1

(1) $F(x)=\int_{-x}^{x}\ln(2+\sin t)\,dt \Rightarrow F'(x)=?$

$F'(x)=\ln(2+\sin x).1-\ln(2+\sin(-x)).(-1)$

$=\ln(2+\sin x)+\ln(2-\sin x)$

(2) $G(x)=\int_{x}^{\pi/4}(t^2-\ln(t))^2\,dt \Rightarrow G'(x)=?$

$G'(x)=\left(\left(\cancel{\frac{\pi}{4}}\right)^2-\ln\frac{\pi}{4}\right)^2\left(\frac{\pi}{4}\right)'-(x^2-\ln x)^2.1$

$=-x^4+2x^2\ln x-\ln^2 x$

(3) $H(x)=\int_{\sin x}^{\cos x}\cosh t^2\,dt \Rightarrow H'(x)=?$

$H'(x)=\cosh((\cos x)^2.(\cos x)'-\cosh(\sin x)^2.(\sin x)'$

$=\cosh((\cos x)^2(-\sin x)-\cosh(\sin x)^2.\cos x$

(4) $P(x)=\int_{x}^{2}[(\cos(t^2)+t]\,dt \Rightarrow P'(x)=?$

$P'(x)=-\int_{2}^{x}(\cos t^2+t)\,dt \Rightarrow P'(x)=-(\cos x^2+x).1$

(5) $g(x)=\int_{1}^{\sqrt{x}}\frac{s^2}{s^2+1}\,ds \Rightarrow g'(x)=?$

$g'(x)=\frac{(\sqrt{x})^2}{(\sqrt{x})^2+1}.(\sqrt{x})'=\frac{x}{x+1}.\frac{1}{2\sqrt{x}}=\frac{\sqrt{x}}{2(x+1)}$

6) $\int \left(\frac{4}{x^2} + 2 - \frac{1}{8x^3}\right) dx = \int \left(4x^{-2} + 2 - \frac{1}{8}x^{-3}\right) dx$

$= 4\,\frac{x^{-1}}{-1} + 2x - \frac{1}{8}\,\frac{x^{-2}}{-2} + C = -\frac{4}{x} + 2x + \frac{1}{16}\,\frac{1}{x^2} + C$

7) $\int \frac{x^4 - \sqrt[3]{x}}{6\sqrt{x}}\, dx = \int \left(\frac{1}{6}\, x^4 \cdot x^{-1/2} - \frac{x^{1/3}}{6} \cdot x^{-1/2}\right) dx$

$= \int \left(\frac{x^{7/2}}{6} - \frac{1}{6} x^{-1/6}\right) dx = \frac{1}{27} x^{9/2} - \frac{1}{5} x^{5/6} + C$

8) $\int 5(x-4)\sqrt[3]{x^2 - 8x}\, dx$

$x^2 - 8x = u$ $\qquad (x-4)\,dx = \frac{du}{2}$

$(2x-8)\,dx = du$

$= \frac{5}{2}\int \sqrt[3]{u}\, du = \frac{5}{2}\int u^{1/3}\, du = \frac{5}{2}\,\frac{u^{4/3}}{4/3} + C = \frac{5}{2}\cdot\frac{3}{4}\,(x^2 - 8x)^{4/3} + C$

9) $\int \frac{dx}{\sqrt{4 - 9x^2}} = \int \frac{1}{\sqrt{4\left(1 - \frac{9}{4}x^2\right)}}\, dx = \frac{1}{2}\int \frac{1}{\sqrt{1 - \frac{9}{4}x^2}}\, dx$

$u = \frac{3x}{2}$ $\qquad du = \frac{3}{2}\,dx \Rightarrow dx = \frac{2}{3}\,du$

$= \frac{1}{2}\cdot\frac{2}{3}\int \frac{1}{\sqrt{1 - u^2}}\, du = \frac{1}{3}\,\mathrm{arcSin}(u) + C = \frac{1}{3}\,\mathrm{arcSin}\,\frac{3x}{2} + C$

10) $\int \frac{3x}{(1+9x^2)^4}\, dx = \frac{1}{6}\int \frac{1}{u^4}\, du = \frac{1}{6}\int u^{-4}\, du = -\frac{1}{18}\cdot u^{-3} + C = -\frac{1}{18}\,\frac{1}{(1+9x^2)^3} + C$

$1 + 9x^2 = u$

$18x\, dx = du$

$3x\, dx = \frac{du}{6}$

(11) $\displaystyle\int_0^{\ln(1+\pi)} e^x \cos(1-e^x)\,dx$

$u = 1-e^x \quad du = -e^x dx$

$x = 0 \Rightarrow u = 1-e^0 = 0$

$x = \ln(1+\pi) \Rightarrow u = 1-e^{\ln(1+\pi)} = 1-(1+\pi) = -\pi$

$$= -\int_0^{-\pi} \cos u\,du = -\sin u \Big|_0^{-\pi} = -(\sin(-\pi) - \sin 0) = 0$$

