1.

$\cancel{10\cdot 11\cdot 12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17\cdot 18}$

$$0 < \frac{10\cdot 11\cdot\ldots\cdot(n+9)}{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)} =$$

$$= \underbrace{\boxed{\frac{10\cdot 11\cdot 12\cdot 13\cdot 14\cdot 15\cdot 16\cdot 17\cdot 18\cdot 19\cdot 20}{1\cdot 3\cdot 5\cdot 7\cdot 9\cdot 11\cdot 13\cdot 15\cdot 17\cdot 19\cdot 21}}}_{S} \cdot \frac{21\cdot 22\cdot\ldots\cdot(n+9)}{23\cdot 25\cdot\ldots\cdot(2n-1)} <$$

$$S\cdot 21\cdot 1\cdot 1\cdot\ldots\cdot 1\cdot \frac{n+9}{(2n-3)(2n-1)} = S\cdot 29\cdot \underset{\underset{0}{\xrightarrow{n\to\infty}}}{\frac{\overset{\overset{1}{\uparrow}}{1}+\overset{\overset{0}{\uparrow}}{\frac{9}{n}}}{(\underset{\underset{2}{\downarrow}}{2}-\underset{\underset{0}{\downarrow}}{\frac{3}{n}})\cdot(2n-1)}}$$

$$0 < \underset{\underset{0}{\downarrow}}{\frac{10\cdot\ldots\cdot(n+9)}{1\cdot\ldots\cdot(2n-1)}} < \underset{\underset{0}{\downarrow}}{S\cdot 29\cdot\frac{1+\frac{9}{n}}{(2-\frac{3}{n})(2n-1)}}$$

$$\underset{0}{\downarrow}$$

2.

~~W nieskończoności~~ $a_0 = 1/2$

$a_{n+1} = a_n^3 - a_n$

$|a_{n+1}| < |a_n|$

$\Updownarrow$

$|a_n^3 - a_n| < |a_n|$

$\Updownarrow$

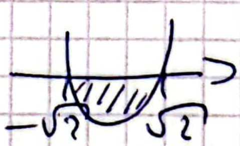
$a_n^6 - 2a_n^4 + a_n^2 < a_n^2$

$\Updownarrow$

$a_n^6 - 2a_n^4 < 0$

$a_n^4(a_n^2 - 2) < 0$

$a_n^4(a_n + \sqrt{2})(a_n - \sqrt{2}) < 0$



$a_n \in (-\sqrt{2}, \sqrt{2})$ $\Uparrow$

a tak jest dla a_0 i będzie dla a_n, bo $|a_n|$ maleje

$|a_n| > 0$, więc granicę

W takim razie w nieskończoności $|a_{n+1}| = |a_n| = g$

$g = g^3 - g$

$0 = g^3 - 2g = g(g - \sqrt{2})(g + \sqrt{2})$

$g = 0 \lor g = \sqrt{2} \lor g = -\sqrt{2}$ czyli $g = 0$, bo $|a_n| < 1 < \sqrt{2}$

czyli jeżeli $|a_n| \to 0$ to $a_n \to 0$