~~Zad~~ 21 $ab = 0 \Rightarrow a = 0 \lor b = 0$

$a \cdot b = 0$

zał, że $a \neq 0$, wtedy istnieje a^{-1}

$a^{-1} \cdot 0 = 0$

$a^{-1} \cdot (a \cdot b) = 0$

$(a^{-1} \cdot a) \cdot b = 0$

$1 \cdot b = 0$

$b = 0$

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$$K = \{ f : f(x) = x^m \cdot \frac{\varphi(x)}{\psi(x)}, m \in \mathbb{Z}, \varphi, \psi \in \mathbb{Q}[x]$$
$$\varphi(0) \neq 0 \quad \psi(0) \neq 0$$

$0 < f \Leftrightarrow \varphi(0) \cdot \psi(0) > 0$

$f < g \Leftrightarrow \cancel{f - g < 0} \quad g - f > 0$

~~$f(x) + g(x) =$~~

$$f(x) + g(x) = x^{m_f} \frac{\varphi_f(x)}{\psi_f(x)} + x^{m_g} \frac{\varphi_g(x)}{\psi_g(x)} =$$

I Jeżeli $m_f > m_g$

$$\cancel{\mathrm{I}\ x^{m_g} \left(x^{m_f - m_g} \frac{\varphi_f(x)\,\psi_g(x)}{\psi_f(x)\,\psi_g(x)} \right)}$$

$$= x^{m_g} \left(x^{m_f - m_g} \frac{\varphi_f(x)}{\psi_f(x)} + \frac{\varphi_g(x)}{\psi_g(x)} \right)$$

czyli ~~we~~ $f + g > 0 \Leftrightarrow g > 0$

analogicznie dla $m_f < m_g$, wtedy $f + g > 0 \Leftrightarrow f > 0$

Weźmy $f(x) = x \quad g(x) = 1$

$f - g = x - 1$ dla $x = 0 \quad x - 1 < 0$

$f - g < 0 \Leftrightarrow f < g$

$n \cdot f(x) - g(x) = nx - 1$ dla $x = 0 \quad nx - 1 < 0$

czyli nie istnieje takie n i warunek Archimedesa nie jest spełniony