Binary Search Tree

- *Recursion Revisited*
- *binary search tree Implementation*
  - *traversal - inorder, preorder, postorder, levelorder*
  - *minimum, maximum,*
  - *predecessor, successor*
  - *height*
  - *clear*
  - *contains*
  - *grow*
  - *trim*

bunnyEars(): counting bunny ears in recursion

```
// each bunny has two ears.
int bunnyEars(int bunnies) {

  return 2 + bunnyEars(bunnies-1);
}
```

funnyEars(): counting funny ears in recursion

```
// even numbered funny has two ears, odd numbered funny three.
int funnyEars(int funnies) {
  if (bunnies == 0) return 0;

  if (funnies % 2 == 0)
    return
  else
    return
}
```

bunnyEars(): counting bunny ears in recursion

```
// each bunny has two ears.
int bunnyEars(int bunnies) {
  if (bunnies == 0) return 0;
  return 2 + bunnyEars(bunnies-1);
}
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funnyEars(): counting funny ears in recursion

```
// even numbered funny has two ears, odd numbered funny three.
int funnyEars(int funnies) {
  if (bunnies == 0) return 0;

  if (funnies % 2 == 0)
    return 2 + funnyEars(funnies - 1);
  else
    return 3 + funnyEars(funnies - 1);
}
```

Binary search trees

size(): in doubly linked list

```
int size(pList p) {
  int count = 0;
  for (pNode c = begin(p); c != end(p); c = c->next)
    count++;
  return count;
}
```

size(): in singly linked list

```
int size(pNode node) {
  if (node->next == nullptr) return 0;
  return 1 + size(node->next);
}
```

Binary search trees

size: Count the number of nodes in the binary tree recursively.

```
int size(tree node) {
  if (node == nullptr) return 0;
  return 
}
```

Binary search trees

size: Count the number of nodes in the binary tree recursively.

```
int size(tree node) {
  if (node == nullptr) return 0;
  return 1 + size(node->left)
}
```

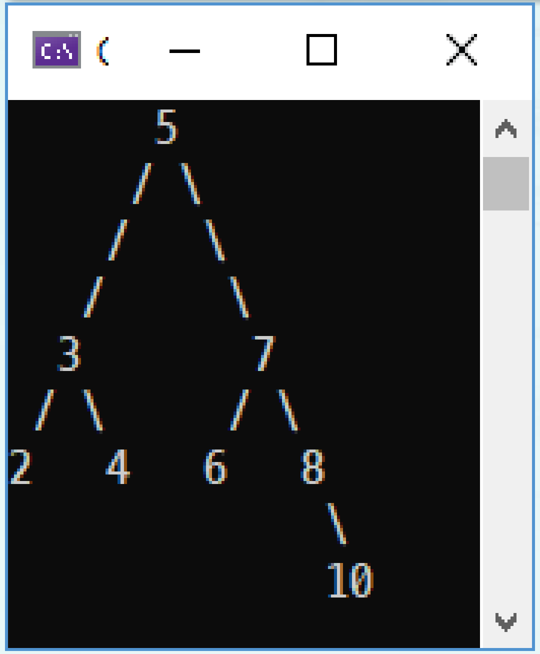
Binary search trees

size: Count the number of nodes in the binary tree recursively.

```
int size(tree node) {
  if (node == nullptr) return 0;
  return 1 + size(node->left) + size(node->right);
}
```

size: Count the number of nodes in the binary tree recursively.

