heap

- complete binary tree (review)
- heap and priority queues (Chapter 9)
- binary heap and min-heap
- max-heap demo
- max-heap coding
- heap sort (Chapter 7)

sink가 젤어렵뎅ㅋㅋ

heap coding: heap.h

Heap ADT: A **one - based** and **one dimensional array** is used to simplify parent and child calculations.

```
struct Heap {
 int *nodes; // an array of nodes
 int capacity; // array size of node or key, item
 int N; // the number of nodes in the heap
 bool (*comp) (Heap*, int, int);
 Heap(int capa = 2) {
   capacity = capa;
   nodes = new int[capacity];
   N = 0;
   comp = nullptr;
 };
 ~Heap() {};
};
using heap = Heap*;
```

heap coding: heap.h

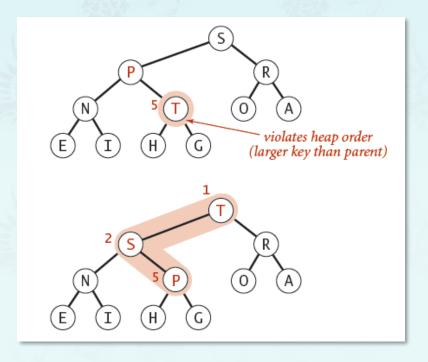
```
void clear(heap hp);
                             // deallocate heap
int size (heap hp);
                          // return nodes in heap currently
int level(int n);
                            // return level based on num of nodes
int capacity (heap hp);
                          // return its capacity (array size)
int reserve (heap hp, int capa); // reserve the array size (= capacity)
                  // return true/false
int full (heap hp);
int empty(heap hp);
                           // return true/false
                       // add a new key
void grow(heap hp, int key);
                          // delete a queue
void trim(heap hp);
int heapify (heap hp);
                             // convert a complete BT into a heap
// helper functions to support grow/trim functions
int less(heap hp, int i, int j); // used in max heap
int more (heap hp, int i, int j); // used in min heap
// helper functions to check heap invariant
```

Promotion in a heap: swim

- To eliminate the violation:
 - Swap key in child with key in parent.
 - Repeat until heap order restored.

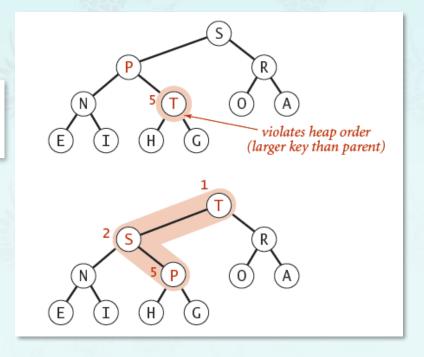
swim up or sink down

This is a maxheap example.



- To eliminate the violation:
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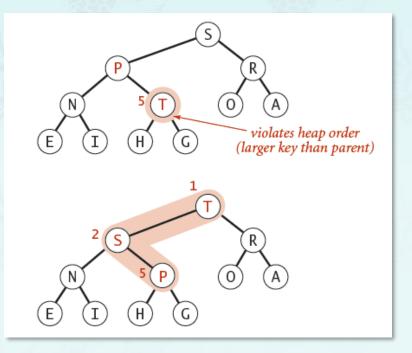
```
bool less(heap h, int p, int c) {
    return h->nodes[p] < h->nodes[c];
}
```



- To eliminate the violation:
 - Swap key in child with key in parent.
 - Repeat until heap order restored.

```
bool less(heap h, int p, int c) {
    return h->nodes[p] < h->nodes[c];
}

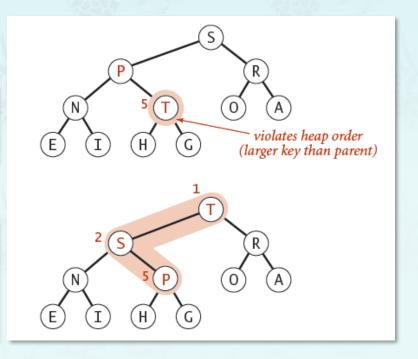
void swap(heap h, int p, int c) {
    Key item = h->nodes[p];
    h->nodes[p] = h->nodes[c];
    h->nodes[c] = item;
}
```



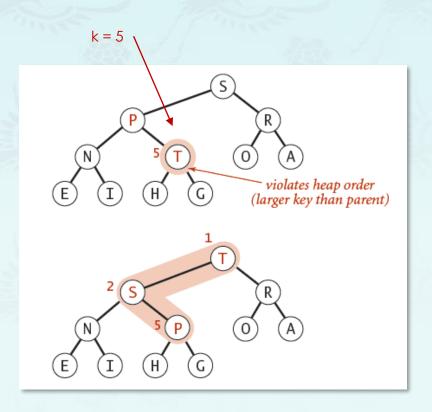
- To eliminate the violation:
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```
bool less(heap h, int p, int c) {
    return h->nodes[p] < h->nodes[c];
}
```

```
void swap(heap h, int p, int c) {
   Key item = h->nodes[p];
   h->nodes[p] = h->nodes[c];
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}
```



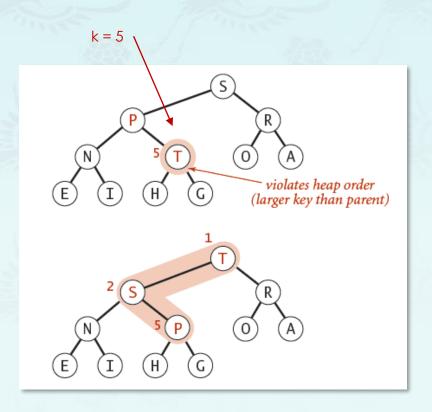
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- To eliminate the violation:
 - Swap key in child with key in parent.
 - Repeat until heap order restored.

