Generating of Music Variations: Dynamical Systems Approach

Rajamangala University of Technology Thanyaburi
Kanatsanun Sub-udom
Wannasa Rianthong
Patipan Somwong

May 1, 2024

Abstract

This paper introduces an innovative procedure for varying musical compositions to address the issue of composer burnout. In addition to utilizing the characteristics of chaotic dynamical systems, well-known for their sensitivity to initial conditions, this approach combines melodic variation with an extended rhythmic structure. This expansion of rhythm is achieved by prolonging the duration of musical notes, resulting in a natural blending of melodic and rhythmic elements. The proposed technique entails the mapping of musical data onto a chaotic attractor, which generates a new variation as the system's trajectories evolve. The objective is to offer composers a systematic and creative tool for exploring creative musical concepts, relieving creative fatigue, and invigorating the compositional process.

1 Introduction

Music variation serves as a catalyst for creative thinking in the songwriting process. It offers flexibility, capable of generating patterns ranging from close replicas to entirely different ones. The outcome depends on the composer's desires. When applied to compositions, it's like creating another version of the same song, making the music open to change every time it's heard. In the past, composers often employed techniques like inversion, retrograde or sections of music to expand upon the original musical content. However, these techniques gradually lost their appeal and were seen as tiresome. Music variation steps in to fill this gap. This technique opens up possibilities for composers to create entirely new musical patterns without being tied to the original framework. Individuals can transform the written notes into their own unique dynamic and fresh music.

Nowadays, artificial intelligence (AI) technologies have significantly advanced, enabling them to create music with ever-increasing proficiency [1]. Well-known AI music generation platforms such as Mubert [2] and Musicity [3] empower users with real-time music generation capabilities, enabling them to effortlessly select their preferred genre or mood and promptly receive a personalized soundtrack tailored to their preferences. On the other hand, Soundraw [4] and Boomy [5] function as AI-driven music creation tools, furnishing a diverse array of features to aid users in sculpting their musical opuses with ease. Meanwhile, AIVA [6] harnesses the power of deep learning to craft original music closely resembling the distinctive style of a particular artist or genre. Users can furnish reference tracks or articulate their desired musical aesthetics, prompting AIVA to generate fresh compositions that align precisely with their specifications. However, these technologies often require high computational resources, making them unable to run on devices with low processing power. Additionally, some AI music composition tools may produce music in limited styles.

Since limitations of AI music technology is an expensive problem to leave unaddressed, the following consequences it may lead to. Firstly, aspiring artists and musicians will miss out on the opportunity to use these tools due to limited access, as most people lack high-performance equipment. Furthermore, all music generated by AI may start to sound similar, potentially leading to a lack of musical diversity. The paper thus aims to address the aforementioned issue by employing a multi-step process. Initially, it utilizes melodic variation with expanded rhythm, which is then translated into numerical values. These numerical values are then input into a chaotic dynamical system, resulting in a new set of numbers different from the original. Finally, these numbers are mapped back to musical notes, resulting in the creation of a new piece of music. This method requires fewer computing resources compared to using AI music composition technology and allows for the creation of diverse musical compositions depending on the original song, initial values, and equations used.

In this work, we explore the theory behind chaotic system equations and the basic music theory in Section 2. In Section 3, we will commence by explaining the theory and examples from [7]. Following that, we will utilize melodic variation with expanded rhythm in Subsection 3.2 to illustrate our subsequent workflow. In Section 4, we will present and exchange ideas, opinions, and relevant information. Finally, we summarize the solutions and insights we have derived to address the issues discussed in this paper succinctly and conclusively, in Section 5

2 Literature Review

we will introduce the necessary mathematical notation and music theory concepts to understanding this paper. We begin by defining the relevant mathematical notation. Let $\mathbb{N}_n$ denote the sequence of natural number with n-elements defined by $\mathbb{N}_n = \{1, 2, \ldots, n\}$ and $\mathbb{R}^n$ be a n-dimensional euclidean space. Additionally, $\dot{x}(t)$ represents a system of ordinary differential equations. Here, $\dot{x}$ represents the derivative of x with respect to time. Furthermore, let x^* is a equilibrium point if $\dot{x}(t) = f(t, x^*) = 0$ for all $t \geq 0$. Lastly, let $\phi_i : \mathbb{R}_+ \to \mathbb{R}$, we can define the domain of the ϕ_i by $\operatorname{dom} \phi_i = \{x | x \in \mathbb{R}_+\}$.