```
int size(tree node) {
  if (node == nullptr) return 0;
  cout << " size at: " << node->key << endl;
  return 1 + size(node->left) + size(node->right);
}
```



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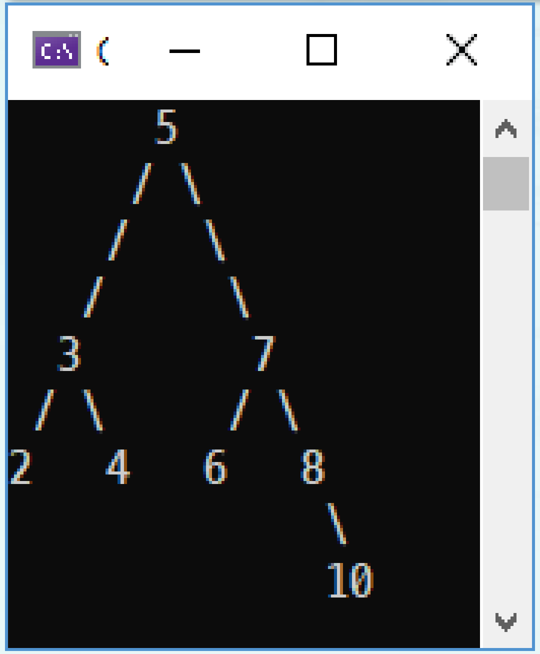

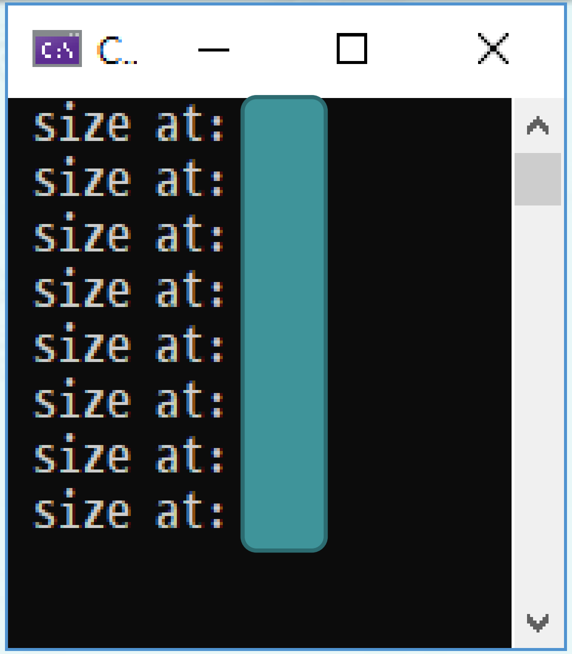

Binary search trees

size: Count the number of nodes in the binary tree recursively.

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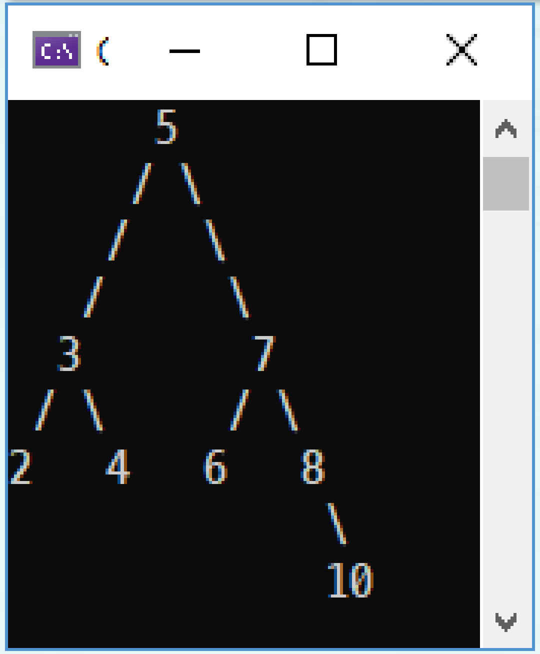
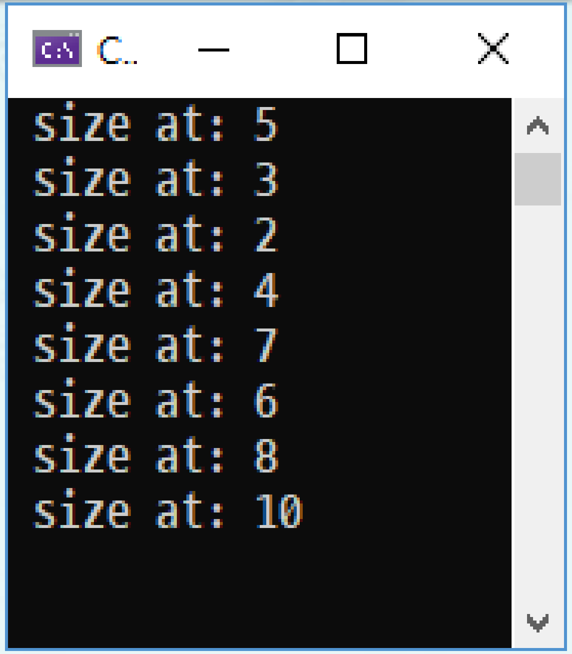




Binary search trees

size: Count the number of nodes in the binary tree recursively.

```
int size(tree node) {
  if (node == nullptr) return 0;
  return 1 + size(node->left) + size(node->right);
}
```

height: compute the max height(or depth) of a tree.
// It is the number of nodes along the longest path from the root node
// down to the farthest leaf node.