```
not reached at root

void swim(h p h, int k)

{
while (
{
}
}
}
```



- To eliminate the violation:
 - Swap key in child with key in parent.
 - Repeat until heap order restored.

```
not reached at root

void swim(h p h, int k)

while (k > 1 &&

{

while (k > 1 &&

{

parent(k/2) is less its child(k)

| the continuous parent(k/2) is less its child(k)

| the continuous parent(k/2) is less its child(k)
```



- To eliminate the violation:
 - Swap key in child with key in parent.
 - Repeat until heap order restored.

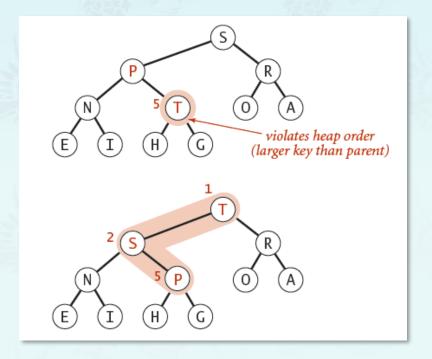
```
not reached at root

void swim(h p h, int k)

while (k > 1 && less(h, k / 2, k))

swap parent(k/2) and its child(k)

swap parent(k/2) and its child(k)
```



- To eliminate the violation:
 - Swap key in child with key in parent.
 - Repeat until heap order restored.

```
not reached at root

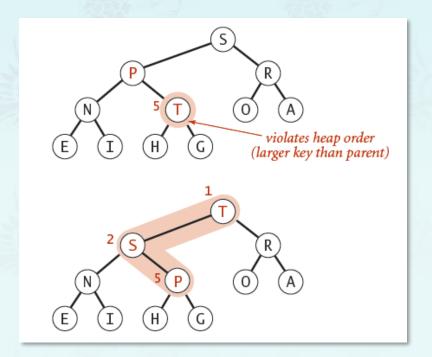
void swim(h p h, int k)

while (k > 1 && less(h, k / 2, k))

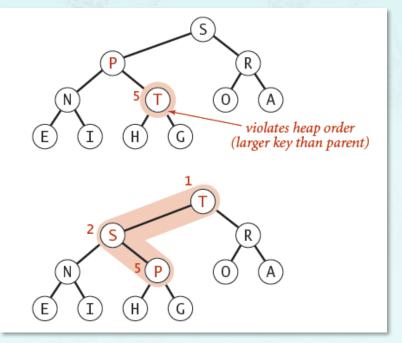
swap(h, k / 2, k);

swap parent(k/2)
and its child(k)

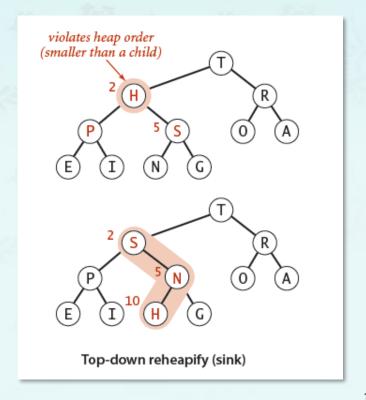
move up
one level
```



- To eliminate the violation:
 - Swap key in child with key in parent.
 - Repeat until heap order restored.



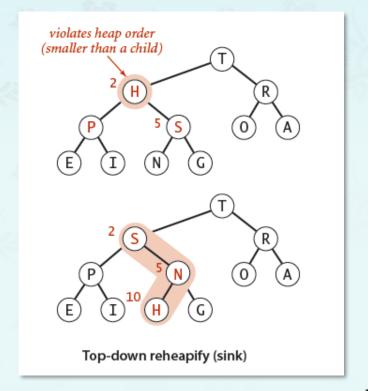
swim up or sink down



Demotion in a heap: sink

- Parent's key becomes smaller than one (or both) of its children's.
- To eliminate the violation:
 - Swap key in parent with key in larger child (of two)
 - Repeat until heap order restored.

This is a maxheap example.

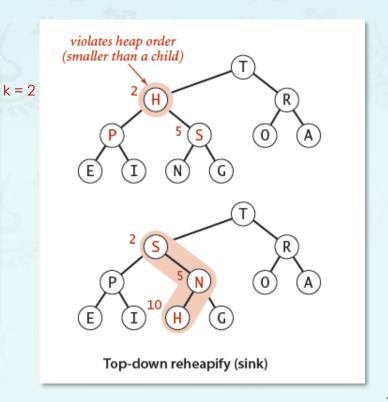


Demotion in a heap: sink

- Parent's key becomes smaller than one (or both) of its children's.
- To eliminate the violation:
 - Swap key in parent with key in larger child (of two)
 - Repeat until heap order restored.

```
void sink(heap h, int k)
{
  while (k's child not reached the last)
  {
    find the larger child of k, let it be j. (j = 5)

    if k's key is not less than j's key, break;
    swap k and j since k's key > j's key
    set k to the next node w
}
}
```

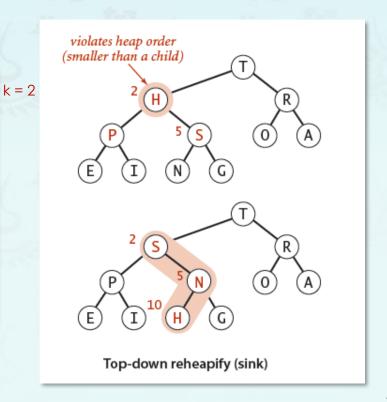


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 - Swap key in parent with key in larger child (of two)
 - Repeat until heap order restored.

