# EE321 Lab2:

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$1-\ h[n] = (7/8)^n \mu[n-4]$

* Is the system causal?

The system is $\rightarrow y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] . h[n-k]$

If the system is causal, to find $y[n_0]$, at an arbitrary instant $n_0$, no parts of $x[n]$ where $n > n_0$ are needed.

Therefore it can be said that if $h[n-k]=0$ when $n<k$ the system is causal, if the system is LTI.

$h[n-k] = \left(\frac{7}{8}\right)^{n-k} . \mu[n-k-4]$ for $n<k$, $n-k<0$
then $n-k-4<0 \Rightarrow \mu[n-k-4]=0$ for $k>n$.
For $k>n$, $(8/9)^n . \mu[n-k-4] . x[n] = 0$ regardless of $x[n]$,
so, values of $x[n]$ for $n > n_0$ are not needed to calculate $y[n_0]$.

$\Rightarrow$ The system is CAUSAL.

* Is the system stable?

$\rightarrow$ If the system, $y[n] = \sum_{k=-\infty}^{\infty} x[k] . h[n-k]$ is stable,

then $|y[n]| < c$ for $|x[n]| < B$, B and c are constants.

$|y[n]| = \left|\sum_{k=-\infty}^{\infty} x[k] . h[n-k]\right| \leq \sum_{-\infty}^{\infty} |h[k]| . \underbrace{|x[n-k]|}_{<B}$ for $\forall n, k$.

then $|y[n]| < B . \sum_{k=-\infty}^{\infty} |h[k]|$ for all $k$.

$\rightarrow$ Now let's look at our system.

$|y[n]| = \left|\sum_{k=-\infty}^{\infty} \underbrace{x[n-k]}_{<B} . (7/8)^k . \mu[k-4]\right| < B . \sum_{k=-\infty}^{\infty} \left|\left(\frac{7}{8}\right)^k . \underbrace{\mu[k-4]}_{\substack{0 \text{ for } k<4 \\ 1 \text{ for } k>4}}\right|$

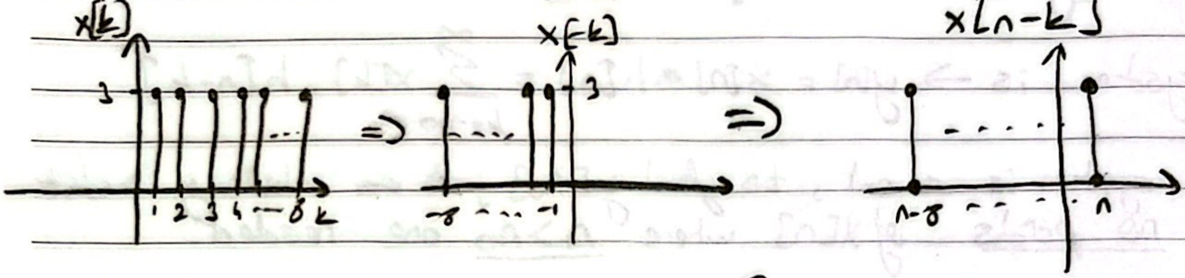
$\hookrightarrow B . \sum_{k=4}^{\infty} |(7/8)^k|$ and since $\sum_{k=3}^{\infty} \left(\frac{7}{8}\right)^k < 1$.

$|y[n]| < B$ for $\forall n, B, y[n]$. $\rightarrow$ System is STABLE.