```
int height(tree node) {

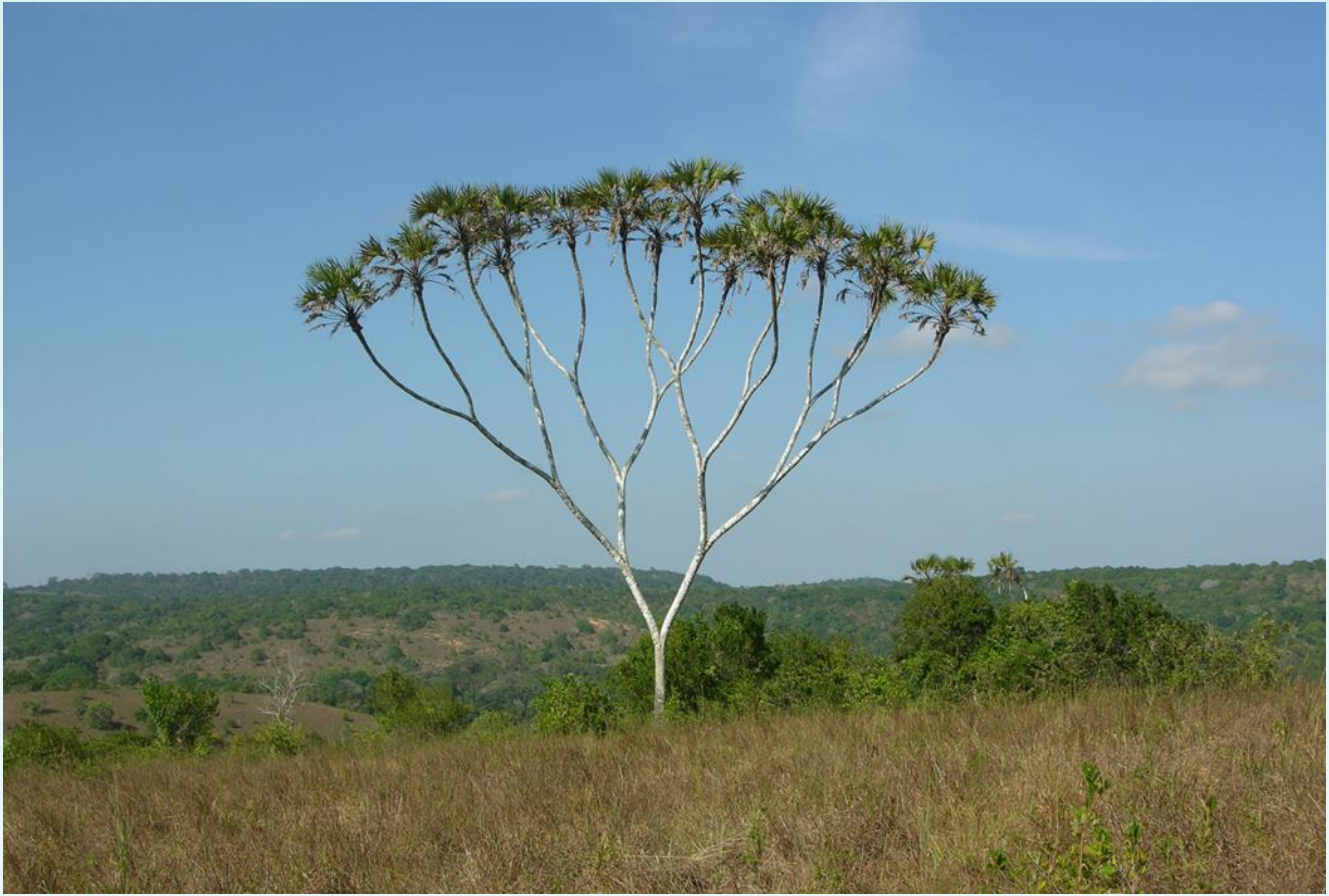

}
```

Binary search trees

BST Node structure:

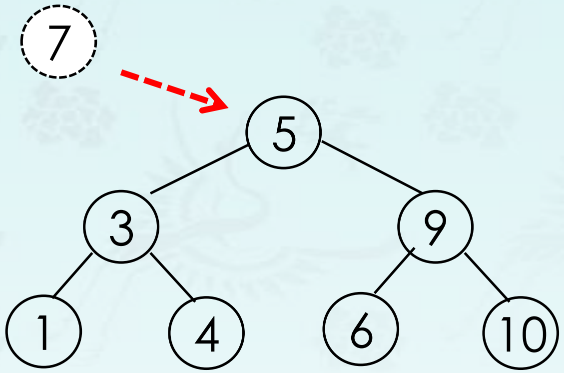
Key	
Left	Right