```
void sink(heap h, int k)
{
  while (k's child not reached the last)
  {
    find the larger child of k, let it be j. (j = 5)

    if k's key is not less than j's key, break;
    swap k and j since k's key > j's key
    set k to the next node which is j.
  }
}
```



Demotion in a heap: sink

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 - Swap key in parent with key in larger child
 - Repeat until heap order restored.

```
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  while (k's child not reached the last)
  {
    find the larger child of k, let it be j. (j = 5)
}
```



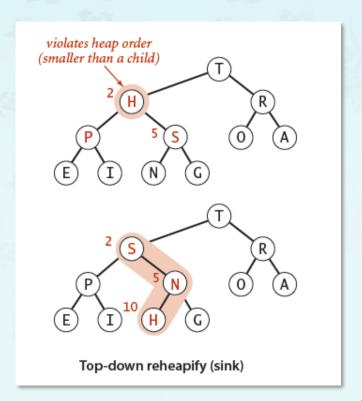
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Demotion in a heap: sink

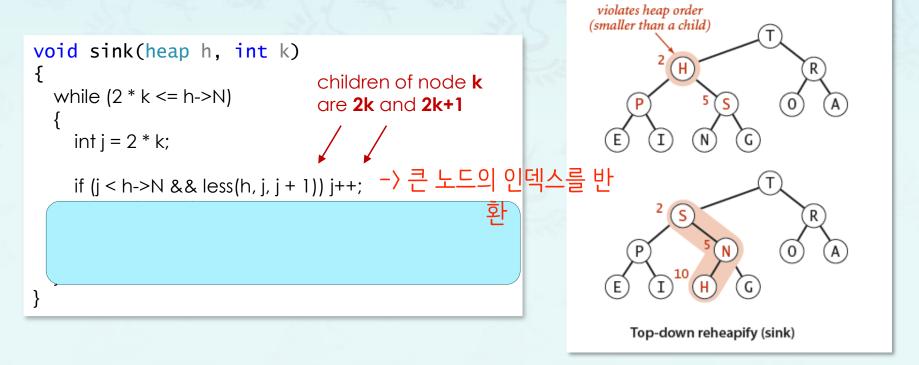
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Demotion in a heap: sink

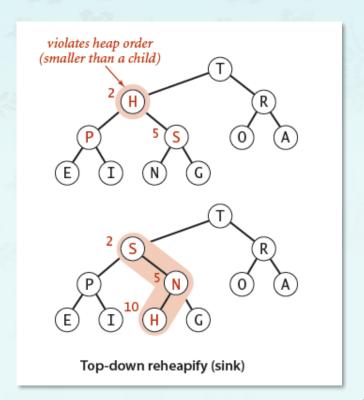
Parent's key becomes smaller than one (or both) of its children's.

- To eliminate the violation:
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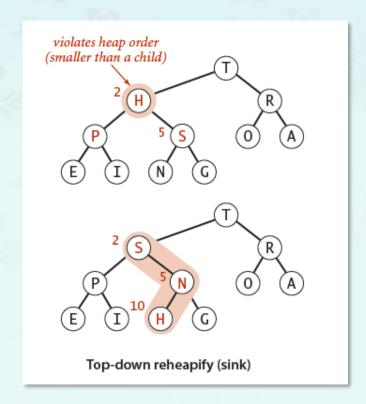
Demotion in a heap: sink

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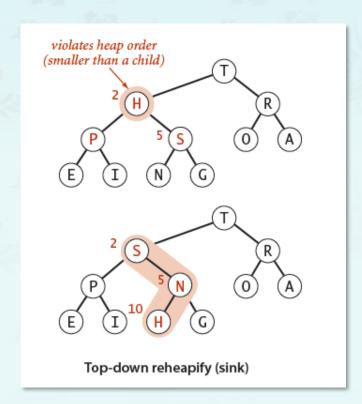
Demotion in a heap: sink

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Demotion in a heap: sink

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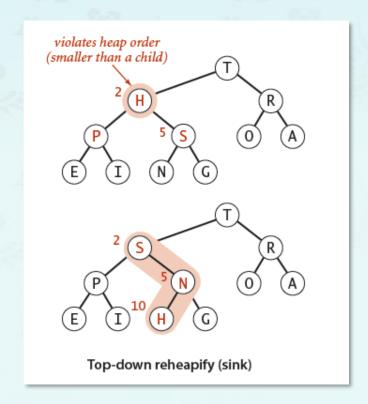


Demotion in a heap: sink

- Parent's key becomes smaller than one (or both) of its children's.
- To eliminate the violation:
 - Swap key in parent with key in larger child
 - Repeat until heap order restored.