(12) $\displaystyle\int_{-\pi}^{\frac{\pi}{2}} \cos x \cos(\sin x)\,dx$

$u = \sin x \Rightarrow du = \cos x\,dx$

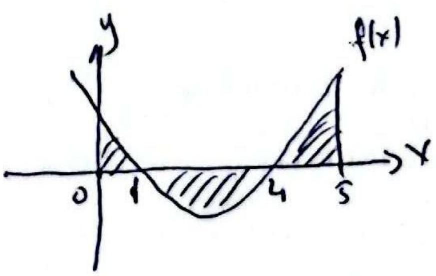
$x = \pi/2 \Rightarrow u = \sin\frac{\pi}{2} = 1$

$x = -\pi \Rightarrow u = \sin(-\pi) = 0$

$$= \int_0^1 \cos u\,du = \sin u \Big|_0^1 = \sin 1 - \sin 0 = \sin 1$$

(13) $f:[0,5] \to \mathbb{R}$ $f(x) = x^2-5x+4$ parabolü ile ve x-ekseninin sınırladığı kapalı bölgenin alanını bulunuz.

$x^2-5x+4 = 0 \Rightarrow x_1 = 1 \in [0,5],\ x_2 = 4 \in [0,5]$



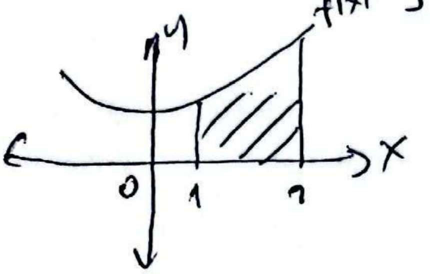
$$A = \int_0^5 |f(x)|dx = \int_0^1 f(x)dx + \int_1^4 -f(x)dx + \int_4^5 f(x)dx$$

$$= \int_0^1 (x^2-5x+4)dx + \int_1^4 -(x^2-5x+4)dx + \int_4^5 (x^2-5x+4)dx$$

$= 49/6$ br^2

14) $y=x^2+1$ eğrisi, $x=1$ ve $x=2$ doğruları ile x-ekseninin sınırladığı bölgenin alanını bulunuz. 4

$y=x^2+1>0$ olduğundan, bölge x-ekseninin üst tarafındadır.



$$A=\int_1^2 (x^2+1)\,dx = 10/3 \text{ br}^2$$

15) $$\int \frac{2\,dx}{1+2\sin x}, \quad \tan\frac{x}{2}=t \qquad \sin x=\frac{2t}{1+t^2} \qquad \cos x=\frac{1-t^2}{1+t^2}$$

$$dx=\frac{2}{1+t^2}\,dt$$

$$=2\int \frac{\frac{2}{1+t^2}}{1+2\frac{2t}{1+t^2}}\,dt = \int \frac{4}{t^2+4t+1}\,dt = 4\int \frac{1}{(t+2)^2-3}\,dt = 4\frac{1}{2\sqrt{3}}\ln\left|\frac{t+2-\sqrt{3}}{t+2+\sqrt{3}}\right|+C$$

$$=\frac{2}{\sqrt{3}}\ln\left|\frac{\tan\frac{x}{2}+2-\sqrt{3}}{\tan\frac{x}{2}+2+\sqrt{3}}\right|+C$$

16) $$\int \frac{1+\cos x}{1-\sin x}\,dx = \int \frac{1+\frac{1-t^2}{1+t^2}}{1-\frac{2t}{1+t^2}}\,\frac{2\,dt}{1+t^2} = \int \frac{4}{(t^2+1)(t-1)^2}\,dt$$

$\tan\frac{x}{2}=t$

$$\frac{4}{(t-1)^2(t^2+1)} = \frac{A}{t-1}+\frac{B}{(t-1)^2}+\frac{Ct+D}{t^2+1}$$

$$4 = A(t-1)(t^2+1)+B(t^2+1)+(Ct+D)(t-1)^2 \qquad A+B=2$$

$t=1$ $\quad 4 = 2A+2B \Rightarrow$

$t=0$ $\quad 4 = -A+B+D$

$t=-1$ $\quad 4 = -4A+2B+(D-C)\cdot 4$

$$=2\int\left\{\frac{t}{t^2+1}-\frac{1}{t-1}+\frac{1}{(t-1)^2}\right\}dt$$

$$=\ln|t^2+1|-2\ln|t-1|-\frac{2}{t-1}+C$$

$$=\ln\left|\tan^2\frac{x}{2}+1\right|-2\ln\left|\tan\frac{x}{2}-1\right|-\frac{2}{\tan\frac{x}{2}-1}+C$$