```
struct TreeNode{
  int       key;      // sorted by key
  TreeNode* left;     // left child
  TreeNode* right;    // right child
  TreeNode(const int k = 0, Tree* l = nullptr, Tree* r = nullptr) {
    key = k; left = l; right =r;
  }
  ~TreeNode() {}
};
using tree = TreeNode*;
```



Operations: grow

- grow(T, k)
  - Insert a node with Key = k into BST **T**
  - Time complexity? O(h)
- **Step 1**:
  if the tree is empty, then Root(**T**) = k
- **Step 2**:
  Pretending we are searching for k in BST, until we meet a nullptr node
- **Step 3**:
  Insert k



Q: Where is it inserted at?

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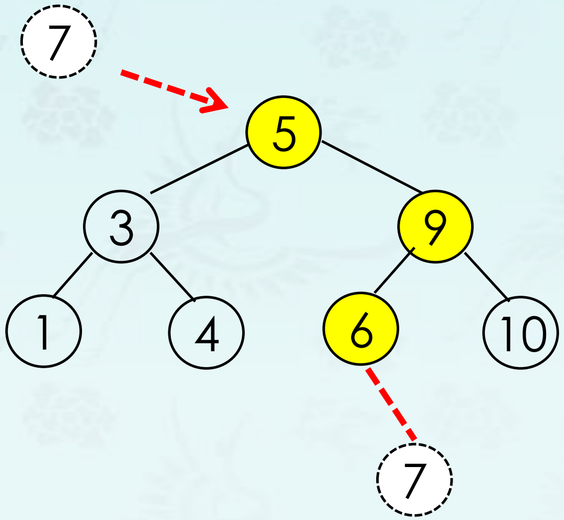

The light nodes are compared with key.

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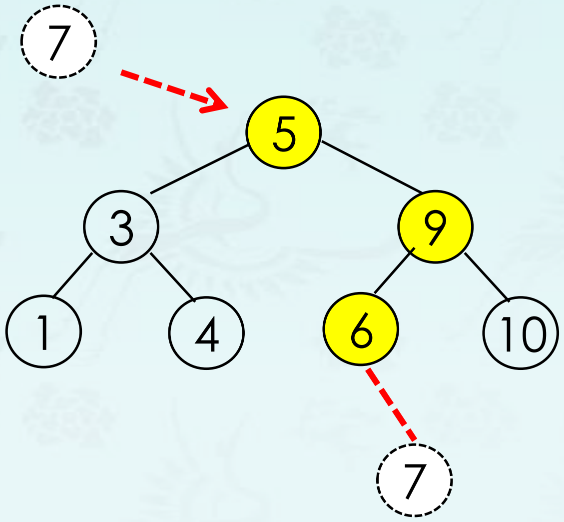

The light nodes are compared with key.

Q: Do you see the difference between the complete binary tree and binary search tree?

grow: insert a new node with given key in BST

Something wrong?

```
tree grow(tree node, int key) {
  if (node == nullptr)
  if (key < node->key)   // recur down the tree
    grow(node->left, key);
  else
    grow(node->right, key);
  else
    cout << "grow: the same key " << key << " is ignored.\n";

  return node;
}
```

grow: insert a new node with given key in BST

Something wrong?

```
tree grow(tree node, int key) {
  if (node == nullptr) return new Tree(key);
  if (key < node->key)   // recur down the tree
    grow(node->left, key);
  else
    grow(node->right, key);
  else
    cout << "grow: the same key " << key << " is ignored.\n";

  return node;
}
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grow: insert a new node with given key in BST

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```
tree grow(tree node, int key) {
  if (node == nullptr) return new Tree(key);
  if (key < node->key)  // recur down the tree
    grow(node->left, key);
  else if (key > node->key)
    grow(node->right, key);
  else
    cout << "grow: the same key " << key << " is ignored.\n";

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}
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  if (node == nullptr) return new Tree(key);
  if (key < node->key)  // recur down the tree
    grow(node->left, key);
  else if (key > node->key)
    grow(node->right, key);
  else
    ;

  return node;
}
```

grow: insert a new node with given key in BST

Something wrong?

```
tree grow(tree node, int key) {
  if (node == nullptr) return new Tree(key);
  if (key < node->key)   // recur down the tree
    node->left = grow(node->left, key);
  else if (key > node->key)
    node->right = grow(node->right, key);
  else
    ;

  return node;
}
```

이게 리컬젼..? 왜죠 왜죠 왜죠 왜죠

inorder traversal: do inorder traversal of BST.

```
void inorder(tree node) {
    if (node == nullptr) return;

    inorder(node->left);
    cout << node->key;
    inorder(node->right);
}
```

inorder traversal: do inorder traversal of BST.

