BILKENT UNIVERSITY

ELECTRICAL AND ELECTRONICS ENGINEERING DEPARTMENT

14/11/2024

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Introduction:

The purpose of this lab is to teach us Fourier Series Expansion with applying it on MATLAB. The lab walks us through the process of discretizing a rectangular waveform, determining and charting its Fourier series expansion, examining its frequency spectrum, and using a small number of Fourier series components to create an approximation of the original signal.

Question 1:

The discretized function $y_a(t)$ with Ts = 1/10 seconds is given in Figure 1 and its plot is given in the Figure 2:

$$y_a(t) = \begin{cases} 0, & t \in [0,7)s \\ 8, & t \in [7,10)s \\ 0, & t \in [10,18)s \end{cases}$$

Figure 1: $y_a(t)$

Question 1.a:

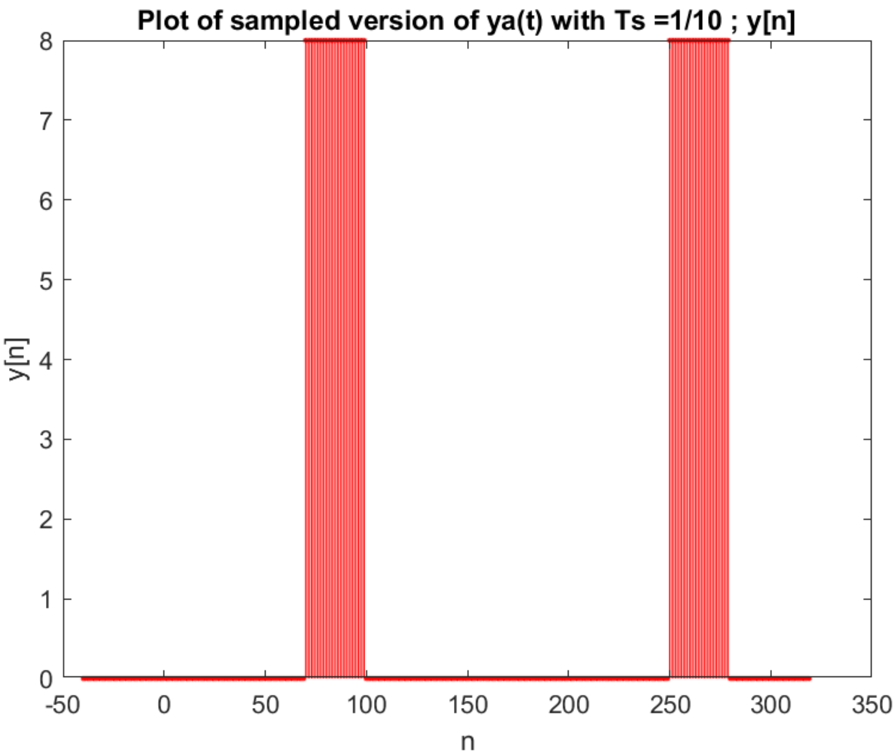

Figure 2: Plot of Discretized $y_a(t)$ with Ts=1/10

Question 1.b:

On the Figure 3, you can see the process of how I found the FSE of $y_a(t)$:

1.b)

$$y_a(t) = \begin{cases} 0, & t \in [0,7)s \\ 8, & t \in [7,10)s \\ 0, & t \in [10,18)s \end{cases}$$

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{18}\int_0^{18} y_a(t)\,dt = \frac{1}{18}\int_7^{10} 8\,dt = 8t\Big|_7^{10} = \frac{24}{18} = \frac{4}{3}$$

$$a_k = \frac{1}{18}\int_0^{18} y_a(t)\,dt\, e^{\frac{-j2\pi}{18}kt}\,dt$$

$$= \frac{1}{18}\int_7^{10} 8 \cdot e^{\frac{-j2\pi kt}{18}}\,dt$$

$$= \frac{-4}{9} \cdot \frac{18}{2\pi kj}\left(e^{\frac{-j20\pi k}{18}} - e^{\frac{-14\pi kj}{18}}\right) = \frac{8}{2\pi kj}\left(e^{\frac{j22\pi k}{18}} - e^{\frac{j16\pi k}{18}}\right)$$

$$a_k = \frac{4}{j\pi k}\left[\cos\left(\frac{11\pi}{9}k\right) - \cos\left(\frac{8\pi}{9}k\right) + j\sin\left(\frac{11\pi}{9}k\right) - j\sin\left(\frac{8\pi}{9}k\right)\right]$$

$$a_k = \frac{4}{\pi k}\left[\sin\left(\frac{11\pi}{9}k\right) - \sin\left(\frac{8\pi}{9}k\right) + j\cos\left(\frac{8\pi}{9}k\right) - j\cos\left(\frac{11\pi}{9}k\right)\right]$$

$$\boxed{\text{FSE of } y_a(t) \text{ is} \rightarrow \frac{4}{3} + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \frac{4\pi tk}{\pi k} \cdot e^{j\pi/9\,kt}}$$

Figure 3: The Process of Finding the FSE of $y_a(t)$

Figure 4 demonstrates the relationship between the coefficients k and their values a_k. a_k values is given in the Figure 4.

$$a_k = \begin{cases} \frac{4}{3}, & k = 0 \\ \frac{4}{j\pi k}\left(e^{j\frac{22}{18}\pi k} - e^{j\frac{16}{18}\pi k}\right), & k \neq 0 \end{cases}$$

Figure 4: The Values of a_k

Question 1.c:

Figure 5 and Figure 6 shows the spectrum of coefficients showing the real and imaginary parts of a_k.

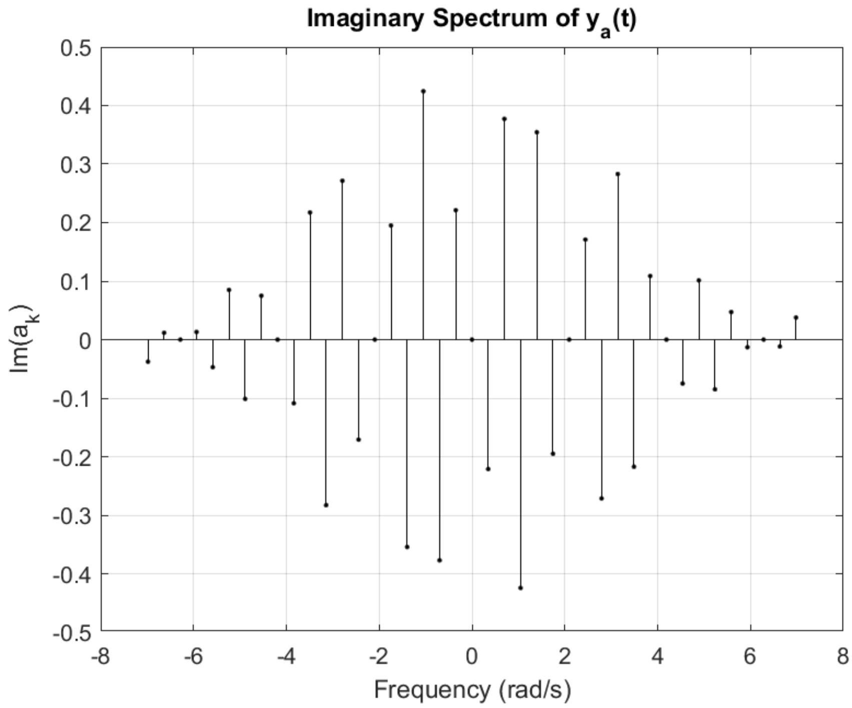

Figure 5: Spectrum of the Imaginary Part of ak

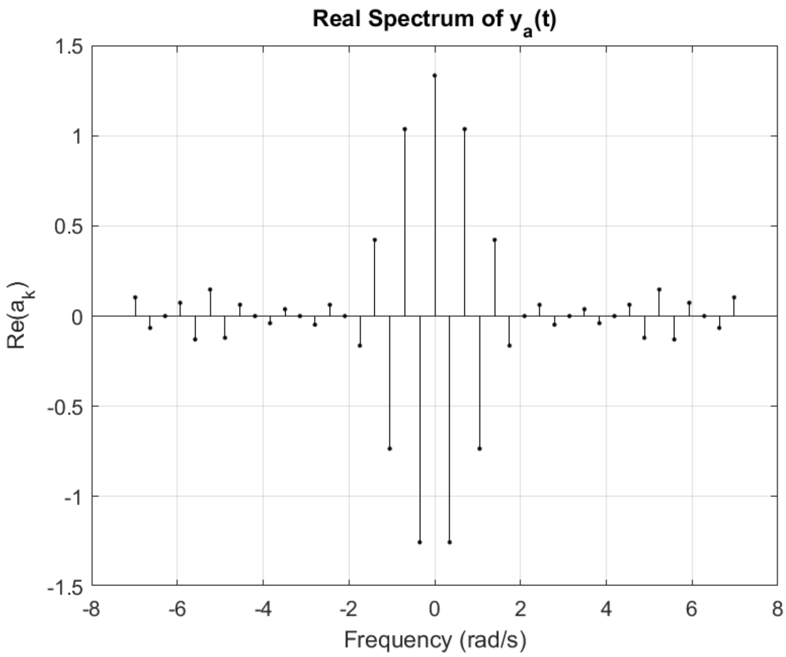

Figure 6: Spectrum of the Real Part of ak

Question 1.d:

In the Part 1.b, the FSE of $y_a(t)$ is found. With using that solution $Z_N(n)$ is found as it is in the Figure 7:

$$Z_N[n] = \frac{10}{\pi} + \sum_{k=1,\, k\neq 0}^{N} 2 \cdot a_k \cdot \cos\left(\frac{2\pi}{9} \cdot k \cdot \frac{n}{9}\right) \text{ for } n \in [-40, 319]$$

Figure 7: $Z_N(n)$

The plot in the Figure 8 is similar to the original function ya(t), though it is not an exact match. For our signal to closely approximate ya(t), the N value would need to extend to infinity. In this case, the N value which is 150, is greater than the other values in the Question 1.e, 1.f, 1.g and others, therefore the plot on the Figure 8 is more similar to ya(t) than in the other questions. So as N decreases toward zero, the approximation increasingly diverges from the original function.

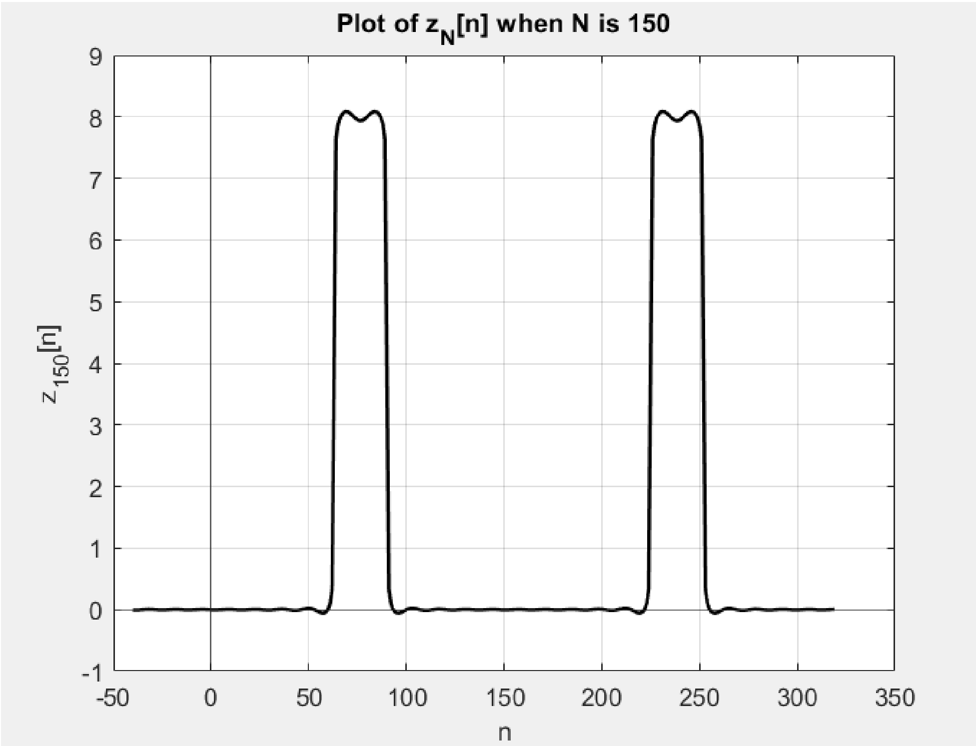

Figure 8: $Z_N(n)$ when N=150

Question 1.e:

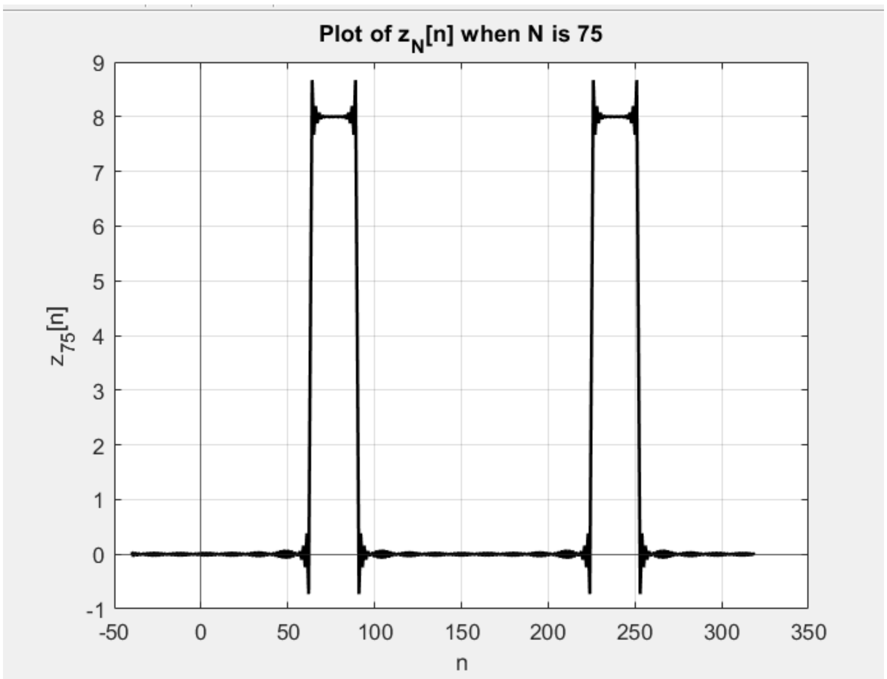

Figure 9: $Z_N(n)$ when N=75

Question 1.f:

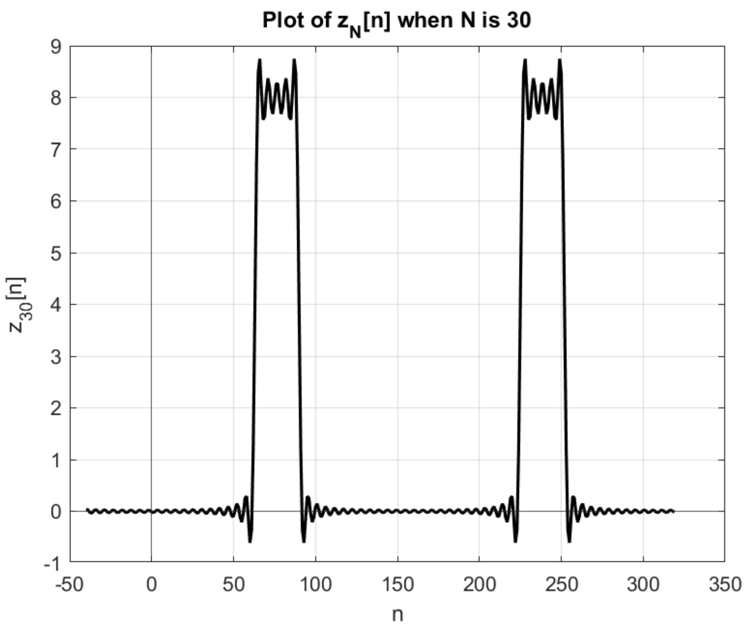

Figure 10: $Z_N(n)$ when N=30

Question 1.g:

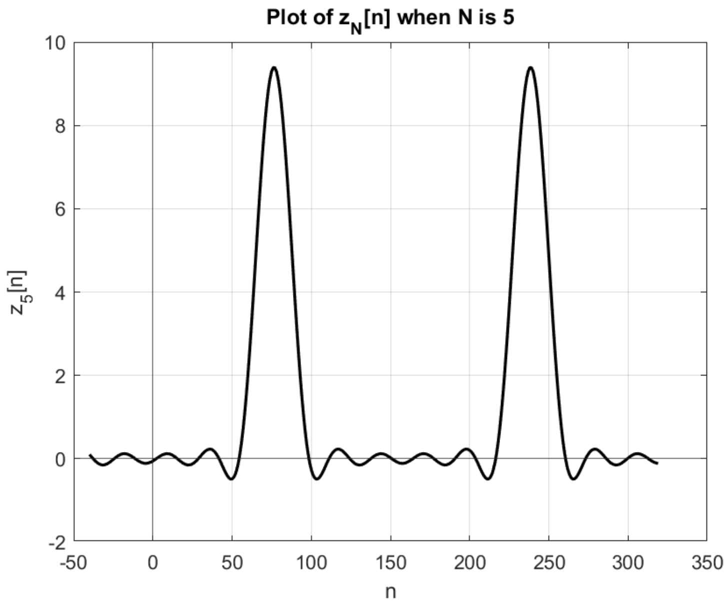

Figure 11: $Z_N(n)$ when N=5

Question 1.h:

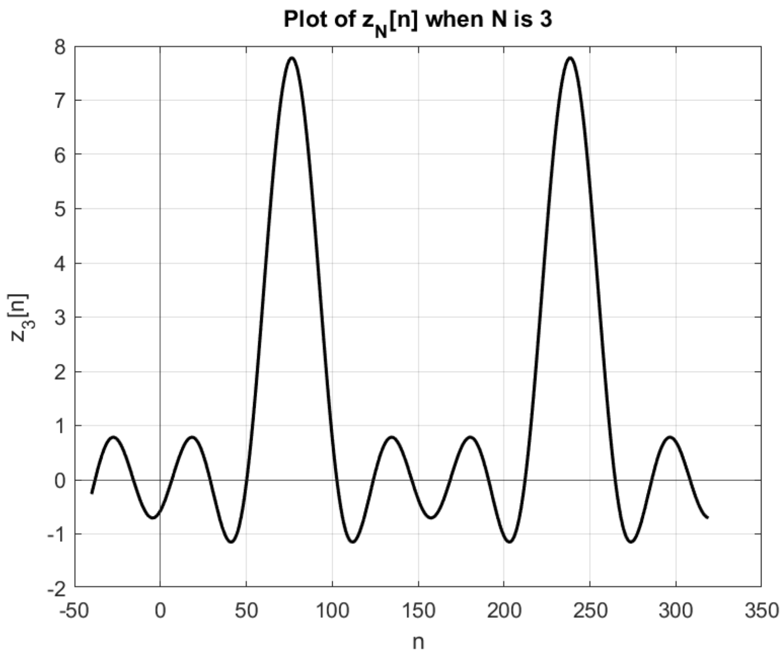

Figure 12: $Z_N(n)$ when N=3

Question 1.i:

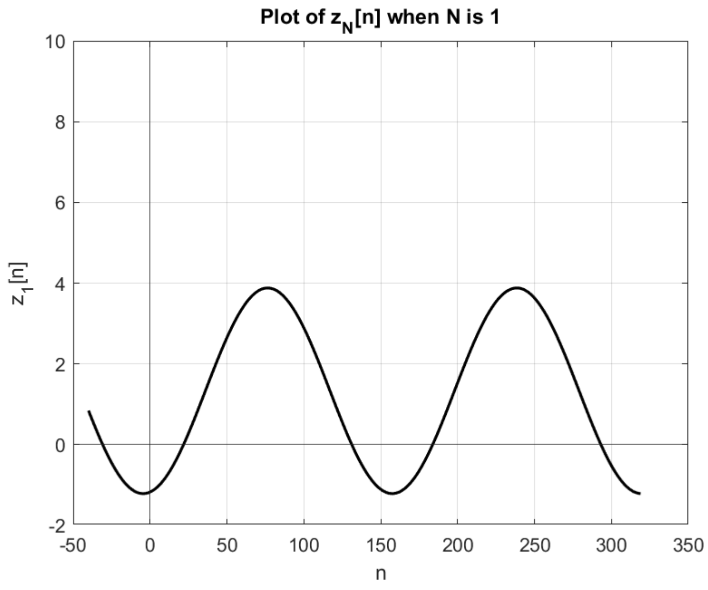

Figure 13: $Z_N(n)$ when N=1

As we observed in part (d), the approximation quality decreases as N approaches zero. This is because, with fewer components, more frequency information is lost, making the approximation less accurate. The components with larger $a_k$values have a greater impact on the shape of the approximation, while those with smaller coefficients contribute less.

Additionally, according to the Gibbs phenomenon, any sudden jumps in the signal can't be completely smoothed out, even as more components are added. In MATLAB, the points are connected smoothly when they are close together, so the plot for N=150 looks smoother than for N=75. As a result, the initial approximation plot may appear smoother in MATLAB than it would naturally.

Question 1.j:

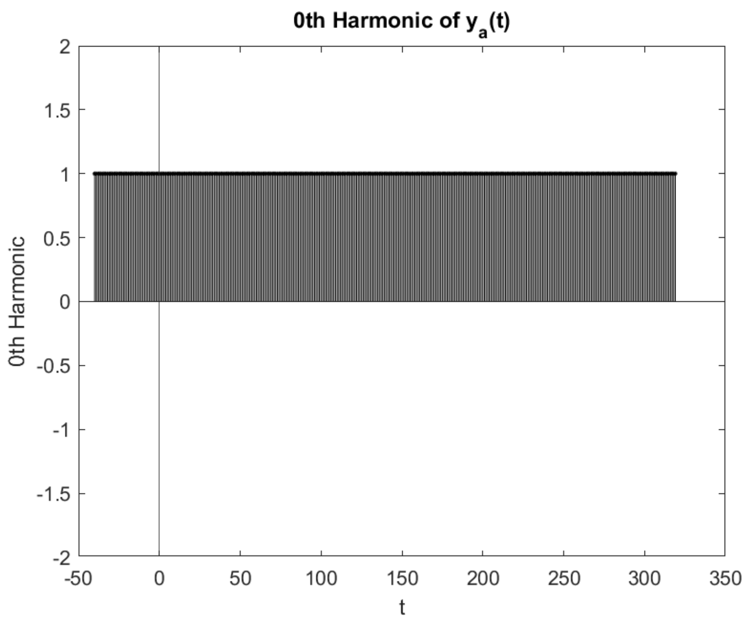

Figure 14: 0^{th} Harmonic of ya(t)

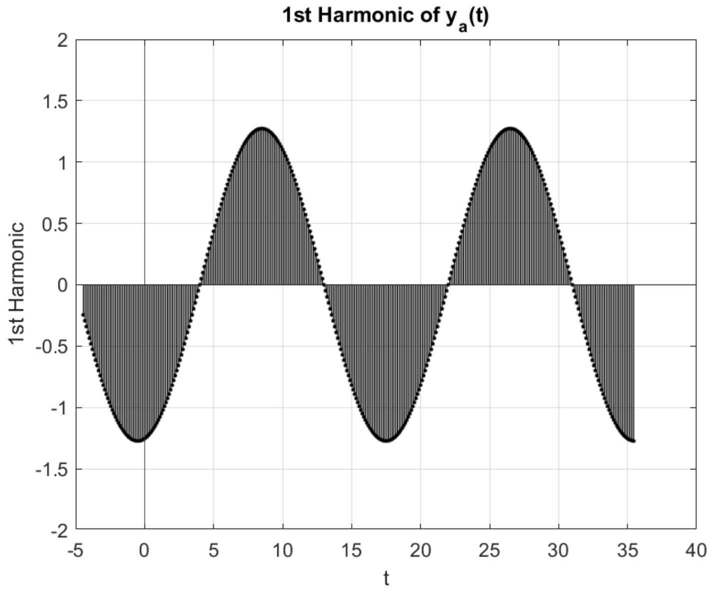

Figure 15: 1^{st} Harmonic of ya(t)