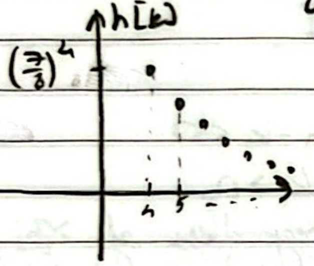
$\rightarrow$ Plot and MatLab code are available in Appendix A.

a) $h[n] = (7/8)^n \cdot u[n-4]$, $\quad x[n] = \begin{cases} 3 & \text{if } 0 \leq n \leq 8 \\ 0 & \text{else} \end{cases}$

$$y_1[n] = x_1[n] * h[n]$$
$$= \sum_{k=-\infty}^{\infty} x_1[k] \cdot h[n-k] = \sum_{k=-\infty}^{\infty} h[k] \cdot x_1[n-k]$$




h[k] is 0 for n<4




$$y_1[n] = \begin{cases} 0 & \text{for } n<4 \\ \sum_{k=4}^{n} 3 \cdot (7/8)^k & \text{for } 4 \leq n < 12 \\ \sum_{k=n-\infty}^{n} 3 \cdot (7/8)^k & \text{for } 12 \leq n \end{cases}$$

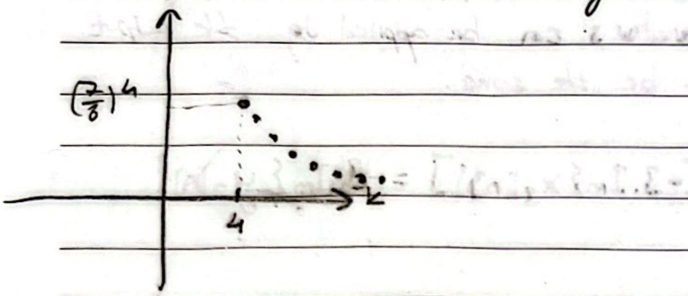
$$y_1[n] = \begin{cases} 0 & \text{for } n<4 \\ \frac{(7/8)^4 - (7/8)^{n+1}}{1-7/8} \cdot 3 & \text{for } 4 \leq n < 12 \\ \frac{(7/8)^{n-8} - (7/8)^{n+1}}{1-7/8} \cdot 3 & \text{for } 12 \leq n \end{cases}$$

→ Plots and the codes of 1.a are available in Appendix B.

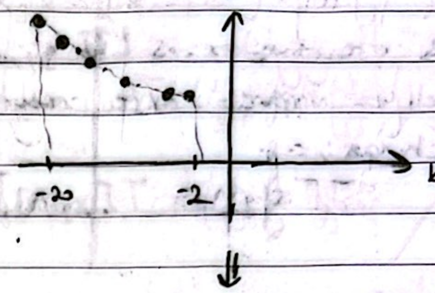
c) $x_3[n] = \begin{cases} e^{j(1/3)n}, & 2 \le n \le 20 \\ 0, & \text{else} \end{cases}$ , $h[n] = (7/8)^n u[n-4]$

$$x_3[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k] \cdot x_3[n-k]$$

① $h[k] = (7/8)^k \cdot u(k-4) \rightarrow 0$ for $n<4$

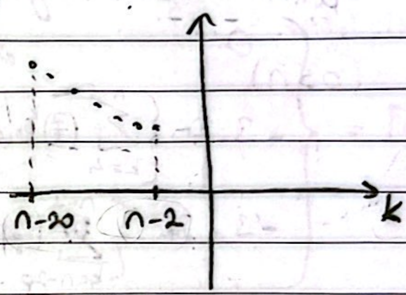



② $x_3(-k) = e^{j(1/3)k}$




$\Downarrow$

③ $x_3(n-k) = e^{j(1/3)(n-k)}$




④

$$y_3[n] = \begin{cases} 0 & \text{for } n<6 \\ \displaystyle\sum_{k=4}^{n-2} \left(\tfrac{7}{8}\right)^k \cdot e^{j(1/3)(n-k)} & \text{for } 6 \le n \le 24 \\ \displaystyle\sum_{k=n-20}^{n-2} \left(\tfrac{7}{8}\right)^k \cdot e^{j(1/3)(n-k)} & \text{for } 24 < n \end{cases}$$