Within music theory, a musical note is a symbol on a staff that represents pitch, which refers to the highness or lowness of a sound. Notes are typically represented by the letters A through G, corresponding to the solfege syllables (Do, Re, Mi, Fa, Sol, La, Ti). Musical notation also incorporates numbers to specify the precise pitch of a musical note. As shown in Figure 1, there are two sets of five horizontal lines, known as the treble clef (the top set of lines) and the bass clef (the bottom set of lines). The treble clef is used to represent higher pitches, while the bass clef is used to represent lower pitches. Each line and space on the staff corresponds to a specific pitch. For example, the note C4 (indicated in Figure 1) is lower in pitch than the note D4, which is located on the line above it. Similarly, the note C6 (shown above the treble clef in Figure 1) represents a higher pitch than the note C5, which is located on the second-highest line of the treble clef. Therefore, notes higher on the staff represent higher pitches, and notes lower on the staff represent lower pitches. It is important to note that the pitches represented on the treble clef are generally higher than those represented on the bass clef. For instance, C5 (located on the treble clef) has a higher pitch than B4 (located on the bass clef). Additionally, each musical note has a duration symbol that specifies how long it should be played. Durations are typically defined using the unit "beat," which refers to a chosen time interval. For example, if we establish from a duration table (see Figure 2) that 1 beat is equivalent to playing a musical note for 1 second, then 2 beats would correspond to playing a note for 2 seconds. For convenience, we can establish a general rule: let t beats equal playing a musical note for t seconds, where t is any positive number. This rule allows us to understand the meaning of fractional beats. For instance, 0.5 beats would signify playing a musical note for 0.5 seconds. This example emphasizes that the relationship of t beats to playing a note for t seconds is a common way to explain the fundamental concept of musical note durations.

Recent advancements in the study of chaotic systems have sparked significant interest in designing systems with diverse characteristics. These systems range from those devoid of equilibrium points [**ren2018new**, **wang2019sbox**] to those with only one stable equilib-

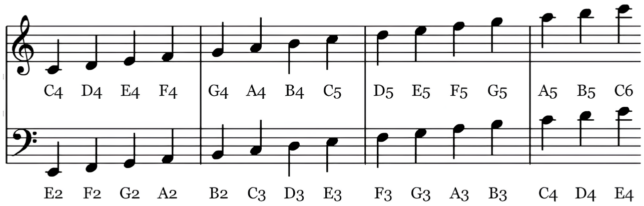
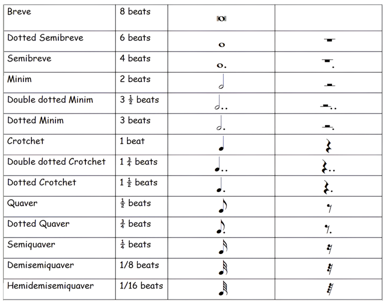


Figure 1: The musical note table start from E2 to C6.



Breve	8 beats		
Dotted Semibreve	6 beats		
Semibreve	4 beats		
Minim	2 beats		
Double dotted Minim	$3\frac{1}{2}$ beats		
Dotted Minim	3 beats		
Crotchet	1 beat		
Double dotted Crotchet	$1\frac{3}{4}$ beats		
Dotted Crotchet	$1\frac{1}{2}$ beats		
Quaver	$\frac{1}{2}$ beats		
Dotted Quaver	$\frac{3}{4}$ beats		
Semiquaver	$\frac{1}{4}$ beats		
Demisemiquaver	1/8 beats		
Hemidemisemiquaver	1/16 beats		

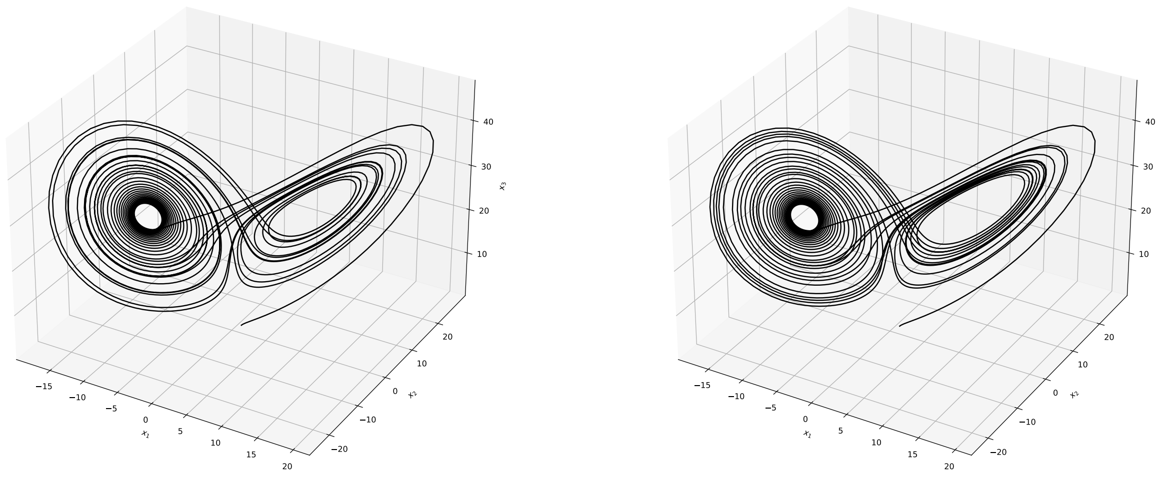
Figure 2: The duration table of musical note.

