

**BILKENT UNIVERSITY**  
**ELECTRICAL AND ELECTRONICS ENGINEERING**  
**DEPARTMENT**

**14/11/2024**



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## Introduction:

The purpose of this lab is to teach us Fourier Series Expansion with applying it on MATLAB. The lab walks us through the process of discretizing a rectangular waveform, determining and charting its Fourier series expansion, examining its frequency spectrum, and using a small number of Fourier series components to create an approximation of the original signal.

### Question 1:

The discretized function  $y_a(t)$  with  $T_s = 1/10$  seconds is given in Figure 1 and its plot is given in the Figure 2:

$$y_a(t) = \begin{cases} 0, & t \in [0, 7)s \\ 8, & t \in [7, 10)s \\ 0, & t \in [10, 18)s \end{cases}$$

Figure 1:  $y_a(t)$

### Question 1.a:

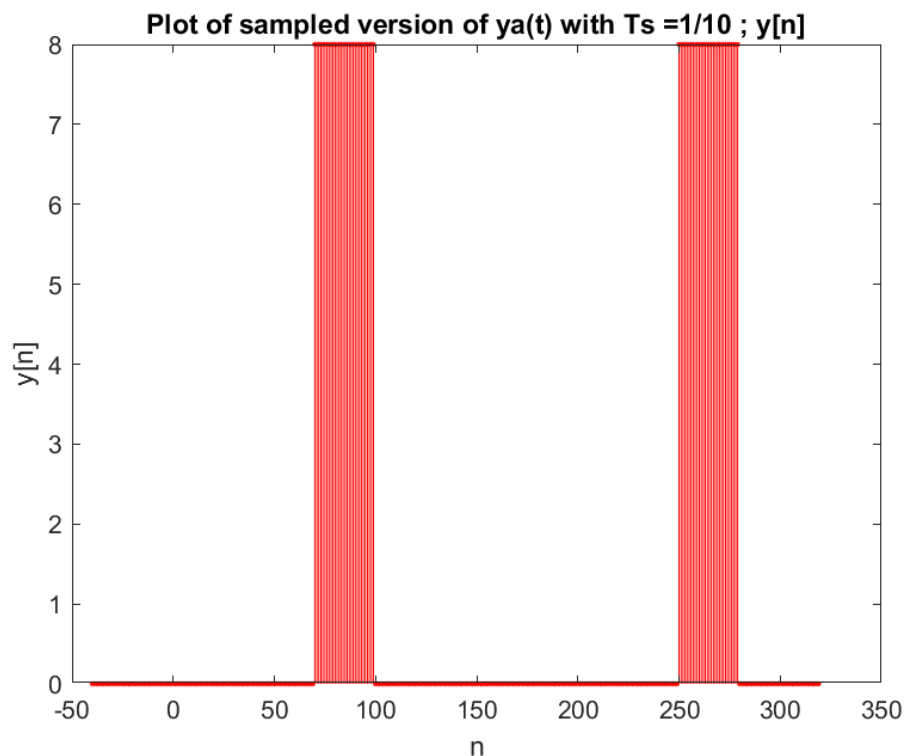


Figure 2: Plot of Discretized  $y_a(t)$  with  $T_s = 1/10$

### Question 1.b:

On the Figure 3, you can see the process of how I found the FSE of  $y_a(t)$ :

$$\begin{aligned}
 1.b) \quad y_a(t) &= \begin{cases} 0, & t \in [0, 7)s \\ 8, & t \in [7, 10)s \\ 0, & t \in [10, 18)s \end{cases} \\
 a_0 &= \frac{1}{T} \int_0^T x(t) dt = \frac{1}{18} \int_0^{18} y_a(t) dt = \frac{1}{18} \int_7^{10} 8 dt \\
 a_k &= \frac{1}{18} \int_0^{18} y_a(t) dt e^{\frac{-j2\pi kt}{18}} dt = 8t \Big|_7^{10} = \frac{24}{18} = \frac{4}{3} \\
 &= \frac{1}{18} \int_7^{10} 8 \cdot e^{\frac{-j2\pi kt}{18}} dt \\
 &= \frac{-4}{9} \cdot \frac{18}{2\pi k j} \left( e^{\frac{-j20\pi k}{18}} - e^{\frac{-14\pi k j}{18}} \right) = \frac{8}{2\pi k j} \left( e^{\frac{j22\pi k}{18}} - e^{\frac{j16\pi k}{18}} \right) \\
 a_k &= \frac{4}{j\pi k} \left[ \cos\left(\frac{11\pi k}{9}\right) - \cos\left(\frac{8\pi k}{9}\right) + j\sin\left(\frac{11\pi k}{9}\right) - j\sin\left(\frac{8\pi k}{9}\right) \right] \\
 a_k &= \frac{4}{\pi k} \left[ \sin\left(\frac{11\pi k}{9}\right) - \sin\left(\frac{8\pi k}{9}\right) + j\cos\left(\frac{8\pi k}{9}\right) - j\cos\left(\frac{11\pi k}{9}\right) \right] \\
 \text{FSE of } y_a(t) \text{ is } &\rightarrow \frac{4}{3} + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \frac{4\pi k}{\pi k} \cdot e^{j\pi/9 kt}
 \end{aligned}$$

Figure 3: The Process of Finding the FSE of  $y_a(t)$

Figure 4 demonstrates the relationship between the coefficients  $k$  and their values  $a_k$ .  $a_k$  values is given in the Figure 4.

$$a_k = \begin{cases} \frac{4}{3}, & k=0 \\ \frac{4}{j\pi k} \left( e^{j\frac{22}{18}\pi k} - e^{j\frac{16}{18}\pi k} \right), & k \neq 0 \end{cases}$$

Figure 4: The Values of  $a_k$

### Question 1.c:

Figure 5 and Figure 6 shows the spectrum of coefficients showing the real and imaginary parts of  $a_k$ .

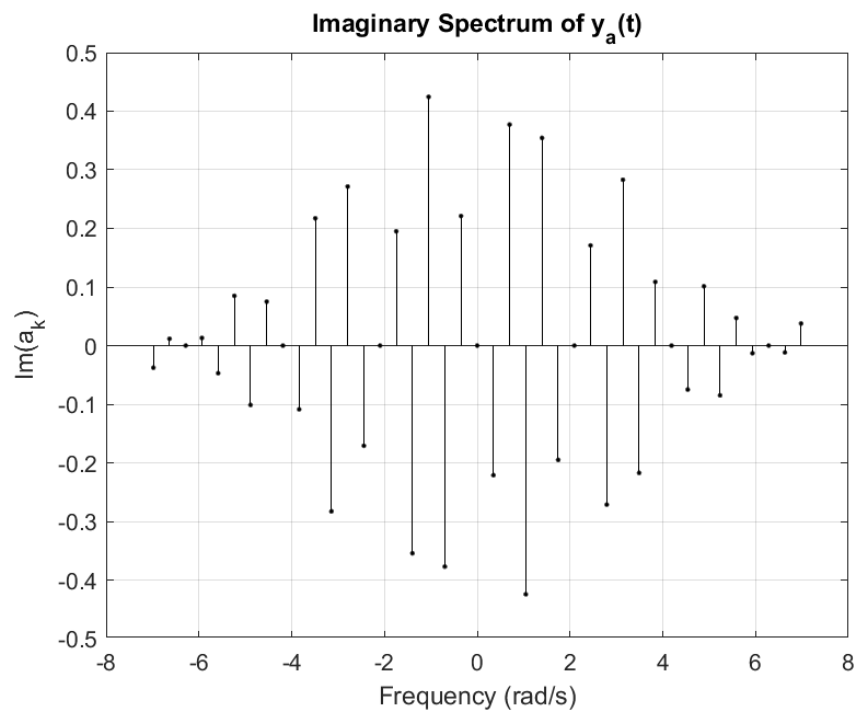


Figure 5: Spectrum of the Imaginary Part of  $a_k$

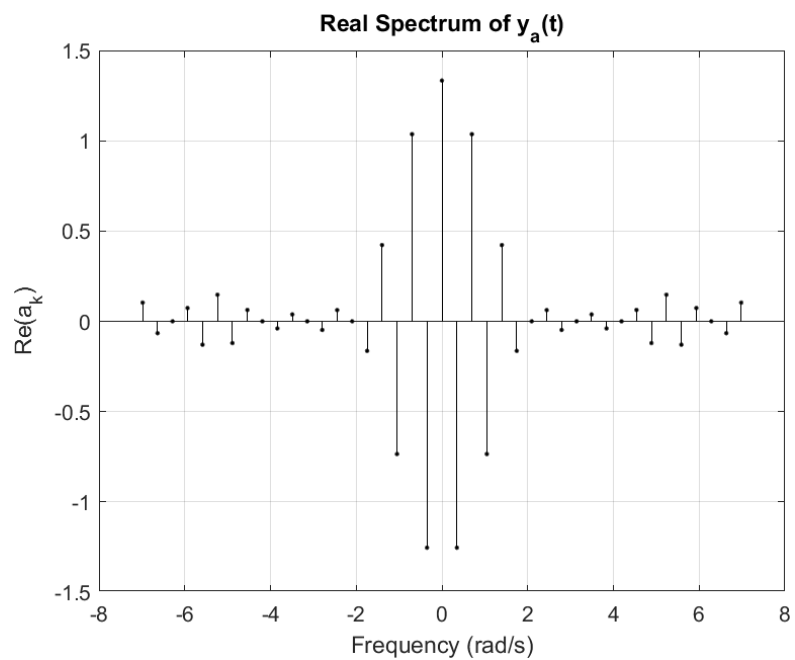


Figure 6: Spectrum of the Real Part of  $a_k$

### Question 1.d:

In the Part 1.b, the FSE of  $y_a(t)$  is found. With using that solution  $Z_N(n)$  is found as it is in the Figure 7:

$$Z_N[n] = \frac{10}{\pi} + \sum_{k=1, k \neq 0}^N 2 \cdot a_k \cdot \cos\left(\frac{2\pi}{9} \cdot k \cdot \frac{n}{9}\right) \text{ for } n \in [-40, 319]$$

Figure 7:  $Z_N(n)$

The plot in the Figure 8 is similar to the original function  $y_a(t)$ , though it is not an exact match. For our signal to closely approximate  $y_a(t)$ , the  $N$  value would need to extend to infinity. In this case, the  $N$  value which is 150, is greater than the other values in the Question 1.e, 1.f, 1.g and others, therefore the plot on the Figure 8 is more similar to  $y_a(t)$  than in the other questions. So as  $N$  decreases toward zero, the approximation increasingly diverges from the original function.