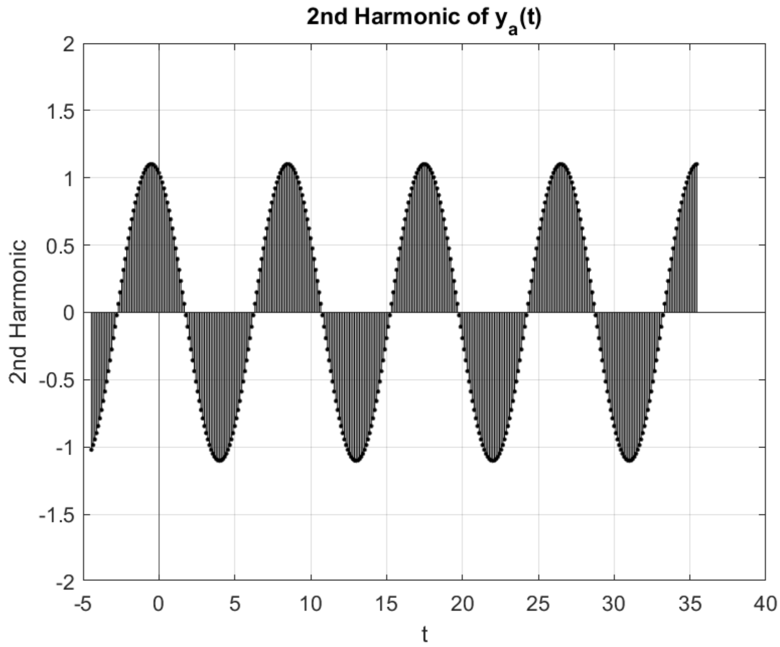

Figure 16: 2^{nd} Harmonic of ya(t)

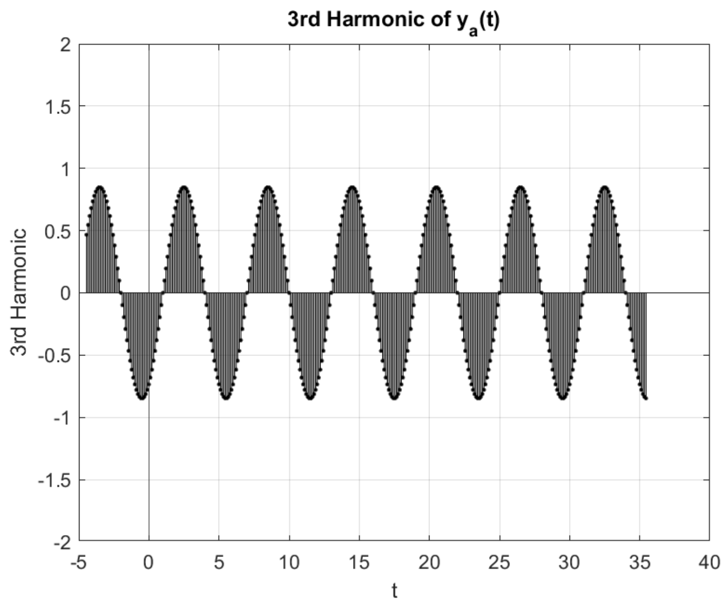

Figure 17: 3rd Harmonic of ya(t)

Question 2:

The discretized signal $y_a(t)$ with Ts = 1/10 seconds is given in Figure 18 and its plot is given in the Figure 19:

$$y_a(t) = \left|5\cos\left(\frac{\pi}{9}t\right)\right|$$

Figure 18: ya(t)

Question 2.a:

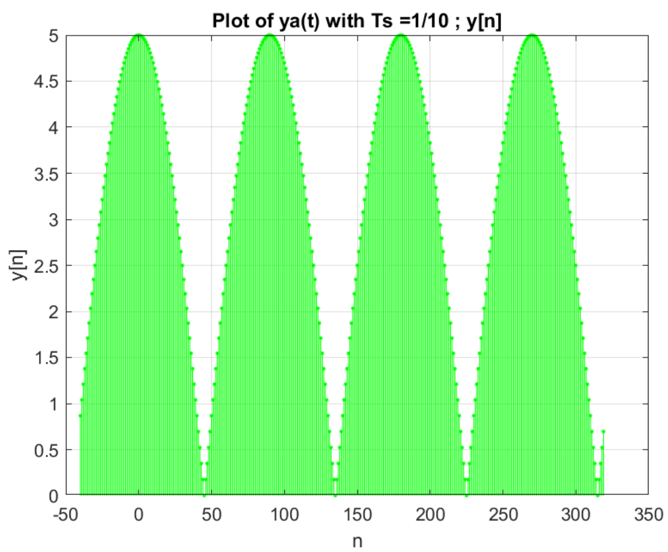

Figure 19: Plot of ya(t) with Ts=1/10

Question 2.b:

The process of finding the FSE of the ya(t) is shown the Figure 20:

2.b) $y_{a_2}(t) = |5\cos(\pi/9\,t)|$. $\underline{\text{Fundamental period is} \rightarrow 9 = T}$

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{9}\int_{-4.5}^{4.5} 5\cos\left(\frac{\pi}{9}t\right)dt = \frac{1}{9}\cdot\frac{9}{\pi}\cdot 5\underbrace{\left(\sin\frac{\pi}{2} - \sin\left(\frac{-\pi}{2}\right)\right)}_{2}$$

$$a_0 = 10/\pi$$

$$a_k = \frac{1}{T}\int_{\text{oop}} y_{a_2}(t)\, e^{-j\frac{2\pi}{T}kt}\,dt = \frac{1}{9}\int_{-4.5}^{4.5} 5.\cos(\pi/9)t.e^{-j\frac{2\pi}{9}kt}\,dt$$

$$= \frac{5}{18}\int_{-4.5}^{4.5}\left(e^{-j\frac{2\pi}{9}kt}.e^{j\pi/9t} + e^{-j\frac{2\pi}{9}kt}.e^{-j\pi/9t}\right)dt$$

$$= \frac{5}{18}\left[\frac{9}{\pi(1-2k)}\cdot\left(e^{j\pi/2)(1-2k)} - e^{-j\frac{\pi}{2}(1-2k)}\right) + \frac{9}{\pi(2k+1)}\cdot\left(e^{-j\frac{\pi}{2}(1+2k)} - e^{j\frac{\pi}{2}(1+2k)}\right)\right]_{-4.5}^{4.5}$$

$$= 5\cdot\left[\frac{\sin\left(\frac{\pi}{2}(1-2k)\right)}{\pi(1-2k)} + \frac{\sin\left(\frac{\pi}{2}(1+2k)\right)}{\pi(2k+1)}\right] \underset{k\neq 0}{=} a_k$$

$$\text{F.S.E}[y_a(t)] = \frac{10}{\pi} + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} a_k.e^{-j\frac{2\pi}{9}kt}$$

since a_k is odd,

$$\text{F.S.E}[y_a(t)] = \frac{10}{\pi} + \sum_{k=1}^{\infty} 2.a_k.\cos\left(\frac{2\pi}{9}kt\right)$$

$$a_k = \begin{cases} \frac{10}{\pi}, & k=0 \\ 5.\left[\frac{\sin\left(\frac{\pi}{2}(1-2k)\right)}{\pi(1-2k)} + \frac{\sin\left(\frac{\pi}{2}(1+2k)\right)}{\pi(1+2k)}\right], & k\neq 0\end{cases}$$

gibbs

$$Z_N[n] = \frac{10}{\pi} + \sum_{k=1,\,k\neq 0}^{N} 2.a_k.\cos\left(\frac{2\pi}{9}.k.\frac{n}{9}\right) \text{ for } n\in[-40,39]$$

Figure 20: The Process of Finding the FSE of a(t)

Question 2.c:

Value of ak is shown in the Figure 21:

$$a_k = \begin{cases} \dfrac{10}{\pi}, & k=0 \\ 5\cdot\left[\dfrac{\sin\left(\frac{\pi}{2}(1-2k)\right)}{\pi(1-2k)} + \dfrac{\sin\left(\frac{\pi}{2}(1+2k)\right)}{\pi(1+2k)}\right], & k \neq 0 \end{cases}$$

$$Z_N[n] = \frac{10}{\pi} + \sum_{k=1,\, k\neq 0}^{N} 2\cdot a_k \cdot \cos\left(\frac{2\pi}{9}\cdot k \cdot \frac{n}{9}\right) \text{ for } n \in [-40, 39]$$

Figure 21: Values of ak

Figure 22 shows the spectrum of coefficients of a_k:

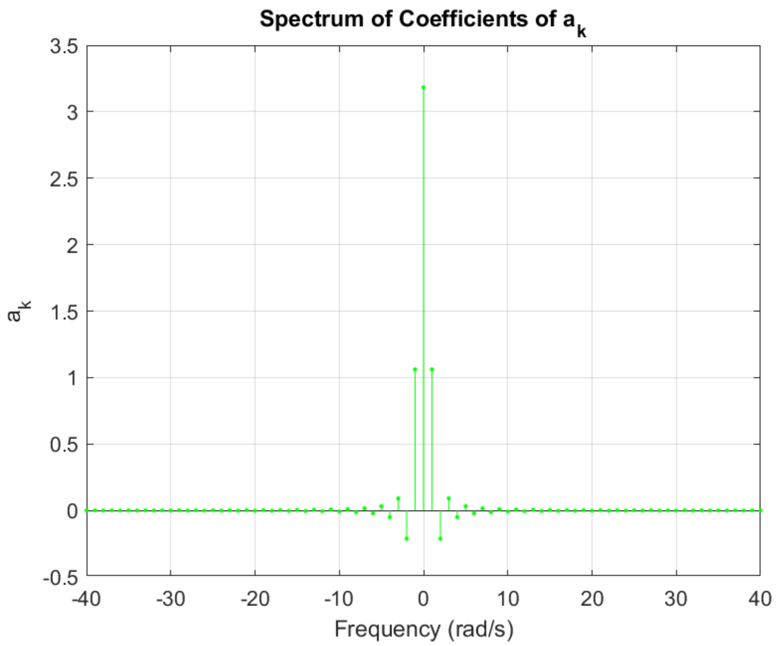

Figure 22: Spectrum of a_k

Question 2.d:

In the Part 2.b, the FSE of $y_a(t)$ is found. With using that solution, $Z_N(n)$ is found as it is in the Figure 23:

$$Z_N[n] = \frac{10}{\pi} + \sum_{k=1,\, k\neq 0}^{N} 2\cdot a_k \cdot \cos\left(\frac{2\pi}{9}\cdot k \cdot \frac{n}{9}\right) \text{ for } n \in [-40, 39]$$

Figure 23: $Z_N(n)$

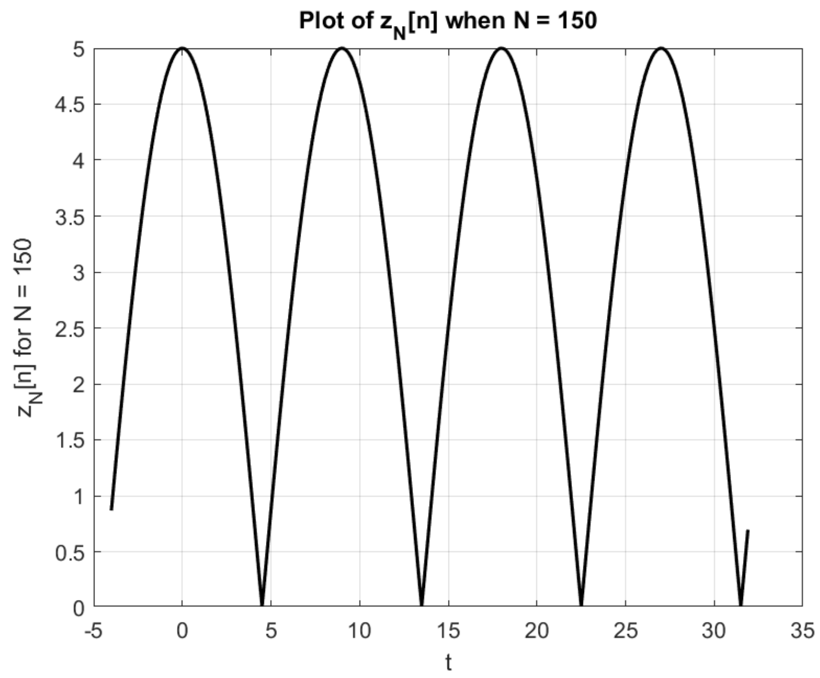

Figure 24: Plot of Zn when N=150

Question 2.e:

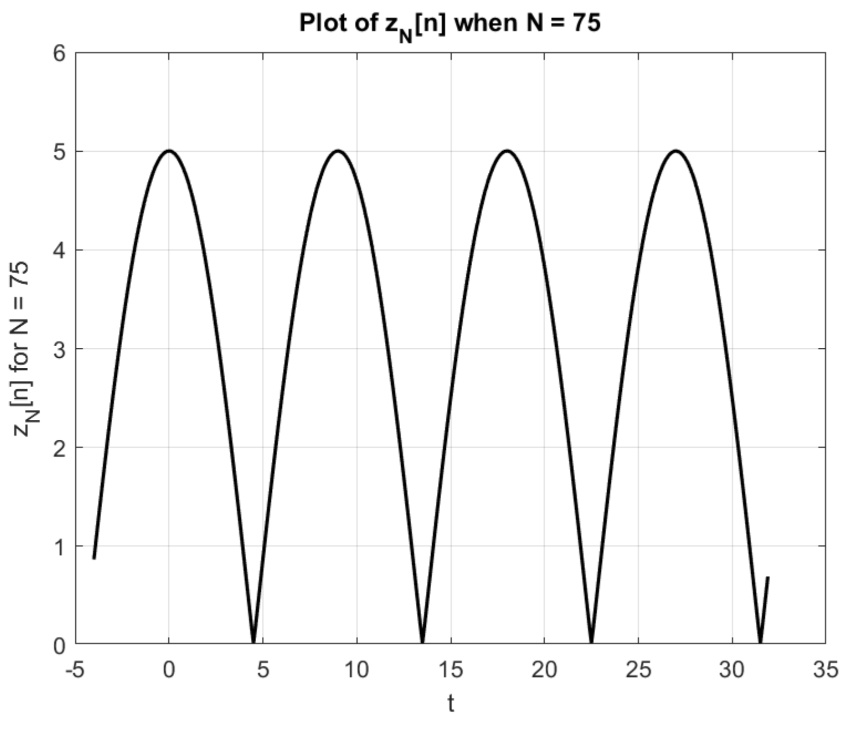

Figure 25: Plot of Zn when N=75

Question 2.f:

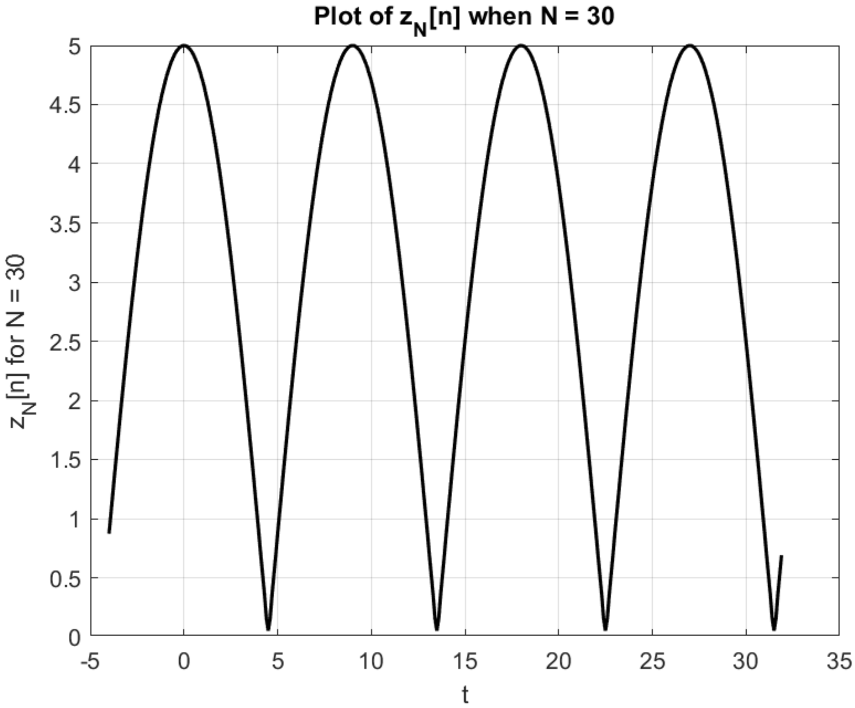

Figure 26: Plot of Zn when N=30

Question 2.g:

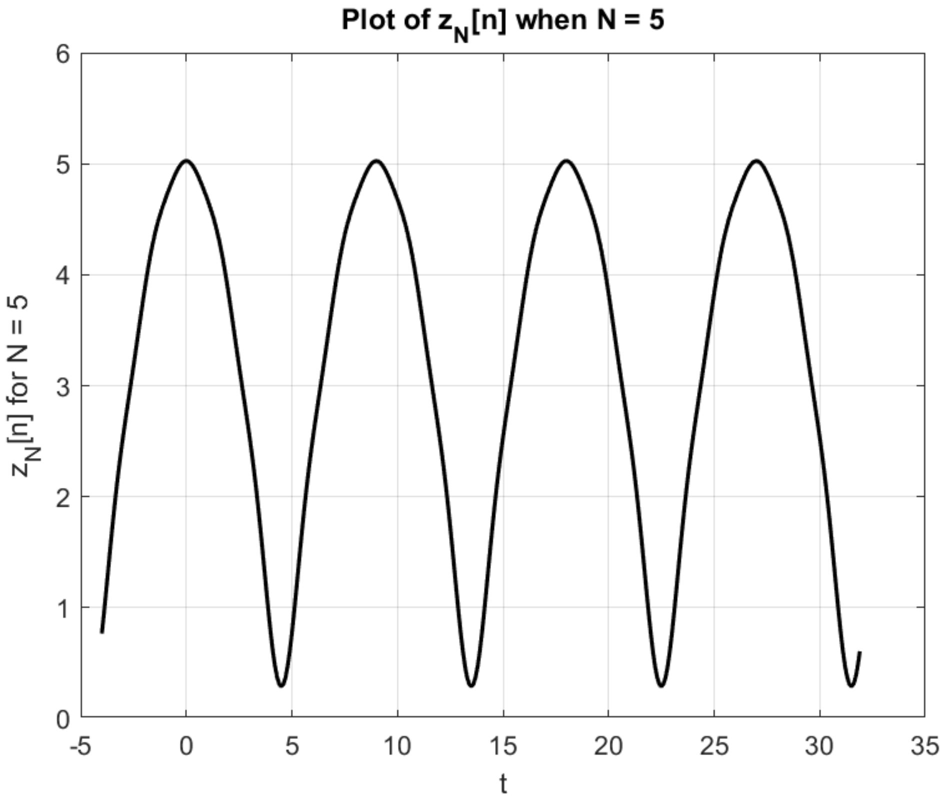

Figure 27: Plot of Zn when N=5

Question 2.h:

Figure 28: Plot of Zn when N=3

Question 2.i:

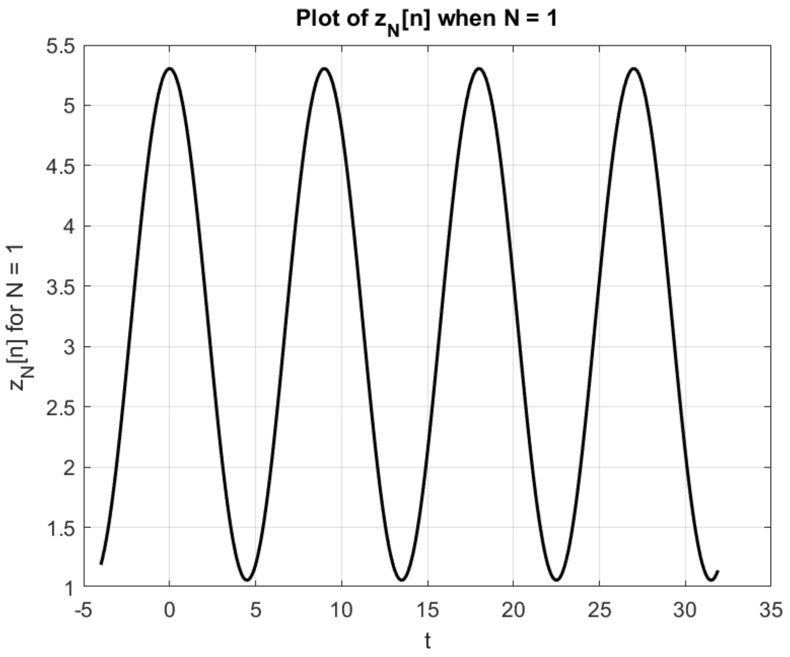

Figure 29: Plot of Zn when N=1

As in the Question 1, as N got closer to zero, more frequency components were lost, lowering the approximation's quality. The approximation's accuracy is decreased by these missing elements. Additionally, the total approximation is more affected by components with comparatively larger ak a k values than by those with smaller coefficients. There were no abrupt jumps or discontinuities in this instance, in contrast to the first question. As a result, the Gibbs phenomenon was not observed in this section of the lab with the original function.

Question 2.j:

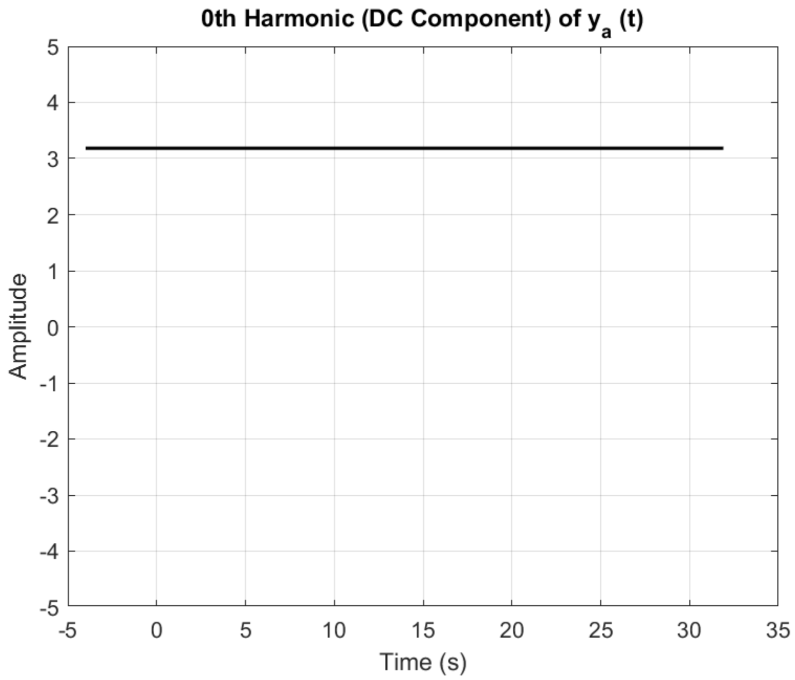

Figure 30: Zeroth Harmonic of ya(t)

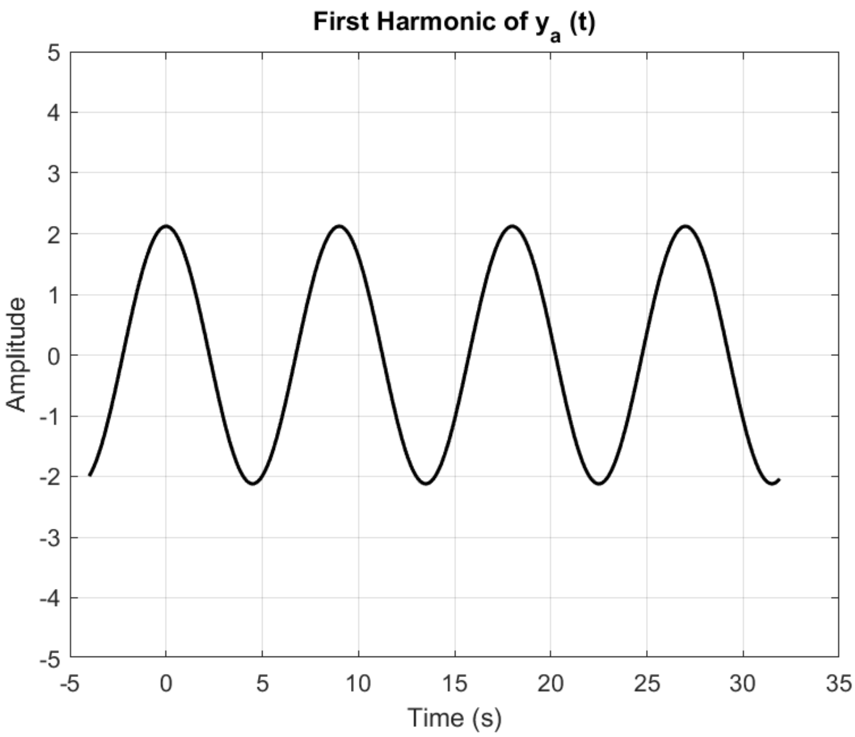

Figure 31: First Harmonic of ya(t)