rium [**wang2012chaotic**], two stable equilibria [**wang2017chaotic**], multi-scroll attractors [**rajagopal2019multiscroll**, **pehlivan2019multiscroll**], various types of symmetry [**field2009symmetry**], and beyond. There are also uses of memristors, memcapacitors, cubic nonlinear resistors, and piecewise linear functions as nonlinear factors to create chaos [**wei2018synchronisation**, **varan2018control**, **rajagopal2019dynamical**, **akgul2019chaotic**, **rajagopal2019simple**]. For example, these properties are often applied to chaotic systems those employed in modeling natural phenomena [8], are exemplified by their capacity to elucidate various natural phenomena, including weather patterns [9], turbulent fluid flows [10], ecological systems [11], and population dynamics [12]. In the domain of finance and economics [13], scholars investigate chaotic dynamics to model stock market fluctuations [14], economic cycles [15], and price dynamics [16], thereby providing insights into the intrinsic unpredictability and nonlinear behavior inherent in financial systems. Likewise, within biomedical systems [17], chaotic systems assume a pivotal role in the modeling and comprehension of intricate biological systems [18], encompassing neural networks [19], cardiac rhythms [20], and gene regulatory networks [21]. The Belousov-Zhabotinsky Reaction, a chaotic system exhibiting spiral wave patterns from its nature, can create cellular automata models with complex behavior based on reactant concentrations and temperature [22, 23, 24]. A chaotic system is characterized by three fundamental properties essential for comprehending its behavior. Firstly, it displays long-term dynamics devoid of periodicity, avoiding convergence towards a fixed point. Secondly, its behavior, though seemingly irregular, stems from deterministic nonlinear interactions rather than random inputs, maintaining intentionality. Thirdly, it exhibits sensitivity to initial conditions, leading to exponential divergence in trajectories situated close to each other. These defining traits of chaos have found extensive applications across various disciplines. However, despite their broad recognition, universally acknowledged theories elucidating these phenomena may still prove elusive.

One of the interesting chaotic systems [25] is the lorenz system, a system of three ordinary differential equations with the derivatives of x_1, x_2 and x_3 with respect to t given by:

$$\begin{aligned}\dot{x}_1 &= \sigma(x_2 - x_1),\\ \dot{x}_2 &= rx_1 - x_2 - x_1x_3,\\ \dot{x}_3 &= x_1x_2 - bx_3,\end{aligned}$$

where $\sigma > 0$, $r > 0$ and $b > 0$ are parameters. If we set the Lorenz system's parameters set



(a) initial condition $(1, 1, 1)$. (b) initial condition $(0.999, 1, 1)$.

Figure 3: Chaotic behavior of the Lorenz system with different initial conditions.

to $\sigma = 10$, $r = 28$ and $b = \dfrac{8}{3}$, it exhibits chaotic behavior, as illustrated in Figure 3. This figure shows the aperiodic long-term behavior (where trajectories do not settle down to fixed points), its deterministic nature (the system has no random or noisy inputs or parameters), and its sensitive dependence on initial conditions (nearby trajectories exhibit different trajectory behaviors). the Lorenz systems have four simple properties. Firstly, their nonlinearity captures the intricate interactions among variables, making them inseparable. Secondly, they exhibit symmetry in time reversal and energy alteration, guaranteeing consistent outcomes despite changes in parameters. Thirdly, they illustrate volume contraction, indicating a continual decrease in system energy and suggesting a tendency for energy or fluid volume to diminish over time Lastly, Lorenz equations have equilibrium point where variable values persist, ensuring long-term stability despite system fluctuations. The Lorentz equation has been extensively applied, particularly in understanding fluid dynamics, extending its reach beyond atmospheric convection [26, 27]. Moreover, it holds promise in cryptography for enhancing the security of communication systems [28] and is indispensable for analyzing chaotic phenomena within engineering systems such as brushless DC motors [29].

Diana S. Dabby [7] pioneered the idea of generating musical variations through chaotic mappings, leveraging chaos theory to create unique renditions of a musical piece. This method capitalizes on the inherent sensitivity of chaotic systems to their initial conditions. In Dabby's work, a sequence of musical pitches, akin to the notes in a melody, is paired with the unpredictable jumps of a specific chaotic system, namely the Lorenz attractor. Imagine a number line where each position represents a note in the original piece. The chaotic system's jumps dictate where on this number line each note from the original sequence falls. By introducing a slightly different starting point for these jumps, a new sequence of notes is generated, resulting in a fascinating variation of the original melody.

3 Main Result

This section presents the musical variations generated from a chaotic mapping method and the melodic variation with expanded rhythm method. In the first method, we will demonstrate how the chaotic mapping can be used to create new variations in musical pitch. In the second method, we will combine the musical variations from the chaotic mapping method with the melodic variation with expanded rhythm method to create more interesting musical variations.

3.1 Musical Variations from a Chaotic Mapping Method

We introduce a modified version of the dabby method [7] for generating musical variations. For a sheet music, let m be a positive integer representing a number of notes, $\{p_k\}_{k=0}^{m-1}$ be a sequence of music pitches and

$$\dot{x}(t) = f(t, x) \tag{1}$$

be a chaotic dynamical system with an initial condition $x(0) \in \mathbb{R}^n$, where $x(t) = (x_1(t), \ldots, x_n(t))$ is differntiable for all $t \geq 0$ and $f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function. Given a sequece $\{\phi_i(kh)\}_{k=0}^{m-1}$ for some $i \in \mathbb{N}_n$ and a step size of h, where $\phi_i : \mathbb{R}_+ \to \mathbb{R}$ is a numerical solution in i-th component to the system (1). Let g be a mapping defined by

$$g(\phi_i(kh)) := p_k \tag{2}$$

for all $k \in \{0\} \cup \mathbb{N}_{m-1}$.