```
void sink(heap h, int k)
{
    while (2 * k <= h->N)
    {
        int j = 2 * k;

        if (j < h->N && less(h, j, j + 1)) j++;
        if (!less(h, k, j)) break;
        swap(h, k, j);
        k = j;
    }
}
```



Insert: Add node at end, then swim it up.

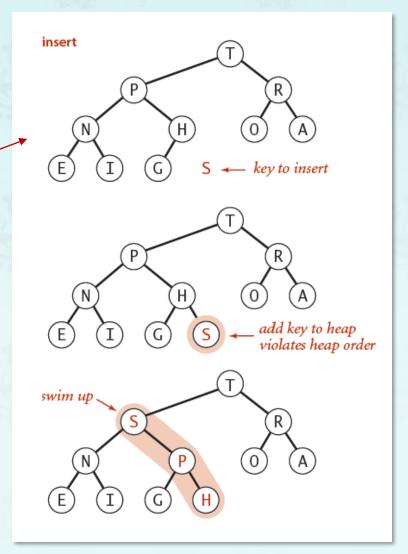
Cost: At most 1 + log N compares.

What is N now?

Insert

Step 1

Step 2



Insertion in a heap: insert Insert: Add node at end, then swim it up. Cost: At most 1 + log N compares. What is N now? (E)void insert(heap h, Key key) add key to heap violates heap order swim up typedef struct heap { *nodes; // an array of node capacity; // array size of node // the number of nodes int } heap;

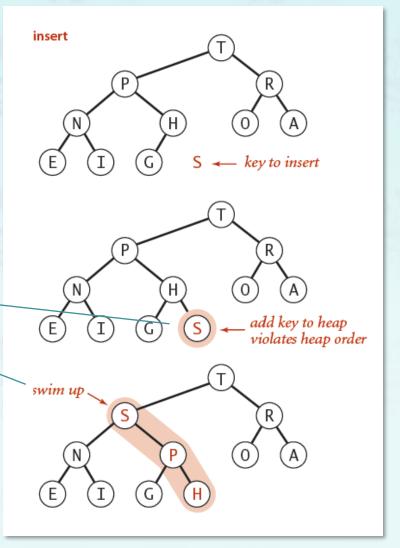
Insertion in a heap:

- Insert: Add node at end, then swim it up.
 - Cost: At most 1 + log N compares.

void swim(heap h, int k)
void sink(heap h, int k)

```
void insert(heap h, Key key)
{
  h->nodes[++h->N] = key;
}

typedef struct heap {
  Key *nodes; // an array of node
  int capacity; // array size of node
  int N; // the number of nodes
} heap;
```

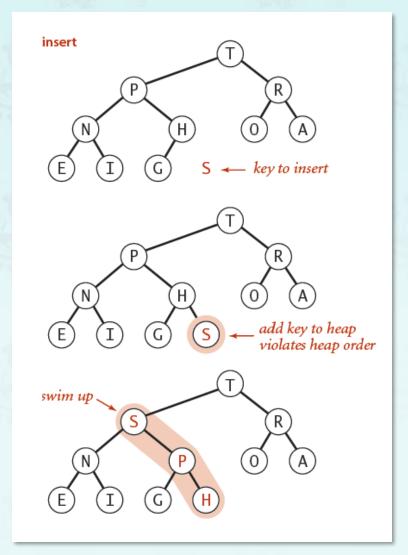


Insertion in a heap:

- Insert: Add node at end, then swim it up.
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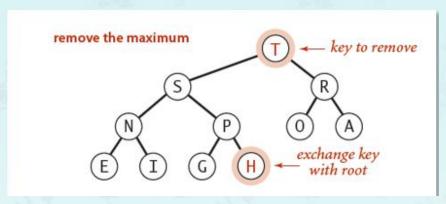
```
void insert(heap h, Key key)
{
  h->nodes[++h->N] = key;
  swim(h, h->N);
}
```

```
typedef struct heap {
   Key *nodes; // an array of node
   int capacity; // array size of node
   int N; // the number of nodes
} heap;
```





- (1) Delete the root (max or min) in a heap:
- (2) How many times do comparisons occur for n nodes ? (select one): n, 2n, n^2, 2 log n, n log n, log n



```
void delete(heap h) {
}
```

2 or 3 lines of code

```
void swim(heap h, int k)
void sink(heap h, int k)
bool less(heap h, int p, int c)
void swap(heap h, int p, int c)
```

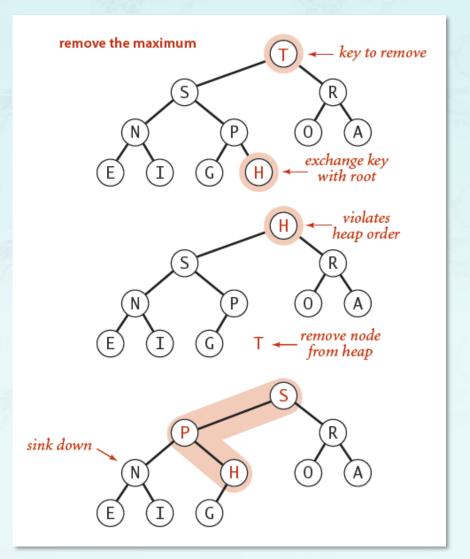
```
typedef int Key;
typedef struct struct_heap *heap;
typedef struct struct_heap {
   Key *nodes;
   int capacity;
   int N;
} struct_heap;
```

- Delete root: Swap root with node at end, then sink it down.
- Cost: At most 2 log N compares.

```
void delete(heap h) {

    swap(h, ..., ...);
    sink(h, ...);
}
```

```
void swim(heap h, int k)
void sink(heap h, int k)
bool less(heap h, int p, int c)
void swap(heap h, int p, int c)
```



- Delete root: Swap root with node at end, then sink it down.
- Cost: At most 2 log N compares.

```
void delete(heap h) {
}
```

```
void swim(heap h, int k)
void sink(heap h, int k)
bool less(heap h, int p, int c)
void swap(heap h, int p, int c)
```



- Delete root: Swap root with node at end, then sink it down.
- Cost: At most 2 log N compares.