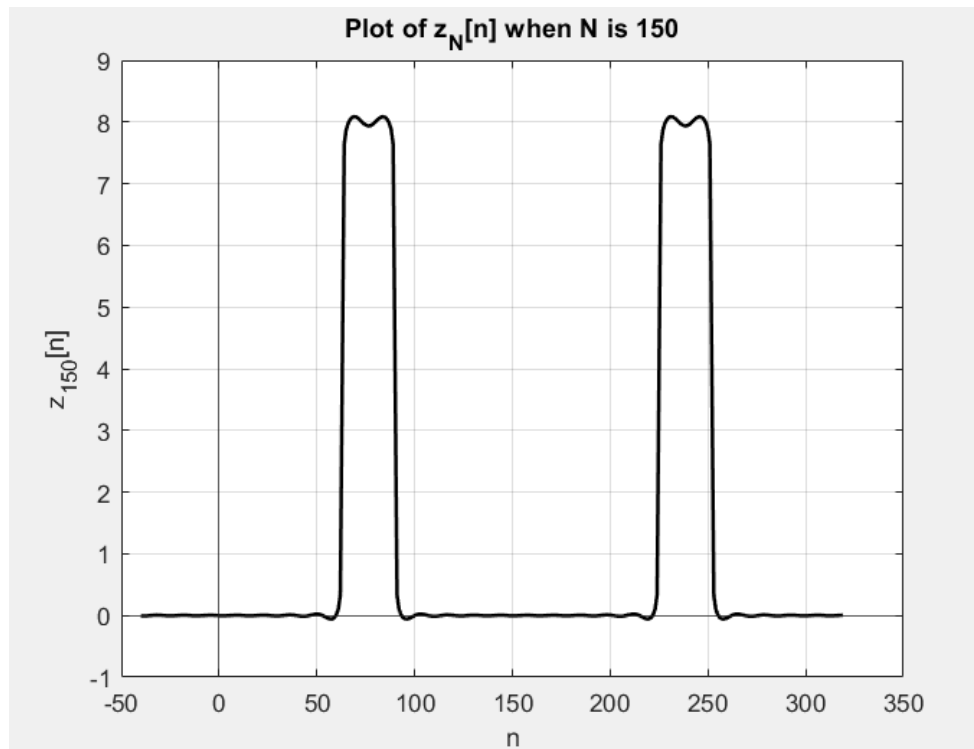


Figure 8:  $Z_N(n)$  when  $N=150$

**Question 1.e:**

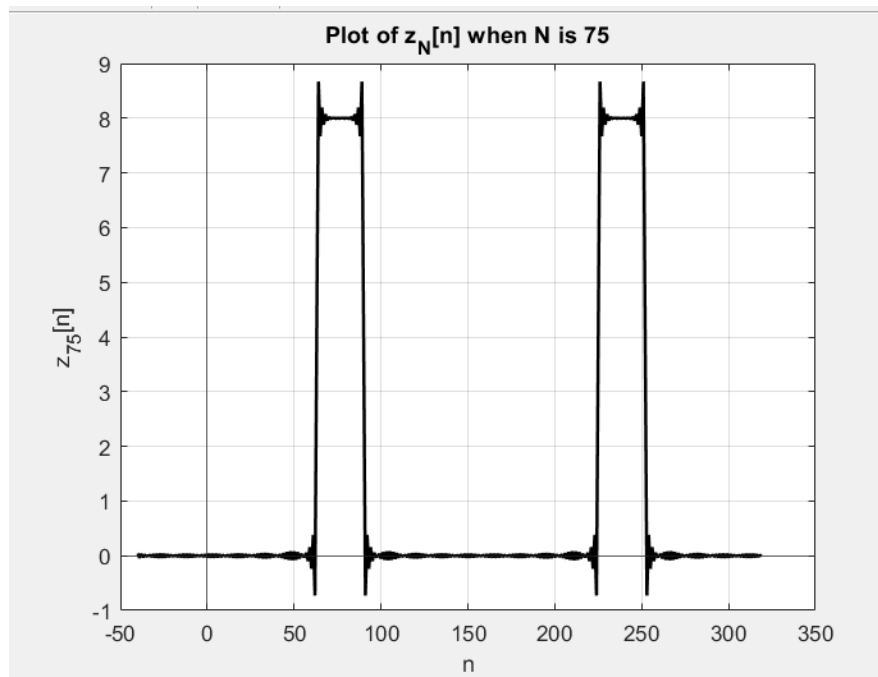


Figure 9:  $Z_N(n)$  when  $N=75$

**Question 1.f:**

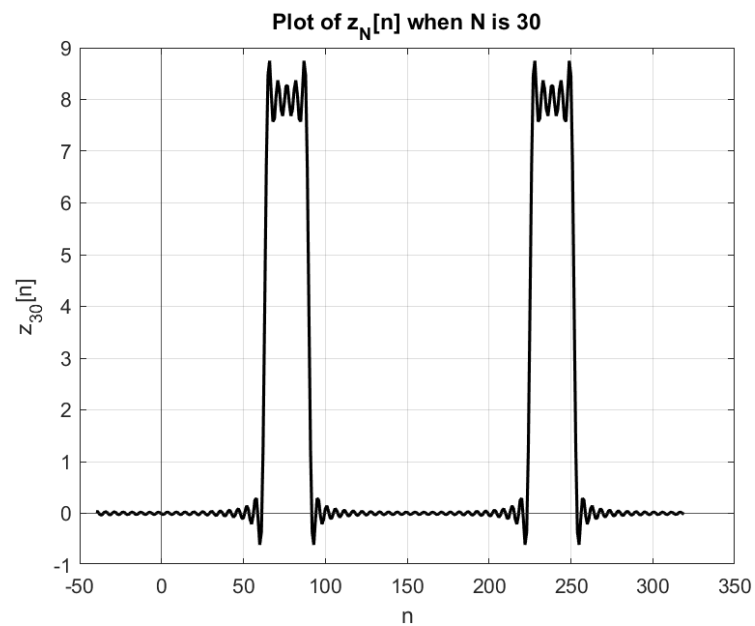


Figure 10:  $Z_N(n)$  when  $N=30$

**Question 1.g:**

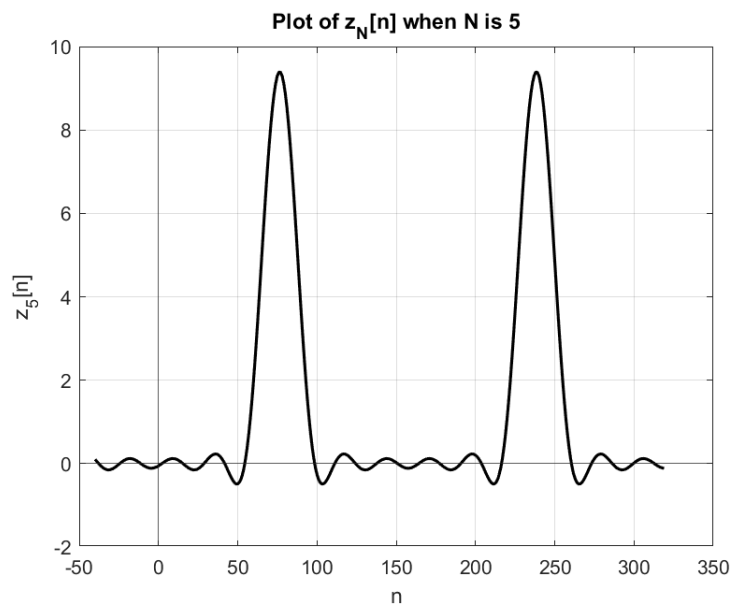


Figure 11:  $Z_N(n)$  when  $N=5$

**Question 1.h:**

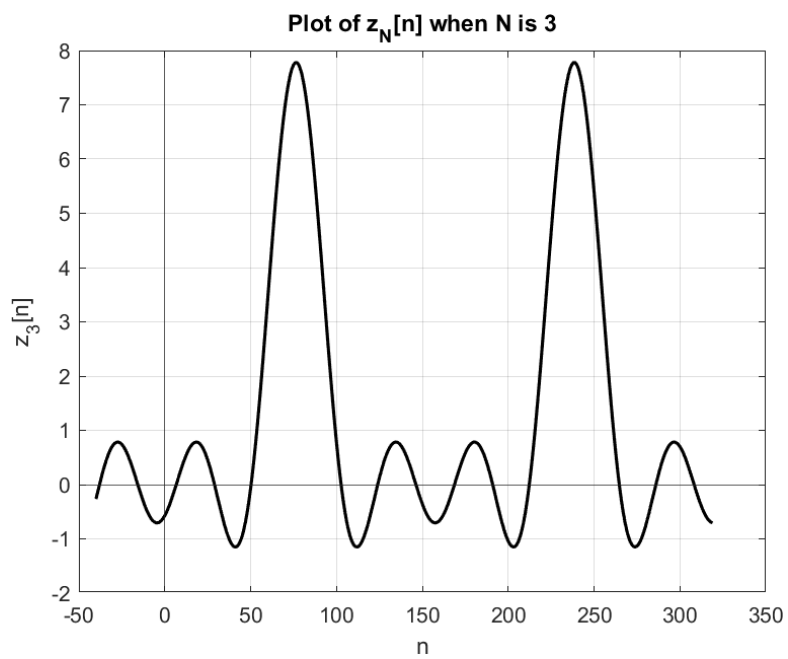


Figure 12:  $Z_N(n)$  when  $N=3$

### Question 1.i:

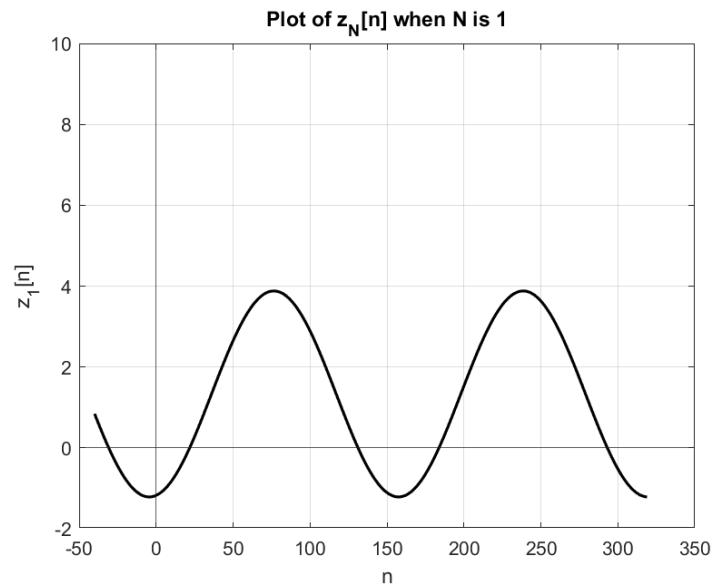


Figure 13:  $Z_N(n)$  when  $N=1$

As we observed in part (d), the approximation quality decreases as  $N$  approaches zero. This is because, with fewer components, more frequency information is lost, making the approximation less accurate. The components with larger  $a_k$  values have a greater impact on the shape of the approximation, while those with smaller coefficients contribute less.

Additionally, according to the Gibbs phenomenon, any sudden jumps in the signal can't be completely smoothed out, even as more components are added. In MATLAB, the points are connected smoothly when they are close together, so the plot for  $N=150$  looks smoother than for  $N=75$ . As a result, the initial approximation plot may appear smoother in MATLAB than it would naturally.

### Question 1.j:

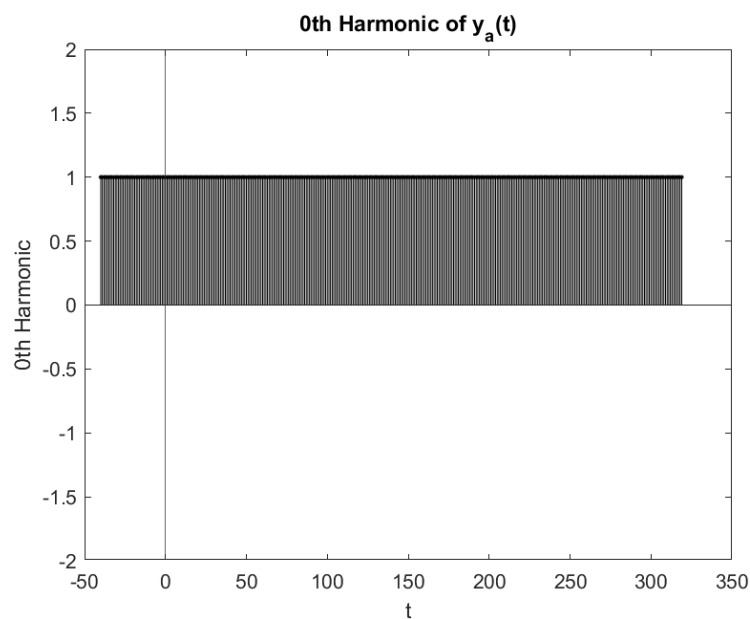




Figure 14: 0<sup>th</sup> Harmonic of  $y_a(t)$

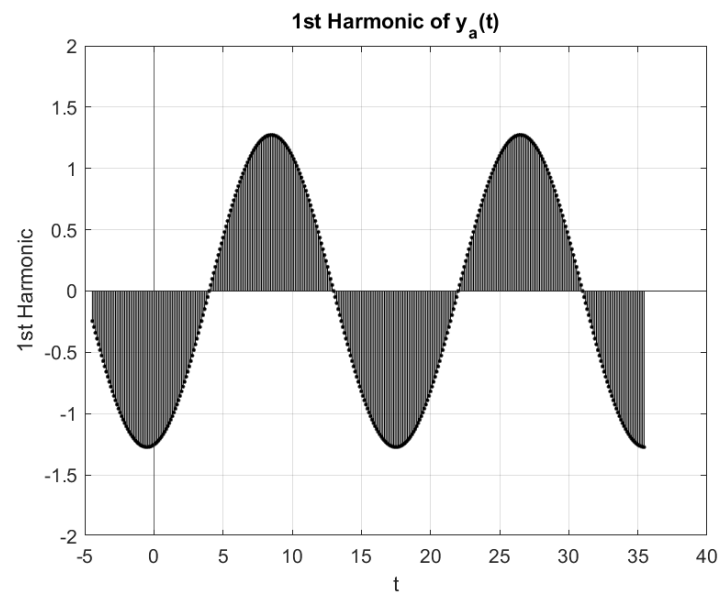


Figure 15: 1<sup>st</sup> Harmonic of  $y_a(t)$

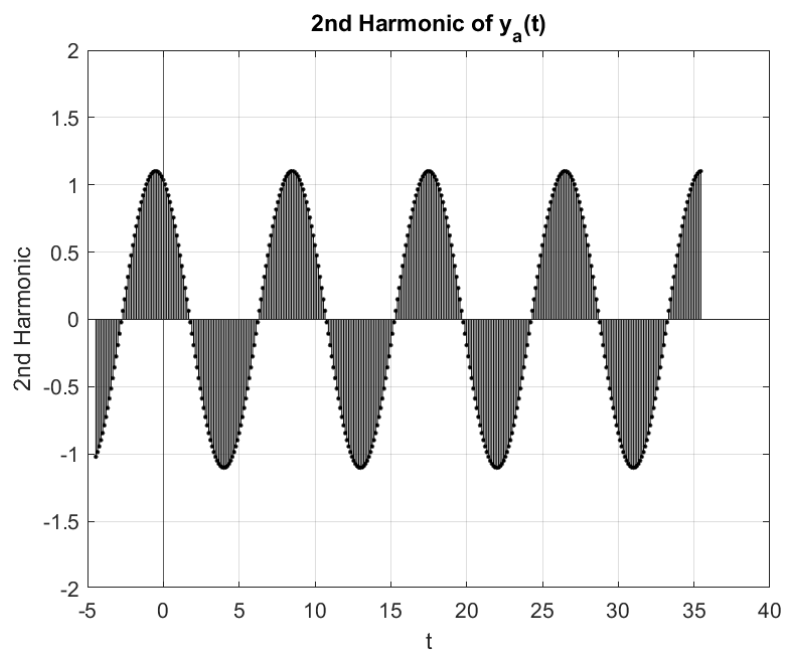


Figure 16: 2<sup>nd</sup> Harmonic of  $y_a(t)$

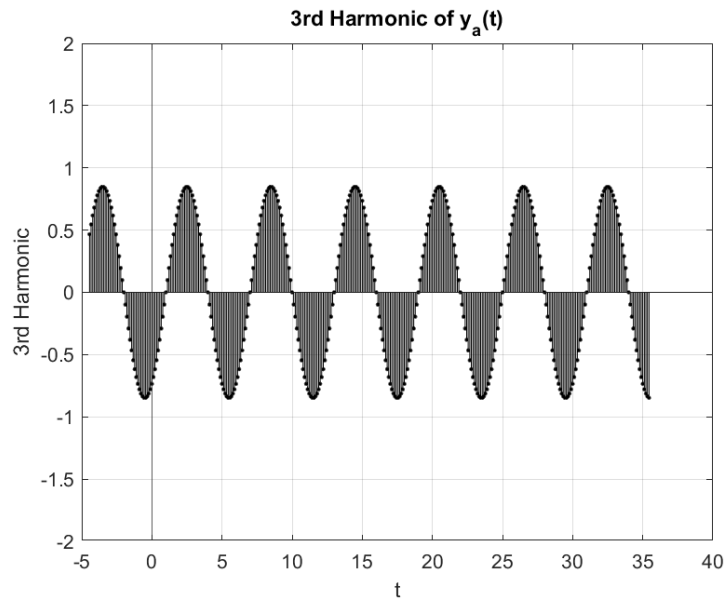


Figure 17: 3<sup>rd</sup> Harmonic of  $y_a(t)$

### Question 2:

The discretized signal  $y_a(t)$  with  $T_s = 1/10$  seconds is given in Figure 18 and its plot is given in the Figure 19:

$$y_a(t) = \left| 5 \cos \left( \frac{\pi}{9} t \right) \right|$$

Figure 18:  $y_a(t)$

### Question 2.a:

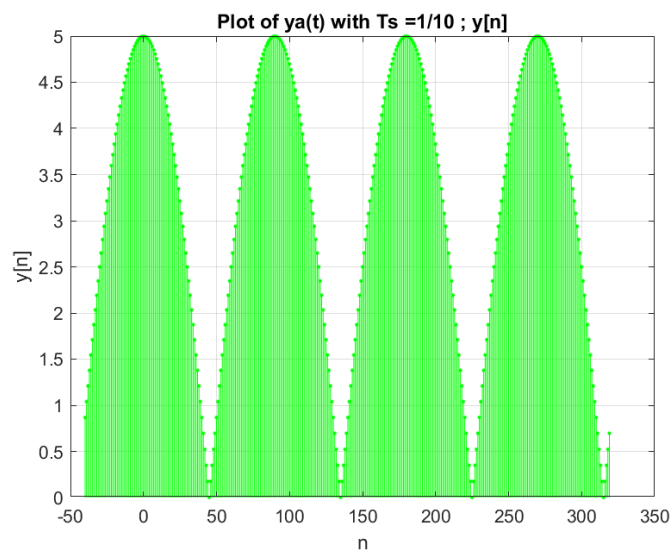


Figure 19: Plot of  $y_a(t)$  with  $T_s=1/10$

### Question 2.b:

The process of finding the FSE of the  $y_a(t)$  is shown the Figure 20:

2.b)  $y_{a2}(t) = |5 \cos(\pi/9 t)|$ . Fundamental period is  $\rightarrow T = 9$

$$a_0 = \frac{1}{T} \int_0^T x(t) dt = \frac{1}{9} \int_{-4.5}^{4.5} 5 \cos\left(\frac{\pi}{9} t\right) dt = \frac{1}{9} \cdot \frac{9}{\pi} \cdot 5 \left( \sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right) \right)$$

$$a_0 = 10/\pi$$

$$a_k = \frac{1}{T} \int_0^T y_{a2}(t) e^{-j \frac{2\pi}{T} k t} dt = \frac{1}{9} \int_{-4.5}^{4.5} 5 \cos(\pi/9 t) e^{-j \frac{2\pi}{9} k t} dt$$

$$= \frac{5}{18} \int_{-4.5}^{4.5} \left( e^{-j \frac{2\pi}{9} k t} \cdot e^{j \pi/9 t} + e^{-j \frac{2\pi}{9} k t} \cdot e^{-j \pi/9 t} \right) dt$$

$$= \frac{5}{18} \left[ \frac{9}{\pi(1-2k)} \left( e^{j \frac{\pi}{2}(1-2k)} - e^{-j \frac{\pi}{2}(1-2k)} \right) + \frac{9}{\pi(2k+1)} \left( e^{-j \frac{\pi}{2}(1+2k)} - e^{j \frac{\pi}{2}(1+2k)} \right) \right]$$

$$= 5 \cdot \left[ \frac{\sin\left(\frac{\pi}{2}(1-2k)\right)}{\pi(1-2k)} + \frac{\sin\left(\frac{\pi}{2}(1+2k)\right)}{\pi(2k+1)} \right] = \frac{a_k}{k \neq 0}$$

$$\text{F.S.E}[y_{a2}(t)] = \frac{10}{\pi} + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} a_k \cdot e^{-j \frac{2\pi}{9} k t}$$

$$\text{F.S.E}[y_{a2}(t)] = \frac{10}{\pi} + \sum_{k=1}^{\infty} 2 \cdot a_k \cdot \cos\left(\frac{2\pi}{9} k t\right)$$

since  $a_k$  is odd,

$$a_k = \begin{cases} \frac{10}{\pi} & , k=0 \\ 5 \cdot \left[ \frac{\sin\left(\frac{\pi}{2}(1-2k)\right)}{\pi(1-2k)} + \frac{\sin\left(\frac{\pi}{2}(1+2k)\right)}{\pi(1+2k)} \right] & , k \neq 0 \end{cases}$$

$$Z_N[n] = \frac{10}{\pi} + \sum_{k=1, k \neq 0}^N 2 \cdot a_k \cdot \cos\left(\frac{2\pi}{9} \cdot k \cdot \frac{n}{9}\right) \text{ for } n \in [-40, 319]$$

Figure 20: The Process of Finding the FSE of  $a(t)$

### Question 2.c:

Value of  $a_k$  is shown in the Figure 21:

$$a_k = \begin{cases} \frac{10}{\pi} & , k=0 \\ 5 \cdot \left[ \frac{\sin\left(\frac{\pi}{2}(1-2k)\right)}{\pi(1-2k)} + \frac{\sin\left(\frac{\pi}{2}(1+2k)\right)}{\pi(1+2k)} \right] & , k \neq 0 \end{cases}$$

$$Z_N[n] = \frac{10}{\pi} + \sum_{k=1, k \neq 0}^N 2 \cdot a_k \cdot \cos\left(\frac{2\pi}{9} \cdot k \cdot \frac{n}{9}\right) \text{ for } n \in [-40, 39]$$

Figure 21: Values of  $a_k$

Figure 22 shows the spectrum of coefficients of  $a_k$ :

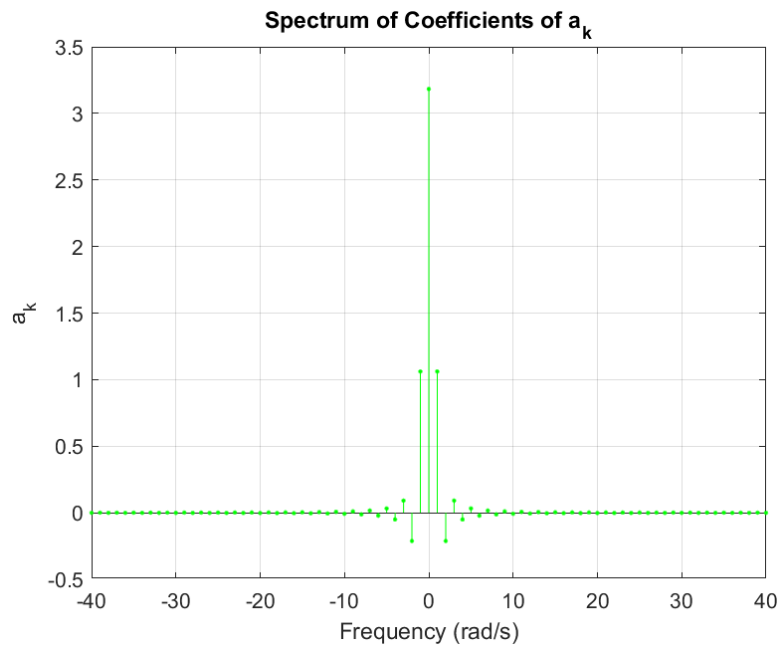


Figure 22: Spectrum of  $a_k$

### Question 2.d:

In the Part 2.b, the FSE of  $y_a(t)$  is found. With using that solution,  $Z_N(n)$  is found as it is in the Figure 23:

$$Z_N[n] = \frac{10}{\pi} + \sum_{k=1, k \neq 0}^N 2 \cdot a_k \cdot \cos\left(\frac{2\pi}{9} \cdot k \cdot \frac{n}{9}\right) \text{ for } n \in [-40, 39]$$