In order to generate a new variation of music, let $\left\{\tilde{\phi}_i(kh)\right\}_{k=0}^{m-1}$ be a sequence of new trajectory with the initial condition $\tilde{x}(0) \in \mathbb{R}^n$, where $\tilde{\phi}_i$ is a numerical solution in i-th component to the system (1) and $\tilde{x}(0)$ located not far from the $x(0)$. i.e., $\|x(0) - \tilde{x}(0)\| \leq d$ for some small positive number $d \in \mathbb{R}$. Consequently, we define another mapping l by:

$$l\left(\tilde{\phi}_i(kh)\right) := \begin{cases} g(\phi_i(b)) & \text{if } \exists a, b \in \operatorname{dom} \phi_i \text{ s.t. } \phi_i(a) < \tilde{\phi}_i(kh) \leq \phi_i(b) \\ & \text{and } \nexists c \in \operatorname{dom} \phi_i \text{ s.t. } \phi_i(a) < \phi_i(c) \leq \phi_i(b), \\ g(\phi_i(a)) & \text{if } \tilde{\phi}_i(kh) < \phi_i(a) \text{ for all } a \in \operatorname{dom} \phi_i, \\ g(\phi_i(b)) & \text{otherwise,} \end{cases} \tag{3}$$

resulting in the sequence $\left\{l\left(\tilde{\phi}_i(kh)\right)\right\}_{k=0}^{m-1}$, which represents a new variation of the original musical pitch $\{p_k\}_{k=0}^{m-1}$.

Example 3.1. Consider the sheet music of Ah vous dirai-je Maman [30] in the first 3 bars, illustrated in Figure 4a. We can convert this sheet music into a sequence $\{p_k\}_{k=0}^{10} = \{C4, C4, G4, G4, A4, A4, G4, F4,$
Let

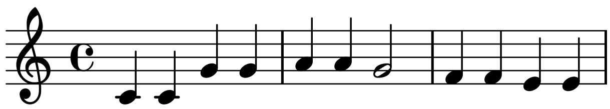
$$\dot{x}(t) = \begin{bmatrix} f_1(t,x) \\ f_2(t,x) \\ f_3(t,x) \end{bmatrix} = \begin{bmatrix} \sigma(x_2 - x_1) \\ rx_1 - x_2 - x_1x_3 \\ x_1x_2 - bx_3 \end{bmatrix} \tag{4}$$

be Lorenz system with parameters $r = 28, \sigma = 10$ and $b = 2.6667$, where $x = (x_1, x_2, x_3)$. The first 11 numerical solution of (4) with the initial condition $x(0) = (1, 1, 1)$ and step size of $h = 0.01$ be a sequence $\{\phi_1(kh)\}_{k=0}^{10} = \{1.00, 1.29, 2.13, 3.74, 6.54, 11.04, 16.69, 19.56, 15.37, 7.55, 1.20\}$, when $\phi_1(kh)$ is a numerical solution in first component to the system (4). We therefore define a mapping g from $\{\phi_1(kh)\}_{k=0}^{10}$ to $\{p_k\}_{k=0}^{10}$ according to the mapping (2), shown as follows:

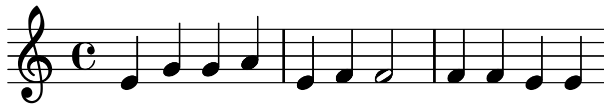
k	0	1	2	3	4	5	6	7	8	9	10
kh	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
$\phi_1(kh)$	1.00	1.29	2.13	3.74	6.54	11.04	16.69	19.56	15.37	7.55	1.20
$g(\phi_1(kh))$	C4	C4	G4	G4	A4	A4	G4	F4	F4	E4	E4

In order to generate a new variation of music, the sequence

$$\left\{\tilde{\phi}_1(kh)\right\}_{k=0}^{10} = \{1.01, 1.30, 2.15, 3.76, 6.58, 11.10, 16.73, 19.55, 15.30, 7.48, 1.15\}$$



(a) The original of Ah vous dirai-je Maman in the first 3 bars.



(b) The new variation of Ah vous dirai-je Maman in the first 3 bars, generated by the Initial Condition $(1.01, 1, 1)$.

Figure 4: The original and new variation of Ah vous dirai-je Maman

is the first 11 numerical solution of (4) but from the initial condition $\tilde{x}(0) = (1.01, 1, 1)$, when $\tilde{\phi}_1(kh)$ is a numerical solution in first component to the system (4). We then define a mapping l from $\left\{\tilde{\phi}_1(kh)\right\}_{k=0}^{10}$ to new musical pitches according to the mapping (3), shown as follows:

k	0	1	2	3	4	5	6	7	8	9	10
kh	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
$\tilde{\phi}_1(kh)$	1.01	1.30	2.15	3.76	6.58	11.10	16.73	19.55	15.30	7.48	1.15
$l(\tilde{\phi}_1(kh))$	E4	G4	G4	A4	E4	F4	F4	F4	F4	E4	E4

this mapping l yields the sequence $\{l(\tilde{\phi}_1(kh))\}_{k=0}^{10} = \{E4, G4, G4, A4, E4, F4, F4, F4, F4, E4, E4\}$, which can be converted to sheet music in Figure 4b (See Figure 5 for the visualization of this method). The new sequence of musical pitches changes the note in index $k = \{1, 2, 4, 5, 6, 7\}$.