then, ↓

$$y_3[n] = \begin{cases} 0 & \text{for } n<6 \\ \left(e^{j1/3}\right)^n \cdot \left[\dfrac{\left(\frac{7}{8} \cdot e^{-j1/3}\right)^4 - \left(\frac{7}{8} \cdot e^{-j1/3}\right)^{n-1}}{1 - \left(\frac{7}{8} \cdot e^{-j1/3}\right)}\right] & \text{for } 6 \le n \le 24 \\ \left(e^{j1/3}\right)^n \cdot \left[\dfrac{\left(\frac{7}{8} \cdot e^{-j1/3}\right)^{n-20} - \left[\frac{7}{8} \cdot e^{-j1/3}\right]^{n-1}}{1 - \left(\frac{7}{8} \cdot e^{-j1/3}\right)}\right] & \text{for } 24 < n \end{cases}$$

→ Plots and MatLab code are available in Appendix D.

d)

$$x_4[n] = \begin{cases} -3\sin[(1/13)n] & \text{if } 2 \le n \le 20 \\ 0 & \text{else} \end{cases}, \quad h[n] = \left(\frac{7}{8}\right)^n \cdot u[n-4]$$

$$x_4[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k] \cdot x_4[n-k]$$

(Hint: Note that $x_4[n] = -3\,\mathrm{Im}\{x_3[n]\}$

Since the system is LTI; linear operators can be applied to the output as well as input. The result must be the same.

Such as:

$$\text{If } y_3[n] = T[x_3[n]] \Rightarrow T[-3.\mathrm{Im}\{x_3[n]\}] = -3.\mathrm{Im}\{y_3[n]\}$$

Therefore,

$$y_4[n] = -3.\mathrm{Im}\{y_3[n]\}.$$

$$y_4[n] = \begin{cases} 0 & \text{for } n<6 \\ -3.\mathrm{Im}\left\{\sum_{k=4}^{n-2} \left(\frac{7}{8}\right)^k \cdot \left(e^{j/13}\right)^{n-k}\right. & \text{for } 6 \le n \le 24 \\ -3.\mathrm{Im}\left\{\sum_{k=n-20}^{n-2} \left(\frac{7}{8}\right)^k \cdot \left(e^{j/13}\right)^{n-k}\right. & \text{for } 24 < n \end{cases}$$

$$\Rightarrow y_4[n] = \begin{cases} 0 & \text{for } n<6 \\ -3\,\mathrm{Im}\left\{\left(e^{j/13}\right)^n \cdot \left[\dfrac{\left(\left(\frac{7}{8}\right)\cdot e^{-j/13}\right)^4 - \left(\frac{7}{8}\cdot e^{-j/13}\right)}{1-\left(\frac{7}{8}e^{-j/13}\right)}\right]\right\} & \text{for } 6 \le n \le 24 \\ -3\,\mathrm{Im}\left\{\left(e^{j/13}\right)^n \cdot \left[\dfrac{\left(\left(\frac{7}{8}\right)\cdot e^{-j/13}\right)^{n-20} - \left[\frac{7}{8}\cdot e^{-j/13}\right]^{n-1}}{1-\left(\frac{7}{8}\cdot e^{-j/13}\right)}\right]\right. & \text{for } 24 < n \end{cases}$$

$\Rightarrow$ Plots and Matlab code are available in Appendix E.

e) $$x_5[n] = \begin{cases} 2\cos[(1/3)n], & \text{if } 2 \le n \le 20 \\ 0 & \text{, else} \end{cases}$$

Hint: Note that $x_5[n] = 2\,\mathrm{Re}\{x_3[n]\}$

Since the system is LTI, if the input and the output of the system is applied with linear operations, the result should be the same. such as:

$$y_3[n] = T[x_3[n]] \Rightarrow 2.\mathrm{Re}\{y_3[n]\} = 2.\mathrm{Re}\{T[x_3[n]]\}$$

since $x_5[n] = 2\,\mathrm{Re}\{x_3[n]\}$, $y_5[n] = 2\,\mathrm{Re}\{T[x_3[n]]\} = 2\,\mathrm{Re}\{y_3[n]\}$

