586

Dla

$A_1 = \{\{1,2\},\{2,3\}\} \subseteq P(\mathbb{N})$

$A_2 = \{\{1,2\},\{1,3\}\} \subseteq P(\mathbb{N})$

$B_1 = B_2 = \{\{1\}\} \subseteq P(\mathbb{N})$

$\bigcap B_1 = \bigcap B_2 = \{1\}$

$\bigcap A_1 = \{2\}$

$\bigcap A_2 = \{1\}$

$$L = \bigcap_{n\in\mathbb{N}} \left(\bigcap A_n \times \bigcap B_n\right) = (\{2\} \times \{1\}) \cap (\{1\} \times \{1\}) = \emptyset$$

$$\bigcap_{n\in\mathbb{N}} (A_n \times B_n) = \left(\{\{1,2\},\{2,3\}\} \times \{\{1\}\}\right) \cap \left(\{\{1,2\},\{1,3\}\} \times \{\{1\}\}\right) =$$

$$= \{\langle\{1,2\},\{1\}\rangle\} \neq \emptyset$$

$$P = \bigcap \{u \times w \mid \langle u,w\rangle \in \bigcap_{n\in\mathbb{N}} (A_n \times B_n)\} = \{\langle 1,1\rangle, \langle 2,1\rangle\} \neq \emptyset$$

czyli A_1, A_2, B_1, B_2 spełniają warunki zadania, ale równość nie zachodzi, bo $\emptyset = L \neq P = \{\langle 1,1\rangle,\langle 2,1\rangle\}$

58c W poniższym zapisie $A \Leftrightarrow B \Leftrightarrow C$ oznacza, że A, B, C są parami równoważne

$$\langle x, y\rangle \in \bigcap_{n\in\mathbb{N}} \left(\bigcap A_n \times \bigcap B_n\right) \iff \forall_{n\in\mathbb{N}} \left(x \in \bigcap A_n \wedge y \in \bigcap B_n\right)$$

$$\iff \forall_{n\in\mathbb{N}}\Big(\big(\forall_\alpha\ \alpha\in A_n \Rightarrow x\in\alpha\big) \wedge \big(\forall_\beta\ \beta\in B_n \Rightarrow y\in\beta\big)\Big)$$

$$\iff \forall_{n\in\mathbb{N}} \forall_{\alpha,\beta} \Big(\big(\alpha\in A_n \Rightarrow x\in\alpha\big) \wedge \big(\beta\in B_n \Rightarrow y\in\beta\big)\Big)$$

$$\iff \forall_{\alpha,\beta} \forall_{n\in\mathbb{N}} \Big(\big(\alpha\in A_n \wedge \beta\in B_n\big) \Rightarrow \big(x\in\alpha \wedge y\in\beta\big)\Big)$$

$$\iff \forall_{\alpha,\beta}\Big(\Big(\exists_{n\in\mathbb{N}}\big(\alpha\in A_n \wedge \beta\in B_n\big)\Big) \Rightarrow \big(x\in\alpha \wedge y\in\beta\big)\Big)$$

$$\iff \forall_{\alpha,\beta}\Big(\langle\alpha,\beta\rangle \in \Big\{u\times w \mid \langle u,w\rangle \in \bigcup_{n\in\mathbb{N}}(A_n\times B_n)\Big\} \Rightarrow \big(x\in\alpha \wedge y\in\beta\big)$$

$$\iff \langle x,y\rangle \in \bigcap \Big\{u\times w \mid \langle u,w\rangle \in \bigcup_{n\in\mathbb{N}}(A_n\times B_n)\Big\}$$

więc LHS = RHS, bo warunki są równoważne