Figure 23:  $Z_N(n)$

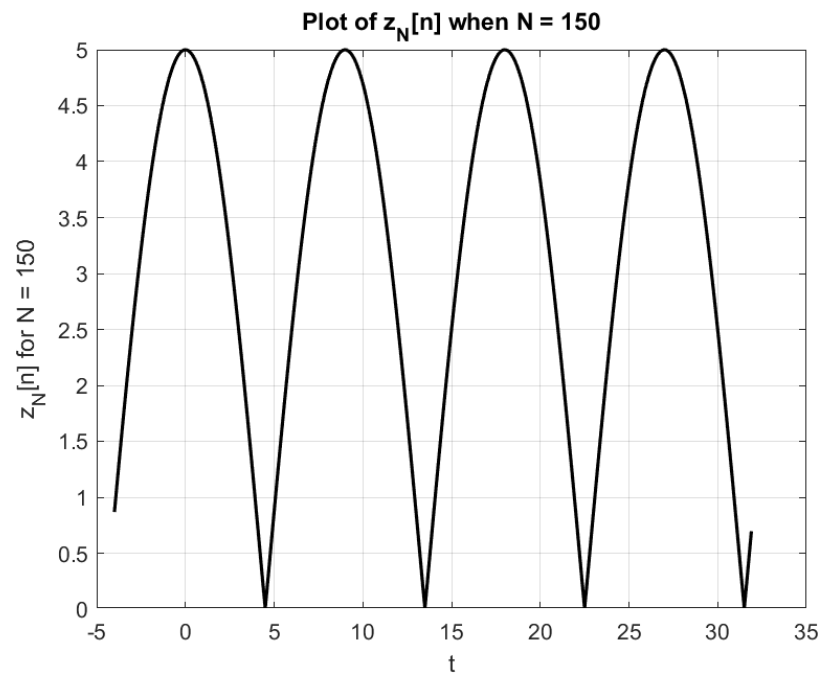


Figure 24: Plot of  $Z_n$  when  $N=150$

**Question 2.e:**

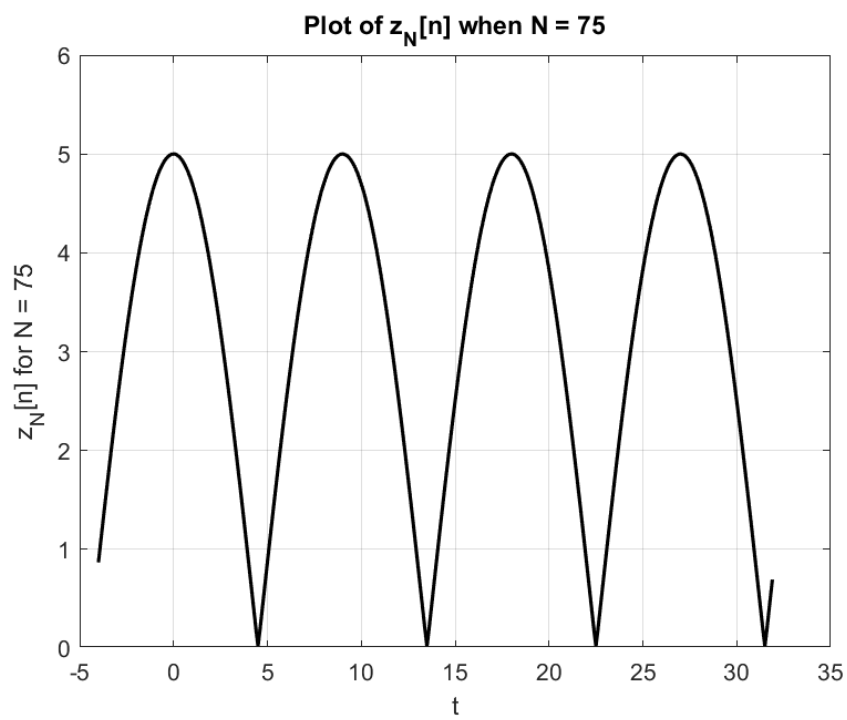


Figure 25: Plot of  $Z_n$  when  $N=75$

**Question 2.f:**

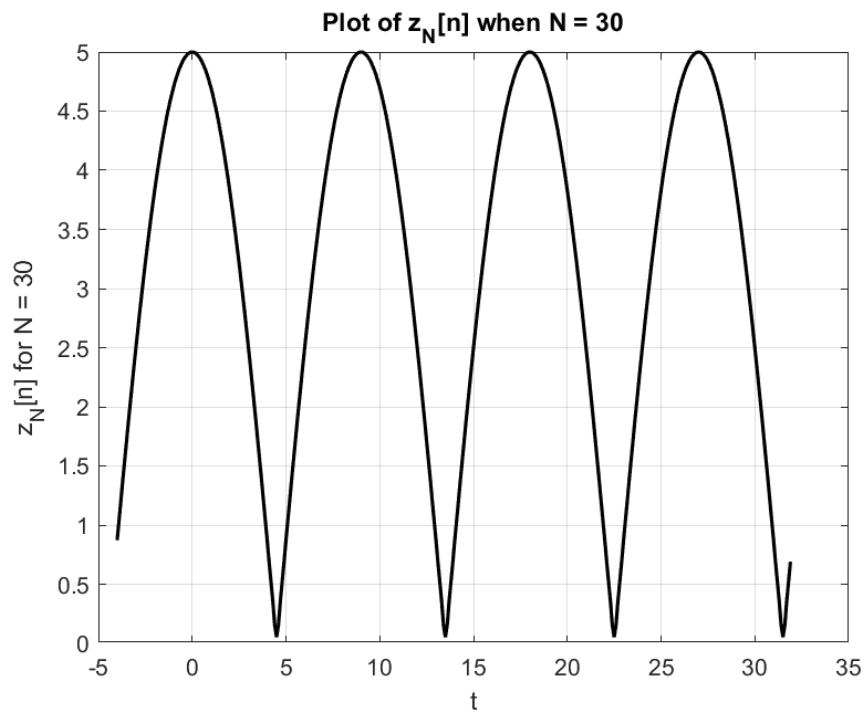


Figure 26: Plot of  $Z_n$  when  $N=30$

**Question 2.g:**

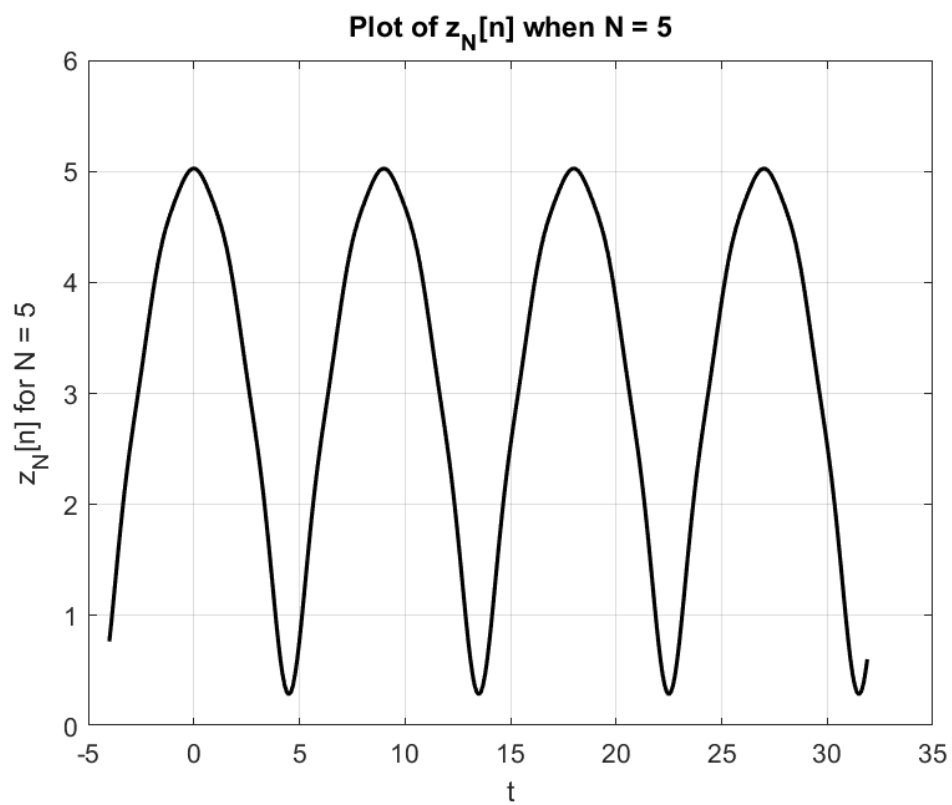


Figure 27: Plot of  $Z_n$  when  $N=5$

### Question 2.h:

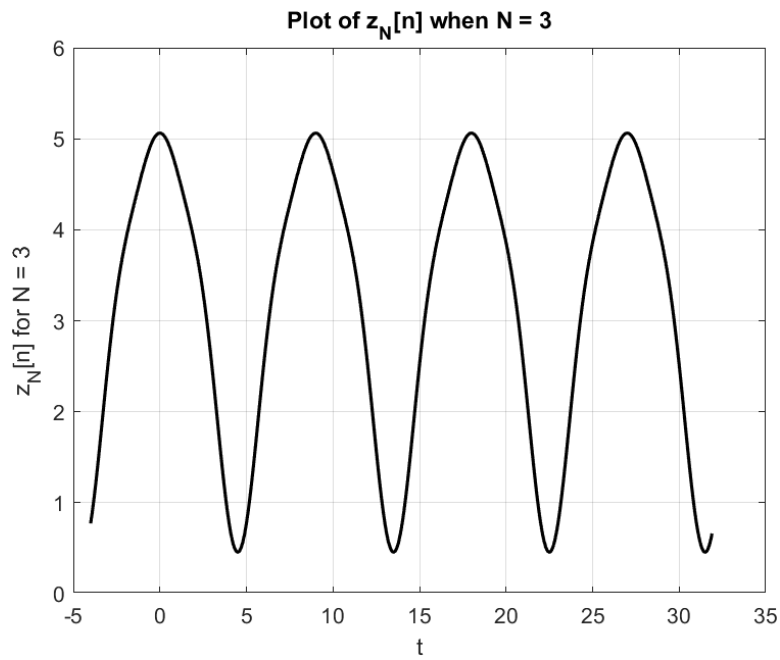


Figure 28: Plot of  $Z_n$  when  $N=3$

### Question 2.i:

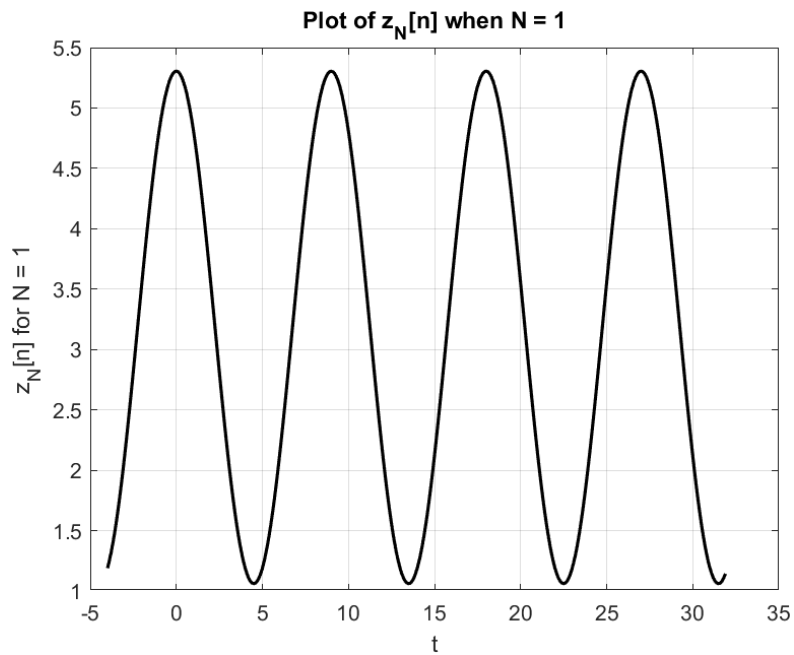


Figure 29: Plot of  $Z_n$  when  $N=1$

As in the Question 1, as  $N$  got closer to zero, more frequency components were lost, lowering the approximation's quality. The approximation's accuracy is decreased by these missing elements. Additionally, the total approximation is more affected by components with comparatively larger  $ak$  a  $k$  values than by those with smaller coefficients. There were no abrupt jumps or discontinuities in this instance, in contrast to the first question. As a result, the Gibbs phenomenon was not observed in this section of the lab with the original function.

**Question 2.j:**

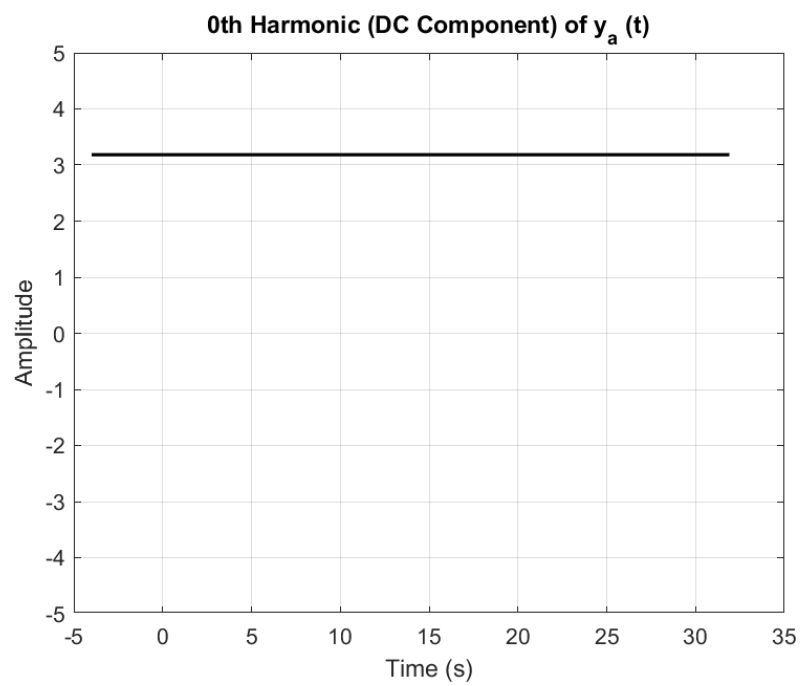


Figure 30: Zeroth Harmonic of  $y_a(t)$

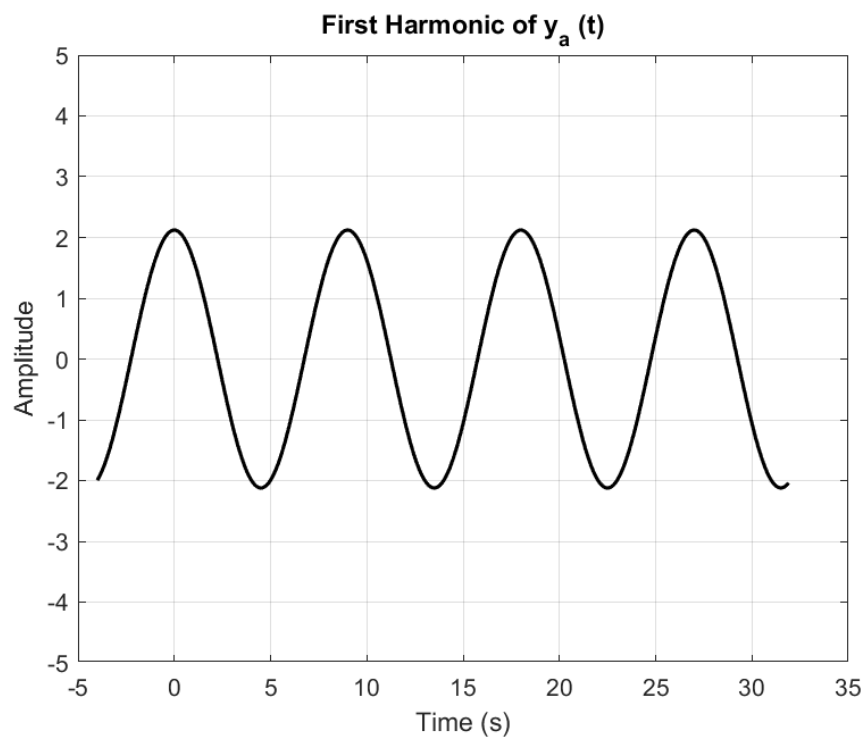


Figure 31: First Harmonic of  $y_a(t)$



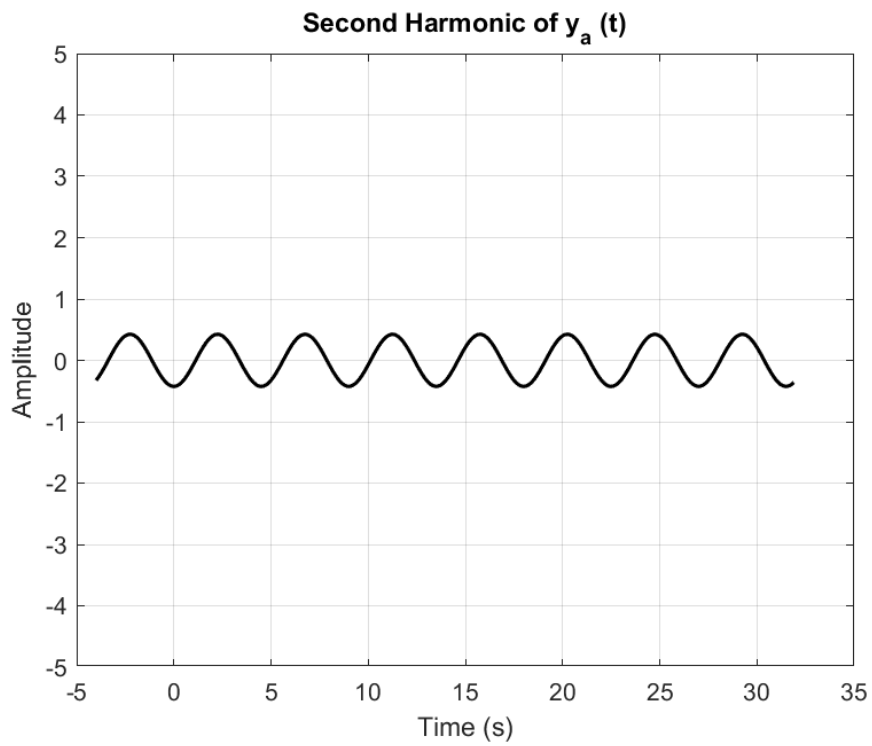


Figure 32: Second Harmonic of  $y_a(t)$

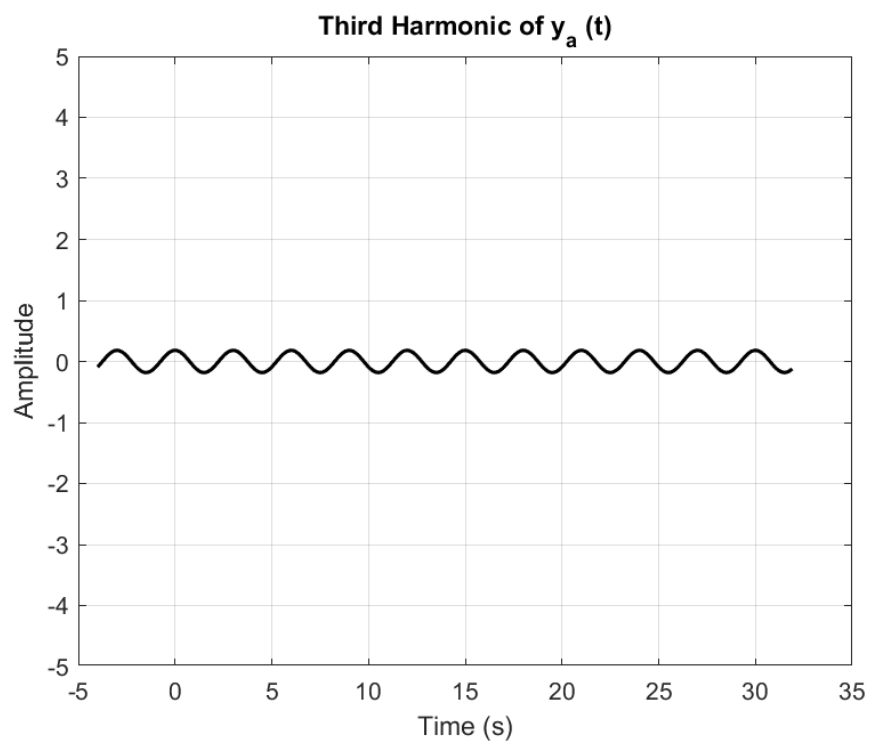


Figure 33: Third Harmonic of  $y_a(t)$

**Question 3:**

The discretized signal  $y_a(t)$  with  $T_s = 1/10$  seconds is given in Figure 34 and its plot is given in the Figure 35:

$$y_a(t) = \begin{cases} |5 \cos(\frac{\pi}{9}t)| & t \in [-4.5, 4.5) \text{ s.} \\ 0 & t \in [4.5, 13.5) \text{ s.} \end{cases}$$