Therefore,

$$y_5[n] = \begin{cases} 0 & \text{for } n<6 \\ 2\,\mathrm{Re}\left\{\sum_{k=4}^{n-2} \left(\frac{7}{8}\right)^k \cdot \left(e^{j/3}\right)^{n-k}\right\} & \text{for } 6 \le n \le 24 \\ 2\,\mathrm{Re}\left\{\sum_{k=n-20}^{n-2} \left(\frac{7}{8}\right)^k \cdot \left(e^{j/3}\right)^{n-k}\right\} & \text{for } 24 < n \end{cases}$$

$$\Big\downarrow \text{then}$$

$$y_5[n] = \begin{cases} 0 & \text{for } n<6 \\ 2\,\mathrm{Re}\left\{\left(e^{j/3}\right)^n \cdot \left[\dfrac{\left(\frac{7}{8}\cdot e^{-j/3}\right)^4 - \left(\frac{7}{8}\cdot e^{-j/3}\right)^{n-1}}{1-\left(\frac{7}{8}\cdot e^{-j/3}\right)}\right]\right\} & \text{for } 6 \le n \le 24 \\ 2\,\mathrm{Re}\left\{\left(e^{j/3}\right)^n \cdot \left[\dfrac{\left(\frac{7}{8}\cdot e^{-j/3}\right)^{n-20} - \left(\frac{7}{8}\cdot e^{-j/3}\right)^{n-1}}{1-\left(\frac{7}{8}\, e^{-j/3}\right)}\right]\right\} & \text{for } 24 < n \end{cases}$$

→ Plots and MatLab code are available in Appendix F.

d) $x_6[n] = x_1[n] + j2x_2[n]$ , $h[n] = (7/8)^n \cdot u[n-4]$

$$\boxed{x_6[n] * h[n] = \left(x_1[n] + j2x_2[n]\right) * h[n]}$$

By using the distributive property of LTI systems:

$$\left(x_1[n] + j2\,x_2[n]\right) * h[n] = \underbrace{x_1[n] * h[n]}_{y_1[n]} + \underbrace{j2\,x_2[n] * h[n]}_{2j\,y_2[n]}$$

From question (1.a), I know "$y_1[n]$", and by using the answer from question (1.b), I can derive "$2j\,y_2[n]$"

$$y_1[n] = \begin{cases} 0 & \text{for } n < 4 \\ \dfrac{(7/8)^4 - (7/8)^{n+1}}{(1-7/8)} \cdot 3 & \text{for } 4 \leq n < 12 \\ \dfrac{(7/8)^{n-8} - (7/8)^{n+1}}{(1-7/8)} & \text{for } 12 \leq n \end{cases}$$

$$2j\,y_2[n] = \begin{cases} 0 & \text{for } n < 4 \\ 2j\left[\dfrac{(7/8)^4 - (7/8)^{n+1}}{1-7/8}\right] \cdot 3 & \text{for } 4 \leq n < 9 \\ 2j\left[\dfrac{(7/8)^4 - (7/8)^{n+1}}{1-7/8} \cdot 3 + \dfrac{(7/8)^4 - (7/8)^{n-4}}{1-7/8}\right] \cdot (-6) & \text{for } 9 \leq n < 12 \\ 2j\left[\dfrac{(7/8)^{n-8} - (7/8)^{n+1}}{1-7/8} \cdot 3 + \dfrac{(7/8)^4 - (7/8)^{n-4}}{1-7/8}\right] \cdot (-6) & \text{for } 12 \leq n < 17 \\ 2j\left[\dfrac{(7/8)^{n-8} - (7/8)^{n+1}}{1-7/8} \cdot 3 + \dfrac{(7/8)^{n-13} - (7/8)^{n-4}}{1-7/8}\right] \cdot (-6) & \text{for } 17 \leq n \end{cases}$$

→ Plots and MatLab code are avaible in Appendix g.