```
void inorder(tree node) {
    if (node == nullptr) return;

    inorder(node->left);
    cout << node->key;
    inorder(node->right);
}
```

```
void inorder(tree node, vector<int>& vec) {
  if (node == nullptr) return;

  inorder(node->left, vec);
          _
  inorder(node->right, vec);
}
```

inorder traversal: do inorder traversal of BST.

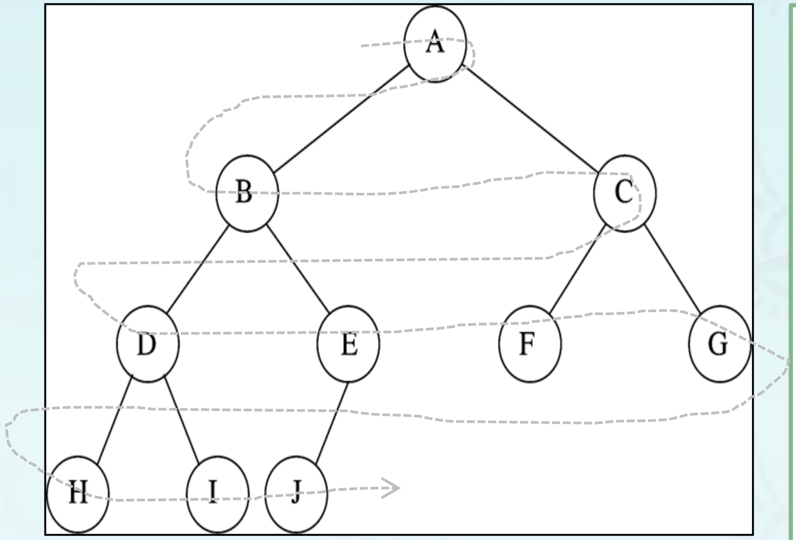
```
void inorder(tree node) {
    if (node == nullptr) return;

    inorder(node->left);
    cout << node->key;
    inorder(node->right);
}
```

```
void inorder(tree node, vector<int>& vec) {
  if (node == nullptr) return;

  inorder(node->left, vec);
  _
  inorder(node->right, vec);
}
```

```
case 'l':
  cout << "\n\tinorder:    ";
  vec.clear();
  inorder(root, vec);
  for (int i : vec)
    cout << i << " ";
```

Level-order traversal

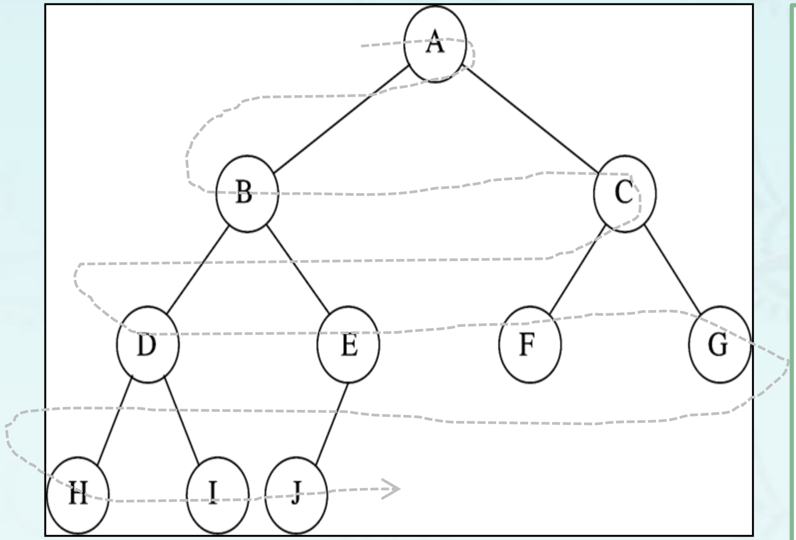
1. **Depth first** search(DFS) – preorder, inorder, postorder traversal
2. **Breadth first** search(BFS) - **level-order** traversal




```
#include <queue>
#include <vector>

void levelorder(tree root, vector<int>& vec)
•  Visit the root. if it is not null, enqueue it.
•  while queue is not empty
    1. que.front() – get the node in the queue
    2. save the key in vec.
    3. if its left child is not null, enqueue it.
    4. if its right child is not null, enqueue it.
    5. que.pop() – remove the node in the queue.
```

Level-order traversal

1. **Depth first** search(DFS) – preorder, inorder, postorder traversal
2. **Breadth first** search(BFS) - **level-order** traversal




```
#include <queue>
#include <vector>

void levelorder(tree root, vector<int>& vec) {
  queue<tree> que;
  if (!root) return;
  que.push(root);
  while ...{


     cout << "your code here\n";


  }
}
```

minimum, maximum:

returns the node with min or max key.
Note that the entire tree does not need to be searched.