3.2 Melodic Variation with Expanded Rhythm Method

Let $\{p_k\}_{k=0}^{m-1}$ be a sequence of music pitches, we can define a sequence of expanded music pitches $\{p'_k\}_{k=0}^{mq-1}$, when q be a positive integer representing a number of expanded notes for each p_k and $p_k = p'_{kq} = p'_{kq+1} = \cdots = p'_{q(k+1)-1}$.

Example 3.2. Let $\{p_k\}_{k=0}^{3} = \{C4, C4, G4, G4\}$ be a sequence of music pitches from the music piece Ah vous dirai-je Maman in the first bars, illustrated in Figure 6a. Given $q = 4$ be the number of expanded notes for each p_k, we can define a sequence of expanded music pitches $\{p'_k\}_{k=0}^{15} = \{C4, C4, C4, C4, C4, C4, C4, C4, G4, G4, G4, G4, G4, G4, G4, G4\}$, illustrated in Figure 6b.

Example 3.3. Consider the sequence $\{p_k\}_{k=0}^{13} = \{C4, C4, G4, G4, A4, A4, G4, F4, F4, E4, E4, D4, D4, C4\}$ from the sheet music of Ah vous dirai-je Maman in the first 4 bars, illustrated in Figure 7a. We can apply the musical variations from a chaotic mapping method described in Section 3.1 to the sequence $\{p'_k\}_{k=0}^{15}$ from example 3.2 by using the initial condition $x(0) = (0.5, 0.5, 0.5)$ and new trajectory initial condition $\tilde{x}(0) = (0.6, 0.5, 0.5)$. This results in a sequence of new pitches

1	2	3	4	5	6	7	8	9	10	11	k
$C4$	$C4$	$G4$	$G4$	$A4$	$A4$	$G4$	$F4$	$F4$	$E4$	$E4$	p_k

(a) The first 11 pitches of the Ah vous dirai-je Maman are marked below the 1D axis.

1	11	2	3	4	5	10	6	9	7	8	k
1.00	1.20	1.29	2.13	3.74	6.54	7.55	11.04	15.37	16.69	19.56	$\phi_1(kh)$

(b) The first 11 components from the numerical solution of Lorenz system with initial condition of $(1, 1, 1)$ are marked below the 1D axis.

1	11	2	3	4	5	10	6	9	6	7	k
$C4$	$E4$	$C4$	$G4$	$G4$	$A4$	$E4$	$A4$	$F4$	$G4$	$F4$	$g(\phi_1(kh))$
1.00	1.20	1.29	2.13	3.74	6.54	7.55	11.04	15.37	16.69	19.56	$\phi_1(kh)$

(c) For each component in $\{\phi_1(kh)\}_{k=0}^{10}$, apply the $g(\phi_1(kh))$ mapping.

1	11	2	3	4	5	10	6	9	7	8	k
1.01	1.15	1.30	2.15	3.76	6.58	7.48	11.10	15.30	16.73	19.55	$\tilde{\phi}_1(kh)$

(d) The first 11 x-components from the numerical solution of Lorenz system with new initial condition of $(1.01, 1, 1)$ are marked below the 1D axis.

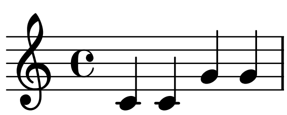
1	11	2	3	4	5	10	6	9	6	7	k
$E4$	$E4$	$G4$	$G4$	$A4$	$E4$	$E4$	$F4$	$F4$	$F4$	$F4$	$l(\tilde{\phi}_1(kh))$
1.01	1.15	1.30	2.15	3.76	6.58	7.48	11.10	15.30	16.73	19.55	$\tilde{\phi}_1(kh)$

(e) For each component in $\{\tilde{\phi}_1(kh)\}_{k=0}^{10}$, apply the $l(\tilde{\phi}_1(kh))$ mapping.

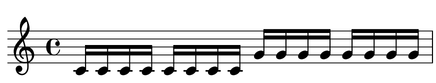
1	2	3	4	5	6	7	8	9	10	11	k
$E4$	$G4$	$G4$	$A4$	$E4$	$F4$	$F4$	$F4$	$F4$	$E4$	$E4$	$l(\tilde{\phi}_1(kh))$

(f) The new variation of the first 11 pitches are marked below the 1D axis.

Figure 5: The figure for visualizes how a chaotic mapping method can be used to generate musical variations

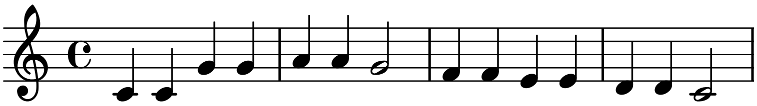


(a) The original of Ah vous dirai-je Maman in the first bars.

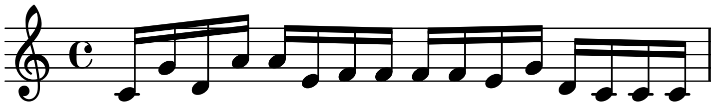


(b) The melodic variation of Ah vous dirai-je, maman in the first bars.