Figure 34:  $y_a(t)$

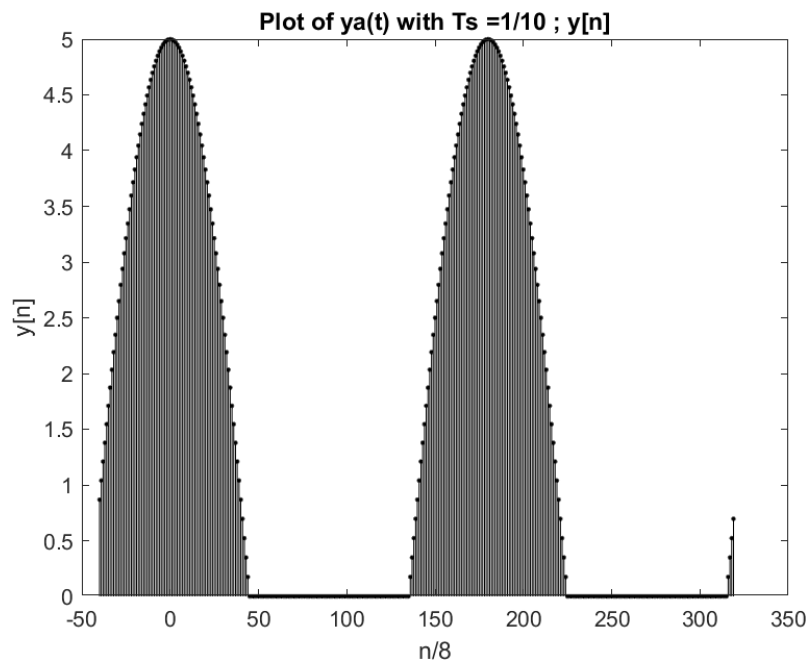
**Question 3.a:**

Figure 35: Plot of Discretized  $y_a(t)$  with  $T_s=1/10$

### Question 3.b:

The process of finding the FSE of the  $y_a(t)$  is shown in Figure 36.

3. b)

$$y_{a3}(t) = \begin{cases} 15 \cos\left(\frac{\pi}{9}t\right), & t \in [-4.5, 4.5]s \\ 0, & t \in [4.5, 13.5]s \end{cases}, \text{Fund. period is } 18 \text{ sec.}$$

$$y_{a3}(t) = y_a(t + 18n), \quad n \in \mathbb{Z}$$

$$y_{a3}(t) = \frac{15 \cos(\pi/9t) + 5 \cos(\pi/9t)}{2} = \frac{y_{a2}(t) + 5 \cos(\pi/9t)}{2}$$

FSE is linear.

$$2 \text{FSE}[y_{a3}(t)] = \text{FSE}[y_{a2}(t)] + \text{FSE}[5 \cos(\pi/9t)]$$

$$2y_{a3}(t) = \frac{10}{\pi}t \left[ \sum_{k=1}^{\infty} 10 \left( \frac{\sin(\pi/2(1-2k))}{\pi(1-2k)} + \frac{\sin(\pi/2(1+2k))}{\pi(1+2k)} \right) \right] \cos\left(\frac{2\pi}{9}kt\right) + \text{FSE}\left[\frac{5}{2}\cos\left(\frac{\pi}{9}t\right)\right]$$

$$a_0 = \frac{5}{\pi}, \quad a_1 = a_{-1} = \frac{1}{18} \int_{-4.5}^{4.5} \frac{y_{a2}(t) + 5.1 \cos(\pi/9t)}{2} e^{-j\pi/9t} dt$$

$$= \frac{5}{18} \int_{-4.5}^{4.5} \cos\left(\frac{\pi}{9}t\right) e^{-j\pi/9t} dt = \frac{5}{18} \int_{-4.5}^{4.5} \frac{1+e^{-j2\pi/9t}}{2} dt = \frac{5}{18} \left( \frac{9}{2} + \frac{1}{2} \cdot \left( \frac{e^{-j\pi} - e^{j\pi}}{-j2\pi/9} \right) \right)$$

$$a_1 = a_{-1} = \frac{5}{18} \cdot \frac{9}{2} = \left( \frac{5}{4} \right)$$

$$a_k = \frac{5}{2} \cdot \left[ \frac{\sin(\pi/2(1-2k))}{\pi(1-2k)} + \frac{\sin(\pi/2(1+2k))}{\pi(1+2k)} \right] = \frac{5}{2} \left[ \frac{\sin(\pi/2(1-k'))}{\pi(1-k')} + \frac{\sin(\pi/2(1+k'))}{\pi(1+k')} \right]$$

$k \neq 0, k \neq 1, k \neq -1$

$$\text{FSE}[y_{a3}(t)] = \frac{5}{\pi} + \frac{5}{4} e^{j\pi/9} + \frac{5}{4} e^{-j\pi/9} + \sum_{k=2}^{\infty} 2a_k \cos\left(\frac{2\pi}{9}kt\right)$$

$$= \frac{5}{2} \cos\left(\frac{\pi}{9}t\right)$$

$$a_k = \begin{cases} \frac{5}{2} \cdot \left( \frac{\sin(\pi/2(1-k))}{\pi(1-k)} + \frac{\sin(\pi/2(1+k))}{\pi(1+k)} \right) & ; k \neq 0, 1, -1 \\ 5/\pi & ; k=0 \\ 5/4 & ; k=1, -1 \end{cases}$$

→ Gi might just be sides choppy, gibbs phenomenon beitrage dazu!

Figure 36: The Process of Finding the FSE of the  $y_a(t)$

### Question 3.c:

Value of  $a_k$  is shown in the Figure 37:

$$a_k = \begin{cases} \frac{5}{2} \cdot \left( \frac{\sin(\pi/2(1-k))}{\pi(1-k)} + \frac{\sin(\pi/2(1+k))}{\pi(1+k)} \right) ; k \neq 0, 1, -1 \\ 5/\pi ; k=0 \\ 5/4 ; k=1, -1 \end{cases}$$

→ Bi-nigher jehle dala sodeh chuyel, gibbs farsman bejrgan deji!

Figure 37: Values of  $a_k$

Figure 38 shows the spectrum of coefficients of  $a_k$ :