```
void delete(heap h) {
    swap(h, 1, h->N--);
}
```

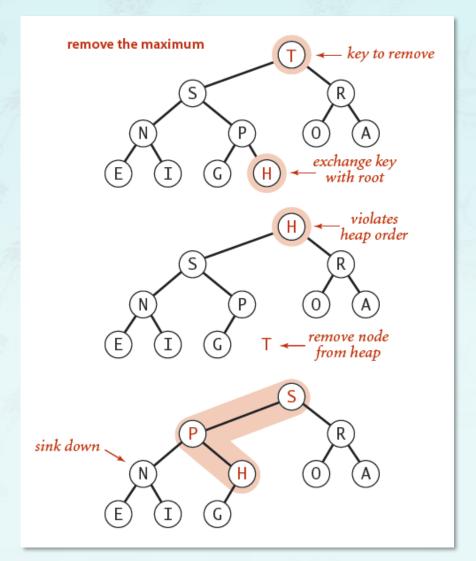
```
void swim(heap h, int k)
void sink(heap h, int k)
bool less(heap h, int p, int c)
void swap(heap h, int p, int c)
```



- Delete root: Swap root with node at end, then sink it down.
- Cost: At most 2 log N compares.

```
void delete(heap h) {
    swap(h, 1, h->N--);
    sink(h, 1);
}
```

```
void swim(heap h, int k)
void sink(heap h, int k)
bool less(heap h, int p, int c)
void swap(heap h, int p, int c)
```



heap coding: heap.h

```
void clear(heap hp);
                         // deallocate heap
int size (heap hp);
                       // return nodes in heap currently
int level(int n);
                       // return level based on num of nodes
                // return its capacity (array size)
int capacity (heap hp);
int reserve (heap hp, int capa); // reserve the array size (= capacity)
              // return true/false
int full (heap hp);
               // return true/false
int empty(heap hp);
void trim(heap hp);
               // delete a queue
int heapify (heap hp);
              // convert a complete BT into a heap
// helper functions to support grow/trim functions
int less(heap hp, int i, int j);  // used in max heap
int more (heap hp, int i, int j); // used in min heap
// helper functions to check heap invariant
```

```
// return the number of items in heap
int size(heap hp) {
   return heap->N;
// Is this heap empty?
int empty(heap hp) {
   return (heap->N == 0) ? true : false;
// Is this heap full?
int full(heap hp) {
   return (heap->N == heap->capacity - 1) ? true : false;
```

```
int less(heap hp, int i, int j) {
   return heap->nodes[i] < heap->nodes[j];
}
```

```
void swap(heap hp, int i, int j) {
   Key t = heap->nodes[i];
   heap->nodes[i] = heap->nodes[j];
   heap->nodes[j] = t;
}
```

```
void swim(heap hp, int k) {
}
```

```
void sink(heap hp, int k) {
}
```

```
void grow(heap hp, int key) {
    cout << "YOUR CODE HERE\n";
    // add key @ ++heap->N
    // swim up @ heap->N
}
```

```
void trim(heap hp) {
   if (empty(heap)) return;

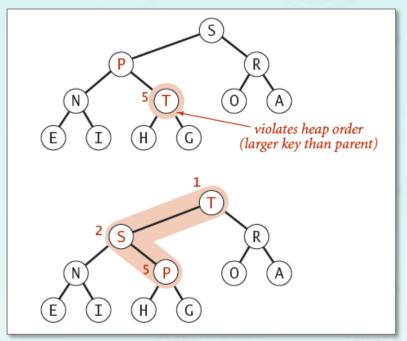
cout << "YOUR CODE HERE\n";
}</pre>
```

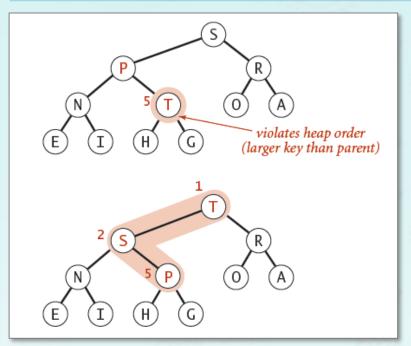
newCBT()	with a given array, instantiate a new complete binary tree its result is neither maxheap nor minheap.
heapify() heapsort()	make a complete binary tree into a (max) heap use max/min-heap to sort elements in heap