Figure 6: The original and melodic variation of Ah vous dirai-je Maman.



(a) The original of Ah vous dirai-je Maman in the first 4 bars.



(b) The new variation with melodic variation of Ah vous dirai-je, maman in the first bars, generated by the Initial Condition $(0.6, 0.5, 0.5)$.

Figure 7: melodic variation method of Ah vous dirai-je Maman.

$\{l(\tilde{\phi}_1(kh))\}_{k=0}^{15} = \{C4, G4, D4, A4, A4, E4, F4, F4, F4, F4, E4, G4, D4, C4, C4, C4\}$ (See Figure 8 for the visualization of this method), which can be converted to musical notation in Figure 7b. The new sequence of musical pitches is significantly different from the original sequence $\{p'_k\}_{k=0}^{15}$.

4 Discussion

This section discusses the potential of combining musical variations from a chaotic mapping and melodic variation with expanded rhythm method for generating musical variations. We will also analyze the impact of this approach on the resulting variations in other sheet music. For convenience, we will refer to this combination of methods as the combined approach throughout this section.

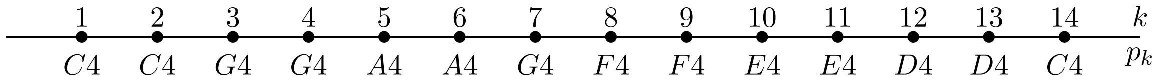
4.1 Strengths of the Combined Approach

The combined approach shows potential in generating interesting musical variations, as shown in the new variations with melodic variation of Pachelbel's Canon [31] (Figure 10) compared to the original melody (Figure 9).

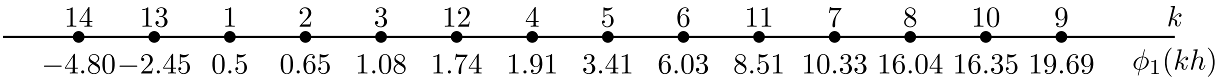
This method appears to be particularly effective for pieces with a larger range of musical pitches, as exemplified by Pachelbel's Canon. The additional pitches provide more material for the chaotic mapping and melodic variation techniques to manipulate, leading to richer and more diverse variations.

4.2 Weaknesses of the Combined Approach

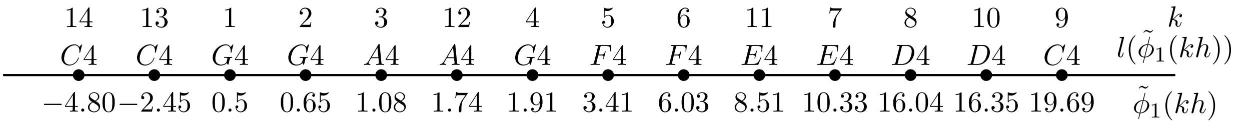
Let's consider the sheet music Vanessa Carlton A Thousand Miles (Figure 11) and the new variation with melodic variation (Figure 12). This result shows that using sheet music with too many musical note durations can lead to new variations with melodies that are difficult to play and listen to. This is a major weakness of this method.



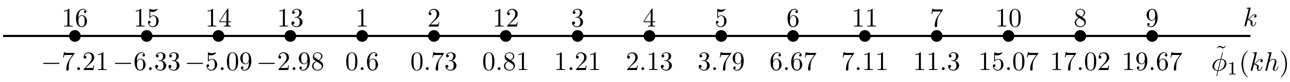
(a) The first 14 pitches of the Ah vous dirai-je Maman are marked below the 1D axis.



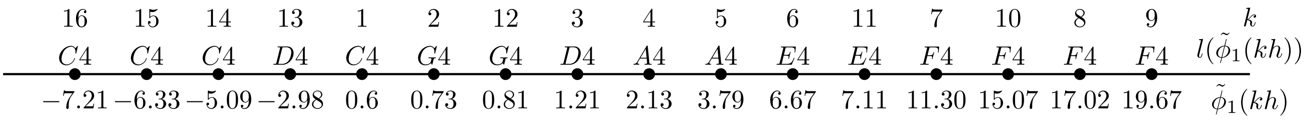
(b) The first 14 components from the numerical solution of Lorenz system with initial condition of $(0.5, 0.5, 0.5)$ are marked below the 1D axis.



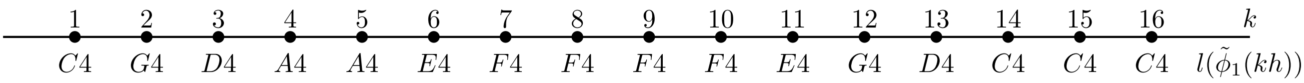
(c) For each component in $\{\phi_1(kh)\}_{k=0}^{13}$, apply the $g(\phi_1(kh))$ mapping.



(d) The first 16 components from the numerical solution of Lorenz system with new initial condition of $(0.6, 0.5, 0.5)$ are marked below the 1D axis.



(e) For each component in $\{\tilde{\phi}_1(kh)\}_{k=0}^{15}$, apply the $l(\tilde{\phi}_1(kh))$ mapping.



(f) The new variation of the first 16 pitches are marked below the 1D axis.