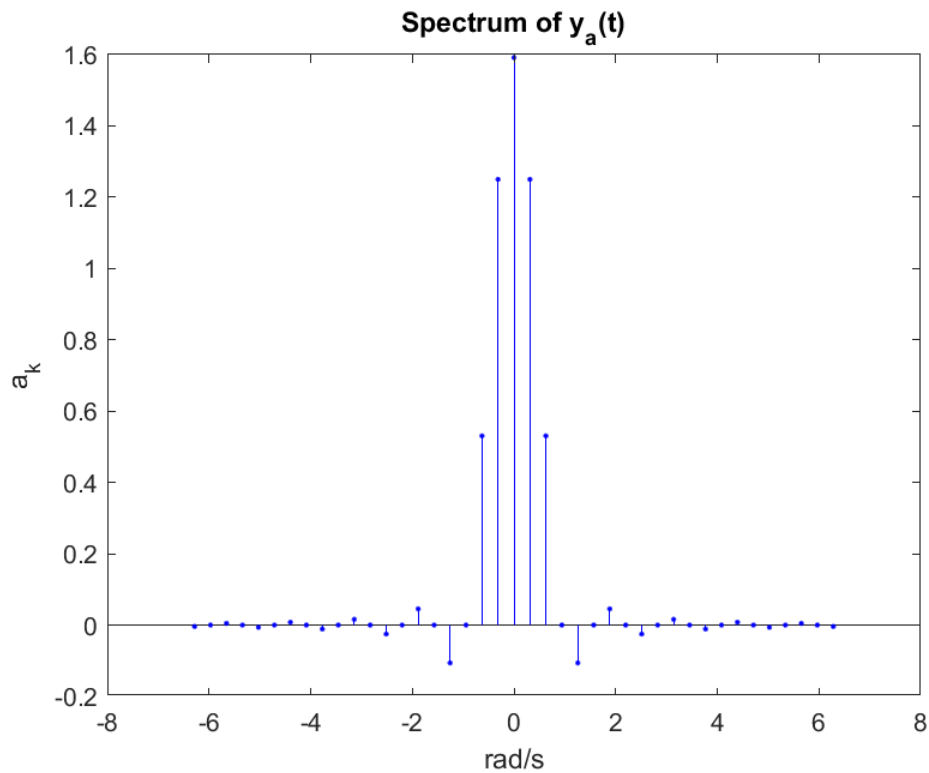


Figure 38: Spectrum of  $a_k$

**Question 3.d:**

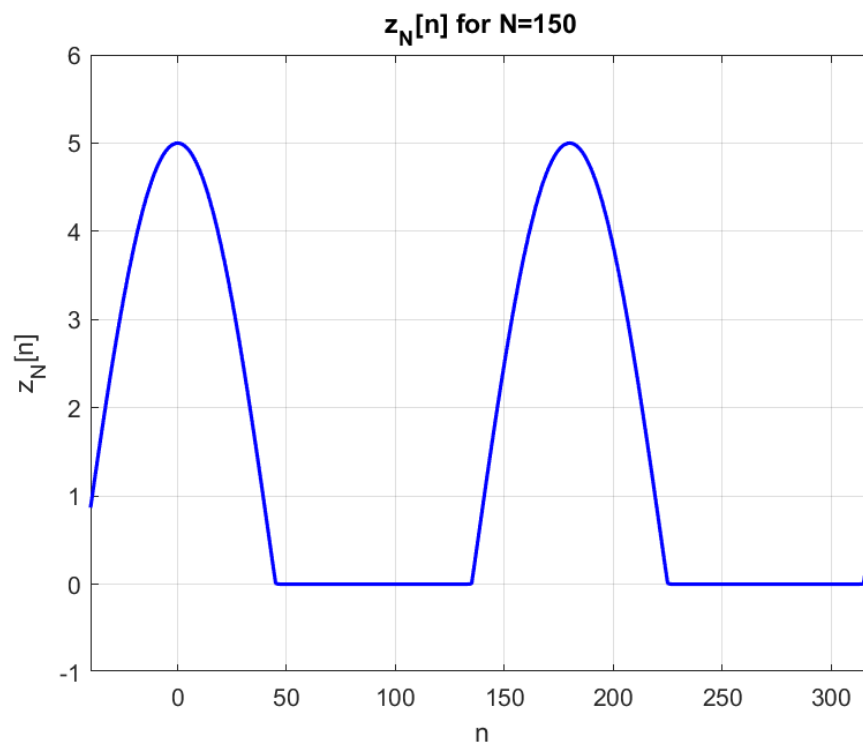


Figure 39: Plot of  $Z_n$  when  $N=150$

**Question 3.e:**

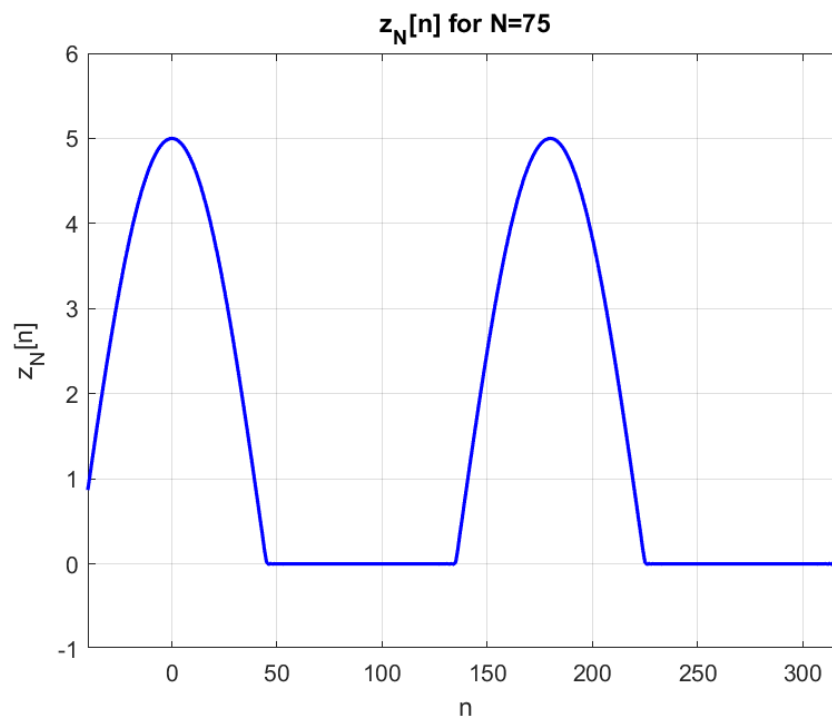


Figure 40: Plot of  $Z_n$  when  $N=75$

**Question 3.f:**

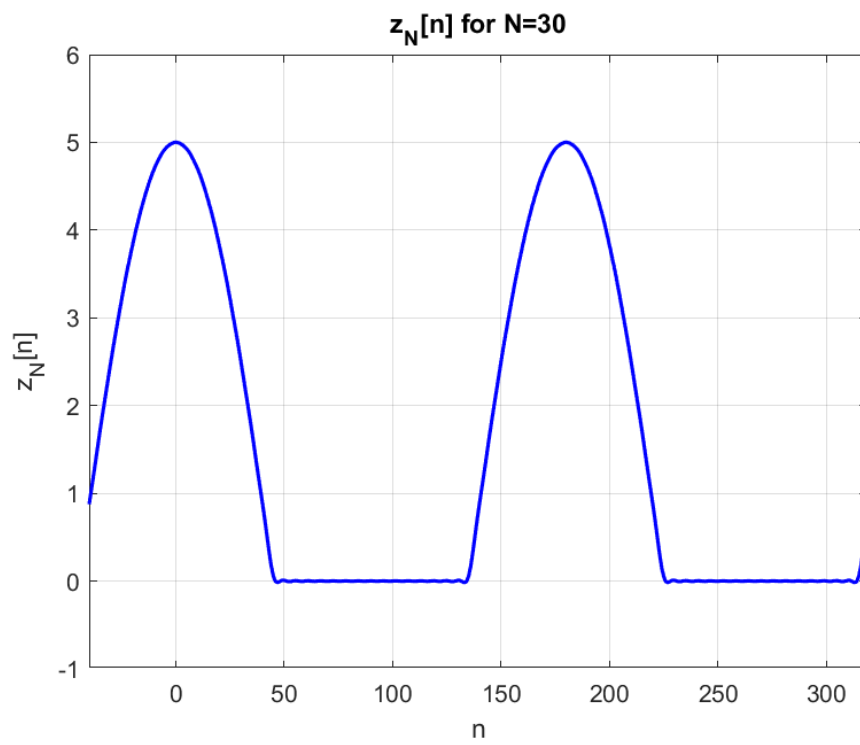


Figure 41: Plot of  $Z_n$  when  $N=150$

**Question 3.g:**

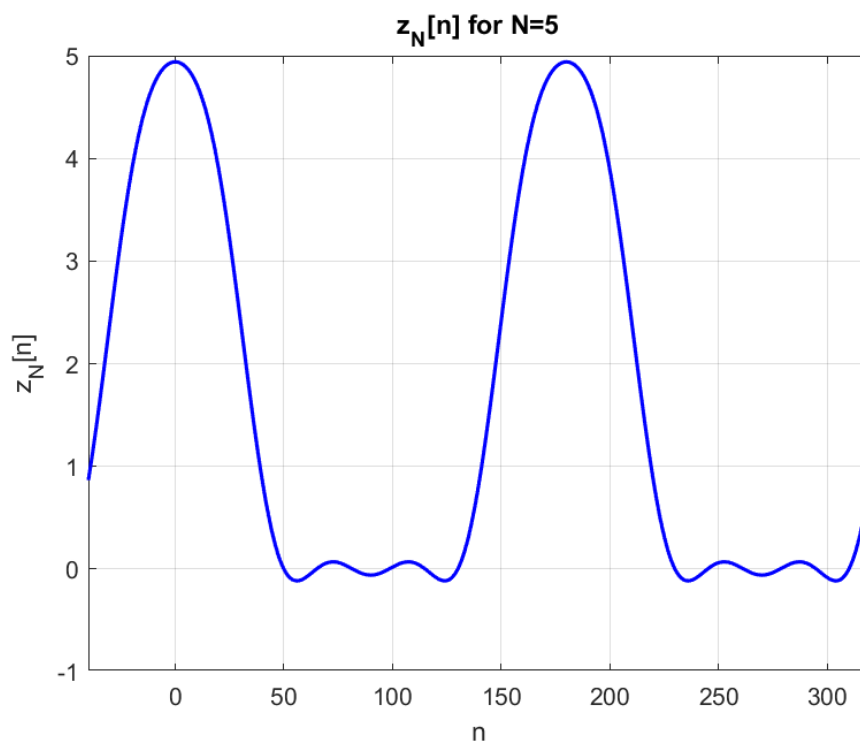


Figure 42: Plot of  $Z_n$  when  $N=5$

**Question 3.h:**

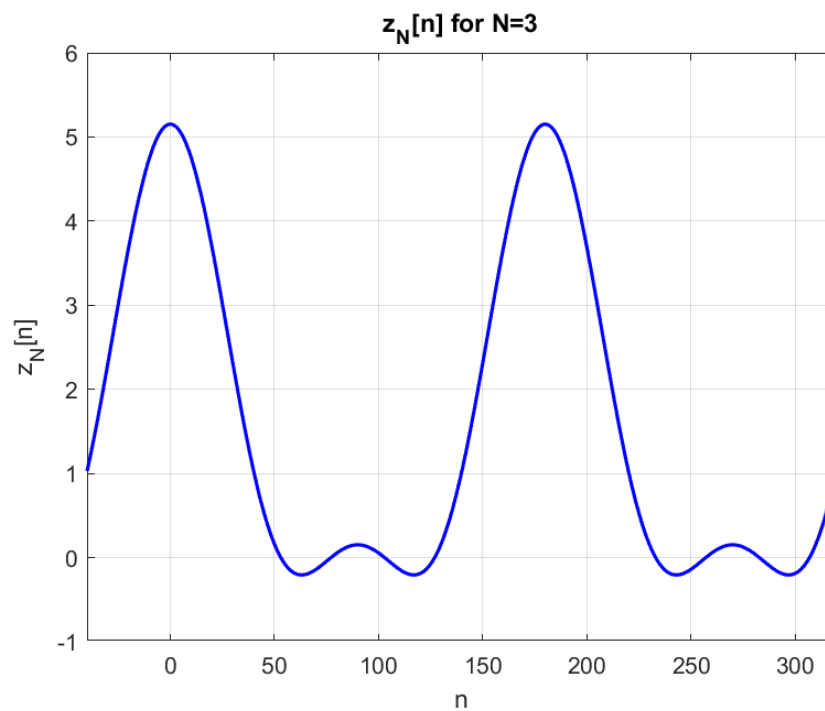


Figure 43: Plot of  $Z_n$  when  $N=3$

**Question 3.j:**

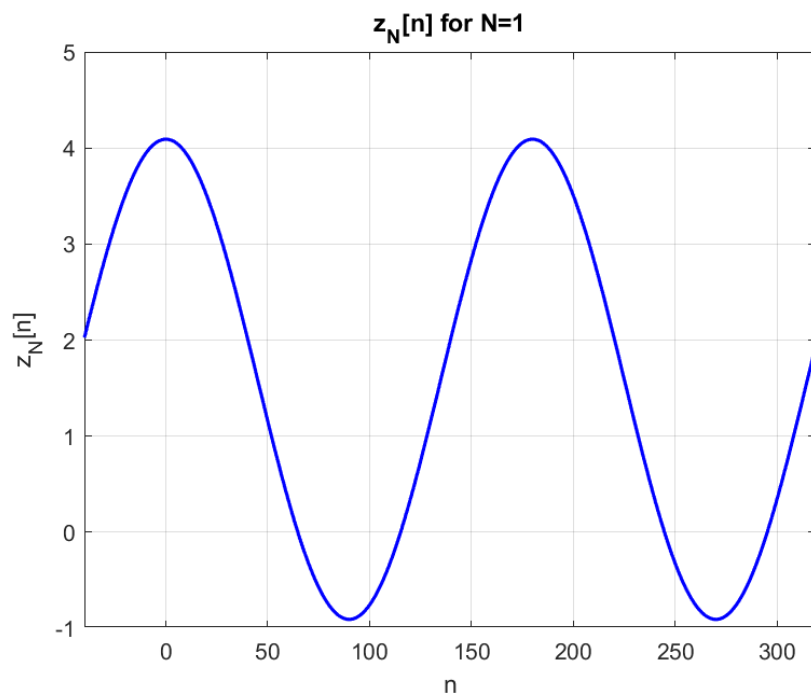


Figure 44: Plot of  $Z_n$  when  $N=1$

The previous two questions can be analyzed similarly to this one. The quality of the approximation declined dramatically as we reduced the number of harmonics used. Higher-

coefficient harmonics had a stronger effect on the approximation than did lower-coefficient harmonics.

**Question 3.k:**

The signal  $y_a(t)$  has only 4 harmonic functions, and they can be seen in the following Figures 45-46-47-48:

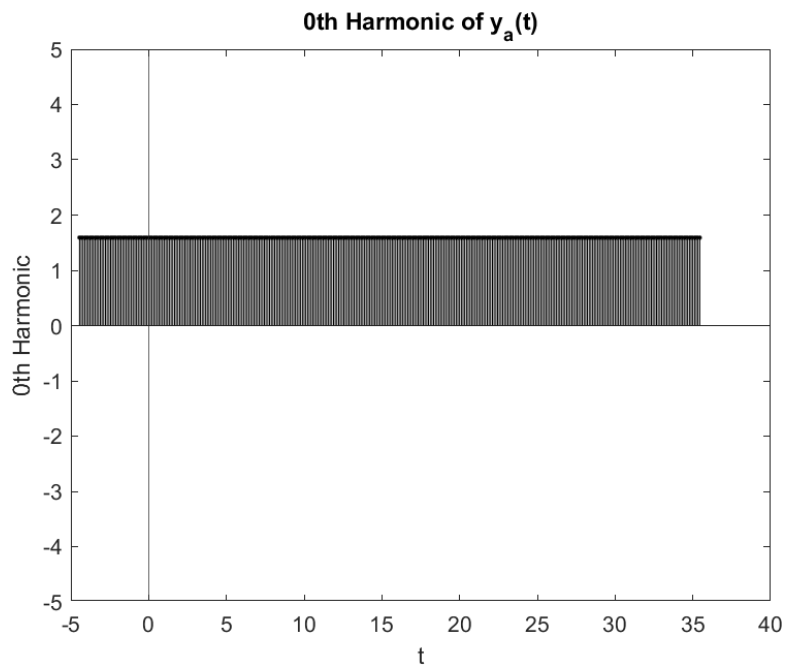


Figure 45: 0<sup>th</sup> Harmonic of  $y_a(t)$

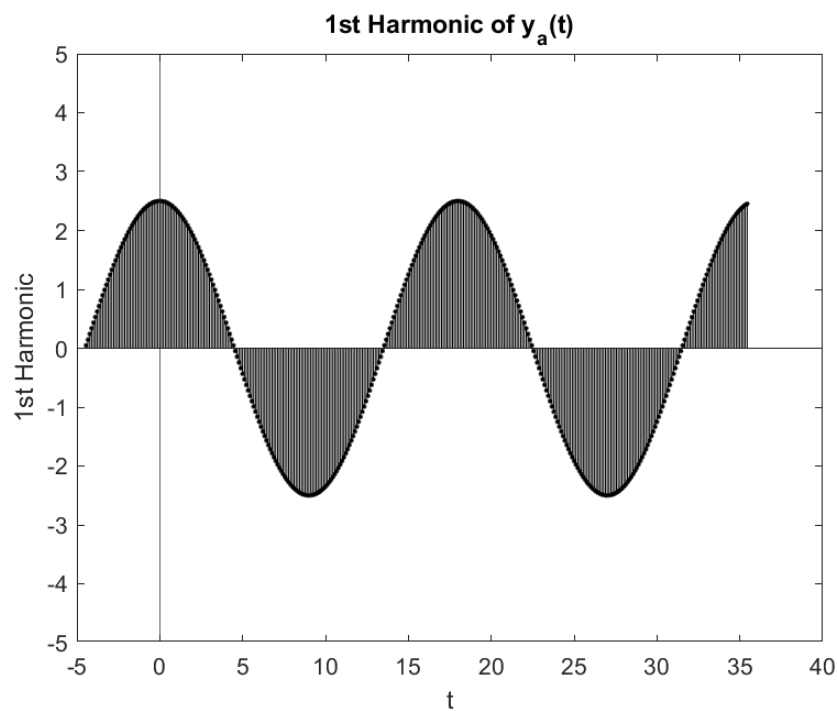


Figure 46: 1<sup>st</sup> Harmonic of  $y_a(t)$



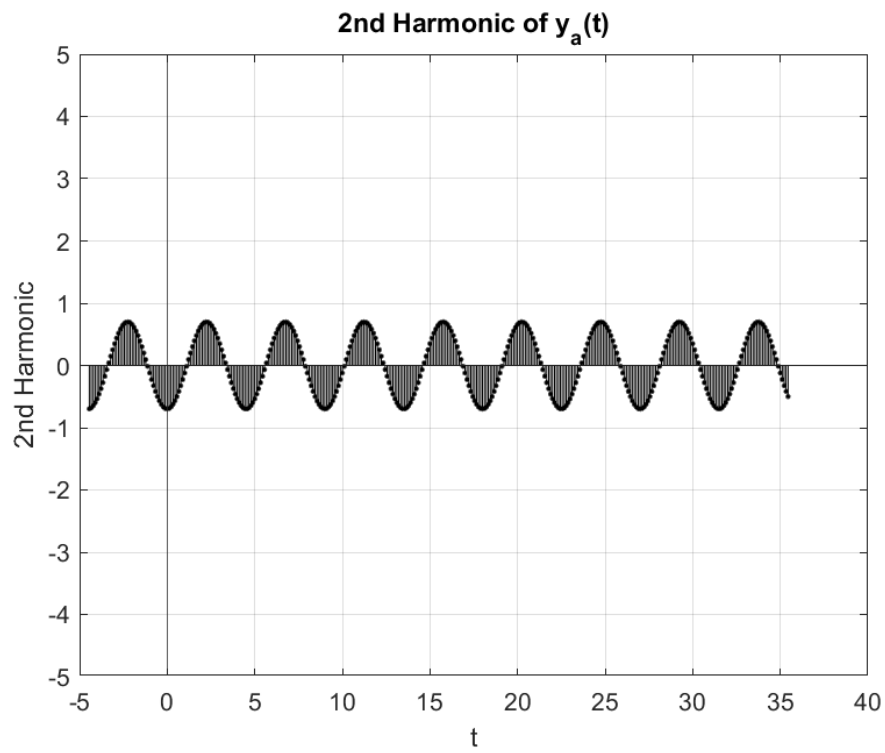


Figure 47: 2<sup>nd</sup> Harmonic of  $y_a(t)$

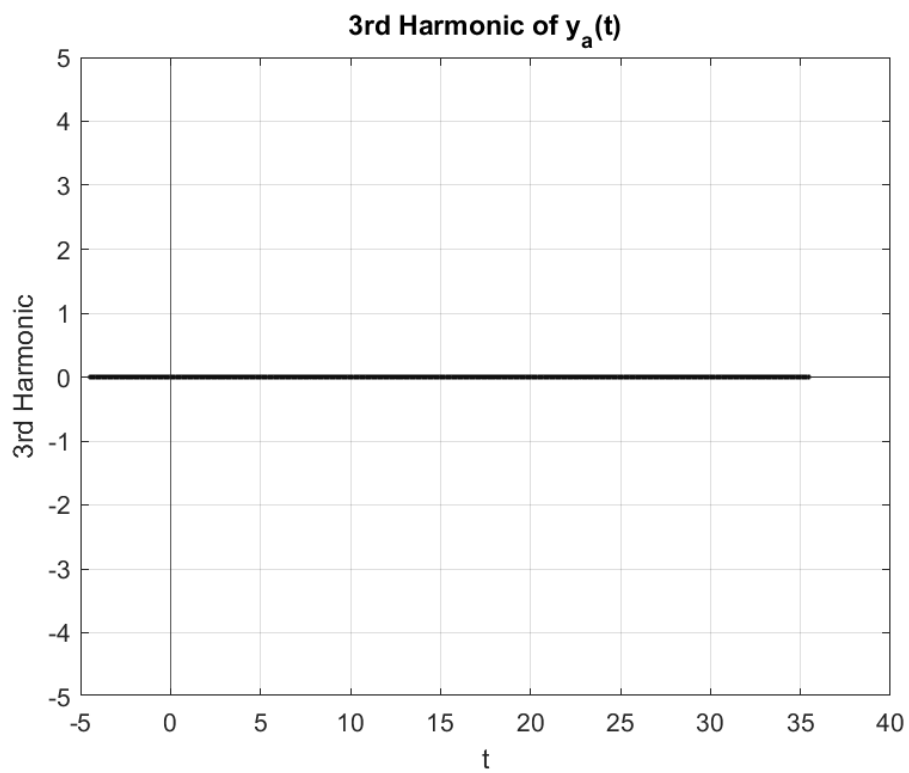


Figure 48: 3<sup>rd</sup> Harmonic of  $y_a(t)$

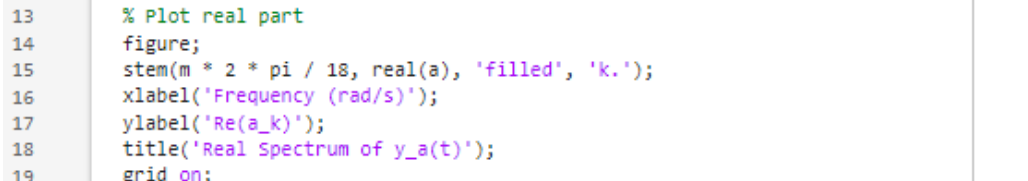
**Conclusion of the Lab:**

All of the questions and requirements are satisfied and they are controlled with the TAs in the Lab hour. This lab thought me a lot about the Fourier Series Expansion. However, writing a report for this lab was exhausting.

## Appendices:

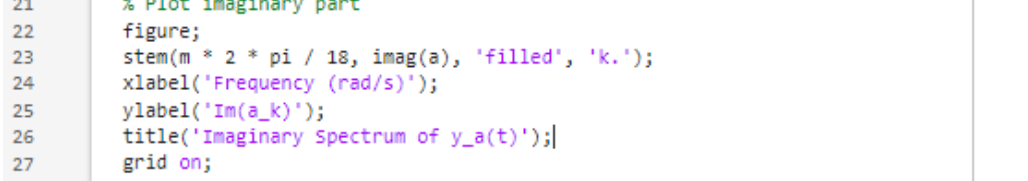
### Question 1:

```
1 % Code to plot the real part of the spectrum
2 m = -20:1:20;
3 a = zeros(size(m));
4
5 a(21) = 4/3;
6
7 for k = 1:length(m)
8     if m(k) ~= 0
9         a(k) = (4/(1i*pi*m(k)))*(exp(1i*22*pi*m(k)/18) - exp(1i*16*pi*m(k)/18));
10    end
11 end
12
13 % Plot real part
14 figure;
15 stem(m * 2 * pi / 18, real(a), 'filled', 'k.');
```



The plot shows the real part of the spectrum. The x-axis is labeled 'Frequency (rad/s)' and ranges from -20 to 20. The y-axis is labeled 'Re(a\_k)' and ranges from -0.1 to 0.1. The plot shows a stem plot with filled circles. The spectrum is zero for all frequencies except at 22 rad/s, where it has a value of approximately 0.08.

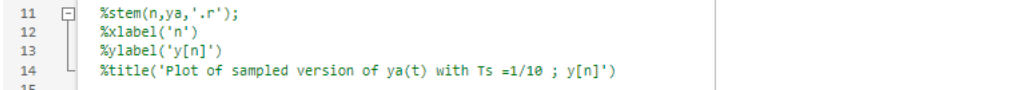
```
16 xlabel('Frequency (rad/s)');
17 ylabel('Re(a_k)');
18 title('Real Spectrum of y_a(t)');
19 grid on;
20
21 % Plot imaginary part
22 figure;
23 stem(m * 2 * pi / 18, imag(a), 'filled', 'k.');
```



The plot shows the imaginary part of the spectrum. The x-axis is labeled 'Frequency (rad/s)' and ranges from -20 to 20. The y-axis is labeled 'Im(a\_k)' and ranges from -0.1 to 0.1. The plot shows a stem plot with filled circles. The spectrum is zero for all frequencies except at 22 rad/s, where it has a value of approximately -0.08.

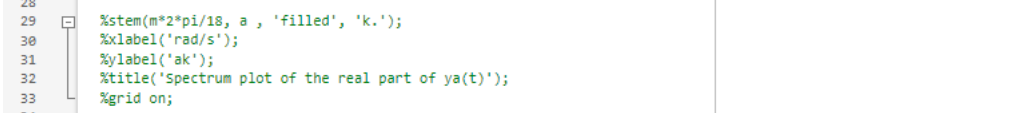
```
24 xlabel('Frequency (rad/s)');
25 ylabel('Im(a_k)');
26 title('Imaginary Spectrum of y_a(t)');
27 grid on;
28
```

```
1 %1.a
2 n = -40:1:319;
3 ya = zeros(size(n));
4 for i = n
5     dum = mod(i/10,18); % T/10 da sample ,period 18
6     if (dum >= 7)&&(dum < 10)
7         ya(i+41) = 8;% m 41 tane deęer aldı
8     end
9 end
10
11 %stem(n,ya,'r.');
```



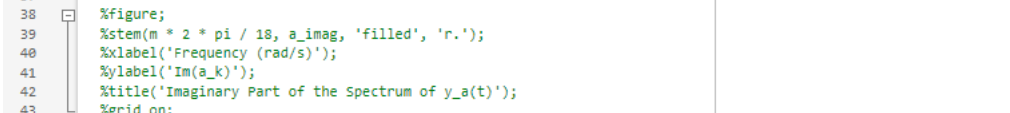
The plot shows the sampled version of ya(t). The x-axis is labeled 'n' and ranges from -40 to 319. The y-axis is labeled 'y[n]' and ranges from 0 to 8. The plot shows a stem plot with red circles. The signal is zero for all n except at n = 41, where it has a value of 8.

```
12 %xlabel('n')
13 %ylabel('y[n]')
14 %title('Plot of sampled version of ya(t) with Ts =1/10 ; y[n]')
15
16 %1.c
17
18 m = -20:1:20;
19 a = zeros(size(m));
20
21 a(21) = 4/3;
22
23 for k = 1:length(m)
24     if m(k) ~= 0
25         a(k) = (4/(1i*pi*m(k)))*(exp(1i*22*pi*m(k)/18) - exp(1i*16*pi*m(k)/18)); %ak katsayılarım elle bulduğum
26     end
27 end
28
29 %stem(m*2*pi/18, a , 'filled', 'k.');
```



The plot shows the spectrum plot of the real part of ya(t). The x-axis is labeled 'rad/s' and ranges from -20 to 20. The y-axis is labeled 'ak' and ranges from -0.1 to 0.1. The plot shows a stem plot with filled circles. The spectrum is zero for all frequencies except at 22 rad/s, where it has a value of approximately 0.08.

```
30 %xlabel('rad/s');
31 %ylabel('ak');
32 %title('Spectrum plot of the real part of ya(t)');
33 %grid on;
34
35
36 %a_imag = imag(a);
37
38 %figure;
39 %stem(m * 2 * pi / 18, a_imag, 'filled', 'r.');
```



The plot shows the imaginary part of the spectrum. The x-axis is labeled 'Frequency (rad/s)' and ranges from -20 to 20. The y-axis is labeled 'Im(a\_k)' and ranges from -0.1 to 0.1. The plot shows a stem plot with red circles. The spectrum is zero for all frequencies except at 22 rad/s, where it has a value of approximately -0.08.

```
40 %xlabel('Frequency (rad/s)');
41 %ylabel('Im(a_k)');
42 %title('Imaginary Part of the Spectrum of y_a(t)');
43 %grid on;
44
45
46 %1.d
47 n = -40:319;
48 N = 150;
```

```

49 N = 150;
50 a = zeros(1, 2*N + 1);
51 a(N + 1) = 4/3;
52
53 for k = -N:N
54     if k ~= 0
55         a(k + N + 1) = (4 / (11 * pi * k)) * (exp(11 * 22 * pi * k / 18) - exp(11 * 16 * pi * k / 18));
56     end
57 end
58
59 zn = zeros(1, length(n));
60 for k = -N:N
61     zn = zn + a(k + N + 1) * exp(11 * pi * k * n / 81); %zn
62 end
63
64 %plot(n, real(zn), 'k','LineWidth', 1.5);
65 %title('Plot of z_N[n] when N is 150');
66 %ylabel('z_{150}[n]');
67 %xlabel('n');
68 %xline(0);
69 %yline(0);
70 %grid on;
71
72 n = -40:319;
73 N = 75;
74 a = zeros(1, 2*N + 1);
75 a(N + 1) = 4/3;
76
77 for k = -N:N
78     if k ~= 0
79         a(k + N + 1) = (4 / (11 * pi * k)) * (exp(11 * 22 * pi * k / 18) - exp(11 * 16 * pi * k / 18));
80     end
81 end
82
83 zn = zeros(1, length(n));
84 for k = -N:N
85     zn = zn + a(k + N + 1) * exp(11 * pi * k * n / 81);
86 end
87
88 %plot(n, real(zn), 'k', 'LineWidth', 1.5);
89 %title('Plot of z_N[n] when N is 75');
90 %ylabel('z_{75}[n]');
91 %xlabel('n');
92 %xline(0);
93 %yline(0);
94 %grid on;
95
96 n = -40:319;
97 N = 30;
98 a = zeros(1, 2*N + 1);
99 a(N + 1) = 4/3;
100
101 for k = -N:N
102     if k ~= 0
103         a(k + N + 1) = (4 / (11 * pi * k)) * (exp(11 * 22 * pi * k / 18) - exp(11 * 16 * pi * k / 18));
104     end
105 end
106
107 zn = zeros(1, length(n));
108 for k = -N:N
109     zn = zn + a(k + N + 1) * exp(11 * pi * k * n / 81);
110 end
111
112 %figure;
113 %plot(n, real(zn), 'k', 'LineWidth', 1.5);
114 %title('Plot of z_N[n] when N is 30');
115 %ylabel('z_{30}[n]');
116 %xlabel('n');
117 %xline(0);
118 %yline(0);
119 %grid on;
120
121 n = -40:319;
122 N = 5;
123 a = zeros(1, 2*N + 1);
124 a(N + 1) = 4/3;
125
126 for k = -N:N
127     if k ~= 0
128         a(k + N + 1) = (4 / (11 * pi * k)) * (exp(11 * 22 * pi * k / 18) - exp(11 * 16 * pi * k / 18));
129     end
130 end
131
132 zn = zeros(1, length(n));
133 for k = -N:N
134     zn = zn + a(k + N + 1) * exp(11 * pi * k * n / 81);
135 end
136
137 %figure;

