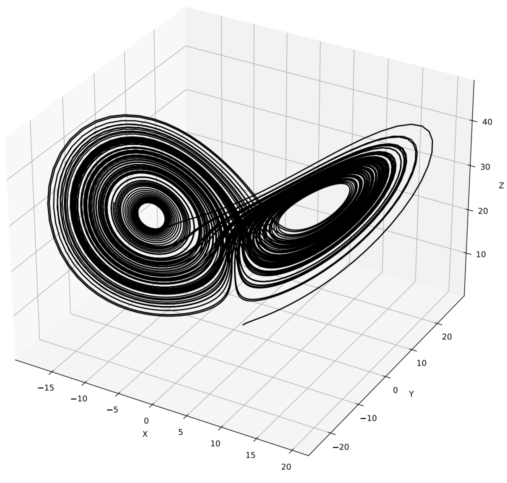

Figure 1: Chaotic behavior of the lorenz equation.

by:

$$
\begin{aligned}
y_{i+1} &= y_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4), \\
k_1 &= f(t_i, y_i), \\
k_2 &= f\left(t_i + \frac{h}{2}, y_i + \frac{h}{2}k_1\right), \\
k_3 &= f\left(t_i + \frac{h}{2}, y_i + \frac{h}{2}k_2\right), \\
k_4 &= f(t_i + h, y_i + hk_3),
\end{aligned}
$$

where $i = 0, 1, 2, ...$ and h is the step size, y is the variable and t is time.

2.3 Melodic Variation

Melodic variation is essential in songwriting, adding depth and diversity to compositions. This article emphasizes its role in enhancing musicality and emotional depth. It cautions against monotony in melodies, stressing the importance of dynamic variation to engage listeners. Additionally, melodic variation contributes to a song's structure by creating contrast between sections. Two main methods are discussed: variations on a theme and countermelodies. Lastly, the article highlights the emotional impact of melodic variation and its ability to evoke a range of feelings in listeners. In conclusion, skillful use of melodic variation enables songwriters to craft compelling compositions that resonate deeply with audiences.

2.4 Musical notes

In music theory [31], a musical note is a symbol on sheet music that represents pitch, which refers to the highness or lowness of a sound. Notes are typically designated by the letters A through G, corresponding to the solfege syllables (Do, Re, Mi, Fa, Sol, La, Ti). Musical notation also incorporates numbers to specify the octave of a pitch. For instance, C4 represents middle C, while C3 is an octave lower and C5 is an octave higher. A deeper understanding of musical notation systems can be found in resources on music theory

2.5 Musical Note Duration

Musical note duration refers to the length of time a musical note is held, which is a fundamental element in creating rhythm within a piece of music. Traditionally, note durations are represented using fractions of a whole note. Common note values include the whole note (4 beats), half note (2 beats), quarter note (1 beat), eighth note (0.5 beats), and sixteenth note (0.25 beats)

3 Main Result

This section explores the musical variations from a chaotic mapping method and combining musical variations from a chaotic mapping and melodic variation with expanded rhythm method. We will demonstrate this method through two examples. In the first example, we will illustrate how a chaotic map can be used to generate variations in musical pitch. In the second example, we will combine this method with another method for expanding rhythm to create more interesting musical variations.

Definition 3.1. Let $\mathbb{N}_n$ denote the sequence of natural number with n-elements defined by:

$$\mathbb{N}_n = \{1, 2, \ldots, n\}$$

3.1 Musical Variations from a Chaotic Mapping

For a musical sheet, let m be a positive integer representing a number of notes, $P = \{p_0, p_1, \ldots, p_{m-1}\}$ be a sequence of music pitches and

$$\dot{x}(t) = f(t, x) \tag{1}$$

be a chaotic dynamical system with an initial condition $x(0) \in \mathbb{R}^n$, where $x(t) = (x_1(t), \ldots, x_n(t))$ is differntiable for all $t \geq 0$. Let $f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function. Given a sequece $V = \{\phi_i(kh)\}_{k=0}^{m-1}$ for some $i \in \mathbb{N}_n$, where ϕ_i is a numerical solution in i-th component to (1) with a step size of h. Let $f : V \to P$ be a mapping defined by $f(v_k) = f(\phi_i(kh)) := p_k$ for all $k \in \{0\} \cup \mathbb{N}_{m-1}$.