```
// instantiates a CBT with given data and its size.
heap newCBT(int *a, int n) {
    int capacity = ?
   heap p = new Heap{ capacity };
    p->N = n;
    for (int i = 0; i < n; i++)
        p->nodes[i + 1] = a[i];
    return p;
```

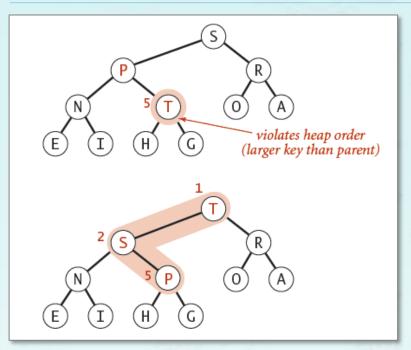
heap

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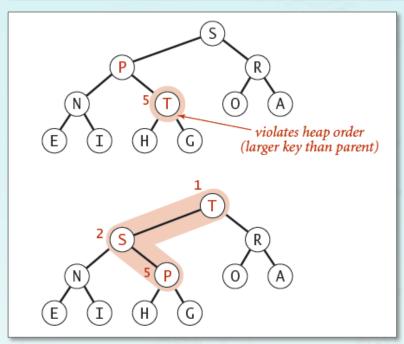




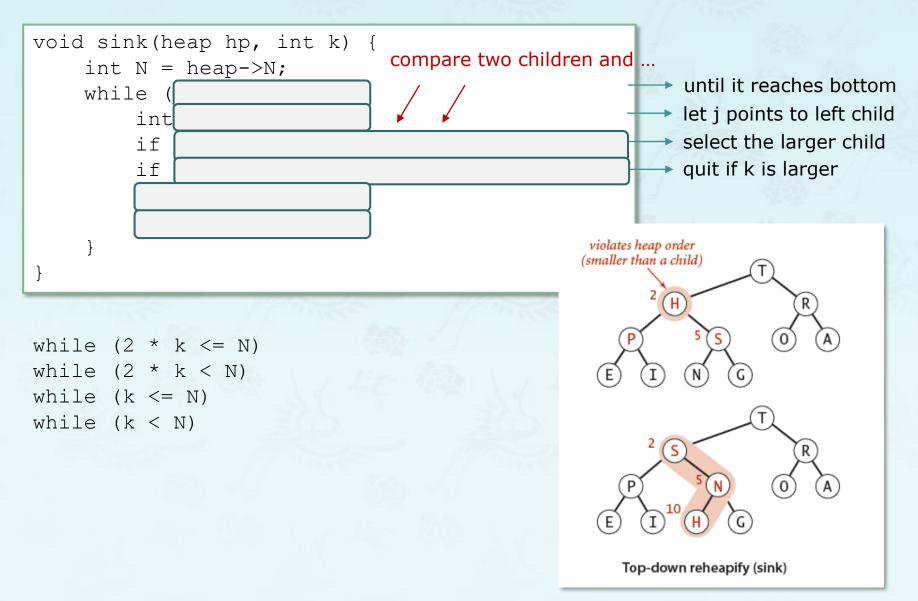
```
void swim(heap hp, int k) {
    while (k > 1 && less(heap, k/2, k)) ←{
        parent of k
}
```

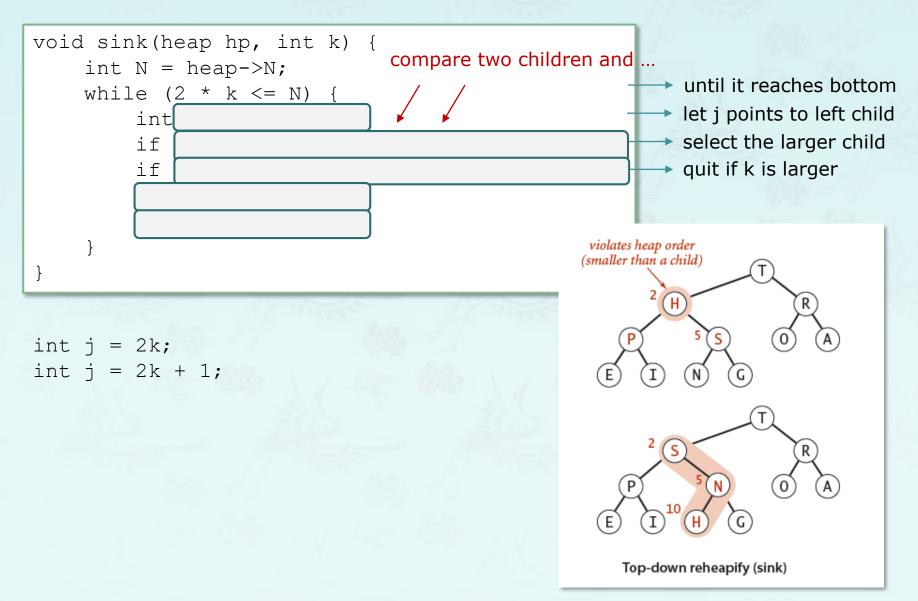


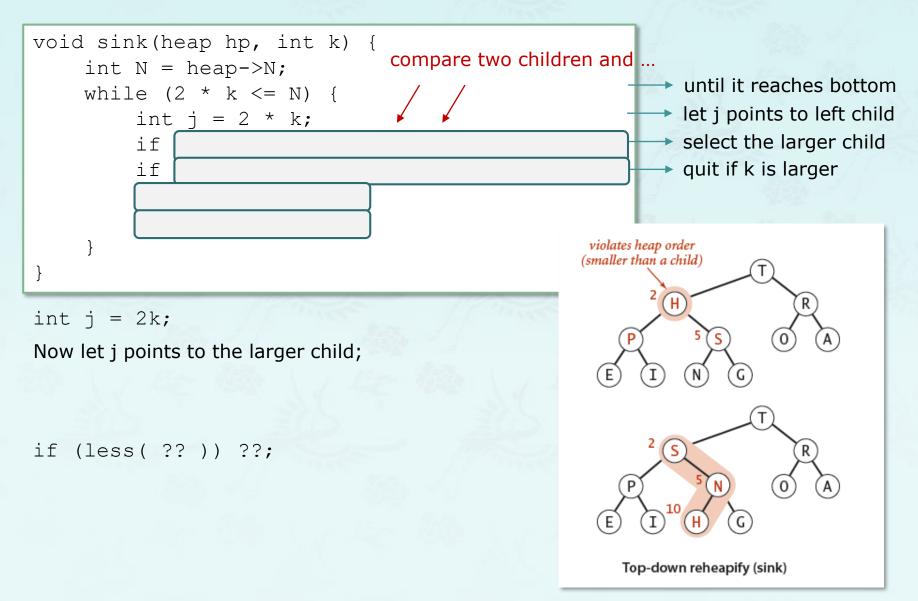
```
void swim(heap hp, int k) {
    while (k > 1 && less(heap, k/2, k)) 
        swap(heap, k/2, k);
    }
    parent of k
}
```

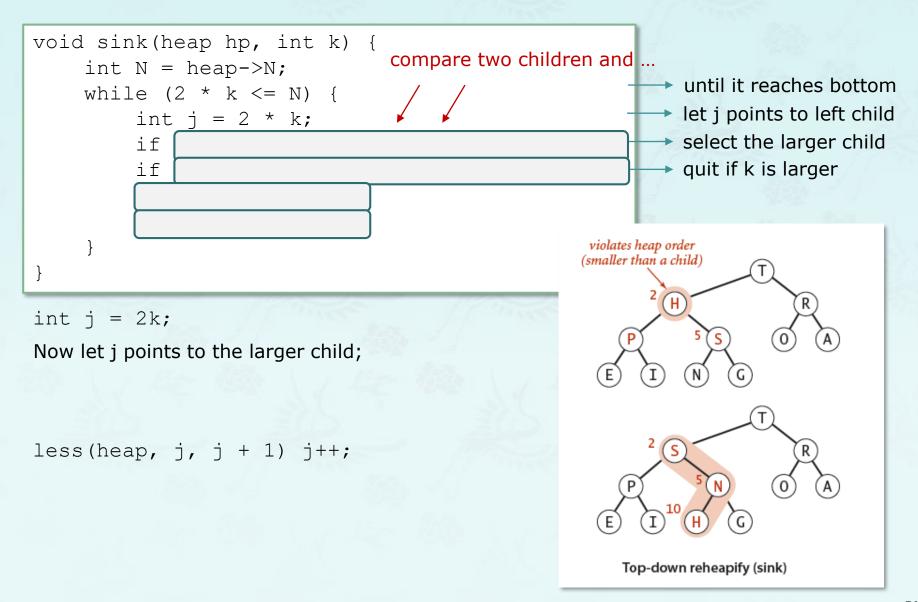


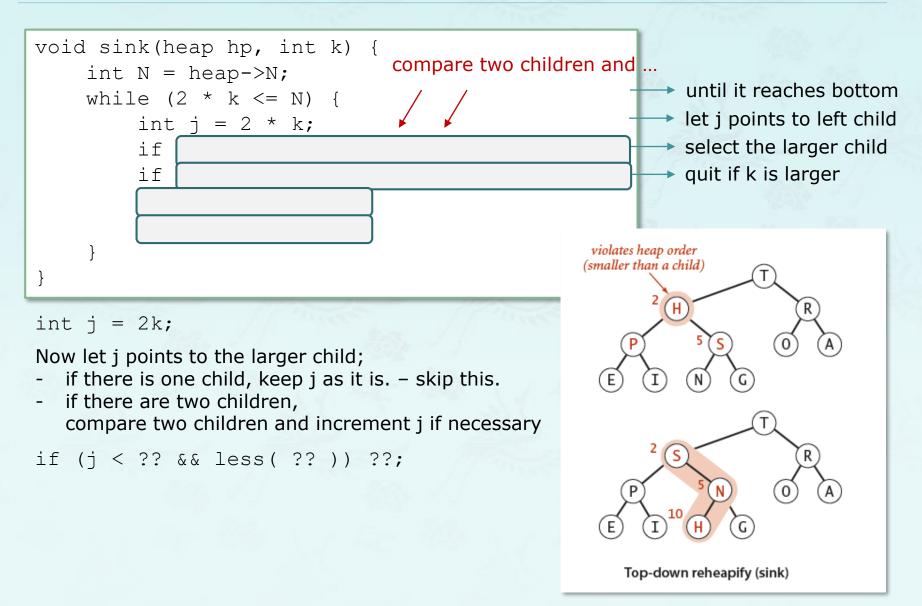
```
void swim(heap hp, int k) {
    while (k > 1 && less(heap, k/2, k)) 
        swap(heap, k/2, k);