```

```

136
137 %figure;
138 %plot(n, real(zn), 'k', 'Linewidth', 1.5);
139 %title('Plot of z_N[n] when N is 5');
140 %ylabel('z_{5}[n]');
141 %xlabel('n');
142 %xline(0);
143 %yline(0);
144 %grid on;
145
146
147 n = -40:319;
148 N = 3;
149 a = zeros(1, 2*N + 1);
150 a(N + 1) = 4/3;
151
152 for k = -N:N
153     if k ~= 0
154         a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
155     end
156 end
157
158 zn = zeros(1, length(n));
159 for k = -N:N
160     zn = zn + a(k + N + 1) * exp(1i * pi * k * n / 81);
161 end
162
163 %figure;
164 %plot(n, real(zn), 'k', 'Linewidth', 1.5);
165 %title('Plot of z_N[n] when N is 3');
166 %ylabel('z_{3}[n]');
167 %xlabel('n');
168 %xline(0);
169 %yline(0);
170 %grid on;
171
172 n = -40:319;
173 N = 1;
174 a = zeros(1, 2*N + 1);
175 a(N + 1) = 4/3;
176
177 for k = -N:N
178     if k ~= 0
179         a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
180     end

```

```

180     end
181 end
182
183 zn = zeros(1, length(n));
184 for k = -N:N
185     zn = zn + a(k + N + 1) * exp(1i * pi * k * n / 81);
186 end
187
188 %figure;
189 %plot(n, real(zn), 'k', 'Linewidth', 1.5);
190 %title('Plot of z_N[n] when N is 1');
191 %ylabel('z_{1}[n]');
192 %ylim([-2 10]);
193 %xlabel('n');
194 %xline(0);
195 %yline(0);
196 %grid on;
197
198 %0 harmonic
199
200 figure;
201 ylim([-2 2]);
202 stem(n, ones(1, length(n))*3/3, 'k.');
203 title('0th Harmonic of y_a(t)');
204 ylabel('0th Harmonic'); xlabel('t'); xline(0); yline(0);
205 ylim([-2 2]);
206 %1. harmonic
207
208 n = -40:319;
209
210
211 k = 1;
212 f_first_harmonic = a(k + N + 1) * exp(1i * pi * k * n / 81);
213
214 figure;
215 ylim([-2 2]);
216 stem(n / 9, real(f_first_harmonic), 'k.');
217 title('1st Harmonic of y_a(t)');
218 ylabel('1st Harmonic');
219 xlabel('t');
220 xline(0);
221 yline(0);
222 grid on;
223 ylim([-2 2]);

```

```

223 ylim([-2 2]);
224 %2.harmonic
225 n = -40:319;
226 N = 2;
227 a = zeros(1, 2 * N + 1);
228 a(N + 1) = 4/3;
229
230
231 for k = -N:N
232     if k ~= 0
233         a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
234     end
235 end
236
237
238 k = 2;
239 f_second_harmonic = a(k + N + 1) * exp(1i * pi * k * n / 81);
240
241 figure;
242 ylim([-2 2]);
243 stem(n / 9, real(f_second_harmonic), 'k.');
244 title('2nd Harmonic of y_a(t)');
245 ylabel('2nd Harmonic');
246 xlabel('t');
247 xline(0);
248 yline(0);
249 grid on;
250 ylim([-2 2]);
251 %3.har
252
253 n = -40:319;
254 N = 3;
255 a = zeros(1, 2 * N + 1);
256 a(N + 1) = 4 / 3;
257
258
259 for k = -N:N
260     if k ~= 0
261         a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
262     end
263 end
264
265
266 k = 3;
267 f_third_harmonic = a(k + N + 1) * exp(1i * pi * k * n / 81);
268
269 figure;
270 ylim([-2 2]);
271 stem(n / 9, real(f_third_harmonic), 'k.');
272 title('3rd Harmonic of y_a(t)');
273 ylabel('3rd Harmonic');
274 xlabel('t');
275 xline(0);
276 yline(0);
277 grid on;
278 ylim([-2 2]);
279

```

## Question 2:

```

1 Ts = 1/10;
2 n = -40:319;
3 t = n * Ts;
4 T_period = 9;
5 omega_0 = 2 * pi / T_period;
6
7 % Discrete signal
8 y = abs(5 * cos(pi * t / 9));
9
10 % Plot the discrete signal y[n]
11 figure;
12 stem(n, y, 'g');
13 xlabel('n');
14 ylabel('y[n]');
15 % title('Plot of ya(t) with Ts =1/10 ; y[n]');
16 grid on;
17
18 % Fourier coefficients a_k
19 a0 = 10 / pi;
20 N_max = 150;
21 ak = zeros(1, 2 * N_max + 1);
22
23 % Compute a_k for k = -N_max to N_max
24 for k = -N_max:N_max
25     if k ~= 0
26         ak(k + N_max + 1) = (5 / pi) * (sin((pi / 2) - pi * k) / (1 - 2 * k) + ...
27                                     sin((pi / 2) + pi * k) / (1 + 2 * k));
28     else
29         ak(k + N_max + 1) = a0;
30     end
31 end
32 %
33 % Plot Fourier series coefficients a_k for k = -40 to 40
34 figure;
35 stem(-40:40, ak(N_max-40+1:N_max+40+1), 'filled', 'g');
36 xlabel('Frequency (rad/s)');
37 ylabel('a_k');
38 %title('Spectrum of Coefficients of a_k');
39 grid on;
40
41 % Array of N values for different reconstructed signals
42 N_values = [150, 75, 30, 5, 3, 1];
43
44 % Plot z_N[n] for different values of N, each in a separate figure
45 for i = 1:length(N_values)
46     N = N_values(i);
47     zN = a0 * ones(size(n));
48
49     % Compute z_N[n] using Fourier series
50     for k = 1:N
51         zN = zN + 2 * ak(k + N_max + 1) * cos(omega_0 * k * n * Ts);
52     end
53
54     % Plot z_N[n] for the current N value in a new figure
55     figure;
56     plot(t, zN, 'k', 'LineWidth', 1.5);
57     xlabel('t');
58     % ylabel(['z_{N}[n] for N = ', num2str(N)]);
59     title(['Plot of z_N[n] when N = ', num2str(N)]);
60     grid on;
61 end
62 %
63
64 zeroth_harmonic = a0 * ones(size(t));
65 figure;
66 plot(t, zeroth_harmonic, 'k', 'LineWidth', 1.5);
67 xlabel('Time (s)');
68 ylabel('Amplitude');
69 ylim([-3 3]);
70 title('0th Harmonic (DC Component) of y_a (t)');
71 grid on;
72 ylim([-5 5]);
73
74 k = 1;
75 first_harmonic = 2 * ak(k + N_max + 1) * cos(omega_0 * k * t);
76 figure;
77 plot(t, first_harmonic, 'k', 'LineWidth', 1.5);
78 xlabel('Time (s)');
79 ylabel('Amplitude');
80
81 title('First Harmonic of y_a (t)');
82 grid on;
83 ylim([-5 5]);
84
85
86 k = 2;
87 second_harmonic = 2 * ak(k + N_max + 1) * cos(omega_0 * k * t);
88 figure;
89 plot(t, second_harmonic, 'k', 'LineWidth', 1.5);
90 xlabel('Time (s)');
91 ylabel('Amplitude');
92 title('Second Harmonic of y_a (t)');
93 grid on;

```

```

93     grid on;
94     ylim([-5 5]);
95
96
97
98
99
100    k = 3;
101    third_harmonic = 2 * ak(k + N_max + 1) * cos(omega_0 * k * t);
102    figure;
103    plot(t, third_harmonic, 'k','LineWidth', 1.5);
104    xlabel('Time (s)');
105    ylabel('Amplitude');
106    title('Third Harmonic of y_a (t)');
107    grid on;
108    ylim([-5 5]);

```

### Question 3:

```

1  % Parameters
2  T = 18;           % Period of the signal (in seconds)
3  omega0 = pi/9;    % Fundamental angular frequency
4  Ts = 0.1;         % Sampling period (in seconds)
5  N = 1;            % Number of harmonics on each side
6  n = -40:319;      % Sample indices
7
8  % Initialize zN for all n
9  zN = zeros(size(n));
10
11 % Precompute Fourier coefficients a_k for k from -N to N
12 a_coeff = zeros(1, 2*N + 1); % Array to store coefficients a_k
13 k_values = -N:N;           % k indices from -N to N
14
15 % Calculate Fourier coefficients a_k
16 for idx = 1:length(k_values)
17     k = k_values(idx);
18     if k == 0
19         a_coeff(idx) = 5/pi; % a_0
20     elseif k == 1 || k == -1
21         a_coeff(idx) = 5/4; % a_1 for both positive and negative k
22     elseif mod(k, 2) == 0
23         % Even harmonic k
24         a_coeff(idx) = (5/pi) * (cos(pi * k / 2)) / (1 - k^2);
25     else
26         % Odd harmonic k >= 3
27         a_coeff(idx) = 0;
28     end
29 end
30
31 % Compute zN[n] using the partial sum of harmonics
32 for i = 1:length(n)
33     zN(i) = sum(a_coeff .* exp(1j * omega0 * k_values * n(i) * Ts));
34 end
35
36 % Plot the real part of zN[n]
37 figure;
38 plot(n, real(zN), 'b', 'LineWidth', 1.5);
39 xlabel('n');
40 ylabel('z_N[n]');
41 title('z_N[n] for N=1');
42 grid on;
43 xlim([-40, 319]);

```



```

1      n = -40:1:319;
2      ya = zeros(size(n));
3      for i = n
4          dum = mod(i/10+4.5,18);
5          if (dum >= 0) && (dum < 9)
6              ya(i+41) = abs(5*cos(pi/9*(dum+4.5)));
7          end
8      end
9      %figure;
10     %stem(n, ya, 'k');
11     %xlabel('n/8');
12     %ylabel('y[n]');
13     %title('Plot of ya(t) with Ts =1/10 ; y[n]')
14
15     m = -20:1:20;
16     a = 2.5 * ((sin(pi/2 * (1 - m)) ./ (pi * (1 - m))) + (sin(pi/2 * (1 + m)) ./ (pi * (1 + m))));
17     a(20) = 5/4;
18     a(21) = 5/pi;
19     a(22) = 5/4;
20
21     %figure;
22     %stem(m * pi / 10, a, 'filled', 'b. ');
23     %xlabel('rad/s');
24     %ylabel('a_k');
25     %title('Spectrum of y_a(t)');
26
27     %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
28
29     figure;
30     stem(n/9,ones(1,length(n))*5/pi,'k. ');
31     title('0th Harmonic of y_a(t)');
32     ylabel('0th Harmonic');xlabel('t');xline(0);yline(0);
33     ylim([-5 5])
34     figure;
35     stem(n/9,5/2*cos(pi/9*n/9),'k. ');
36     title('1st Harmonic of y_a(t)');
37     ylabel('1st Harmonic');xlabel('t');xline(0);yline(0);
38     ylim([-5 5])
39     % Define the range for n
40     n = -40:319;
41
42     % Initialize the array for ya
43     ya = zeros(size(n));
44
45     % Calculate ya values for each n
46     for i = n
47         dum = mod(i / 10, 9); % Get the modulo value for the current n
48         ya(i + 41) = abs(5 * cos(pi / 9 * dum)); % Calculate the corresponding value of ya
49     end
50
51     % Define the range for m (Fourier coefficients)
52     m = -20:1:20;
53     a = zeros(size(m));
54
55     a = 5 * ((sin(pi / 2 * (1 - 2 * m)) ./ (pi .* (1 - 2 * m))) + (sin(pi / 2 * (1 + 2 * m)) ./ (pi .* (1 + 2 * m)))); % Fourier series coefficients
56
57     % Second Harmonic (k = 2)
58     k = 2;
59     f_second_harmonic = 3.3 * a(k + 21) * cos(2 * pi / 9 * k * n / 9);
60
61     % Plot the second harmonic
62     figure;
63     stem(n / 9, f_second_harmonic, 'k. ');
64     title('2nd Harmonic of y_a(t)');
65     ylabel('2nd Harmonic');
66     xlabel('t');
67     xline(0); % Adds a vertical line at t = 0
68     yline(0); % Adds a horizontal line at y = 0
69     grid on; % Enables grid for better readability
70     ylim([-5 5])
71
72     % Define the range for n
73     n = -40:319;
74
75     % Initialize the array for ya
76     ya = zeros(size(n));
77
78     % Calculate ya values for each n
79     for i = n
80         dum = mod(i / 10, 9); % Get the modulo value for the current n
81         ya(i + 41) = abs(5 * cos(pi / 9 * dum)); % Calculate the corresponding value of ya
82     end
83
84     % Define the range for m (Fourier coefficients)
85     m = -20:1:20;
86     a = zeros(size(m));
87
88     a = 5 * ((sin(pi / 2 * (1 - 2 * m)) ./ (pi .* (1 - 2 * m))) + (sin(pi / 2 * (1 + 2 * m)) ./ (pi .* (1 + 2 * m)))); % Fourier series coefficients
89
90     % third Harmonic (k = 3)
91     k = 3;
92     f_second_harmonic = 0 * a(k + 21) * cos(2 * pi / 9 * k * n / 9);
93
94     f_second_harmonic = 0 * a(k + 21) * cos(2 * pi / 9 * k * n / 9);
95
96     % Plot the second harmonic
97     figure;
98     stem(n / 9, f_second_harmonic, 'k. ');
99     title('3rd Harmonic of y_a(t)');
100    ylabel('3rd Harmonic');
101    xlabel('t');
102    xline(0); % Adds a vertical line at t = 0
103    yline(0); % Adds a horizontal line at y = 0
104    grid on; % Enables grid for better readability
105    ylim([-5 5])

```