```
tree minimum(tree node) {   // returns left-most node key


}
```

```
tree maximum(tree node) {   // returns right-most node key


}
```

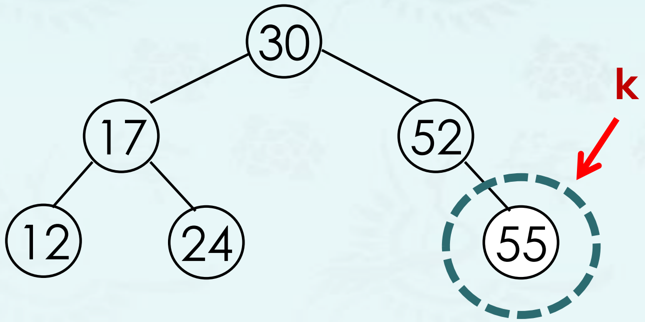
pred(), succ() – predecessor, successor:

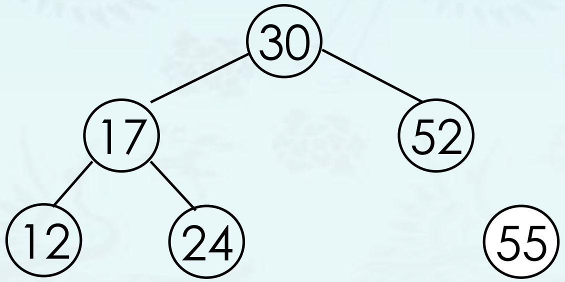
```
Input: root node, key
output: predecessor node, successor node

1. If root is nullptr, then return

2. if key is found then
    a. If its left subtree is not nullptr
        Then predecessor will be the right most
        child of left subtree or left child itself.
    b. If its right subtree is not nullptr
        The successor will be the left most child
        of right subtree or right child itself.
    return
```

Operations: trim

- trim(**T**, k)
  - trim a node with Key = k into BST **T**
  - Time complexity: O(h)

Case 1: k has no child



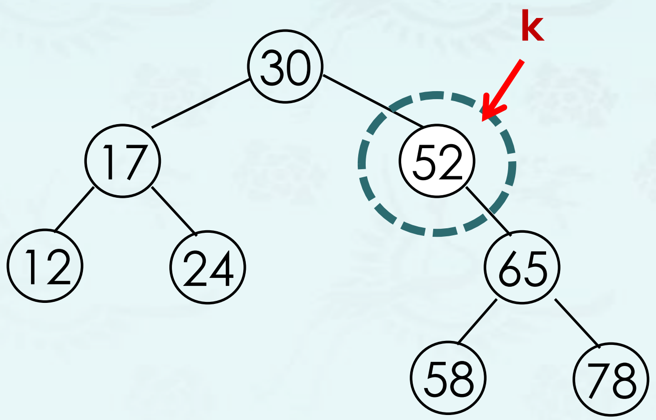


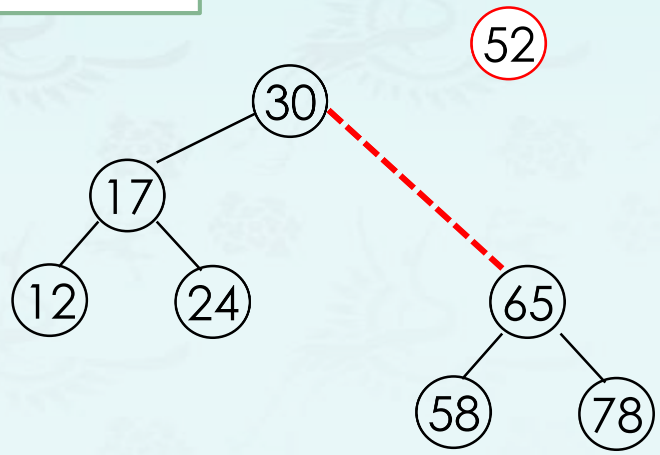
We can simply trim it from the tree