Consequently, we introduce another sequence $\widetilde{V}_i = \left\{\tilde{\phi}_i(kh)\right\}_{k=0}^{m-1}$, where $\tilde{\phi}_i$ is a numerical solution with a new initial condition $\tilde{x}(0) \in \mathbb{R}^n$ i-th component to (1), when $\tilde{x}(0)$ start not far from $x(0)$, i.e., $\|x(0) - \tilde{x}(0)\| \leq d$ for some small positve number $d \in \mathbb{R}$. Then, we define another mapping $g : \widetilde{V} \to P$ by:

$$g(\tilde{v}_k) = g\left(\tilde{\phi}_i(kh)\right) := \begin{cases} f(\phi_i(b)) & \text{if } \exists a, b \in \operatorname{dom}\phi_i \text{ s.t. } \phi_i(a) < \tilde{\phi}(kh) \leq \phi_i(b) \\ f(\phi_i(a)) & \text{if } \tilde{\phi}(kh) < \phi_i(a) \text{ for all } a \in \operatorname{dom}\phi_i \\ f(\phi_i(b)) & \text{otherwise.} \end{cases}$$

This procedure yields, a new sequence of music pitches $\widetilde{P} = \{\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_n\}$.

Example 3.2. Let a sequence of pitches of 12 variations on Ah vous dirai-je Maman [32] in the first 3 bars in Figure 2 denoted by $P = \{C4, C4, G4, G4, A4, A4, G4, F4, F4, E4, E4\}$ in Figure 4a and Lorenz system be a dynamical system with chaotic behavior by giving Lorenz parameters $r = 28, \sigma = 10$ and $b = 2.6667$. Then the numerical solution of Lorenz system using fourth-order Runge–Kutta method with initial condition of $(1, 1, 1)$ is denoted by $X = \{1.00, 1.29, 2.13, 3.74, 6.54, 11.04, 16.69, 19.56, 15.37, 7.55, 1.20\}$ in Figure 4b, when X is a sequence of x-value from numerical solution of Lorenz system. Then, a mapping from music pitch to real value denoted by f result in Figure 4c as follows:

$f(x_i)$	$f(1.00)$	$f(1.29)$	$f(2.13)$	$f(3.74)$	$f(6.54)$	$f(11.04)$
p_i	C4	C4	G4	G4	A4	A4
$f(x_i)$	$f(16.69)$	$f(19.56)$	$f(15.37)$	$f(7.55)$	$f(1.20)$	
p_i	G4	F4	F4	E4	E4	

Figure 2: The original of 12 variations on Ah vous dirai-je Maman in the first 3 bars.

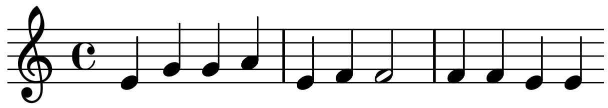

Figure 3: The new variation of 12 variations on Ah vous dirai-je Maman in the first 3 bars, generated by the Initial Condition $(1.01, 1, 1)$.

Next, We generating a new trajectory with an initial condition of $(1.01, 1, 1)$ and $X' = \{1.01, 1.30, 2.15, 3.76, 6.58, 11.10, 16.73, 19.55, 15.30, 7.48, 1.15\}$ is a sequence of x-value from new trajectory in Figure 4d. Then, a mapping from real value to music pitch denoted by g result in Figure 4e as follows:

$g(x_i')$	$g(1.01)$	$g(1.30)$	$g(2.15)$	$g(3.76)$	$g(6.58)$	$g(11.10)$
p_i	E4	G4	G4	A4	E4	F4
$g(x_i')$	$g(16.73)$	$g(19.55)$	$g(15.30)$	$g(7.48)$	$g(1.15)$	
p_i	F4	F4	F4	E4	E4	

This procedure yields, as shown in Figure 4, a new sequence of music pitches $\hat{P} = \{E4, G4, G4, A4, E4, F4, F4, F4, F4, E4, E4\}$ in Figure 4f. Which can be converted to sheet music in Figure 3. Since this method uses the same note duration and musical notes to create a new variation, so the resulting changes in the sequence compared to the original sequence might seem relatively small.