Figure 8: The figure for visualizes how a chaotic mapping with melodic variation method can be used to generate musical variations

Figure 9: The original of Pachelbel's Canon in the first 10 bars.

Figure 10: The new variation with melodic variation of Pachelbel's Canon in the first 10 bars (including 2 additional bars at the end of the 10th bar).

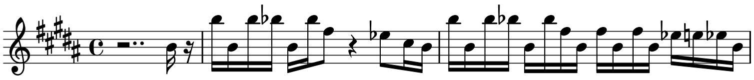


Figure 11: The original of Vanessa Carlton - A Thousand Miles in the first 3 bars.

Figure 12: The new variation with melodic variation of Vanessa Carlton - A Thousand Miles in the first 3 bars.

4.3 Limitations and Considerations

Further evaluation with a wider range of musical pieces is necessary to determine the generalizability of these observations. The effectiveness of the combined approach might vary depending on the musical style and characteristics of the original piece.

The computational complexity of the chaotic mapping technique should also be considered. While it offers a powerful approach for variation, it might require more processing power compared to simpler melodic variation methods.

4.4 Future Directions

Exploring different parameters and configurations within the chaotic mapping and melodic variation techniques could potentially lead to a wider range of variation styles and outcomes.

Investigating methods for user control over the variation process would be valuable. This could allow musicians to tailor the generated variations to their specific creative goals.

Integrating this combined approach into music composition tools could provide composers with a valuable tool for generating new musical ideas and exploring creative possibilities.

5 Conclusion

The proposed method, leveraging the power of chaotic systems, offers a promising approach to music composition, addressing the limitations of existing AI techniques and providing composers with a tool to generate novel, unpredictable, and diverse musical compositions while reducing computational resource demands. This method has the potential to stimulate creativity, alleviate composer's burnout, and expand the boundaries of musical expression, paving the way for a new era of music creation driven by chaos and innovation.

References

[1] Alexandra Bonnici et al. *Music and AI*. Frontiers Media SA, 2021. 170 pp. ISBN: 978-2-88966-602-7.

[2] Mubert Inc. *Mubert - Human and AI music generator for your video content, podcasts and apps*. 2024. URL: https://mubert.com/ (visited on 04/24/2024).

[3] Musicfy Inc. *Musicfy - Change Your Voice with AI*. 2023. URL: https://musicfy.lol/ (visited on 04/27/2024).

[4] SOUNDRAW Inc. *SOUNDRAWTHE MUSIC TOOL FOR CREATORS AND ARTISTS*. 2020. URL: https://soundraw.io (visited on 04/24/2024).

[5] Boomy Corporation. *Boomy - Unleash your creativity. Make music with Boomy AI*. 2023. URL: https://boomy.com/ (visited on 04/24/2024).

[6] Aiva Technologies SARL. *AIVA Your personal AI music generation assistant*. 2024. URL: https://www.aiva.ai/ (visited on 04/24/2024).

[7] Diana S. Dabby. "Musical variations from a chaotic mapping". In: *Chaos: An Interdisciplinary Journal of Nonlinear Science* 6.2 (1996), pp. 95–107. ISSN: 1054-1500. DOI: 10.1063/1.166171.

[8] L. Sanabria. "Modelling natural phenomena". In: Oct. 2020, p. 61.

[9] *Weather Pattern - an overview — ScienceDirect Topics*. URL: https://www.sciencedirect.com/topics/earth-and-planetary-sciences/weather-pattern (visited on 04/28/2024).

[10] *Turbulent Flow - an overview — ScienceDirect Topics*. URL: https://www.sciencedirect.com/topics/engineering/turbulent-flow (visited on 04/28/2024).

[11] Marcus Crawford. "Ecological Systems Theory: Exploring the Development of the Theoretical Framework as Conceived by Bronfenbrenner". en. In: *Journal of Public Health Issues and Practices* 4.2 (2020). ISSN: 2581-7264. DOI: 10.33790/jphip1100170. URL: https://gexinonline.com/archive/journal-of-public-health-issues-and-practices/JPHIP-170 (visited on 04/28/2024).

[12] *Population Dynamics - an overview — ScienceDirect Topics*. URL: https://www.sciencedirect.com/topics/social-sciences/population-dynamics (visited on 04/28/2024).

[13] Yi Liao et al. "A Study on the Complexity of a New Chaotic Financial System". en. In: *Complexity* 2020 (Oct. 2020). Publisher: Hindawi, e8821156. ISSN: 1076-2787. DOI: 10.1155/2020/8821156. URL: https://www.hindawi.com/journals/complexity/2020/8821156/ (visited on 04/28/2024).

[14] Markus Vogl. "Chaos measure dynamics in a multifactor model for financial market predictions". In: *Communications in Nonlinear Science and Numerical Simulation* 130 (Mar. 2024), p. 107760. ISSN: 1007-5704. DOI: 10.1016/j.cnsns.2023.107760. URL: https://www.sciencedirect.com/science/article/pii/S1007570423006810 (visited on 04/28/2024).