Operations: trim

- trim(**T**, k)
  - trim a node with Key = k into BST **T**
  - Time complexity: O(h)

Case 2: k has one child



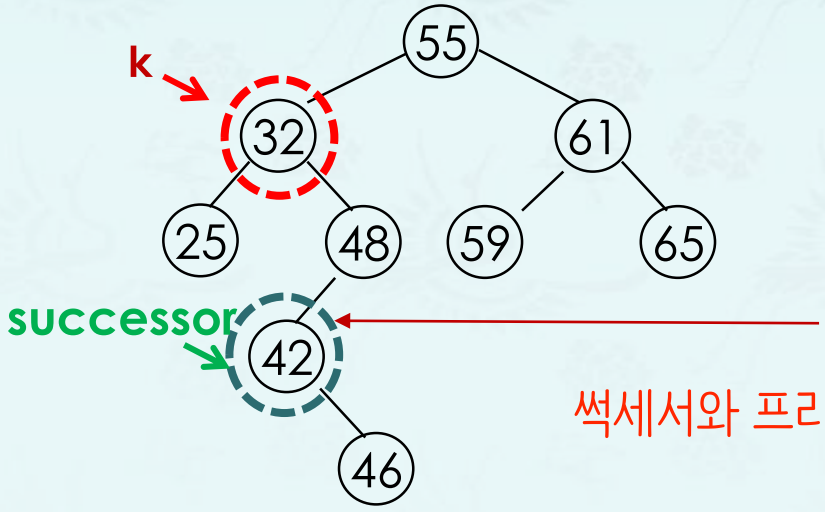


After removing it, connect it's subtree to it's parent node.

Operations: trim

- trim(**T**, k)
  - trim a node with Key = k into BST **T**
  - Time complexity: O(h)

Case 3: k has two children



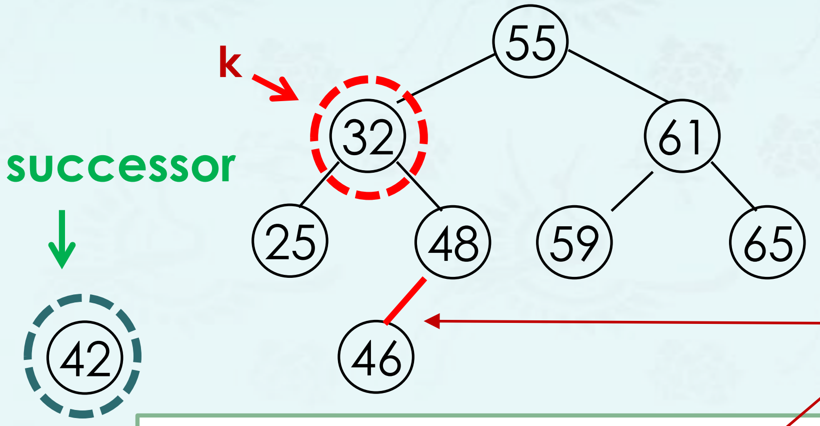
Find it's successor

썩세서와 프리디세서는 childern이 하나!!

Operations: trim

- trim(**T**, k)
  - trim a node with Key = k into BST **T**
  - Time complexity: O(h)

Case 3: k has two children



This is done by calling another trim() with succ key, recursively.

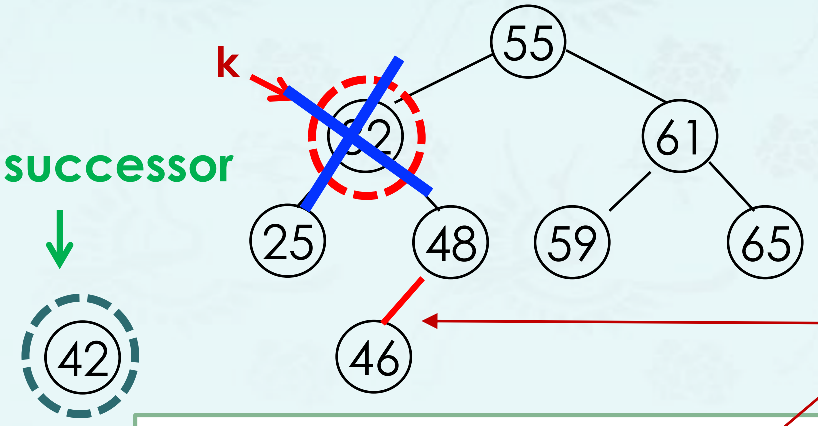
Pull out successor,
and connect the tree with it's child

Operations: trim

- trim(**T**, k)
  - trim a node with Key = k into BST **T**
  - Time complexity: O(h)

Case 3: k has two children

오른쪽은 successor 왼쪽은predessor(?)