3.2 Melodic Variation with Expanded Rhythm Method

Given a note duration denoted by ϕ and divided into D equal parts, the duration, R, of each individual division is defined by the following equation:

$$R = \frac{\phi}{D}.$$

Note: In musical theory, equal parts refers to divisions that all have the same duration.

Example 3.3. Consider the music piece 12 variations on Ah vous dirai-je Maman, illustrated in Figure 5. The Figure shows that we already have 6 quarter notes and 1 half note.

If we want to divide each musical note into 4 parts, we can find the duration of each individual division as follows:

- Half note to 4 parts:

$$R = \frac{\phi}{D} = \frac{2}{4} = 0.5$$

i	1	2	3	4	5	6	7	8	9	10	11
p_i	$C4$	$C4$	$G4$	$G4$	$A4$	$A4$	$G4$	$F4$	$F4$	$E4$	$E4$

(a) The first 11 pitches of the 12 variations on Ah vous dirai-je Maman are marked below the pitch axis.

i	1	11	2	3	4	5	10	6	9	7	8
x_i	1.00	1.20	1.29	2.13	3.74	6.54	7.55	11.04	15.37	16.69	19.56

(b) The first 11 x-components of the numerical solution of Lorenz system with initial condition of $(1, 1, 1)$ are marked below the x axis.

i	1	11	2	3	4	5	10	6	9	6	7
$f(x_i)$	$C4$	$E4$	$C4$	$G4$	$G4$	$A4$	$E4$	$A4$	$F4$	$G4$	$F4$
x_i	1.00	1.20	1.29	2.13	3.74	6.54	7.55	11.04	15.37	16.69	19.56

(c) For each x-component x_i, apply the $f(x_i)$ mapping.

i	1	11	2	3	4	5	10	6	9	7	8
x'_i	1.01	1.15	1.30	2.15	3.76	6.58	7.48	11.10	15.30	16.73	19.55

(d) The first 11 x-components of the numerical solution of Lorenz system with new initial condition of $(1.01, 1, 1)$ are marked below the x axis.

i	1	11	2	3	4	5	10	6	9	6	7
$g(x'_i)$	$E4$	$E4$	$G4$	$G4$	$A4$	$E4$	$E4$	$F4$	$F4$	$F4$	$F4$
x'_i	1.01	1.15	1.30	2.15	3.76	6.58	7.48	11.10	15.30	16.73	19.55

(e) For each x-component x'_i, apply the $g(x'_i)$ mapping.

i	1	2	3	4	5	6	7	8	9	10	11
$\hat{p}_i$	$E4$	$G4$	$G4$	$A4$	$E4$	$F4$	$F4$	$F4$	$F4$	$E4$	$E4$

(f) The new variation of the first 11 pitches are marked below the pitch axis.

Figure 4: The visualizes how a chaotic mapping method can be used to generate musical variations

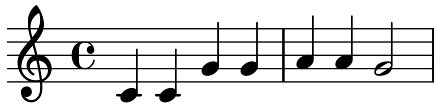

Figure 5: The original of 12 variations on Ah vous dirai-je Maman in the first 2 bars.

Figure 6: The melodic variation of Ah vous dirai-je, maman in the first 2 bars.

- Quarter note to 4 parts:
$$R = \frac{\phi}{D} = \frac{1}{4} = 0.25$$

Following this calculation, a half note can be divided into 4 eighth notes, and a quarter note can be divided into 4 sixteenth notes. This division is represented by the sequence $P = \{C4, C4, C4, C4, C4, C4, C4, C4, G4, G4, G4, G4, G4, G4, G4, G4, A4, A4, A4, A4, A4, A4, A4, A4, G4, G4, G4, G4\}$ which can be converted to sheet music, as shown in Figure 6.