[15] Angelo M. Tusset et al. "Dynamic Analysis and Control of a Financial System with Chaotic Behavior Including Fractional Order". en. In: *Fractal and Fractional* 7.7 (July 2023). Number: 7 Publisher: Multidisciplinary Digital Publishing Institute, p. 535. ISSN: 2504-3110. DOI: 10.3390/fractalfract7070535. URL: https://www.mdpi.com/2504-3110/7/7/535 (visited on 04/28/2024).

[16] Driss Ait Omar et al. "Chaotic Dynamics in Joint Price QoS Game with Heterogeneous Internet Service Providers". In: *Journal of Computer Networks and Communications* 2022 (Jan. 2022). ISSN: 2090-7141. DOI: 10.1155/2022/9541887. URL: https://doi.org/10.1155/2022/9541887 (visited on 04/28/2024).

[17] V. V. Grigorenko et al. "Study of chaotic dynamics of the parameters of biomedical systems". In: *AIP Conference Proceedings* 2467.1 (June 2022), p. 060037. ISSN: 0094-243X. DOI: 10.1063/5.0093011. URL: https://doi.org/10.1063/5.0093011 (visited on 04/28/2024).

[18] Feng Li. "Incorporating fractional operators into interaction dynamics of a chaotic biological model". In: *Results in Physics* 54 (Nov. 2023), p. 107052. ISSN: 2211-3797. DOI: 10.1016/j.rinp.2023.107052. URL: https://www.sciencedirect.com/science/article/pii/S2211379723008458 (visited on 04/28/2024).

[19] Hairong Lin et al. "Chaotic dynamics in a neural network with different types of external stimuli". In: *Communications in Nonlinear Science and Numerical Simulation* 90 (Nov. 2020), p. 105390. ISSN: 1007-5704. DOI: 10.1016/j.cnsns.2020.105390. URL: https://www.sciencedirect.com/science/article/pii/S1007570420302227 (visited on 04/28/2024).

[20] Augusto Cheffer and Marcelo A. Savi. "Biochaos in cardiac rhythms". en. In: *The European Physical Journal Special Topics* 231.5 (June 2022), pp. 833–845. ISSN: 1951-6401. DOI: 10.1140/epjs/s11734-021-00314-7. URL: https://doi.org/10.1140/epjs/s11734-021-00314-7 (visited on 04/28/2024).

[21] Abicumaran Uthamacumaran. "A review of dynamical systems approaches for the detection of chaotic attractors in cancer networks". In: *Patterns* 2.4 (Apr. 2021), p. 100226. ISSN: 2666-3899. DOI: 10.1016/j.patter.2021.100226. URL: https://www.sciencedirect.com/science/article/pii/S2666389921000404 (visited on 04/28/2024).

[22] Artur Karimov et al. “Empirically developed model of the stirring-controlled Belousov–Zhabotinsky reaction”. In: *Chaos, Solitons & Fractals* 176 (Nov. 2023), p. 114149. ISSN: 0960-0779. DOI: 10.1016/j.chaos.2023.114149. URL: https://www.sciencedirect.com/science/article/pii/S0960077923010512 (visited on 04/28/2024).

[23] Jiraporn Luengviriya et al. “Meandering spiral waves in a bubble-free Belousov–Zhabotinsky reaction with pyrogallol”. In: *Chemical Physics Letters* 588 (Nov. 2013), pp. 267–271. ISSN: 0009-2614. DOI: 10.1016/j.cplett.2013.10.025. URL: https://www.sciencedirect.com/science/article/pii/S0009261413012906 (visited on 04/28/2024).

[24] Bastien Chopard et al. *Cellular Automata: 15th International Conference on Cellular Automata for Research and Industry, ACRI 2022, Geneva, Switzerland, September 12–15, 2022, Proceedings*. en. Springer Nature, Aug. 2022. ISBN: 978-3-031-14926-9.

[25] *Lorenz Equation - an overview — ScienceDirect Topics*. URL: https://www.sciencedirect.com/topics/engineering/lorenz-equation (visited on 04/24/2024).

[26] “Compact Modeling of Nonlinear Analog Circuits Using System Identification via Semidefinite Programming and Incremental Stability Certification”. In: 29 (Sept. 2010), pp. 1149–1162. DOI: 10.1109/TCAD.2010.2049155.

[27] Zeraoulia Elhadj. *Models and Applications of Chaos Theory in Modern Sciences*. Journal Abbreviation: Models and Applications of Chaos Theory in Modern Sciences Publication Title: Models and Applications of Chaos Theory in Modern Sciences. Sept. 2011. ISBN: 978-1-57808-722-8. DOI: 10.1201/b11408.

[28] Amalia Orue et al. “Lorenz System Parameter Determination and Application to Break the Security of Two-channel Chaotic Cryptosystems”. In: (July 2006).

[29] Guoyuan Qi. “Energy Cycle of Brushless DC Motor Chaotic System”. In: *Applied Mathematical Modelling* 51 (July 2017). DOI: 10.1016/j.apm.2017.07.025.

[30] Maurice Hinson. *12 Variations on Ah, Vous Dirai-Je, Maman, K. 265*. ALFRED PUBN, 1987. 16 pp. ISBN: 978-0-7390-2032-6.

[31] Johann Pachelbel and Willard A. Palmer. *Canon in D: Variations on the Theme for Piano*. Alfred Music, 2005. 8 pp. ISBN: 978-0-7390-7134-2.