

$$z1 \quad V_0 = 2 \quad V_1 = 0$$

$$V_{n+1} = aV_n - bV_{n-1}$$

~~Wyswietlacz dobra dla 2n~~

$$V_{2n} = (V_n)^2 - 2b^n$$

$$V_{2n+1} = V_n V_{n+1} - ab^n$$

~~Wyswietlacz dobra dla 2n+1~~

Pokazie ze rachodzi tez dla  $n+1$  jest to prawda

$$V_{2n+2} = aV_{2n+1} - bV_{2n}$$

$$V_{2n+2} = a(aV_n - bV_{n-1}) - bV_{2n}$$

$$V_{2n+2} = a^2 V_{2n} - abV_{2n-1} - bV_{2n}$$

zero:

dla  $n=1$  OK

$n=2$  OK.

Zest. ie dla ~~wyswietlacza 2n~~ 2n:

$$\begin{aligned} V_{2n+2} &= a(aV_n - bV_{n-1}) - bV_{2n} \\ &= a^2 V_{2n} - abV_{2n-1} - bV_{2n} \end{aligned}$$

$$V_{2n+2} = a^2 V_{2n} - ab V_{2n-1} - b(aV_{2n-1} - bV_{2n-2}) = \\ = a^2 V_{2n} - 2ab V_{2n-1} + b^2 V_{2n-2} =$$

haben

$$a^2(V_n^2 - 2b^n) - 2ab \cdot \text{mit } (V_{n-1}V_n - ab^{n-1}) + b^2 V_{2n-2}$$

$$a^2 V_n^2 - 2a^2 b^n - 2ab V_{n-1} V_n + 2a^2 b^{n-1} + b^2 V_{2n-2}$$

$$a^2 V_n^2 - 2ab V_{n-1} V_n + b^2 (V_{n-1}^2 - 2b^{n-1}) =$$

$$a^2 V_n^2 - 2ab V_{n-1} V_n + b^2 V_{n-1}^2 - 2b^{n+1} = V_{n+1}^2 - 2b^{n+1}$$

$$\text{S. 2} \quad V_{2n+3} = aV_{2n+2} - bV_{2n+1} =$$

$$= a((V_{n+1})^2 - 2b^{n+1}) - b(V_n V_{n+1} - (V_{n+1} V_n -$$

$$= a(V_{n+1})^2 - 2ab^{n+1} - bV_{n+1} V_n + ab^{n+1}$$

$$= V_{n+1} (aV_{n+1} - bV_n) + ab^{n+1}$$

$$= V_{n+1} V_{n+2} - ab^{n+1}$$

$$\begin{aligned}
 & \text{Richtig} \quad \text{zu zeigen: } \left(1 + \frac{1}{n}\right)^n = \binom{n}{0} + \binom{n}{1} \frac{1}{n} + \binom{n}{2} \frac{1}{n^2} + \dots + \binom{n}{n} \frac{1}{n^n} = \\
 & = \frac{n!}{0!} + \frac{n}{1! \cdot n} + \frac{n(n-1)}{2! \cdot n^2} + \dots + \frac{n(n-1)(n-2)\dots \cdot 1}{n! \cdot n^n} \\
 & \text{Beweis: } \left\langle 2 + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!} \right\rangle \stackrel{(*)}{\leq} 2 + \frac{1}{2^1} + \frac{1}{2^2} + \dots + \frac{1}{2^n} \\
 & \text{Durch} \quad \left\langle 3 \right\rangle
 \end{aligned}$$

Dann ist (\*)

$$n! \geq 2^{n-1}$$

Basis: dla  $n=1$

$$2! \geq 2^1 \quad \text{OK}$$

~~W~~ Zweit. ze  $n! \geq 2^{n-1}$  dla  $n \geq 1$

Pokusze, ze  $n!(n+1) \geq 2^{n-1} \cdot 2$

$$\begin{aligned}
 n!(n+1) &\geq 2^{n-1}(n+1) \geq 2^{n-1} \cdot 2 = 2^n \\
 &\square
 \end{aligned}$$