3.3 Combining Musical Variations from a Chaotic Mapping and Melodic Variation with Expanded Rhythm

For a musical sheet, let m be a positive integer representing a number of notes, $P = \{p_0, p_1, \ldots, p_{m-1}\}$ be a sequence of music pitches and
$$\dot{x}(t) = f(t, x) \tag{2}$$
be a chaotic dynamical system with an initial condition $x(0) \in \mathbb{R}^n$, where $x(t) = (x_1(t), \ldots, x_n(t))$ is differntiable for all $t \geq 0$. Next, let q be a positive integer representing a number of expanded notes, $P' = \{p'_0, p'_1, \ldots, p'_{q-1}\}$ be a sequence of expanded music pitches and $f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function. Given a sequece $V = \{\phi_i(kh)\}_{k=0}^{m-1}$ for some $i \in \mathbb{N}_n$, where ϕ_i is a numerical solution in i-th component to (2) with a step size of h. Let $f : V \to P$ be a mapping defined by $f(v_k) = f(\phi_i(kh)) := p_k$ for all $k \in \{0\} \cup \mathbb{N}_{m-1}$.

Consequently, we introduce another sequence $\widetilde{V}_i = \left\{\tilde{\phi}_i(kh)\right\}_{k=0}^{q-1}$, where $\tilde{\phi}_i$ is a numerical solution with a new initial condition $\tilde{x}(0) \in \mathbb{R}^n$ i-th component to (2), when $\tilde{x}(0)$ start not far from $x(0)$, i.e., $\|x(0) - \tilde{x}(0)\| \leq d$ for some small positve number $d \in \mathbb{R}$. Then, we define another mapping $g : \widetilde{V} \to P$ by:

$$g(\tilde{v}_k) = g\left(\tilde{\phi}_i(kh)\right) := \begin{cases} f(\phi_i(b)) & \text{if } \exists a, b \in \operatorname{dom} \phi_i \text{ s.t. } \phi_i(a) < \tilde{\phi}(kh) \leq \phi_i(b) \\ f(\phi_i(a)) & \text{if } \tilde{\phi}(kh) < \phi_i(a) \text{ for all } a \in \operatorname{dom} \phi_i \\ f(\phi_i(b)) & \text{otherwise.} \end{cases}$$

This procedure yields, a new sequence of music pitches $\widetilde{P} = \{\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_q\}$.

Example 3.4. Let a sequence of pitches of 12 variations on Ah vous dirai-je Maman in the first 4 bars in Figure 7 denoted by $P = \{C4, C4, G4, G4, A4, A4, G4, F4, F4, E4, E4, D4, D4, C4\}$ in

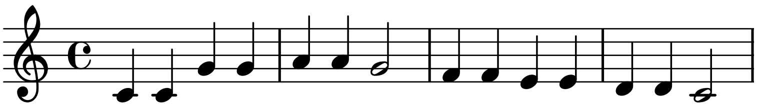

Figure 7: The original of 12 variations on Ah vous dirai-je Maman in the first 4 bars.

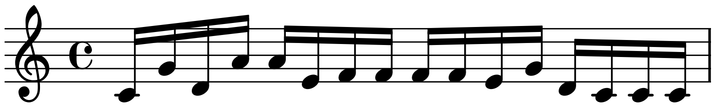

Figure 8: The new variation with melodic variation of Ah vous dirai-je, maman in the first bars, generated by the Initial Condition $(0.6, 0.5, 0.5)$.

Figure 9a and Lorenz system be a dynamical system with chaotic behavior by giving Lorenz parameters $r = 28, \sigma = 10$ and $b = 2.6667$. If we consider the first bar of 12 variations on Ah vous dirai-je Maman by melodic variation with expanded rhythm, $P' = \{C4, C4, C4, C4, C4, C4, C4, C4, G4, G4, G4, G4, G4, G4, G4, G4\}$ will be a sequence of expanded music pitches. Then the numerical solution of Lorenz system using fourth-order Runge–Kutta method with initial condition of $(0.5, 0.5, 0.5)$ is denoted by $X = \{0.50, 0.65, 1.08, 1.91, 3.41, 6.03, 10.33, 16.04, 19.69, 16.35, 8.51, 1.74, -2.45, -4.80\}$ in Figure 9b, when X is a sequence of x-value from numerical solution of Lorenz system. Then, a mapping from music pitch to real value denoted by f result in Figure 9c as follows:

$f(x_i)$	$f(0.50)$	$f(0.65)$	$f(1.08)$	$f(1.91)$	$f(3.41)$	$f(6.03)$	$f(10.33)$
p_i	C4	C4	G4	G4	A4	A4	G4
$f(x_i)$	$f(16.04)$	$f(19.69)$	$f(16.35)$	$f(8.51)$	$f(1.74)$	$f(-2.45)$	$f(-4.80)$
p_i	F4	F4	E4	E4	D4	D4	C4

Next, We generating a new trajectory with an initial condition of $(0.6, 0.5, 0.5)$ and $X' = \{0.60, 0.73, 1.21, 2.13, 3.79, 6.67, 11.30, 17.02, 19.67, 15.07, 7.11, 0.81, -2.98, -5.09, -6.33, -7.21\}$ is a sequence of x-value from new trajectory in Figure 9d. Then, a mapping from real value to music pitch denoted by g result in Figure 9e as follows:

$g(x'_i)$	$g(0.60)$	$g(0.73)$	$g(1.21)$	$g(2.13)$	$g(3.79)$	$g(6.67)$	$g(11.30)$	$g(17.02)$
p_i	C4	G4	D4	A4	A4	E4	F4	F4
$g(x'_i)$	$g(19.67)$	$g(15.07)$	$g(7.11)$	$g(0.81)$	$g(-2.98)$	$g(-5.09)$	$g(-6.33)$	$g(-7.21)$
p_i	F4	F4	E4	G4	D4	C4	C4	C4

This procedure yields, as shown in Figure 9, a new sequence of music pitches $\hat{P} = \{C4, G4, D4, A4, A4, E4, F4, F4, F4, F4, E4, G4, D4, C4, C4, C4\}$ in Figure 9f. Which can be converted to sheet music in Figure 8.

4 Discussion

This section explores the potential of Combination of Musical Variations from a Chaotic Mapping and Melodic Variation with Expanded Rhythm technique for generating musical variations.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	i
$C4$	$C4$	$G4$	$G4$	$A4$	$A4$	$G4$	$F4$	$F4$	$E4$	$E4$	$D4$	$D4$	$C4$	p_i

(a) The first 14 pitches of the 12 variations on Ah vous dirai-je Maman are marked below the pitch axis.

14	13	1	2	3	12	4	5	6	11	7	8	10	9	i
−4.80	−2.45	0.5	0.65	1.08	1.74	1.91	3.41	6.03	8.51	10.33	16.04	16.35	19.69	x_i

(b) The first 14 x-components of the numerical solution of Lorenz system with initial condition of $(0.5, 0.5, 0.5)$ are marked below the x axis.

14	13	1	2	3	12	4	5	6	11	7	8	10	9	i
$C4$	$C4$	$G4$	$G4$	$A4$	$A4$	$G4$	$F4$	$F4$	$E4$	$E4$	$D4$	$D4$	$C4$	$f(x_i)$
−4.80	−2.45	0.5	0.65	1.08	1.74	1.91	3.41	6.03	8.51	10.33	16.04	16.35	19.69	x_i

(c) For each x-component x_i, apply the $f(x_i)$ mapping.

16	15	14	13	1	2	12	3	4	5	6	11	7	10	8	9	i
−7.21	−6.33	−5.09	−2.98	0.6	0.73	0.81	1.21	2.13	3.79	6.67	7.11	11.3	15.07	17.02	19.67	x'_i

(d) The first 16 x-components of the numerical solution of Lorenz system with new initial condition of $(0.6, 0.5, 0.5)$ are marked below the x axis.