```
int succ = value(minimum(root->right));
root->key = succ;
root->right = trim(root->right, succ);
```

This is done by calling another trim() with succ key, recursively.

Pull out successor,
and connect the tree with it's child

Operations: trim

- trim(**T**, k)
  - trim a node with Key = k into BST **T**
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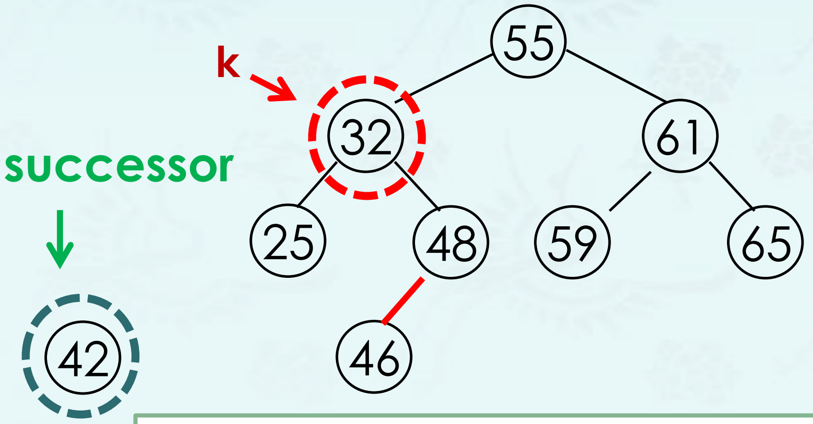
Case 3: k has two children



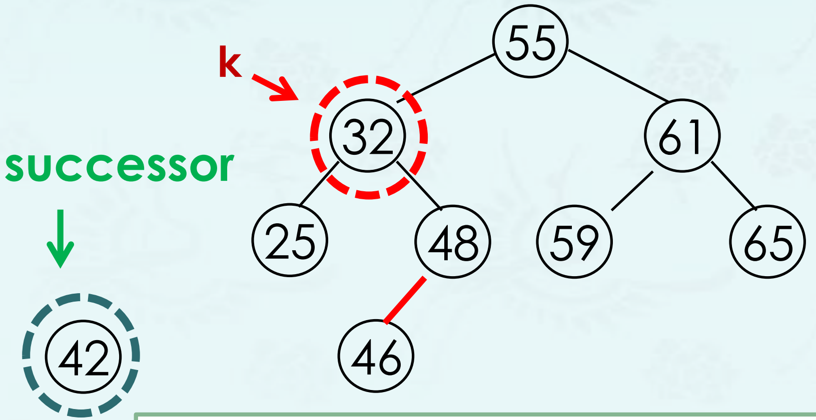
Pull out successor,
and connect the tree with it's child

Q: What if successor has **two** children?

Operations: trim

- trim(**T**, k)
  - trim a node with Key = k into BST **T**
  - Time complexity: O(h)

Case 3: k has two children



Pull out successor,
and connect the tree with it's child

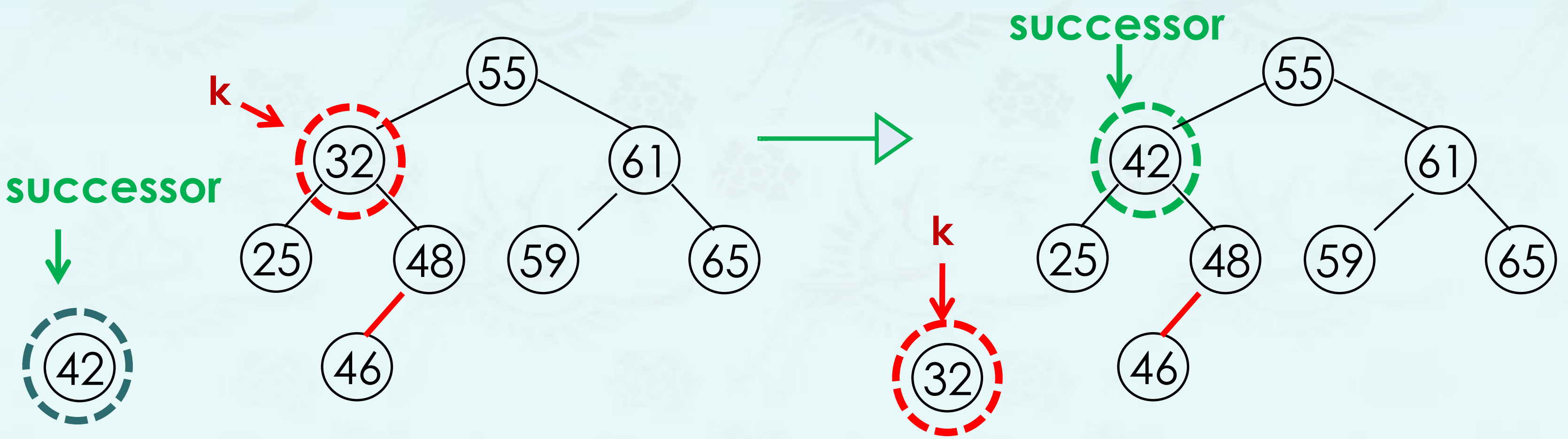
Q: What if successor has **two** children?

A: Not possible !
Because if it has two nodes, at least one of them is less than it, then in the process of finding successor, we won't pick it !

Operations: trim

- trim(**T**, k)
  - trim a node with Key = k into BST **T**
  - Time complexity: O(h)

Case 3: k has two children



Replace the **key** with it's successor

trim**: trim node with the key and return the new root.

장고?

```
tree trim(tree root, int key) {
  if (root == nullptr) return root;// base case
  if (key < root->key)
    root->left = trim(root->left, key);
  else if (key > root->key) {
    root->right = trim(root->right, key);
  else {
    if (root->left == nullptr) {
       // your code here - trim the right one, return root
    }
    else if (root->right == nullptr) {
       // your code here - trim the left one, return root
    }
    else {// two children case
      // get the successor: smallest in right subtree
      // copy the successor's content to this "root" node
      // trim the successor recursively, which has one or no child case
    }
  }
  return root;
}
```

Binary search trees

http://visualgo.net/bst