        k = k/2;
    }
    parent of k
}
```

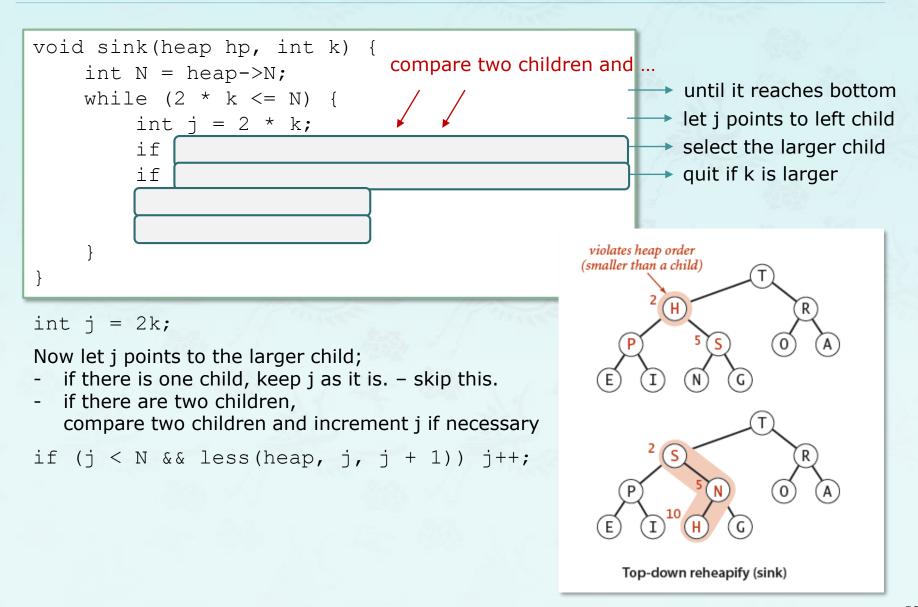


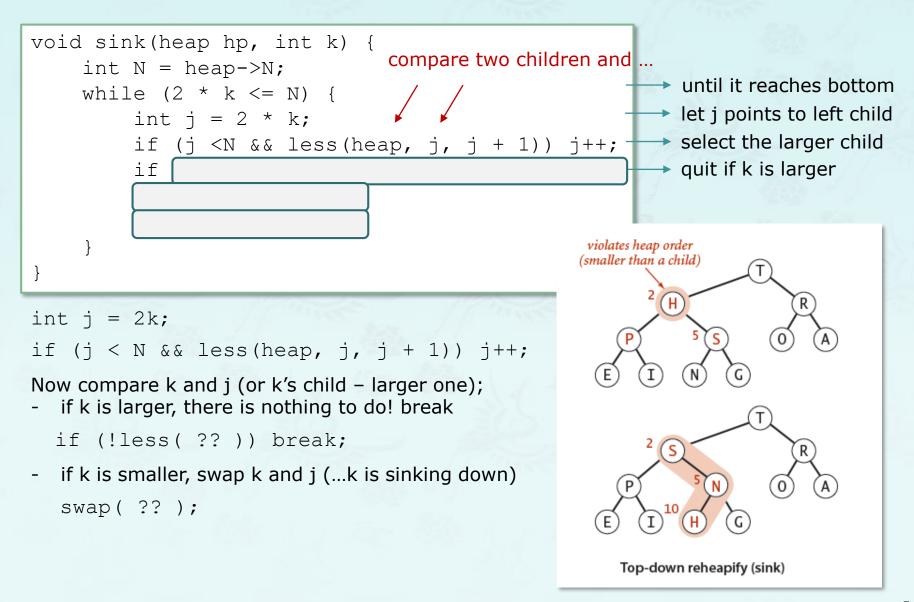




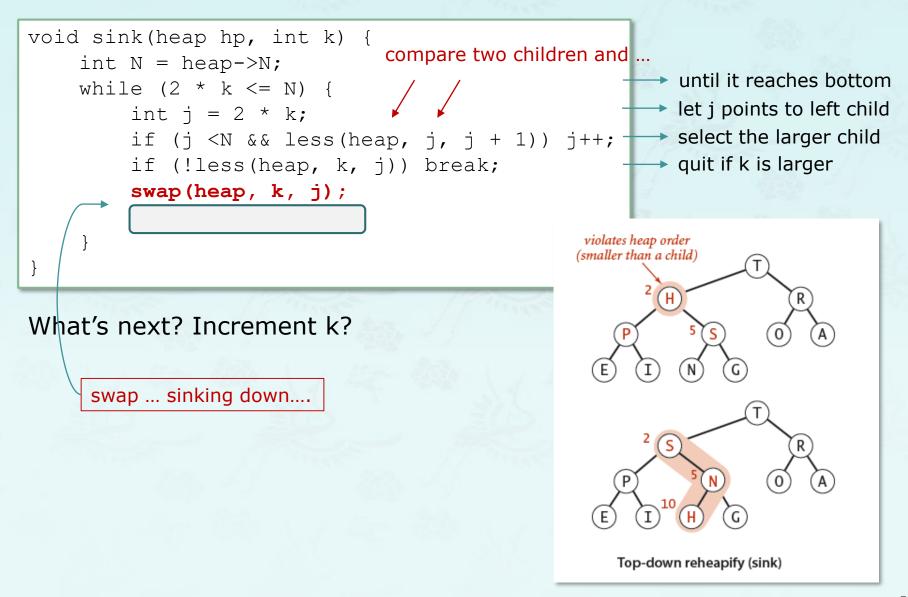








```
void sink(heap hp, int k) {
                                   compare two children and ...
     int N = heap -> N;
                                                             until it reaches bottom
     while (2 * k \le N) {
                                                              let j points to left child
          int i = 2 * k;
                                                               select the larger child
          if (j <N && less(heap, j, j + 1)) j++;
          if (!less(heap, k, j)) break;
                                                             quit if k is larger
          swap(heap, k, j);
                                                      violates heap order
                                                      (smaller than a child)
int j = 2k;
if (j < N \&\& less(heap, j, j + 1)) j++;
Now compare k and j (or k's child – larger one);
- if k is larger, there is nothing to do! break
  if (!less(heap, k, j)) break;
- if k is smaller, swap k and j (...k is sinking down)
  swap(heap, k, j);
                                                         Top-down reheapify (sink)
```



```
void sink(heap hp, int k) {
    int N = heap -> N;
    while (2 * k \le N) {
         int j = 2 * k;
         if (j <N && less(heap, j, j + 1)) j++;
         if (!less(heap, k, j)) break;
         swap(heap, k, j);
         k = j;
                                                     violates heap order
                                                    (smaller than a child)
      swap ... sinking down....
                                                        Top-down reheapify (sink)
```