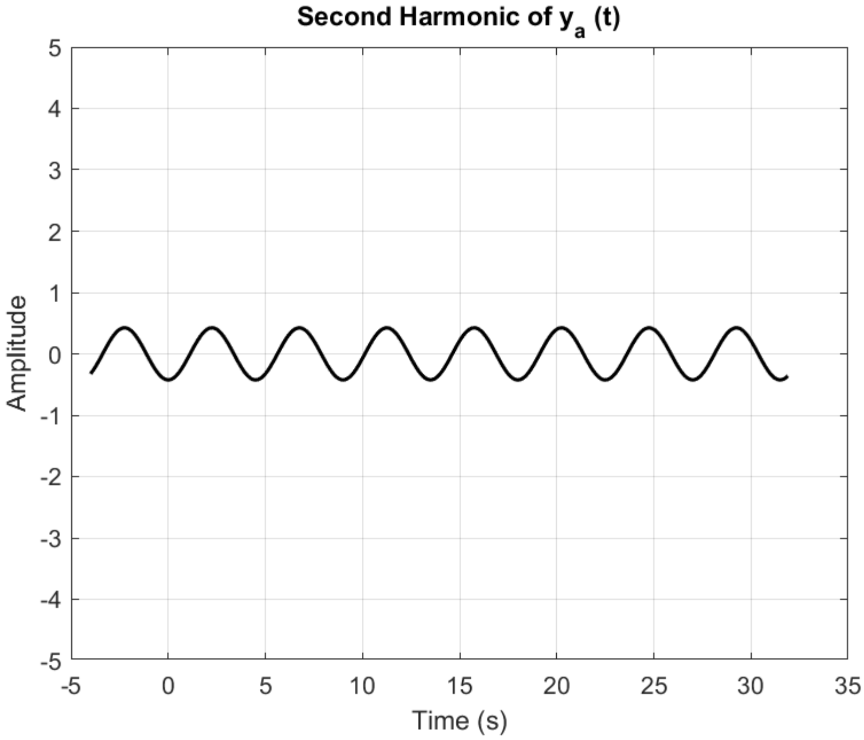

Figure 32: Second Harmonic of ya(t)

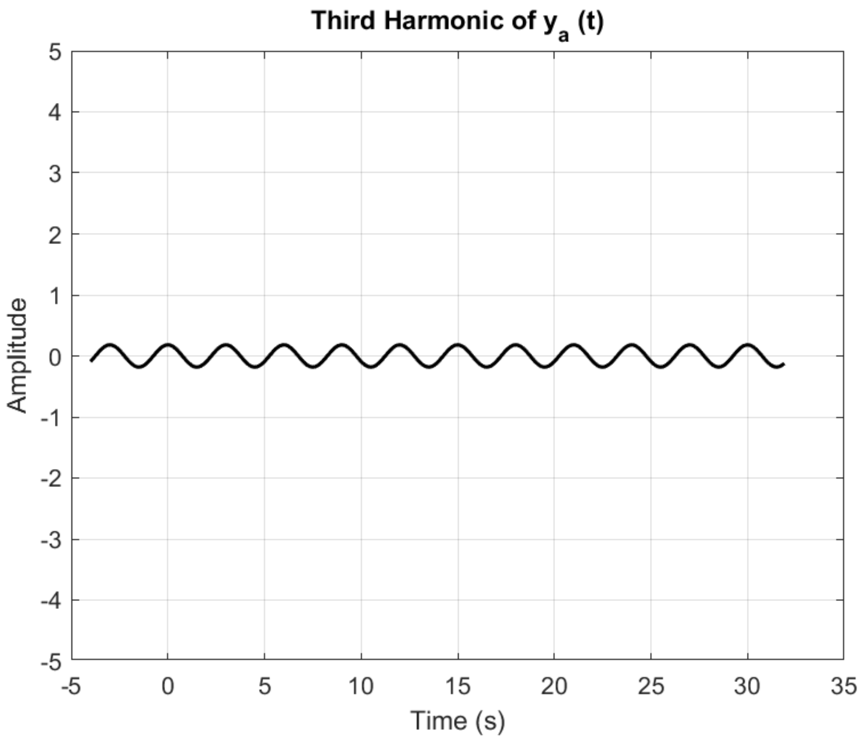

Figure 33: Third Harmonic of ya(t)

Question 3:

The discretized signal $y_a(t)$ with Ts = 1/10 seconds is given in Figure 34 and its plot is given in the Figure 35:

$$y_a(t) \ = \ \begin{cases} \left|5\cos\left(\frac{\pi}{9}t\right)\right| & t \in [-4.5, 4.5)\,\text{s}\,. \\ 0 \quad t \in [4.5, 13.5)\text{s}. \end{cases}$$

Figure 34: $y_a(t)$

Question 3.a:

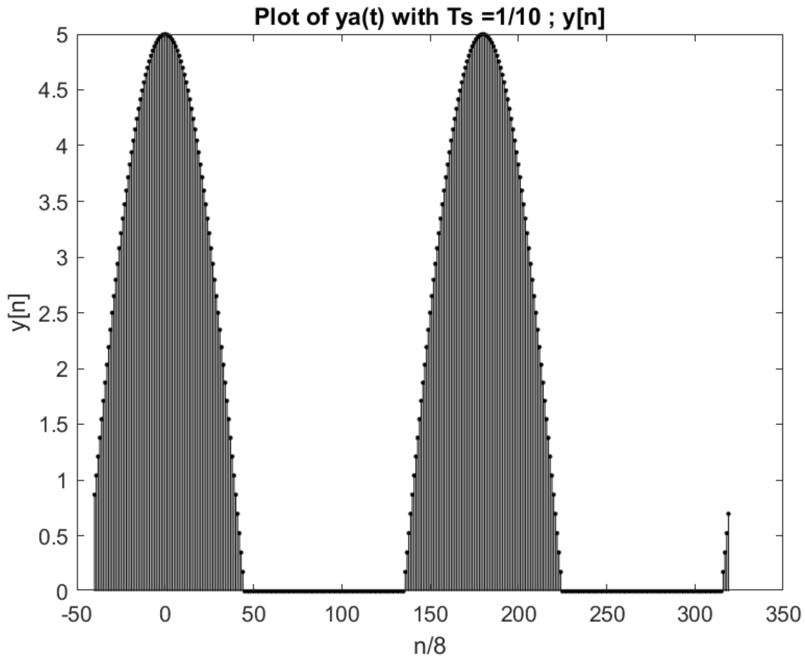

Figure 35: Plot of Discretized $y_a(t)$ with Ts=1/10

Question 3.b:

The process of finding the FSE of the $y_a(t)$ is shown in Figure 36.

3.b)

$$y_{a_3}(t) = \begin{cases} |5\cos(\frac{\pi}{9}t)|, & t \in [-4.5, 4.5)\,s \\ 0, & t \in [4.5, 13.5)\,s \end{cases}$$, Fund. period is 18 sec.

$y_{a_3}(t) = y_a(t+18n)$, $n \in \mathbb{Z}$

$$y_{a_3}(t) = \frac{|5\cos(\pi/9\,t)| + 5\cos(\pi/9\,t)}{2} = \frac{y_{a2}(t) + 5\cos(\pi/9\,t)}{2}$$

FSE is linear.

$$2\,FSE[y_{a3}(t)] = FSE[y_{a2}(t)] + FSE(5\cos(\pi/9\,t))$$

$$2y_{a3}(t) = \frac{10}{\pi} + \left[\sum_{k=1}^{\infty} 10\left(\frac{\sin(\pi/2(1-2k))}{\pi(1-2k)} + \frac{\sin(\pi/2(1+2k))}{\pi(1+2k)}\right)\cos\left(\frac{2\pi}{9}kt\right)\right] + FSE\left[5\cos\left(\frac{\pi}{9}t\right)\right]$$

$$\frac{5}{2}e^{j\frac{\pi}{9}t} + \frac{5}{2}e^{-j\frac{\pi}{9}t}$$

$$a_0 = \frac{5}{\pi}, \quad a_1 = a_{-1} = \frac{1}{18}\int_{-4.5}^{4.5} \frac{y_{a2}(t) + 5\cos(\pi/9\,t)}{2} e^{-j\pi/9\,t}\,dt$$

$$= \frac{5}{18}\int_{-4.5}^{4.5} \cos\left(\frac{\pi}{9}t\right)e^{-j\frac{\pi}{9}t}\,dt = \frac{5}{18}\int_{-4.5}^{4.5}\frac{1+e^{-j\frac{2\pi}{9}t}}{2}\,dt = \frac{5}{18}\left(\frac{9}{2} + \cancel{\frac{1}{2}\cdot\left(\frac{e^{-j\pi} - e^{j\pi}}{-j\frac{2\pi}{9}}\right)}\right)$$

$$a_1 = a_{-1} = \frac{5}{18}\cdot\frac{9}{2} = \frac{5}{4}$$

$$a_k = \frac{5}{2}\cdot\left[\frac{\sin(\frac{\pi}{2}(1-2k))}{\pi(1-2k)} + \frac{\sin(\frac{\pi}{2}(1+2k))}{\pi(1+2k)}\right] = \frac{5}{2}\left[\frac{\sin(\frac{\pi}{2}(1-k'))}{\pi(1-k')} + \frac{\sin(\frac{\pi}{2}(1+k'))}{\pi(1+k')}\right]$$

$k \neq 0, k \neq 1, k \neq -1$

$$FSE[y_{a3}(t)] = \frac{5}{\pi} + \frac{5}{4}e^{j\frac{\pi}{9}} + \frac{5}{4}e^{-j\frac{\pi}{9}} + \sum_{k=2}^{\infty} 2a_k \cdot \cos\left(\frac{\pi}{9}t\right)$$

$$\cancel{= \frac{5}{2}\cos\left(\frac{\pi}{9}t\right)}$$

$$a_k = \begin{cases} \frac{5}{2}\cdot\left(\frac{\sin(\pi/2(1-k))}{\pi(1-k)} + \frac{\sin(\pi/2(1+k))}{\pi(1+k)}\right) & ; k \neq 0, 1, -1 \\ 5/\pi & ; k = 0 \\ 5/4 & ; k = 1, -1 \end{cases}$$

→ Çi high freq data sudden change, gibbs fenomeni belirgin değil.

Figure 36: The Process of Finding the FSE of the $y_a(t)$

Question 3.c:

Value of ak is shown in the Figure 37:

$$a_k = \begin{cases} \frac{5}{2} \cdot \left(\frac{\sin(\pi/2(1-k))}{\pi(1-k)} + \frac{\sin(\pi/2(1+k))}{\pi(1+k)} \right) & ; k \neq 0, 1, -1 \\ 5/\pi & ; k = 0 \\ 5/4 & ; k = 1, -1 \end{cases}$$

→ Gi high freq daha sıdır çıkıyor, gibbs fenomeni belirgin değil.

Figure 37: Values of ak

Figure 38 shows the spectrum of coefficients of a_k:

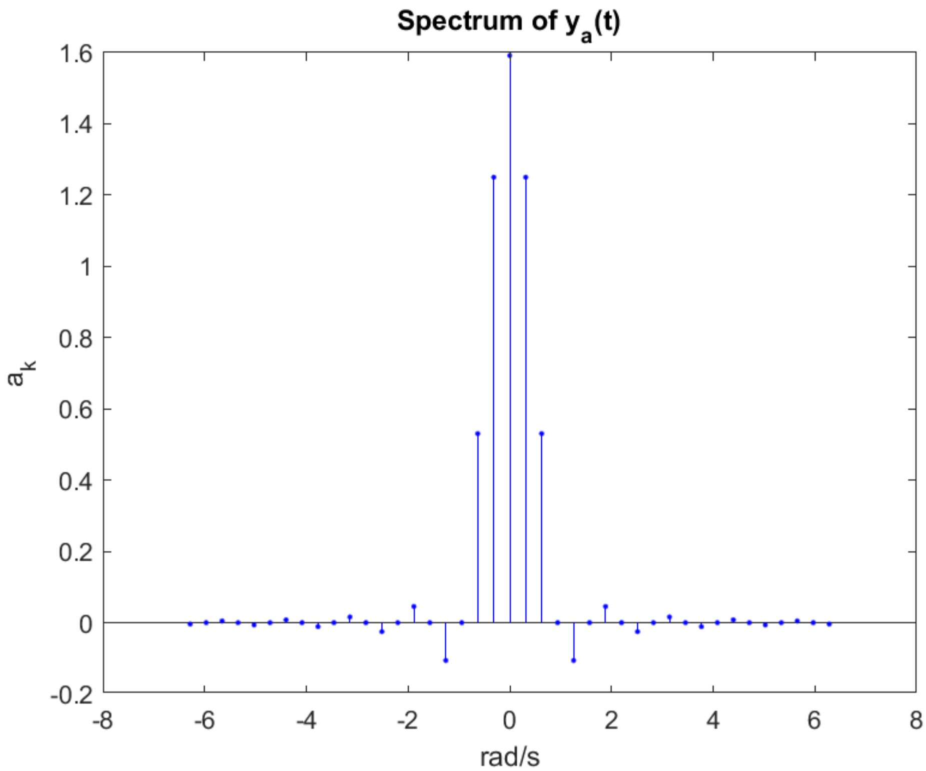

Figure 38: Spectrum of a_k

Question 3.d:

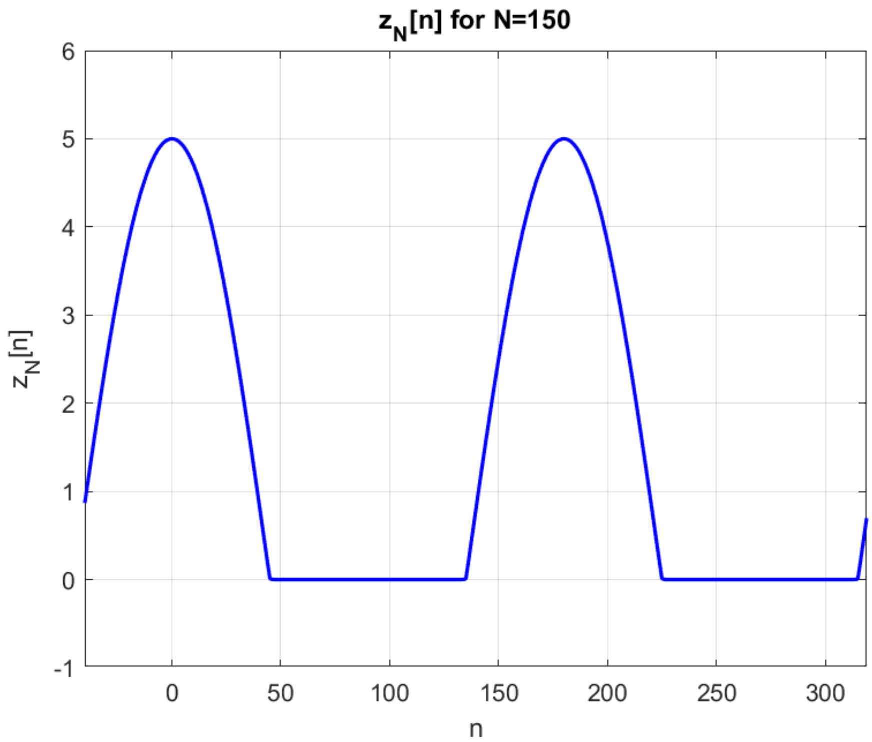

Figure 39: Plot of Zn when N=150

Question 3.e:

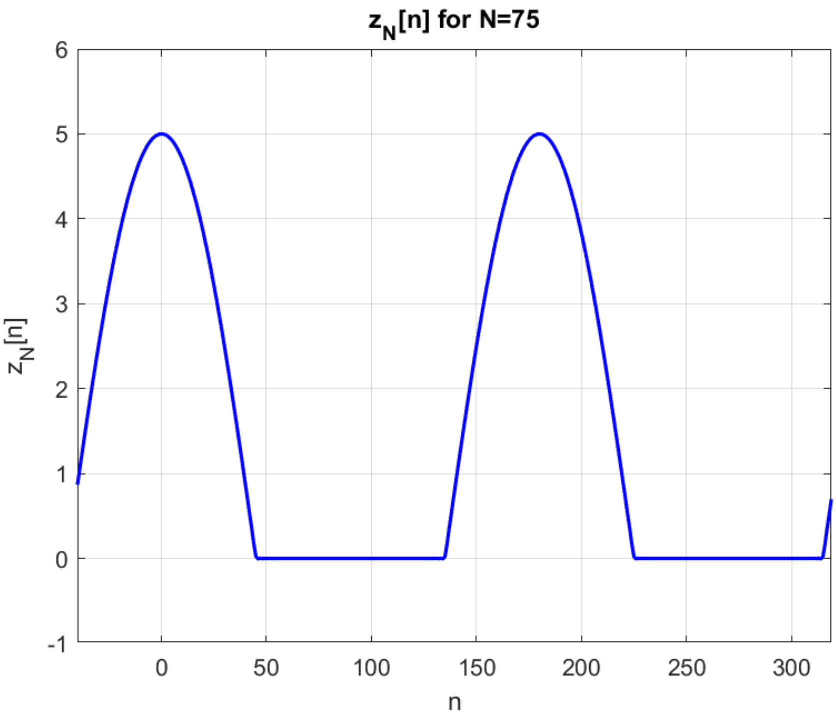

Figure 40: Plot of Zn when N=75

Question 3.f:

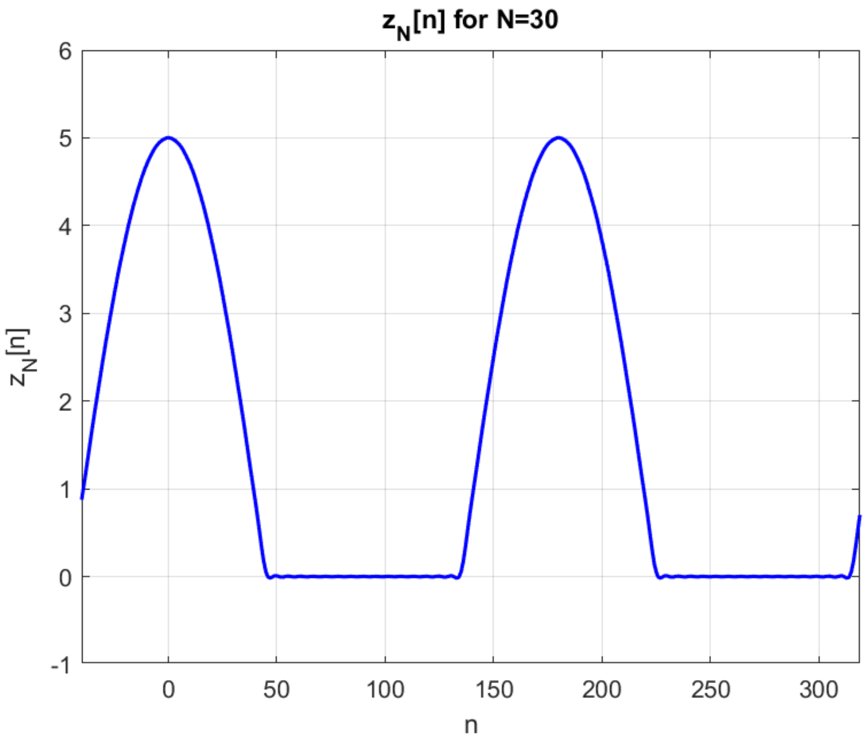

Figure 41: Plot of Zn when N=150

Question 3.g:

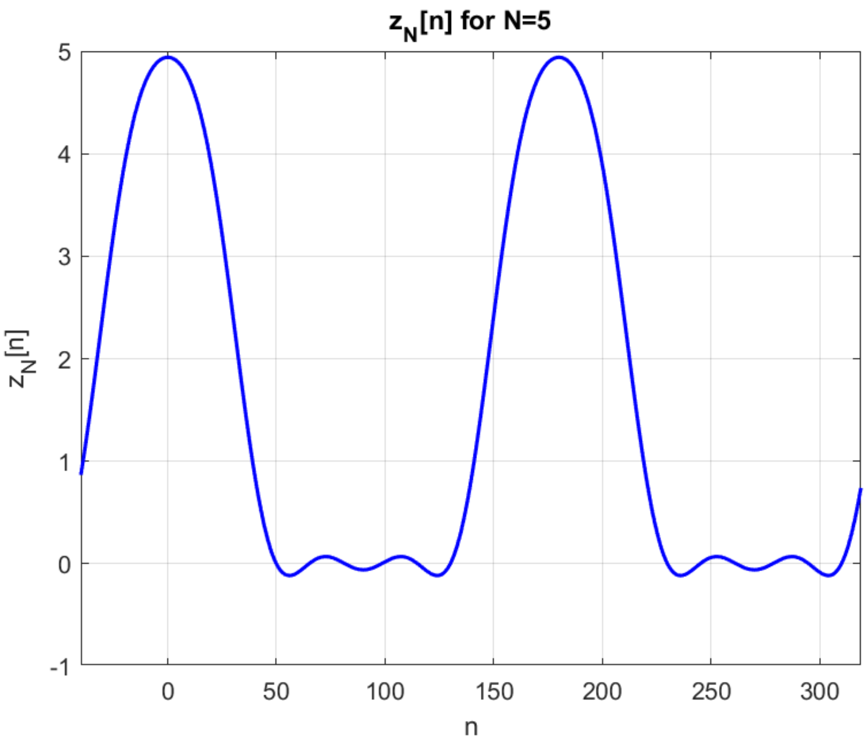

Figure 42: Plot of Zn when N=5

Question 3.h:

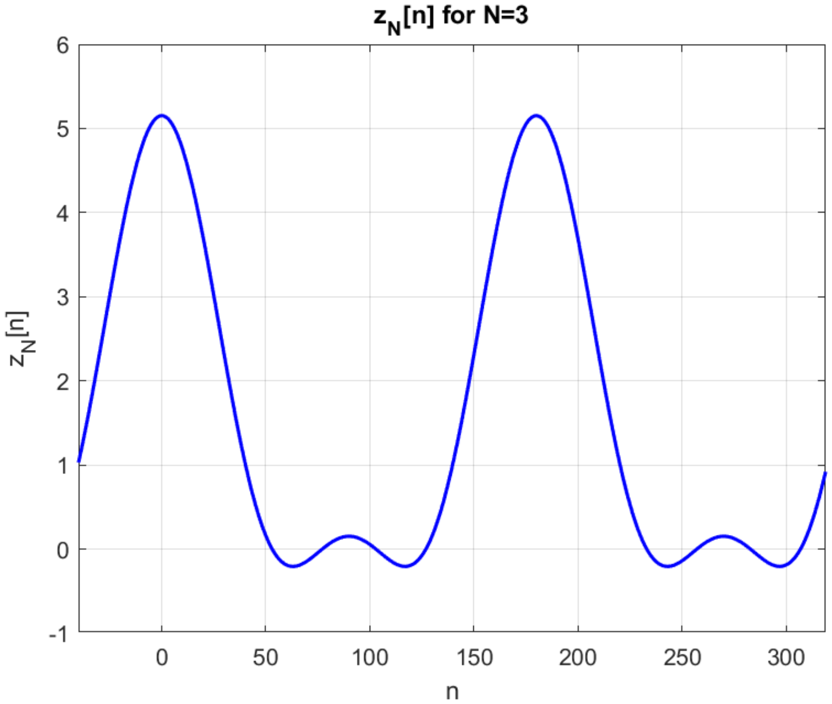

Figure 43: Plot of Zn when N=3

Question 3.j:

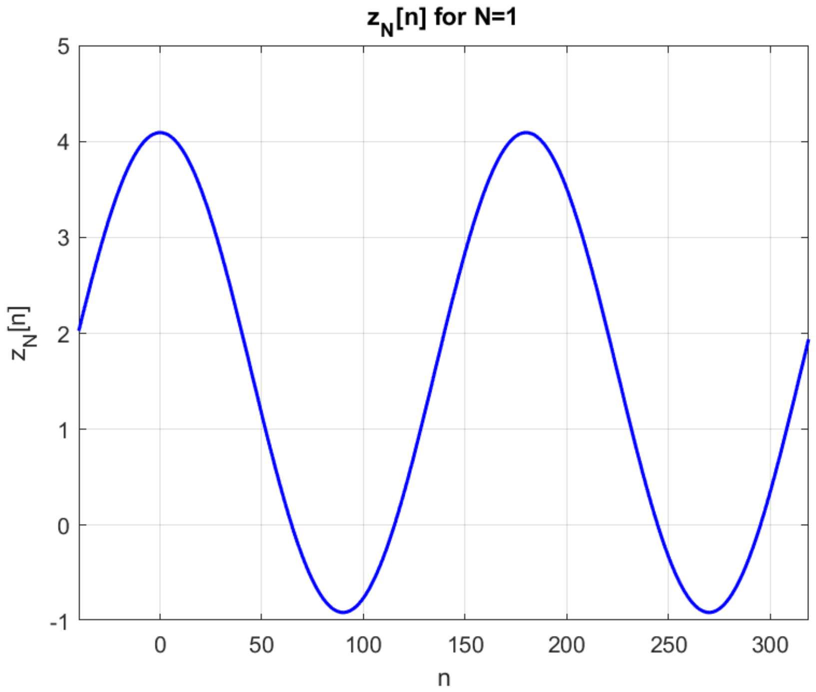

Figure 44: Plot of Zn when N=1

The previous two questions can be analyzed similarly to this one. The quality of the approximation declined dramatically as we reduced the number of harmonics used. Highercoefficient harmonics had a stronger effect on the approximation than did lower-coefficient harmonics.