16	15	14	13	1	2	12	3	4	5	6	11	7	10	8	9	i
$C4$	$C4$	$C4$	$D4$	$C4$	$G4$	$G4$	$D4$	$A4$	$A4$	$E4$	$E4$	$F4$	$F4$	$F4$	$F4$	$g(x'_i)$
−7.21	−6.33	−5.09	−2.98	0.6	0.73	0.81	1.21	2.13	3.79	6.67	7.11	11.30	15.07	17.02	19.67	x'_i

(e) For each x-component x'_i, apply the $g(x'_i)$ mapping.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	i
$C4$	$G4$	$D4$	$A4$	$A4$	$E4$	$F4$	$F4$	$F4$	$F4$	$E4$	$G4$	$D4$	$C4$	$C4$	$C4$	$\hat{p}_i$

(f) The new variation of the first 11 pitches are marked below the pitch axis.

Figure 9: The visualizes how a chaotic mapping with melodic variation method can be used to generate musical variations

Figure 10: The original of Pachelbel's Canon in the first 10 bars.

Figure 11: The new variation with melodic variation of Pachelbel's Canon in the first 10 bars (including 2 additional bars at the end of the 10th bar).

It analyzes the impact of this approach on the resulting variations compared to traditional melodic variation techniques.

4.1 Strengths of the Combined Approach

The combined approach shows potential in generating interesting musical variations, as shown in the new variations with melodic variation of Pachelbel's Canon [33] (Figure 11) compared to the original melody (Figure 10).

This method appears to be particularly effective for pieces with a larger range of musical pitches, as exemplified by Pachelbel's Canon. The additional pitches provide more material for the chaotic mapping and melodic variation techniques to manipulate, leading to richer and more diverse variations.

4.2 Weaknesses of the Combined Approach

Let's consider the music sheet for Vanessa Carlton's A Thousand Miles (Figure 12) and the new variation with melodic variation (Figure 13). This result shows that using music sheets with too many musical note durations can lead to new variations with melodies that are difficult to play and listen to. This is a major weakness of this method.

4.3 Limitations and Considerations

Further evaluation with a wider range of musical pieces is necessary to determine the generalizability of these observations. The effectiveness of the combined approach might vary depending

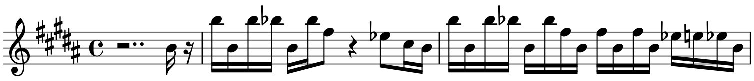

Figure 12: The original of Vanessa Carlton - A Thousand Miles in the first 3 bars.

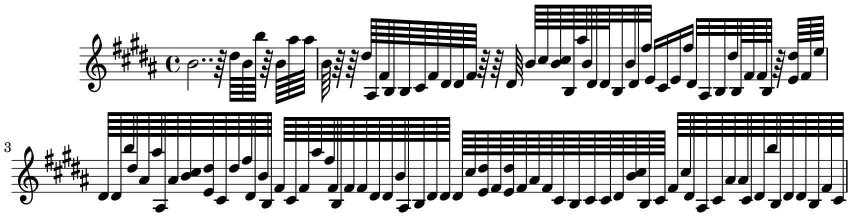

Figure 13: The new variation with melodic variation of Vanessa Carlton - A Thousand Miles in the first 3 bars.

on the musical style and characteristics of the original piece.

The computational complexity of the chaotic mapping technique should also be considered. While it offers a powerful approach for variation, it might require more processing power compared to simpler melodic variation methods.

4.4 Future Directions

Exploring different parameters and configurations within the chaotic mapping and melodic variation techniques could potentially lead to a wider range of variation styles and outcomes.

Investigating methods for user control over the variation process would be valuable. This could allow musicians to tailor the generated variations to their specific creative goals.

Integrating this combined approach into music composition tools could provide composers with a valuable tool for generating new musical ideas and exploring creative possibilities.

5 Conclusion

The proposed method, leveraging the power of chaotic systems, offers a promising approach to music composition, addressing the limitations of existing AI techniques and providing composers with a tool to generate novel, unpredictable, and diverse musical compositions while reducing computational resource demands. This method has the potential to stimulate creativity, alleviate composer's burnout, and expand the boundaries of musical expression, paving the way for a new era of music creation driven by chaos and innovation.

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