What is Calculus?

Introduction

Geometry is the study of shapes.

Algebra is the study of operations and their application to solving equations.

Calculus is the mathematical study of motion and change.

Dictionary defines calculus as **"a particular method of mathematical calculation or reasoning".** While being literally correct, this definition grossly understates the sweeping influence that this branch of mathematics has had on the scientific and technological advances that have come about in the last 300 years.

Great thinkers from ancient times were fascinated by giant planets of our solar system and countless stars from galaxies, which were all in a state of perpetual motion or in a state of **constant change** in vast space and time. Life on earth reflected this cosmic reality as everything in it including the living beings was characterized by **movement and change**, be it the movement of rain clouds across the continents, sprinting athletes, galloping horses, flowing rivers or change in seasons.

Thus, any sense of control over, or intelligent coexistence with this dynamic world would rest on man's ability to clearly understand this phenomenon of **movement and change** and acquire the knowledge to predict its behavior. The search for a universal order behind all **motion and change** started as mere philosophical contemplation, but evolved over several centuries of research by many distinguished mathematicians, until **Isaac Newton,** from England and **Gottfried Leibniz,** from Germany**,** independently developed a **'new mathematical method'** around the year 1665, **which explained with precision and clarity, the phenomenon of motion and change.**

This 'new mathematical method' is what we all know as 'Calculus'.

We may trace calculus to its earliest roots with the 5th century B.C. **Greek philosopher Zeno**, who postulated his famous **paradoxes about motion**. These paradoxes were the earliest **seeds** of the **concepts of calculus**. Zeno questioned the universal observation that when an arrow is shot from a bow, it starts moving rapidly through the air and ultimately hits its target. He argued that "the fast moving arrow is however **stationary** in one place at any **instant of time**. How could one say then that the arrow is also moving at those instants of time?" He surmised theoretically that the **arrow can never hit its target** as it is stationary at any instant of time.

But it is also true that the arrow does move through the air very rapidly and reaches its target. **That the arrow has a certain speed or velocity at all the instants where it appears to be stationary is a concept developed in Calculus.** Speed or velocity was the only term used earlier to completely describe all the aspects of motion. However, a new concept of the **'Instantaneous velocity'** was developed to describe the **'rate of change'** in motion. Calculus thus resolved Zeno's paradoxes after nearly two millenniums.

The **entire concept of Calculus** is based upon **understanding of its two main ideas, Derivative** and **Integral.** These fundamental ideas of Calculus can be understood by **observing** and **analyzing** common **day-to-day examples**, as explained in the following chapters. **Calculus is all around us**. When we are driving a car and see how fast we are moving, there is **Calculus**. When we hit a cricket ball and see where it lands, there is **Calculus**. When we observe planets moving around the Sun, there is **Calculus**. And when

we launch a satellite into the orbit, there is **Calculus**.

Calculus has been extremely effective as it not only enables us to understand our ever changing world, but also enables us to predict the changes and control them. We can calculate where the cricket ball will land when we hit it, or launch a satellite with predetermined speed to place it into a geostationary orbit.

Calculus gave mankind a **scientific tool** that perhaps has been **most influential** in setting off the **spectacular advances in science and technology**, **from physics to engineering, chemistry to biology and from business to economics and in many other fields**.

In fact, it can be said that 'Calculus' is indeed one of the greatest intellectual triumphs of humanity.

Note :– Calculus makes extensive use of basic mathematics. Hence a revision of basic mathematics will help a reader to understand the concepts of calculus better.

With this in mind, a brief review of a few related topics which are frequently used in study of calculus is covered in the first chapter of **'Pre-calculus review'**.