Question 3.k:

The signal ya(t) has only 4 harmonic functions, and they can be seen in the following Figures 45-46-47-48:

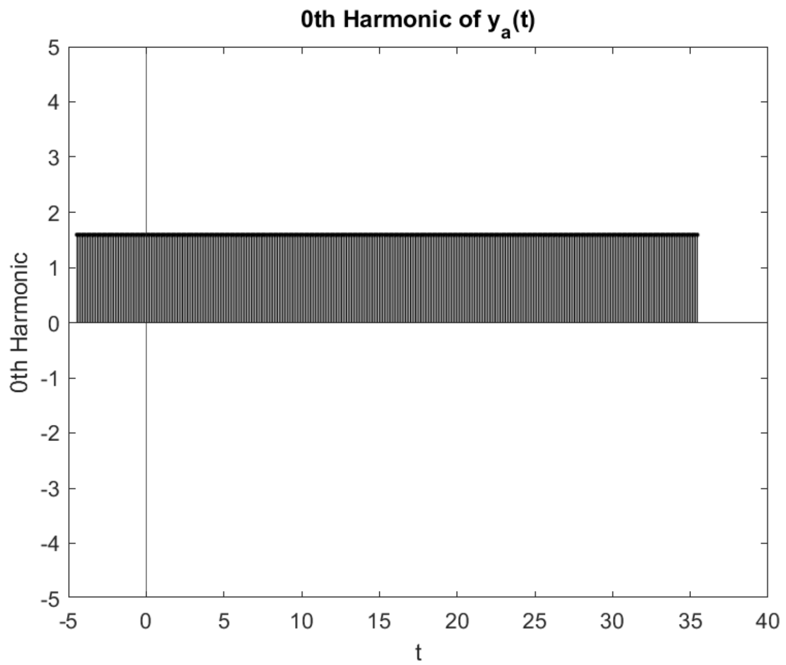

Figure 45: 0^{th} Harmonic of ya(t)

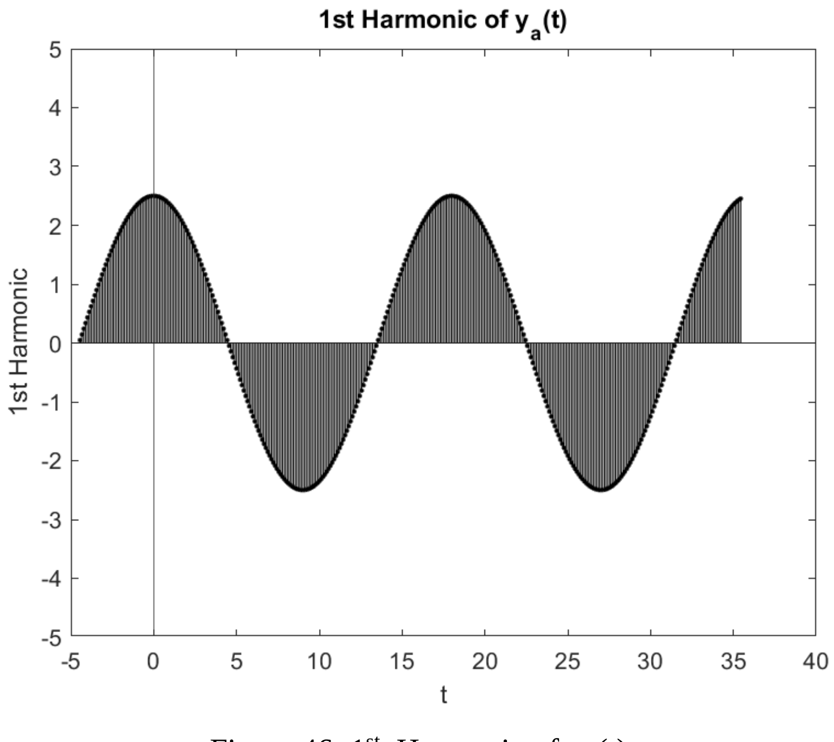

Figure 46: 1^{st} Harmonic of ya(t)

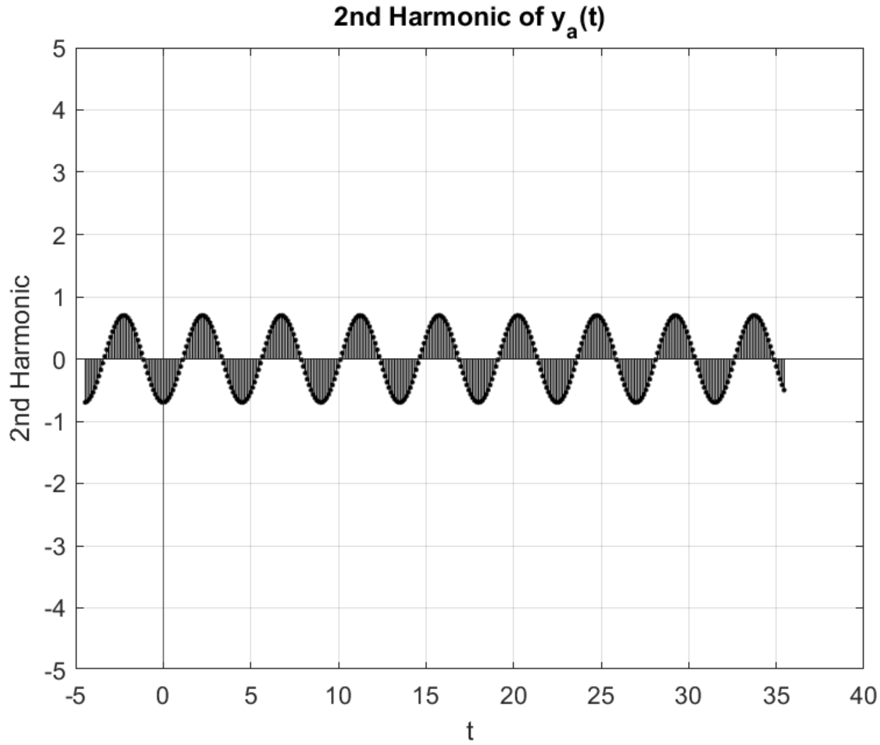

Figure 47: 2nd Harmonic of ya(t)

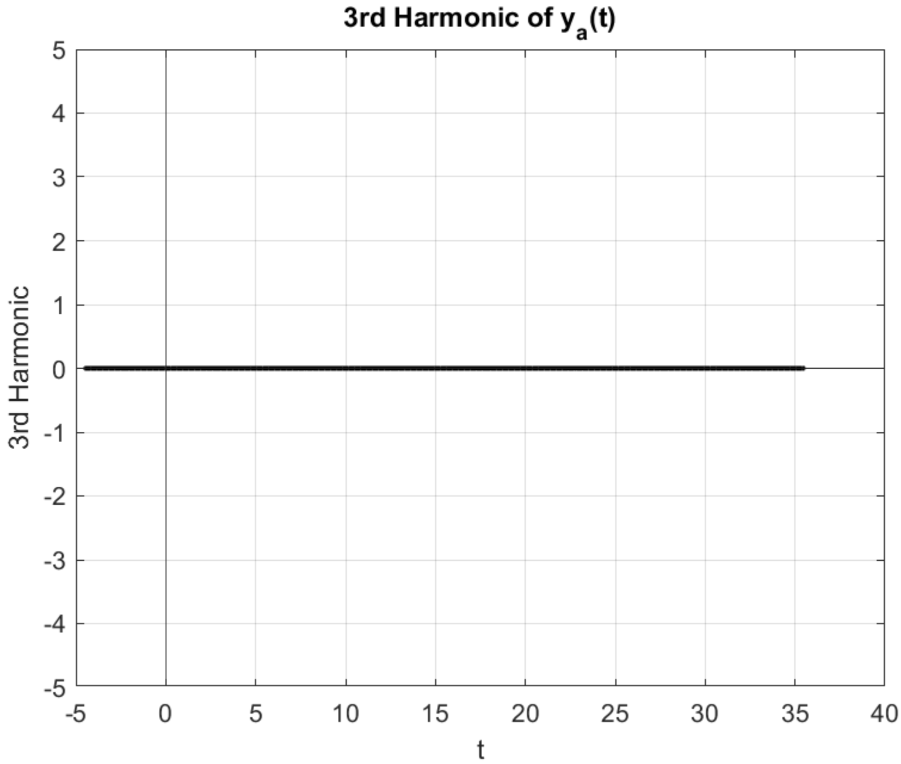

Figure 48: 3rd Harmonic of ya(t)

Conclusion of the Lab:

All of the questions and requirements are satisfied and they are controlled with the TAs in the Lab hour. This lab thought me a lot about the Fourier Series Expansion. However, writing a report for this lab was exhausting.

Appendices:

Question 1:

```
% Code to plot the real part of the spectrum
m = -20:1:20;
a = zeros(size(m));

a(21) = 4/3;

for k = 1:length(m)
    if m(k) ~= 0
        a(k) = (4/(1i*pi*m(k)))*(exp(1i*22*pi*m(k)/18) - exp(1i*16*pi*m(k)/18));
    end
end

% Plot real part
figure;
stem(m * 2 * pi / 18, real(a), 'filled', 'k.');
xlabel('Frequency (rad/s)');
ylabel('Re(a_k)');
title('Real Spectrum of y_a(t)');
grid on;

% Plot imaginary part
figure;
stem(m * 2 * pi / 18, imag(a), 'filled', 'k.');
xlabel('Frequency (rad/s)');
ylabel('Im(a_k)');
title('Imaginary Spectrum of y_a(t)');
grid on;
```

```
%1.a
n = -40:1:319;
ya = zeros(size(n));
for i = n
    dum = mod(i/10,18);% T/10 da sample ,period 18
    if (dum >= 7)&&(dum < 10)
        ya(i+41) = 8;% m 41 tane değer aldı
    end
end

%stem(n,ya,'.r');
%xlabel('n')
%ylabel('y[n]')
%title('Plot of sampled version of ya(t) with Ts =1/10 ; y[n]')

%1.c

m = -20:1:20;
a = zeros(size(m));

a(21) = 4/3;

for k = 1:length(m)
    if m(k) ~= 0
        a(k) = (4/(1i*pi*m(k)))*(exp(1i*22*pi*m(k)/18) - exp(1i*16*pi*m(k)/18)); %ak katsayılarım elle bulduğum
    end
end

%stem(m*2*pi/18, a , 'filled', 'k.');
%xlabel('rad/s');
%ylabel('ak');
%title('Spectrum plot of the real part of ya(t)');
%grid on;


%a_imag = imag(a);

%figure;
%stem(m * 2 * pi / 18, a_imag, 'filled', 'r.');
%xlabel('Frequency (rad/s)');
%ylabel('Im(a_k)');
%title('Imaginary Part of the Spectrum of y_a(t)');
%grid on;


%1.d

n = -40:319;
N = 150;
```

```
N = 150;
a = zeros(1, 2*N + 1);
a(N + 1) = 4/3;

for k = -N:N
    if k ~= 0
        a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
    end
end

zn = zeros(1, length(n));
for k = -N:N
    zn = zn + a(k + N + 1) * exp(1i * pi * k * n / 81);  %Zn
end

%plot(n , real(zn), 'k','LineWidth', 1.5);
%title('Plot of z_N[n] when N is 150');
%ylabel('z_{150}[n]');
%xlabel('n');
%xline(0);
%yline(0);
%grid on;

n = -40:319;
N = 75;
a = zeros(1, 2*N + 1);
a(N + 1) = 4/3;

for k = -N:N
    if k ~= 0
        a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
    end
end

zn = zeros(1, length(n));
for k = -N:N
    zn = zn + a(k + N + 1) * exp(1i * pi * k * n / 81);
end

%plot(n , real(zn), 'k', 'LineWidth', 1.5);
%title('Plot of z_N[n] when N is 75');
%ylabel('z_{75}[n]');
%xlabel('n');
%xline(0);
%yline(0);
```

```
%yline(0);
%grid on;

n = -40:319;
N = 30;
a = zeros(1, 2*N + 1);
a(N + 1) = 4/3;

for k = -N:N
    if k ~= 0
        a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
    end
end

zn = zeros(1, length(n));
for k = -N:N
    zn = zn + a(k + N + 1) * exp(1i * pi * k * n / 81);
end

%figure;
%plot(n , real(zn), 'k', 'LineWidth', 1.5);
%title('Plot of z_N[n] when N is 30');
%ylabel('z_{30}[n]');
%xlabel('n');
%xline(0);
%yline(0);
%grid on;

n = -40:319;
N = 5;
a = zeros(1, 2*N + 1);
a(N + 1) = 4/3;

for k = -N:N
    if k ~= 0
        a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
    end
end

zn = zeros(1, length(n));
for k = -N:N
    zn = zn + a(k + N + 1) * exp(1i * pi * k * n / 81);
end
```

```
%figure;
%plot(n , real(zn), 'k', 'LineWidth', 1.5);
%title('Plot of z_N[n] when N is 5');
%ylabel('z_{5}[n]');
%xlabel('n');
%xline(0);
%yline(0);
%grid on;


n = -40:319;
N = 3;
a = zeros(1, 2*N + 1);
a(N + 1) = 4/3;

for k = -N:N
    if k ~= 0
        a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
    end
end

zn = zeros(1, length(n));
for k = -N:N
    zn = zn + a(k + N + 1) * exp(1i * pi * k * n / 81);
end

%figure;
%plot(n, real(zn),  'k','LineWidth', 1.5);
%title('Plot of z_N[n] when N is 3');
%ylabel('z_{3}[n]');
%xlabel('n');
%xline(0);
%line(0);
%grid on;

n = -40:319;
N = 1;
a = zeros(1, 2*N + 1);
a(N + 1) = 4/3;

for k = -N:N
    if k ~= 0
        a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
    end
```

```
    end
end

zn = zeros(1, length(n));
for k = -N:N
    zn = zn + a(k + N + 1) * exp(1i * pi * k * n / 81);
end

%figure;
%plot(n , real(zn), ss, 'LineWidth', 1.5);
%title('Plot of z_N[n] when N is 1');
%ylabel('z_{1}[n]');
%ylim([-2 10]);
%xlabel('n');
%xline(0);
%yline(0);
%grid on;

%0 harmonic

figure;
ylim([-2 2]);
stem(n,ones(1,length(n))*3/3,'k.');
title('0th Harmonic of y_a(t)');
ylabel('0th Harmonic');xlabel('t');xline(0);yline(0);
ylim([-2 2]);
%1. hrmonik

n = -40:319;


k = 1;
f_first_harmonic = a(k + N + 1) * exp(1i * pi * k * n / 81);

figure;
ylim([-2 2]);
stem(n / 9, real(f_first_harmonic), 'k.');
title('1st Harmonic of y_a(t)');
ylabel('1st Harmonic');
xlabel('t');
xline(0);
yline(0);
grid on;
ylim([-2 2]);
```

```
ylim([-2 2]);
%2.harmonic
n = -40:319;
N = 2;
a = zeros(1, 2 * N + 1);
a(N + 1) = 4/3;


for k = -N:N
    if k ~= 0
        a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
    end
end


k = 2;
f_second_harmonic = a(k + N + 1) * exp(1i * pi * k * n / 81);

figure;
ylim([-2 2]);
stem(n / 9, real(f_second_harmonic), 'k.');
title('2nd Harmonic of y_a(t)');
ylabel('2nd Harmonic');
xlabel('t');
xline(0);
yline(0);
grid on;
ylim([-2 2]);
%3.har

n = -40:319;
N = 3;
a = zeros(1, 2 * N + 1);
a(N + 1) = 4 / 3;


for k = -N:N
    if k ~= 0
        a(k + N + 1) = (4 / (1i * pi * k)) * (exp(1i * 22 * pi * k / 18) - exp(1i * 16 * pi * k / 18));
    end
end


k = 3;
f_third_harmonic = a(k + N + 1) * exp(1i * pi * k * n / 81);
```

```
f_third_harmonic = a(k + N + 1) * exp(1i * pi * k * n / 81);

figure;
ylim([-2 2]);
stem(n / 9, real(f_third_harmonic), 'k.');
title('3rd Harmonic of y_a(t)');
ylabel('3rd Harmonic');
xlabel('t');
xline(0);
yline(0);
grid on;
ylim([-2 2]);

```

Question 2:

```matlab
Ts = 1/10;
n = -40:319;
t = n * Ts;
T_period = 9;
omega_0 = 2 * pi / T_period;

% Discrete signal
y = abs(5 * cos(pi * t / 9));

% % Plot the discrete signal y[n]
 %figure;
 %stem(n, y, '.g');
 %xlabel('n');
 %ylabel('y[n]');
% title('Plot of ya(t) with Ts =1/10 ; y[n]');
 %grid on;

% Fourier coefficients a_k
a0 = 10 / pi;
N_max = 150;
ak = zeros(1, 2 * N_max + 1);

% Compute a_k for k = -N_max to N_max
for k = -N_max:N_max
    if k ~= 0
        ak(k + N_max + 1) = (5 / pi) * (sin((pi / 2) - pi * k) / (1 - 2 * k) + ...
                                        sin((pi / 2) + pi * k) / (1 + 2 * k));
    else
        ak(k + N_max + 1) = a0;
    end
end
%
 % Plot Fourier series coefficients a_k for k = -40 to 40
 %figure;
%stem(-40:40, ak(N_max-40+1:N_max+40+1), 'filled', '.g');
%xlabel('Frequency (rad/s)');
%ylabel('a_k');
 %title('Spectrum of Coefficients of a_k');
%grid on;

% Array of N values for different reconstructed signals
 N_values = [150, 75, 30, 5, 3, 1];

% Plot z_N[n] for different values of N, each in a separate figure
 for i = 1:length(N_values)
    N = N_values(i);
   zN = a0 * ones(size(n));

     % Compute z_N[n] using Fourier series
     for k = 1:N
         zN = zN + 2 * ak(k + N_max + 1) * cos(omega_0 * k * n * Ts);
     end

    %  Plot z_N[n] for the current N value in a new figure
   %  figure;
   %  plot(t, zN,'k', 'LineWidth', 1.5);
   %  xlabel('t');
    % ylabel(['z_{N}[n] for N = ', num2str(N)]);
    % title(['Plot of z_N[n] when N = ', num2str(N)]);
    % grid on;
 end
%

zeroth_harmonic = a0 * ones(size(t));
figure;
plot(t, zeroth_harmonic, 'k','LineWidth', 1.5);
xlabel('Time (s)');
ylabel('Amplitude');
ylim([-3 3]);
title('0th Harmonic (DC Component) of y_a (t)');
grid on;
ylim([-5 5]);

k = 1;
first_harmonic = 2 * ak(k + N_max + 1) * cos(omega_0 * k * t);
figure;
plot(t, first_harmonic, 'k','LineWidth', 1.5);
xlabel('Time (s)');
ylabel('Amplitude');

title('First Harmonic of y_a (t)');
grid on;
ylim([-5 5]);


k = 2;
second_harmonic = 2 * ak(k + N_max + 1) * cos(omega_0 * k * t);
figure;
plot(t, second_harmonic, 'k','LineWidth', 1.5);
xlabel('Time (s)');
ylabel('Amplitude');
title('Second Harmonic of y_a (t)');
grid on;
```

```
grid on;
ylim([-5 5]);





k = 3;
third_harmonic = 2 * ak(k + N_max + 1) * cos(omega_0 * k * t);
figure;
plot(t, third_harmonic, 'k','LineWidth', 1.5);
xlabel('Time (s)');
ylabel('Amplitude');
title('Third Harmonic of y_a (t)');
grid on;
ylim([-5 5]);
```

Question 3:

```
% Parameters
T = 18;            % Period of the signal (in seconds)
omega0 = pi/9;     % Fundamental angular frequency
Ts = 0.1;          % Sampling period (in seconds)
N =1;           % Number of harmonics on each side
n = -40:319;       % Sample indices

% Initialize zN for all n
zN = zeros(size(n));

% Precompute Fourier coefficients a_k for k from -N to N
a_coeff = zeros(1, 2*N + 1);  % Array to store coefficients a_k
k_values = -N:N;              % k indices from -N to N

% Calculate Fourier coefficients a_k
for idx = 1:length(k_values)
    k = k_values(idx);
    if k == 0
        a_coeff(idx) = 5/pi;  % a_0
    elseif k == 1 || k == -1
        a_coeff(idx) = 5/4;   % a_1 for both positive and negative k
    elseif mod(k, 2) == 0
        % Even harmonic k
        a_coeff(idx) = (5/pi) * (cos(pi * k / 2)) / (1 - k^2);
    else
        % Odd harmonic k >= 3
        a_coeff(idx) = 0;
    end
end

% Compute zN[n] using the partial sum of harmonics
for i = 1:length(n)
    zN(i) = sum(a_coeff .* exp(1j * omega0 * k_values * n(i) * Ts));
end

% Plot the real part of zN[n]
figure;
plot(n, real(zN),'b', 'LineWidth', 1.5);
xlabel('n');
ylabel('z_N[n]');
title('z_N[n] for N=1');
grid on;
xlim([-40, 319]);
```

```
n = -40:1:319;
ya = zeros(size(n));
for i = n
    dum = mod(i/10+4.5,18);
    if (dum >= 0) && (dum < 9)
        ya(i+41) = abs(5*cos(pi/9*(dum+4.5)));
    end
end
%figure;
%stem(n, ya, '.k');
%xlabel('n/8')
%ylabel('y[n]')
%title('Plot of ya(t) with Ts =1/10 ; y[n]')

m = -20:1:20;
a = 2.5 * ((sin(pi/2 * (1 - m)) ./ (pi * (1 - m))) + (sin(pi/2 * (1 + m)) ./ (pi * (1 + m))));
a(20) = 5/4;
a(21) = 5/pi;
a(22) = 5/4;

%figure;
%stem(m * pi / 10, a, 'filled', 'b.');
%xlabel('rad/s');
%ylabel('a_k');
%title('Spectrum of y_a(t)');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

figure;
stem(n/9,ones(1,length(n))*5/pi,'k.');
title('0th Harmonic of y_a(t)');
ylabel('0th Harmonic');xlabel('t');xline(0);yline(0);
ylim([-5 5])
figure;
stem(n/9,5/2*cos(pi/9*n/9),'k.');
title('1st Harmonic of y_a(t)');
ylabel('1st Harmonic');xlabel('t');xline(0);yline(0);
ylim([-5 5])
% Define the range for n
n = -40:319;

% Initialize the array for ya
ya = zeros(size(n));

% Calculate ya values for each n
for i = n
    dum = mod(i / 10, 9);  % Get the modulo value for the current n
    ya(i + 41) = abs(5 * cos(pi / 9 * dum));  % Calculate the corresponding value of ya
end
```

```

% Define the range for m (Fourier coefficients)
m = -20:1:20;
a = zeros(size(m));

a = 5 * ((sin(pi / 2 * (1 - 2 * m)) ./ (pi .* (1 - 2 * m))) + (sin(pi / 2 * (1 + 2 * m)) ./ (pi .* (1 + 2 * m))));  % Fourier series coefficients

% Second Harmonic (k = 2)
k = 2;
f_second_harmonic = 3.3 * a(k + 21) * cos(2 * pi / 9 * k * n / 9);

% Plot the second harmonic
figure;
stem(n / 9, f_second_harmonic, 'k.');
title('2nd Harmonic of y_a(t)');
ylabel('2nd Harmonic');
xlabel('t');
xline(0);  % Adds a vertical line at t = 0
yline(0);  % Adds a horizontal line at y = 0
grid on;   % Enables grid for better readability
ylim([-5 5])


% Define the range for n
n = -40:319;

% Initialize the array for ya
ya = zeros(size(n));

% Calculate ya values for each n
for i = n
    dum = mod(i / 10, 9);  % Get the modulo value for the current n
    ya(i + 41) = abs(5 * cos(pi / 9 * dum));  % Calculate the corresponding value of ya
end

% Define the range for m (Fourier coefficients)
m = -20:1:20;
a = zeros(size(m));

a = 5 * ((sin(pi / 2 * (1 - 2 * m)) ./ (pi .* (1 - 2 * m))) + (sin(pi / 2 * (1 + 2 * m)) ./ (pi .* (1 + 2 * m))));  % Fourier series coefficients

% third Harmonic (k = 3)
k = 3;
f_second_harmonic = 0 * a(k + 21) * cos(2 * pi / 9 * k * n / 9);
```

```
f_second_harmonic = 0 * a(k + 21) * cos(2 * pi / 9 * k * n / 9);

% Plot the second harmonic
figure;
stem(n / 9, f_second_harmonic, 'k.');
title('3rd Harmonic of y_a(t)');
ylabel('3rd Harmonic');
xlabel('t');
xline(0);  % Adds a vertical line at t = 0
yline(0);  % Adds a horizontal line at y = 0
grid on;   % Enables grid for better readability
ylim([-5 5])
```